ANALYSIS AND EVALUATION OF INTERWELL SEISMIC LOGGING TECHNIQUES FOR RESERVOIR CHARACTERIZATION

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OBJECTIVE

The objective of this three-year research program is to investigate interwell seismic logging techniques for indirectly interpreting oil and gas reservoir geology and pore fluid permeability. This work involves a balanced study of advanced theoretical and numerical modeling of seismic waves transmitted between pairs of reservoir wells combined with experimental data acquisition and processing of measurements at controlled sites as well as in full-scale reservoirs. This reservoir probing concept is aimed at demonstrating unprecedented high-resolution measurements and detailed interpretation of heterogeneous hydrocarbon-bearing formations.

SUMMARY OF TECHNICAL PROGRESS

Task 3 - Conduct Full-Scale Experimental Field Test

Interwell seismic experiments were conducted in the month of September at the University of Oklahoma Gypsy test site which is located in Pawnee County, Oklahoma. During the field test a full suite of interwell seismic data were acquired and will be used to extract rock porosity and permeability. In particular, interwell seismic experiments were conducted using two borehole hydrophone arrays (streamers) consisting of twelve detector channels (i.e., simultaneous source-to-detector measurements were made in two boreholes pairs having different separation distances) for source-independent seismic attenuation and dispersion studies.

Task 4 - Data Processing Studies

DATA PROCESSING AND INTERPRETATION OF INTERWELL SEISMIC DATA ACQUIRED AT THE BUCKHORN TEST SITE IN ILLINOIS

INTRODUCTION

High-resolution subsurface seismic experiments were conducted to demonstrate the capabilities of the interwell seismic method and to evaluate the experimental seismic system overall operation and performance. The interwell seismic experiments were conducted at the Western Kentucky Petroleum, Buckhorn test site near Quincy, Illinois, which is documented by well records and in which the well spacings are typical of shallow hydrocarbon reservoirs.

Interwell seismic has become an extremely efficient technique for determining rock physical parameters and to detect heterogeneities in hydrocarbon-bearing formations. In particular, to predict lateral changes in porosity within the Kankakee limestone formation which is horizontally distributed at a depth of about 200 m, and less than 8 m thickness. The Kankakee formation is considered as a thin high-velocity layer in the low-velocity background shale (Figure 1). The results of several kinds of logging measurements conducted just after drilling five boreholes indicate a good correlation between well-to-well (Saito, 1991).
A plan view of the location of the wells is shown in Figure 2. The diameter of each borehole is 6 inches. A 24-element hydrophone array (hydrophone streamer) was used to acquire interwell seismic data using a 1000-Joule arc discharge borehole seismic source. This source was placed in well A, and the streamers were placed in wells D, B, and E. In addition, a wall lock probe (OYO shuttle) was used to record 3-component waveforms in Kankakee limestone formation.

THE EXPERIMENTAL INTERWELL SEISMIC SYSTEM

An experimental high-frequency system was devised to read pressure seismic data. The system included an arc discharge source, a cylindrical bender source, multiconductor cables, winches, 24-element hydrophone arrays, a 3-component wall lock probe containing accelerometers, and a 3-component pneumatic probe (also containing accelerometers).

The seismic data acquired with the cylindrical bender source was used as a reference source for evaluating the prototype high-frequency arc discharge source. The cylindrical bender transducer is a monopole source designed to operate in the frequency range of 500-4000 Hz. On the other hand, the arc discharge source employs a 1000 Joule electric arc discharge to generate a bipolar pressure impulse having a time duration of approximately 1 ms, and a half-power spectral bandwidth of about 200-2000 Hz.

The pressure interwell seismic data were recorded using the OYO McSeis 160, 24-channel seismograph, with a 62 µsec sample rate, used-controller fixed gain, 2048 sample per trace, and a variable time delay depending on the source-receiver offset. A common source gather consisted of 24 traces corresponding to the 24 receivers in the 24-element hydrophone array.

