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RATIONAL CHOICES FOR THE WAVELENGTHS
OF A TWO COLOR INTERFEROMETER

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Rational Choices for the Wavelengths of a Two Color Interferometer.

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Abstract

If in a two color interferometer for plasma density measurements, the two wavelengths are chosen to have a ratio that is a rational number, and if the signals from each of the wavelengths are multiplied in frequency by the appropriate

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and selecting the desired Fourier component. The Fourier components have an integer relationship to the fundamental (frequency and phase); hence the need for a rational number.

On the face of it, the rational number 1:1 is worthless for a standard interferometer, because the two signals cancel; in this case, one would use Faraday rotation², and the two "colors" are the right and left hand polarized components of a linearly polarized laser source. In order for this to work at all, one needs to have and know a magnetic field along the line of sight. This is often the case in magnetic confinement plasma research, especially for sightlines along the confining field. The disadvantage of this technique is that the effect is small: the phase shift is reduced by the ratio $\omega_{ce}/\omega_{laser}$ ($\sim 10^{-3}$) compared to the phase shift of a standard interferometer. (ω_{ce} is the electron cyclotron frequency.)

The rational number 2:1 has not been used much in plasma devices^{3,4,5}; however, it is a most sensible choice. In such a device, the plasma is probed by both a laser beam and its second harmonic. After passing through the plasma the fundamental laser beam is frequency doubled a second time; the interference is between the two frequency doubled components. In a plasma, the phase shift is proportional to wavelength, and the harmonic has half the phase shift of the fundamental. After doubling, the formerly fundamental phase shift is twice as big, and is compared to a half; the net phase shift is 3/2 the phase shift one would get with the fundamental alone. The motion of optical components, on the other hand, is proportional to $1/\lambda$, and the fundamental has half the phase shift of the harmonic; after doubling, they are exactly equal, and cancel completely. (Because there is a difference of index of refraction between the fundamental and the harmonic, there is a systematic window phase shift.) Frequency doubling works very well for YAG lasers at 1064 nm, and suitable components are available off the shelf. This technique was originally developed for optical metrology^{6,7,8}.

The Rational Interferometer

The third choice, a non-harmonic rational number such as 14:13 might not be realized exactly; however, with some lasers, particularly CO₂ lasers, one can at least come close. It is good sense to do so. Consider a two color Michelson interferometer, in which both colors go in both legs, as in Fig. 1. Suppose also, that in the reference leg, there is a Bragg cell which frequency shifts both colors by some frequency (near 40 MHz) before remixing with the measurement beam, and, finally, that the two colors are separated and detected independently. In the detectors, there are RF signals with phase shifts (now with respect to the Bragg cell

driver frequency) appropriate to each color.

[As drawn, Fig. 1 shows the reference beam passing through the Bragg cell twice; this would result in a double frequency shift. Also, because Bragg cells deviate the frequency shifted beams by an amount proportional to λ , the box marked "Bragg Cell" would have to include either a dispersive wedge or a grating to provide compensation, or else there would have to be two retro-reflectors.]

Consider, further, that one wavelength, λ_1 , is chosen for reasons of intensity, ease of use, etc., and that the second wavelength is: $\lambda_2 = \Lambda_2 + \epsilon$, where $\Lambda_2 = (I/J)\lambda_1$, I and J are integers, and ϵ (hopefully) is small. Suppose that in the detector electronics the RF signal for λ_2 is frequency multiplied by I and the λ_1 signal by J . Then, there exists in the electronics two high frequency signals which have a relative phase relationship in which the motion component is small. These two high frequency signals can be heterodyned against one another to produce one signal with a phase shift which is only (or predominately) the plasma density.

The phase contribution from the two colors can be written:

$$\phi_1 = P\lambda_1 + L/\lambda_1,$$

$$\phi_2 = P\lambda_2 + L/\lambda_2$$

with P the plasma contribution, L the unbalanced length contribution, and $\lambda_2 = (I/J)\lambda_1 + \epsilon$. $P\lambda_1$ is the plasma phase shift one would get with an interferometer at λ_1 , L/λ_1 is the phase shift error from unbalanced lengths in that interferometer. Thus:

$$\begin{aligned} P\lambda_1 &= \lambda_1 \left(\frac{\lambda_1 \phi_1 - \lambda_2 \phi_2}{\lambda_1^2 - \lambda_2^2} \right) = J \frac{(J\phi_1 - I\phi_2 - J\eta\phi_2)}{(J^2 - I^2 - 2J\eta - (J\eta)^2)} \\ &= \frac{\Delta\phi - J\eta\phi_2}{2n - 2\eta - J\eta^2} \end{aligned}$$

where,

$\Delta\phi \equiv J\phi_1 - I\phi_2$, $n \equiv J - I$, and $\eta \equiv \epsilon/\lambda_1$. $\Delta\phi$ is the parameter of interest in the rational interferometer. It is given by:

$$\Delta\phi \equiv J\phi_1 - I\phi_2 = P\lambda_1 \left(\frac{J^2 - I^2}{J} + I\eta \right) + \eta \frac{L}{\lambda_2} \left(\frac{1}{(1 + \frac{J}{I}\eta)} \right)$$

$$\Delta\phi \approx P\lambda_1 \left(2n \left(1 - \frac{n}{2J} \right) \right) + \eta \frac{L}{\lambda_2}.$$

