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**SOLAR VARIABILITY OBSERVED THROUGH
CHANGES IN SOLAR LIMB-DARKENING FUNCTION
AND MEAN DIAMETER**

Final Report

for the period
February 1, 1987 - August 31, 1993

Henry A. Hill

THE UNIVERSITY OF ARIZONA
Tucson, Arizona 85721

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FINAL REPORT

1. SUMMARY OF WORK PERFORMED ON SOLAR VARIABILITY

(For the period February 1987 through August 1993)

1.1. Scope of the 1987-1993 Research Program

Climatically significant solar variability can be monitored both from space and from a ground-based observatory. Changes in solar luminosity have been detected from space by the Solar Maximum Mission spacecraft (Willson 1987). So far, the detected changes are too small to be climatically significant because the time base for these measurements has been short -- too short, in fact, to detect long-term trends. SCLERA determines changes in solar luminosity indirectly through ground-based observations of changes in the solar limb-darkening function, shape, and diameter. Advances at SCLERA during the last 20 years in the accurate measurement of these properties have yielded evidence that the study of the solar limb-darkening function, shape, and diameter offers an indirect probe of the solar atmosphere and interior. This technique is proving to be both highly sensitive and cost-effective in relation to more direct methods (Beardsley *et al.* 1988).

The advantages of a dedicated, ground-based observatory over measurements from spacecraft are its relative adaptability, ease of maintenance, and low cost. However, ground-based observations must contend with problems introduced by observing through the Earth's atmosphere and by changes in the long-term stability of the observing instrument. Both of these problems have been addressed at SCLERA and currently pose no limitation of solar diameter measurements at the parts-per-million level. The atmospheric problems of seeing and differential refraction are managed by separate procedures. For the former, a technique is used for the definition of the edge of a solar limb which exhibits a greatly reduced sensitivity to atmospheric seeing (Hill, Stebbins, and Oleson 1975). For the latter, diameters measured at several solar latitudes are used to yield a solar oblateness and diameter with reduced sensitivity to differential refraction. Differential radius measurements are used to detect changes in the solar limb-darkening function with a reduced sensitivity to differential refraction. The long-term stability of the telescope is monitored with an interferometric technique which is itself stable over long periods of time.

Exploration of information contained in the global oscillations of the Sun is the basis of solar seismology programs. Such exploration permits the examination of the internal structure of a star at an unprecedented level of detail and accuracy. Changes in the internal structure of the Sun relevant to changes in energy output can be detected through observing changes in the mode frequency of oscillation. This approach is also used at SCLERA in its study of long-term variability in solar irradiance.

* SCLERA is an acronym for the Santa Catalina Laboratory for Experimental Relativity by Astrometry, an interdisciplinary research and educational program operating from both the Solar Studies Division and the Department of Physics in the Faculty of Science at the University of Arizona.

1.2. Long-Term Changes in the Apparent Solar Diameter/Limb-Darkening Function

Satellite observations have demonstrated that the total solar irradiance changes by about 0.1% on a time scale akin to the 11-year sunspot cycle. In fact, much of the irradiance variability is explained by the manifestations of magnetic activity - sunspots and faculae. However, models which attempt to reproduce total irradiance using observed sunspot sizes and locations and various proxies for the presence of faculae tend to fail at reproducing the observed irradiance during solar maxima. By better understanding properties of sunspots and employing better measures of facular contributions to irradiance, refinements to these models may well improve their performance. There are, however, mechanisms other than solar magnetic activity which could influence total irradiance, such as global or local changes in the quiet photosphere. The flux of radiant energy from the quiet photosphere depends on the temperature gradient and variations in the photospheric temperature gradient can be determined by monitoring the solar limb-darkening function. Limb-darkening changes could serve as an indication that the quiet photosphere is contributing to solar irradiance variability. Measurements of the solar diameter are sensitive to the solar limb darkening, and thus the solar diameter can also be used as a proxy for total irradiance changes. One of the main efforts of the SCLERA program is to monitor the solar limb-darkening and diameter in order to determine the degree to which the quiet photosphere plays a part in total irradiance variability.

1.2.1. Differential Radius Measurements and the Limb-Darkening Function

The work of SCLERA, which involves long-term solar variability has focused on observations of the solar limb darkening (Beardsley *et al.* 1988). This is implemented by observing the radial intensity distribution of the outermost 32 arcseconds of the solar limb at various solar latitudes. Observations at the extreme limb of the Sun have the advantage of sampling a wide range of photospheric layers without sampling large distances across the solar surface. This minimizes the effects of surface features, such as sunspots, faculae and granulation, which interfere with the intensity of the quiet photosphere. Observations can be made at constant solar latitudes, eliminating possible intensity variations introduced by a latitude-dependent limb-darkening function which affect the observations using drift scanning. At the limb of the Sun, however, seeing effects can be deleterious. This can be overcome by using the Finite Fourier Transform Definition (FFTD) (Hill, Stebbins, and Oleson 1975) of the solar edge. The edge is defined to be a distance ρ from the center of the Sun such that