Figures 3 and 4 illustrate common-source gathers with detectors at 2 m intervals from 154 to 200 m in well B. Figure 3 shows the seismogram produced using the cylindrical bender source stationed in well A at a depth of 160 m. Similarly, Figure 4 shows the seismogram produced using the prototype arc discharge source stationed in well A also located at the depth of 160 m. Both seismograms exhibit identical first break time and they show sharp reflections caused by the presence of the shale/Kankakee limestone interface. However, secondary events associated with major reflections and tube waves can be better identified when the arc discharge source is used rather than the cylindrical bender source. In addition, cylindrical bender source data show wave distortions and feedover waveforms caused by inductive coupling between the source and detector cables. For further evaluation, pulses produced by both sources, the bender and arc discharge, are selected from zero-vertical offset seismic traces which are given in Figures 5 and 6. Although the first break times are the same in both full waveforms, the time break associated with the arc discharge source was easier to identify than the time break associated with the cylindrical bender source. The corresponding spectra of the first 27 ms of the traces are shown in Figures 7 and 8. The maximum power spectrum for the bender was 35 dB and for the arc discharge was about 55 dB. The spectra show that the arc discharge source is approximately 100 times more powerful than the cylindrical bender source (i.e., about 20 dB). The energy produced by the cylindrical bender source was sharply reduced at about 1500 Hz and the energy generated by
the arc discharge slowly decreases by reaching about 15 dB at 4000 Hz (i.e., about 27% of its maximum energy).

**DISPERSION AND ATTENUATION PARAMETERS FROM INTERWELL SEISMIC DATA**

In order to extract dispersion and attenuation curves from interwell seismic data, we selected zero-vertical offset gathers recorded in wells D and B at source depths ranging from 154 m to 200 m, respectively, in well A. Figures 9a and 9b illustrate zero-vertical offset seismograms produced at 20 m and 46 m separation distances from the source well A. Both seismic sections show strong first arrivals for waves traveling in the shale formation, and weak first arrivals for waves traveling in the Silurian Kankakee formation. The shale/Kankakee limestone interface is clearly defined by a sharp reflection event observed in both seismograms. The horizontal compressional wave velocity in the shale varies between 2.7 km/s and 3.5 km/s, and the compressional wave velocity in the Kankakee limestone formation varies between 4.0 km/s to 5.0 km/s. Several waveforms were selected to determine attenuation and dispersion effects in the shale and Kankakee limestone formation. Figures 10 and 11 illustrate in detail the waveforms that are used to extract quality factors $Q$, and dispersions between wells D and B. The P-wave events associated with the limestone formation are clearly shown by the detectors in well D. Alternatively, the P-wave events observed in well B have been decreased in amplitude and broadened by the disturbances that exist between wells D and B. Those events correlate with a low-velocity zone delineated using travel-time tomography by Saito (1991).

**Time-Frequency Analysis**

The P-wave events were determined from the seismic traces using time-frequency analysis given in Appendix A. As an example, Figure 12a shows time-frequency plots for the first 30 ms of a zero-vertical offset seismic trace which was recorded at a depth of 186 m within the shale formation. In this figure the first contour plot represents a P-wave event having a peak-frequency of 750 Hz, the second contour is a reflection event which arrives at 15 ms, and the last contour is a low-frequency tube wave which arrives after 27 ms at the detector in well D. The P-wave event time-frequency plot for the first 10 m recorded in well D at a depth of 186 m also exhibits a peak-frequency of 750 Hz as shown in Figure 12b.

The time-window of the P-wave events indicated by the time-frequency plots were used to extract the P-wave pulses recorded at 20 m and 46 m as illustrated in Figure 13. The peak-to-peak amplitude of the compressional waveform observed in well B is approximately four times smaller than the amplitude of the P-wave event recorded by the detector in well D.