In a rational interferometer, $\Delta\phi$ is generated before phase detection, in appropriate frequency multiplication circuits or crystals. This signal is mostly plasma signal, with a small contribution from mirror motion. For instance, $P\lambda_1 \sim 1 \rightarrow 10$ fringe for a typical plasma + interferometer situation, (often ~ 1 fringe for a CO₂ laser setup) and $L/\lambda_2 \sim 100$ fringe for 1 mm motion. η can be $< 10^{-4}$ ($\epsilon < 1$ nm), so that the motional noise is

<0.01 fringe.

On the other hand, in a standard two color interferometer^{9,10,11,12,13}, one measures the phases of the two signals independently, and determines the motional and plasma contributions by appropriate sums of the two signals. In both channels, the phase signal is completely dominated by the motion of mirrors and other optical components. For a 10.6 μm line, the motional noise would be hundreds of fringes; a line density of $2 \times 10^{20} \text{ m}^{-2}$ would produce only 1 fringe. This, of course, requires the difference between two large numbers, both of which need to be measured very carefully.

Table 1 shows a set of wavelength¹⁴ pairs possible in CO₂ lasers, which have a wavelength ratio near some rational number, along with the transition names, rational numbers, ϵ expressed as phase error (in fringes/cm = $\epsilon L / \lambda_1 \lambda_2$, $L = 1 \text{ cm}$), and a signal to noise figure. This S/N figure is the ratio of the detected phase shift for a density which, in a standard two color interferometer, would give 1 fringe, to the motional phase shift given by one cm of mirror motion. (That is, the signal is $\sim 2 \times 10^{20} \text{ m}^{-2}$, and the total length change is 1 cm.) S/N would be ~ 0.001 in a standard two color interferometer. The final column, named tunability, is the fraction of the natural line width (Doppler broadening of room temperature CO₂, $\delta\lambda/\lambda \sim 10^{-6}$) each line would have to be detuned in order that the wavelength ratio would be exactly a rational number. (If both lines are extracted from a single laser, and if the laser length $< \sim 10^5 \lambda$, the lines will be in the identified rational relationship; then the motional noise = 0, and S/N = ∞ .)

Table 1: Selected CO₂ Laser wavelengths.

$\lambda_1: \lambda_2$ (microns)	Transition 1 (in 00°1→10°0 band: Transition 2 + band	J:I	Motional Noise in fringes/ cm	S/N (cm)	Tun- ability
10.44059: 9.69483	P(4): P(36) 00°1→02°0	14:13	9.3×10^{-4}	2078	0.0
10.57105: 9.20073	P(18): R(34) 00°1→02°0	54:47	5.2×10^{-2}	251	0.5
10.61136: 9.30539	P(22): R(14) 00°1→02°0	65:57	6.7×10^{-2}	224	0.5
10.51312: 10.31843	P(12): R(10) 00°1→10°0	54:53	1.4×10^{-2}	144	0.1
10.69639: 10.17033	P(30): R(32) 00°1→10°0	61:58	6.6×10^{-2}	88	0.6
10.71857: 10.22001	P(32): R(24) 00°1→10°0	43:41	6.6×10^{-2}	60	0.8
10.65316: 9.51981	P(26): P(16) 00°1→02°0	47:42	1.9×10^{-1}	51	2.1

Table 2: Selected CO₂ line paired with He-Ne line.

10.12535: 0.632816	P(42) 00°1→10°0: He-Ne	16:1	2.8×10^{-2}	37	14
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Table 1 is a very small sampling of possible wavelengths: λ_1 is

restricted to the $00^{\circ}1 \rightarrow 10^{\circ}0$ P band, and only transitions near the center of the selected bands were looked for (P(n) or R(n), $4 \leq n \leq 40$). The selected bands are: $00^{\circ}1 \rightarrow 10^{\circ}0$ P and R, $00^{\circ}1 \rightarrow 02^{\circ}0$ P and R. The table lists the transitions with the largest S/N. These transitions give a factor of about 10^5 improvement over irrationally chosen lines.

[Table 2 shows a comparison between the common He-Ne line at 633 nm, and a weak CO₂ line with a wavelength 16 times longer. There are also CO₂ lines in the vicinity of 10 μ m which are near the rational number J/3, for $43 \leq J \leq 52$.]