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} I(\rho + a \sin \pi s) \cos 2\pi s \, ds = 0 \quad , \quad (1)$$

where I is the observed solar intensity as a function of radius and a is the scan amplitude, the parameter indicating the amount of the solar limb being scanned. It has been shown (Hill, Stebbins, and Oleson 1975) that the FFTD has reduced sensitivity to atmospheric seeing when the parameter a is at least as large as the width of the seeing function, but that it is quite sensitive to the shape of the limb-darkening function. It is also apparent that Eq. (1) is unchanged by any constant scaling of I and is, therefore, insensitive to spatially large-scale changes in Earth's atmospheric transparency.

Equation (1) shows that the edge position ρ and scan amplitude a are related through the limb darkening exhibited by I . Therefore, for a particular limb intensity profile and latitude on the solar disk θ ,

$$\rho = \rho(a, \theta) \quad . \quad (2)$$

This dependence of limb position on scan amplitude makes it possible to investigate the shape of the limb-darkening function. Limb positions can be determined for different scan amplitudes a_i and a_j , and the differential radius is defined as

$$\Delta r(a_i, a_j, \theta) = \rho(a_i, \theta) - \rho(a_j, \theta) \quad . \quad (3)$$

When the solar limb profile is a step function, $\Delta r = 0$. For $a_i < a_j$, the more limb darkening exhibited in an intensity profile, the greater the differential radius value. Because of this dependence of Δr on the limb-darkening function, it can be used to monitor changes in the shape of the limb-darkening function over time and, hence, to infer concurrent changes in the photospheric temperature gradient.

1.2.2. Observations

The limb-darkening observations were obtained with the SCLERA astrometric telescope (Oleson *et al.* 1974), which is configured to project the solar image onto a horizontal focal plane while continuously tracking the Sun. In the focal plane is the detector assembly, which consists of two slit blocks positioned near the edge of the solar image at opposite ends of the image diameter. Both slit blocks contain three apertures each seeing 5.7° of the solar circumference and 1 arcsecond of the radius. The centers of the slits in each block are offset by 7.2° along the circumference. (Prior to 1979 each slit block contained only one slit.) The entire detector assembly can rotate in order that each slit will see the same part of the limb as the image rotates during the day and to sample the limb at selected heliographic latitudes.

The solar image is scanned sinusoidally along the diameter defined by the detector assembly, so that about 32 arcseconds of the solar limb is seen by each slit. The scan frequency is 1.6 Hz and thus as many as 10^5 scans can be made in a 10-hour observing day. The light passing through the slits encounters an interference filter which passes light in an 8 nm band of the continuum centered at 550 nm. The intensity of this radiation is sampled 256 times for each slit during each scan. Eleven consecutive intensity profiles are averaged and recorded for each slit. These intensity profiles are used to determine the differential radius value at each limb location. Table 1 indicates the years that observations were obtained and the selected heliographic latitudes that the detector assembly was positioned for the observations.

TABLE 1
Summary of SCLERA observations for long-term variability.

Year	Number of Days	Latitude Angles Observed	Measured Scan Amplitude (arcseconds)
1973	7	$\pm 15^\circ, \pm 105^\circ$	27.2, 20.4, 13.6, 6.8
1979	16	0°	25.6, 19.2, 12.8, 6.4
1981	13	$\pm 13^\circ, \pm 39^\circ, \pm 64^\circ, \pm 90^\circ$	22.5, 16.9, 11.3, 6.3
1983	59	$0^\circ, \pm 45^\circ, 90^\circ$	21.5, 16.1, 10.8, 5.4
1984	21	$0^\circ, \pm 45^\circ, 90^\circ$	21.5, 16.1, 10.8, 5.4
1986	65	0°	33.0, 24.8, 16.5, 8.3
1987	82	0°	31.0, 23.3, 15.5, 7.8
1988	59	$0^\circ, \pm 45^\circ, \pm 90^\circ$	31.2, 23.4, 13.6, 7.8
1989	42	$0^\circ, \pm 45^\circ, \pm 90^\circ$	31.0, 23.3, 15.5, 7.8
1990a	16	$0^\circ, \pm 45^\circ, \pm 90^\circ$	31.7, 23.7, 15.9, 7.9
1990b	20	$0^\circ, \pm 22.5^\circ, \pm 45^\circ, \pm 90^\circ$	30.9, 23.2, 15.5, 7.7
1991a	26	$0^\circ, \pm 22.5^\circ, \pm 45^\circ, \pm 90^\circ$	31.3, 23.5, 15.7, 7.8
1991b	13	$0^\circ, \pm 22.5^\circ, \pm 45^\circ, \pm 90^\circ$	30.8, 23.1, 15.4, 7.7
1992a	20	$0^\circ, \pm 22.5^\circ, \pm 45^\circ, \pm 90^\circ$	30.5, 22.9, 15.3, 7.6
1992b	10	$0^\circ, \pm 22.5^\circ, \pm 45^\circ, \pm 90^\circ$	30.5, 22.9, 15.3, 7.6