In a similar fashion as above, we applied time-frequency analysis to the zero-vertical offset traces recorded within the Kankakee limestone formation. Figure 14a shows the contour plots of a 30 ms trace and Figure 14b shows the plot for the first 10 ms. Time windows of 10 ms were used to plot the P-wave events as time-frequency contours for
detectors in wells D and B (see Figures 14b and 15b). In this case, the high frequency content in the waveform has been reduced by the presence of the attenuated limestone formation within the region between wells D and B. The P-wave pulses extracted from the zero-vertical offset traces are shown in Figure 16. Those waveforms indicate a reduction of amplitude about nine times for the waves traveling within the target zone of interest in the limestone. The lower amplitude waveform has been broadened by the presence of the low-velocity zone which was delineated by travel-time tomography (Saito, 1991). Such low-velocity inhomogeneity within the limestone formation appears to be a zone of low-saturation, which effect have reduced the compressional wave velocity.

Attenuation and Dispersion Curves

The next step in the interpretation was to calculate attenuation and dispersion from the selected seismic waveforms using the spectral ratio method given in Appendix B. The waveforms given in Figure 13 associated with the shale formation are used to extract quality factor and phase velocity at a depth of 186 m (see Figure 4). The quality factor was determined to be $Q = 28$ and the phase velocity was found to reach its asymptotic value at 3 km/s, which corresponds to the horizontal velocity of the shale, which was estimated using travel-time tomography. The dispersion characteristics are clearly observed in the phase-velocity curve by showing an increase of the velocity between 200 Hz and 500 Hz. In addition, we calculated attenuation curves from seismic traces recorded at the depths of 180 m, 184 m, and 190 m. These attenuation curves with the corresponding quality factors $Q = 50, 29, and 28$, are shown in Figure 18. These results show that the attenuation increases as a function of depth in the shale formation.

Similarly, we calculated attenuation and phase velocities for the Kankakee limestone formation which are obtained from the traces recorded at 196 m, 198 m, and 200 m and are given in Figure 19 and 20 together with the corresponding quality factors $Q = 4.5, 7, and 4.4$. As was expected, the low-quality factors and low-phase velocities of the limestone formation are associated with the high-porosity and low-velocity disturbance (heterogeneity) delineated by Saito (1991).

Interpretation of Three-Component Interwell Seismic Data

In order to characterize the Kankakee formation even further, we conducted three-component interwell seismic experiments between well A and B. A three-component wall lock probe was placed in the Kankakee limestone formation in well B at a depth of 194 m, and the arc discharge source was stationed at 0.5 m intervals.

Figure 21 shows the x-component and y-component seismograms, which indicate unusually high-frequency pulses associated with P-wave and S-waves. The shear wave events are shown in most of the traces. The shear-wave velocity is about 3 km/s and corresponds to the shear-wave velocity derived from sonic logs. In order to study the high-frequency content in the seismograms we selected the 3-component common-detector traces (source was placed at 198.5 m and detector at 194 m) shown in Figure 22. The corresponding spectra are shown in Figure 23. To determine the peak-frequency of each event in the
seismic trace we use the time-frequency representation of the x-component trace as shown in Figure 24. The first contour corresponds to a P-wave event having a peak frequency of 4000 Hz and arrives in 8.5 ms at the detector in well B. The second contour is a shear wave event having about the same peak-frequency than that of the P-wave event and arrives in about 15 ms at the detector in well B, and the last contours correspond to reflections and tube waves which arrive at different times, and they have peak frequencies between 250 Hz and 750 Hz. The presence of unusually high-frequency events in the time-frequency plots may be the result of resonant vibrations of the 3-component detector shuttle and high-frequency modes trapped in the low-velocity zone within the Kankakee limestone formation.