All the CO₂ transition pairs in Table 1 have horrendous looking integers, except for the first pair, with the best S/N, which has 14 and 13. Unfortunately both lines in this pair are far from the center of their bands, and consequently likely to be very weak. The lines of the pair in the third row are near the band center, but the integers are 65 and 57. However, the situation is not quite as bad as it looks, because these integers can be obtained by addition and subtraction (via mixing) as well as multiplication, and the phase differencing can be done at any point in the process. Thus, $65\phi_1 - 57\phi_2$ can be achieved by $65\delta\phi + 8\phi_2$ or by $57\delta\phi + 8\phi_1$, where $|\delta\phi| = |\phi_1 \pm \phi_2|$. The integers 65 and 57 can be achieved by many routes.

For instance suppose, in this case, and for the interferometer of Fig. 1, that the signals are at 80 MHz, and that the signal for λ_2 is heterodyned to both 85 MHz and 40 MHz. If these signal at 85 MHz is heterodyned against the λ_1 signal at 80 MHz, and the difference signal at 5 MHz multiplied by 65, we would end at 325 MHz with a phase shift $65(\phi_1 - \phi_2)$. If the 40 MHz λ_2 signal is multiplied by 8 to 320 MHz, and that heterodyned with the 325 MHz signal, we would end up with a signal at 5 MHz with the desired phase relation $(65\phi_1 - 57\phi_2)$ and we would have used a fairly small part count; 1 tripler, 1 quintupler, 2 quadruplers, 1 doubler and 4 heterodyne circuits.

The line density resolution that can be achieved is:

$$\delta(P\lambda_1)_r = \left(\frac{J}{J^2 - I^2} \right) \Delta\phi, \text{ where } \Delta\phi \text{ is the resolution of the phase}$$

detector, and the line density is expressed in terms of fringes of the longer wavelength of a two color interferometer, and the subscript 'r' refers to the technique using rational numbers. By comparison, the density resolution of a standard two color interferometer (subscript '2c'), would be:

$$\delta(P\lambda_1)_{2c} = \left(\frac{\lambda_1}{\lambda_1^2 - \lambda_2^2} \right) (\lambda_1 \delta\phi_1 - \lambda_2 \delta\phi_2) = \delta\phi \left(\frac{\lambda_1}{\lambda_1^2 - \lambda_2^2} \right) \sqrt{\lambda_1^2 + \lambda_2^2}.$$

The second expression on the right considers the two phase detectors to have the same resolution; they are added in quadrature. If $\Delta\phi = \delta\phi$, and

J/I is a good approximation to λ_1/λ_2 , then the ratio of the line density resolutions is:

$$\frac{\delta(P\lambda_1)_r}{\delta(P\lambda_1)_{2c}} = \frac{1}{J\sqrt{1 + \frac{\lambda_2^2}{\lambda_1^2}}} = \frac{1}{\sqrt{J^2 + I^2}}$$

This ratio is 1/86 for the case $J = 65$, $I = 57$; for 14:13 it is 1/19. For situations in which $\lambda_2 \sim \lambda_1$, $I \sim J$, the ratio is $\sim 1/J\sqrt{2}$; the factor of $\sim\sqrt{2}$ coming from having 1 phase detector, instead of 2 in quadrature, and the factor of J coming from multiplying the phase (one of them, anyway) by J , but presuming the phase detector has the same resolution as in the irrational case.

Rationalizing the Irrational

If the choice of laser lines cannot be made on the basis of a rational number, then the problem can be inverted: what rational number is close enough to the actual ratio to make the motional noise negligible. There is always such a rational number; the practical problem is to find usable integers (just a few stages of doublers, triplers, mixers, etc.) which would nevertheless reduce the motional error to a small correction. At worst, one would have a small error which would have to be determined from a phase measurement of the λ_2 signal. Then, one would have one measurement which is predominantly the plasma density, and another measurement which is predominantly the correction. As an added benefit, this second phase detector could be quite wide band, to look at high frequency effects, which don't need any correction for motional noise.

If one already has a two color interferometer with Bragg cells incorporated in it, then one could rationalize it, or at least make it less irrational, by merely adding new electronics circuitry, which look at the RF signals in parallel with the existing circuitry, and which performs the necessary multiplications and mixing. This can be done, even if the wavelengths are not adjusted to be more suitable to the rational interferometer.

Advantages

The advantages of using this rational number technique for two-color interferometers are that the effects of motion can be nearly canceled out, and thereby reduced to a small correction to the measured phase; the smallest resolvable line density, for a given ability to resolve phase, is smaller (by a factor of $\sim\sqrt{J^2 + I^2}$); the dynamic range of the phase detector is much smaller, and devoted almost completely to the desired

plasma phase measurement; there is no need for a massive support structure; and, because the interferometer is insensitive to vibration, optical components (such as retro-reflectors) can be mounted to the vacuum vessel.

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Fig. 1

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