CONCLUSIONS

The presence of the low-velocity zone previously detected and delineated using travel-time tomography has been characterized by performing attenuation measurements (using hydrophone detectors), and three-component interwell seismic measurements (using a wall lock detector probe containing accelerometers). The low-velocity zone is located in the Kankakee formation along borehole D at a depth between 196 m to 198 m where the Suspension P-S log also showed low velocities. The core information from well D indicates that the Kankakee limestone is formed by vertical fractures, vugs, and a very porous matrix which is partially saturated with oil and water. In addition, the degree of saturation varies between wells A and B, from fully saturated (wells A and D) to partially saturated formation (wells D and B). Thus, the high-attenuation of the seismic pulses traveling in the Kankakee formation is caused by the heterogeneous conditions of the porous and fracture rock matrix which is partially saturated with oil and water.

REFERENCE

STRATIGRAPHY AT THE BUCKHORN TEST SITE

FIGURE 1
WELL LAYOUT AT THE BUCKHORN TEST SITE

Well Layout

A

D

B

C

E

64.45m
50.65m
40.95m
57.55m
19.69m
26.08m
45.50m

Diameter: 6"
COMMON-SOURCE WAVEFORMS IN WELLS SPACED 46 m.
Cylindrical Bender

FIGURE 3
COMMON-SOURCE WAVEFORMS IN WELLS SPACED 46 m.

Arc Discharge

FIGURE 4
ZERO-VERTICAL OFFSET WAVEFORM FOR SHALE FORMATION
Cylindrical Bender

FIGURE 5
ZERO-VERTICAL OFFSET WAVEFORM FOR SHALE FORMATION
Arc Discharge

FIGURE 6
ZERO-VERTICAL OFFSET SPECTRUM FOR SHALE FORMATION

Cylindrical Bender

![Graph showing the spectrum with frequency on the x-axis and magnitude (dB) on the y-axis. The graph peaks around 1000 Hz and shows a decrease in magnitude as frequency increases.]
ZERO-VERTICAL OFFSET SPECTRUM FOR SHALE FORMATION

Arc Discharge

Frequency (Hz)
Magnitude (dB)

FIGURE 8
ZERO-VERTICAL OFFSET SEISMOGRAMS

A Well Spaced 20 m
B Well Spaced 46 m

Time (ms)  Time (ms)

Source Depth (m)  Receiver Depth (m)

FIGURE 9
ZERO-VERTICAL OFFSET WAVEFORMS
Well Spaced 20 m

FIGURE 10
ZERO-VERTICAL OFFSET WAVEFORMS
Well Spaced 46 m

FIGURE 11
TIME-FREQUENCY ANALYSIS OF ZERO-VERTICAL OFFSET WAVEFORMS FOR THE SHALE FORMATION BETWEEN WELLS A AND D. SOURCE AND DETECTOR AT A DEPTH OF 186 m.

Time-Window of 30 ms

Time-Window of 10 ms

FIGURE 12
ZERO-VERITCAL OFFSET P-WAVES FOR THE SHALE FORMATION. SOURCE AND DETECTORS AT A DEPTH OF 186 m.

**Detector in Well D**

**Detector in Well B**

FIGURE 13
TIME-FREQUENCY ANALYSES OF ZERO-VERTICAL OFFSET WAVEFORMS FOR THE KANKAKEE FORMATION BETWEEN WELLS A AND D. SOURCE AND DETECTOR AT A DEPTH OF 198 m.

**Time-Window of 30 ms**

**Time-Window of 10 ms**

**FIGURE 14**
TIME-FREQUENCY ANALYSIS OF ZERO-VERTICAL OFFSET WAVEFORMS FOR THE KANKAKEE FORMATION BETWEEN WELLS A AND B. SOURCE AND DETECTOR AT A DEPTH OF 198 m.

Figure 15

(a) Time-Window of 30 ms

(b) Time-Window of 10 ms
ZERO-VERTICAL OFFSET P-WAVES FOR THE KANKAKEE FORMATION. SOURCE AND DETECTORS AT A DEPTH OF 198 m.

**FIGURE 16**

- **Detector in Well D**
- **Detector in Well B**
DISPERSION AND ATTENUATION CURVES FOR THE SHALE FORMATION BETWEEN WELLS D AND B AT A DEPTH OF 186 m.

FIGURE 17
ATTENUATION CURVES FOR THE SHALE FORMATION BETWEEN WELLS D AND B AT SEVERAL DEPTHS

- Shale Depth = 180 m, Q = 50
- Shale Depth = 184 m, Q = 29
- Shale Depth = 190 m, Q = 26

**FIGURE 18**
DISPERSION CURVES FOR THE KANKAKEE FORMATION BETWEEN WELLS D AND B AT A DEPTH OF 198 m.

FIGURE 19
ATTENUATION CURVES FOR THE KANKAKEE FORMATION BETWEEN WELLS D AND B.
COMMON-DETECTOR SEISMOGRAMS. WALL LOCK VSP PROBE AT A DEPTH OF 194 m. WELL SPACED AT 46 m.

X-Component

Y-Component

(FIGURE 21)
THREE-COMPONENT WAVEFORMS RECORDED USING THE SHUTTLE WALL LOCK DETECTOR.
SPECTRA RECORDED USING THE SHUTTLE WALL LOCK DETECTOR. SOURCE AT A DEPTH OF 198.5 m AND DETECTOR AT A DEPTH OF 194 m.

FIGURE 23
TIME-FREQUENCY ANALYSIS OF X-COMPONENT WAVEFORMS RECORDED USING THE SHUTTLE WALL LOCK DETECTOR. SOURCE AT A DEPTH OF 198.5 m AND DETECTOR AT A DEPTH OF 194 m.
APPENDIX A

TIME-FREQUENCY ANALYSIS
All the time-frequency analysis for this presentation was done using the spectrogram, which is based upon the Short-Term Fourier Transform (STFT). I define the STFT to be

\[
\text{STFT}(t, f) = \int_{-\infty}^{\infty} x(t') w(t' - t) e^{-j2\pi ft'} dt'
\]

(1)

where \(x(t)\) is the time series to be analyzed and \(w(t)\) is a Gaussian analysis window:

\[
w(t) \equiv \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}
\]

(2)

Note that \(w(t)\) is normalized:

\[
\int_{-\infty}^{\infty} w(t) dt = 1
\]

(3a)

\[
\int_{-\infty}^{\infty} t^2 w(t) dt = \sigma^2
\]

(3b)

The spectrogram is just the squared magnitude of the STFT, hence it is a measure of power:

\[
\text{SPEC}(t, f) \equiv |\text{STFT}(t, f)|^2
\]

(4)

In practice, this process is done with discrete time-sampled series. The Fourier transform in (1) is then done with Fast Fourier Transform (FFT). The relevant parameters are:

- \(N_x\) = Number of elements in the \(x()\) time series
- \(N_w\) = Number of elements in \(w(t)\)
- \(\Delta t\) = Sample time (seconds) of both \(x()\) and \(w()\)
- \(N_{\text{fft}}\) = Number of points for (zero-padded) FFT
- \(\Delta f\) = Frequency resolution \(= \frac{1}{N_{\text{fft}} \cdot \Delta t}\)
For all of our presented results, we had

\[
\begin{align*}
N_w &= 150 \\
\sigma &= 10 \cdot \Delta t \\
\Delta t &= 62 \mu\text{sec} \\
N_{\text{fft}} &= 2048 \\
\Rightarrow \Delta f &= 7.88 \text{ Hz}
\end{align*}
\]

The choice of these parameters bears discussing. The sample time \(\Delta t\) is fixed by the data being analyzed. The parameter \(\sigma\) determines the width of the Gaussian analysis window. If the window is too narrow (small \(\sigma\)), the time-frequency distribution (TFD) is "smeared" in the frequency direction -- the signal appears to have an artificially large bandwidth. On the other hand, if the window is too wide (large \(\sigma\)), the TFD is spread in the time direction; in the extreme, this becomes an ordinary spectrum, with no time information. The \(N_w\) parameter is just to truncate the window, for computational purposes. It should be large enough that the window is already near zero at the edge. Choosing \(N_w > 10\sigma/\Delta t\) should be sufficient. The \(N_{\text{fft}}\) parameter must be a power of 2 and it should be large enough to have good frequency resolution. For the Illinois data, \(N_{\text{fft}}\) was chosen larger than it needed to be in order to get better quality contour plots.

In all cases, time gating was done first in order to reduce the size of the spectrogram calculation. An example would be in order. If I specify examining the portion of the signal \(s(t)\) from 10 msec to 20 msec:

\[
\begin{align*}
\text{index1} &= 10 \text{ msec/}\Delta t \quad = \quad 161 \text{ (rounded down)} \\
\text{index2} &= 20 \text{ msec/}\Delta t \quad = \quad 322 \text{ (rounded down)} \\
N_x &= \text{index2} + 1 \quad = \quad 162 \\
x(1) &= s(\text{index1}) \\
x(2) &= s(\text{index1} + 1) \\
\vdots \\
x(N_x) &= s(\text{index2})
\end{align*}
\]

This effective time shift creates a phase modulation in the STFT, but since we only care about the magnitude (spectrogram), this phase modulation can be ignored.

While writing this report, we have noticed a potential problem with my time gating. We are currently chopping the signal at the ends of the region of interest, whereas we should include the adjacent area using the windowing function. As currently implemented, there could be Gibbs phenomena coming into play. The effects are probably minimal, as we have not observed them. And it is possible the effects are largely (completely?) masked by the Gaussian windowing function. An analysis is not worth doing -- better to just fix the program next time.
APPENDIX B

ATTENUATION - SPECTRAL RATIO METHOD
APPENDIX B

ATTENUATION - SPECTRAL RATIO METHOD

A. Computational Flow

(1) We start with a time series extracted from a trace. It is defined by a few parameters:

\( \Delta t = \text{sampling time in seconds} \)

\( N = \text{number of points} \)

\( x[i] = \text{the time series data, for } 0 \leq i < N \)

\( T_{\text{init}} = \text{(travel) time, in seconds, corresponding to } x[0] \)

\( R = \text{distance (meters) from source to receiver.} \)

We actually have two series, \( x_1[] \) and \( x_2[] \), each with their own set of other parameters \( (N, \Delta t, R, T_{\text{init}}) \). \( x_1[] \) is for the close receiver (reference waveform) and \( x_2[] \) is the one farther away.

(2) Find the spectrum of each waveform.

a. Remove DC level: \( x[i] = x[i] - \text{SUM}(x[]) \)

b. Apply \( N \)-point window, which goes to zero at each end. I could use Hamming, or others, but I prefer Blackman-Harris window. Choice of window is probably insignificant

\[
    w[i] = 0.35875 - 0.48829 \cos\left(\frac{2\pi i}{N}\right) + 0.14128 \cos\left(\frac{4\pi i}{N}\right) - 0.01168 \cos\left(\frac{6\pi i}{N}\right) \quad 0 \leq i < N
\]

\( x[i] = w[i] \times x[i] \)

c. Zero-pad the end of \( x[] \) out to \( N_{\text{fft}} \) points (power of 2). Take the complex FFT:

\[
    z[i] = \sum_{k=0}^{N_{\text{fft}}-1} x[k] e^{-j \frac{2\pi ik}{N_{\text{fft}}}} \quad 0 \leq i < N_{f}
\]
\[ N_f \equiv 1 + \frac{N_{ft}}{2} = \text{Number of frequencies (0 to Nyquist)} \]

d. Separate into phase and magnitude (in decibels), for non-negative frequencies:

\[ \log_{ampl}[i] = 0.5 \cdot 20 \log_{10}\left[ \frac{\text{Re}(Z[i])}{Z + \text{Im}(Z[i])};_Z \right] \quad 0 \leq i < N_f \]

\[ \text{phase}[i] = i \cdot 2\pi \Delta f \text{T}_{\text{init}} + \text{atan2}(\text{Im}(Z[i]), \text{Re}(Z[i])) \quad 0 \leq i < N_f \]

where

\[ N_f \equiv 1 + \frac{N_{ft}}{2} = \text{Number of frequencies (0 to Nyquist)} \]

\[ \Delta f \equiv \frac{1}{N_{ft} \Delta t} \]

\[ \text{atan2}(y, x) \equiv \tan^{-1}\left(\frac{y}{x}\right) \text{ corrected for quadrant to give } [-\pi, \pi]. \text{ A C/Fortran library function.} \]

e. Unwrap phase to get rid of $2\pi$ discontinuities caused by $\tan^{-1}(\cdot)$. Notice the phase is dominated by the linear $T_{\text{init}}$ term, because we zoom in on the signal of interest. This linear domination also makes phase-unwrapping easier, since the method utilizes linear extrapolation.

3. Take the spectral ratio. If the two spectra have different frequency sampling $\Delta f$, then one of them is interpolated to match the other. This is not needed for any Illinois data.

\[ \log_{ampl}[i] = \log_{ampl2}[i] - \log_{ampl1}[i] \quad 0 \leq i < N_f \]

\[ \text{phase}[i] = \text{phase2}[i] - \text{phase1}[i] \quad 0 \leq i < N_f \]

4. Find the linear fit of the spectral ratio vs. frequency in the frequency range of interest:

\[ \text{slope} = \frac{\Delta \log_{ampl}}{\Delta \text{frequency}} \text{ This should be negative.} \]

5. Find Q value
Q = \frac{20 \log_{10}(e) \pi \Delta r}{-\text{slope} \cdot v}

where \( \Delta r = R_2 - R_1 \) = difference in distances

\[ v = \text{user-input guess to velocity} = \begin{cases} 
3000 \text{ m/sec in shale} \\
4100 \text{ m/sec in limestone}
\end{cases} \]

(6) Find the phase velocity curve. Phase velocity is defined by

\[ c = \frac{\omega \Delta r}{\phi} \]

where \( \phi \) is the phase difference of the spectral ratio

and \( \frac{\partial c}{\partial \omega} = -\frac{\Delta r}{\phi} \left[ \frac{\omega \partial \phi}{\partial \omega} - 1 \right] \)

a. We need \( \frac{\partial c}{\partial \omega} > 0 \) for very low frequency. But the phase we found in the spectral ratio is ambiguous by multiples of \( 2\pi \). So we add \( 2\pi \) to all the phases until \( \frac{\partial c}{\partial \omega} > 0 \). This usually takes 0-2 iterations. Note CDERIV is an approximation to \( \frac{\partial c}{\partial \omega} \).

\[
\text{CDERIV}(i) \equiv \frac{\Delta r}{\text{phase}[i]} \left( \frac{i \cdot \Delta \omega \left( \text{phase}[i + 1] - \text{phase}[2] \right) - 1}{\Delta \omega \cdot \text{phase}[i]} \right)
\]

test_point = 4
test_deriv = CDERIV (test_point)
while (test_deriv <= 0) do
    add \( 2\pi \) to all elements of phase[]
    test_deriv = CDERIV (test_point)
endwhile

For each i:

\[
\text{if phase}[i] \neq 0 \\
\quad c[i] = \frac{i \Delta w \Delta r}{\text{phase}[i]}
\]

else,

\[
c[i] = \begin{cases} 
0 & \text{if } i = 0 \\
\text{MaxVelocity} & \text{if } i \neq 0
\end{cases}
\]

B. Mathematical Formulation

The mathematics look like this.

Let

\[
\begin{align*}
E(t) &= \text{signal at receiver} \\
G(t) &= \text{source signal} \\
A(t) &= \text{recorded signal} \\
r &= \text{distance traveled} \\
T &= \text{initial time for recorded signal} \\
\tilde{E}(\omega) &= \text{Fourier transform of } E(t)
\end{align*}
\]

Relationships:

\[
E(t) = A(t - T) \Rightarrow \tilde{E}(\omega) = \tilde{A}(\omega) e^{j\omega T}
\]

\[
\tilde{E}(\omega) = \tilde{G}(\omega) e^{jkr - \omega r}.
\]

\[
\Rightarrow \tilde{A}(\omega) = \tilde{G}(\omega) e^{-j\omega T + jkr - \omega r}
\]

Spectral ratio \(\tilde{S}(\omega)\) is

\[
\tilde{S}(\omega) = \frac{\tilde{A}_2(\omega)}{\tilde{A}_1(\omega)} = e^{-\alpha \Delta r} e^{j\Delta r - j\omega(T_2 - T_1)}
\]

In simplifying equation (6), we have made some assumptions inherent in the method. We are assuming that the basic seismic wavelet is contained in the source spectrum \(\tilde{G}(\omega)\). Furthermore, we are assuming that the only changes that occur between the boreholes can be written
in the exponential propagation terms. In particular, multipath effects and media inhomogeneities are ignored.

Let \( z(\omega) = 20 \log_{10}(|\tilde{S}(\omega)|) \) \hspace{1cm} (7)

\[ \phi_{sr}(\omega) = \phi_2(\omega) - \phi_1(\omega) \] \hspace{1cm} (8)

where \( \phi_1(\omega) = \text{Arg}(\tilde{A}_1(\omega)) + \omega T_1 \) \hspace{1cm} (9a)

\[ \phi_2(\omega) = \text{Arg}(\tilde{A}_2(\omega)) \] \hspace{1cm} (9b)

\( \phi_1(\omega) \) and \( \phi_2(\omega) \) are the phases found when taking the spectra. \( z(\omega) \) and \( \phi_{sr}(\omega) \) are the spectral ratio values used in the program. Notice \( z(\omega) \) is in dB.

Then \( \alpha \) in dB/m is given by

\[ \alpha (\text{dB/m}) = -\frac{z(\omega)}{\Delta r} \] \hspace{1cm} (10)

\( Q \) is defined by

\[ \alpha \Delta r = \frac{\pi f \Delta r}{Q v} \] \hspace{1cm} (11)

when \( \alpha \) is in nepers/m. So

\[ \alpha \Delta r = -\ln(|\tilde{S}(\omega)|) \]

\[ = -\frac{z(\omega)}{20 \log_{10}(e)} \]

\[ \alpha \Delta r = \frac{\pi f \Delta r}{Q v} \]

or

\[ z(\omega) = -\frac{20 \log_{10}(e) \pi f \Delta r}{Q v} \] \hspace{1cm} (13)
If the slope of $z(\omega)$ vs frequency $f$ is denoted by $m$ (which is negative), then

$$m = -\frac{20 \log_{10}(e) \pi \Delta r}{Q \nu} \Rightarrow Q = \frac{20 \log_{10}(e) \pi \Delta r}{-m \nu} \quad (13)$$

Now look at phase velocity. Starting with (6) and using (8) and (9):

$$\text{Arg}(\mathcal{S}(\omega)) = k \Delta r - \omega (T_2 - T_1)$$
$$= \frac{\omega}{c} \Delta r - \omega (T_2 - T_1)$$
$$= \text{Arg} \left( \frac{\hat{A}_2(\omega)}{\hat{A}_1(\omega)} \right)$$
$$= \text{Arg} \left( \hat{A}_2(\omega) \right) - \text{Arg} \left( \hat{A}_1(\omega) \right)$$
$$= \phi_2(\omega) - \omega T_2 - \phi_1(\omega) + \omega T_1$$
$$\text{Arg}(\mathcal{S}(\omega)) = \phi_{sr}(\omega) - \omega (T_2 - T_1) \quad (14b)$$

Comparing (14a) and (14b)

$$\phi_{sr}(\omega) = \frac{\omega}{c} \Delta r \quad (15)$$

or

$$c = \frac{\omega \Delta r}{\phi_{sr}(\omega)} \quad (16)$$

which is the expression used in the program.