As indicated in Eq. (3), the differential radius depends on the amplitude of the image scan, and thus the actual scan amplitude must be determined. This measurement is done several times during the observing run and an average value is found. Table 1 also lists this as the measured scan amplitude. Limb positions corresponding to fractional parts of this measured scan amplitude (also listed in Table 1) are also calculated with the FFTD. Differences between certain pairs of these limb locations constitute a differential radius measurement. In order to meaningfully compare the differential radius values over different observing runs, a fixed pair of scan amplitudes must be used. The values of 8 and 24 arcseconds were chosen for this purpose since actual limb locations were calculated with scan amplitudes close to these values, so the corrections that need to be made to get differential radius values $\Delta r(8,24)$ are relatively small. This correction is based on the dependence of limb position on scan amplitude using the mean limb-darkening function measured in 1987 and 1988. An additional correction is made to account for changes in the actual amount of limb scanned due to the changing earth-sun distance. Differential radius values are normalized to correspond to the amount of limb that would be scanned when the earth is at aphelion in early July. Since many of the observations were made close to this time, these corrections should also be minimized.

The data reduction procedure results in an average differential radius value for each slit at each of its assigned limb positions, on each observing day. For each of these time series the effects of clouds and tracking errors are removed. Then by subjecting the time series to certain criteria the effects of sunspots and faculae are also eliminated. The remaining data then reflect the limb darkening of the quiet photosphere. For the analysis of long-term limb-darkening variability all of the daily differential values at all of the observed limb locations are combined into a single value for each observing run. Although different limb locations were observed during different observing runs, this single quantity is the best indicator of the globally averaged limb-darkening function. These quantities are shown versus the corresponding time of the observing run in Figure 1.

Figure 1. Differential radius results, corrected to standard scan amplitudes of 8 and 24 arcseconds, from observations made from 1973 to 1992.

1.2.3. Comparison of Differential Radius and ACRIM Observations

Kroll, Hill, and Beardsley (1990) have discussed the differential radius results in the context of the ACRIM observations presented by Willson and Hudson (1988). What follows is a summary of their discussion.

The ACRIM data with which Willson and Hudson (1988) are concerned was obtained during two time periods when the SMM satellite was in the "precision solar-pointed mode." The first of these periods was in 1980 just after the launch, and the second during 1984-1986. The irradiance measurements during these times were corrected for the presence of sunspots and

shown as a function of the Ca-plage area, which serves as a crude measure of facular concentration. The data from the two time periods falls into two distinct groups, and linear fits were made for both groups of data. What Willson and Hudson (1988) conclude is that the irradiance change observed from one time period to the other, cannot be accounted for by sunspot blocking and facular enhancement as measured by Ca-plage area.

Kroll, Hill, and Beardsley (1990) address this conclusion by noting that between the two time periods in question, the measured value of the differential radius also changed, and hence the irradiance observations may include a contribution from the quiet photosphere. To investigate this possibility, they determine a residual irradiance change from the Willson and Hudson (1988) data by extrapolating both of the linear fits to zero and differencing these two values. This value of δL is -1.65 W/m^2 and signifies the irradiance change not accounted for by sunspots and faculae. Kroll, Hill, and Beardsley (1990) also quantify the differential radius change by first averaging $\Delta r(8,24)$ for 1979 and 1981 and also averaging the 1984 and 1986 values. The differences of these averages is $\delta[\Delta r(8,24)] = 0.14 \text{ arcseconds}$. Using the average solar irradiance $L = 1367.72 \text{ W/m}^2$ and the aphelion value of the apparent solar radius $R = 945 \text{ arcseconds}$, and assuming that all of the residual irradiance change is correlated with a change in the differential radius, it is found that $(\delta L/L)/[\delta(\Delta r)/R] \approx -7$.

Kroll, Hill, and Beardsley (1990) point out certain *caveats* that accompany this correlation. The first is that the corrections made for the sunspot blocking and facular enhancement could be improved. Indeed, models using the strength of the He 10830 line as a proxy for the facular contribution are an improvement over models using Ca-plage areas, although a discrepancy with observations persists. This correlation does not allow for additional mechanisms to be relevant. With this in mind, the magnitude of the correlation stated above should be considered an upper limit.

The decade from 1973 to 1983 is a time when the differential radius values showed a significant change, which has not been seen repeated in further observations. The possibility exists that this is an artifact of the data analysis procedure, but there is no justification for this being the case. For the differential radius change from 1976 to 1980, the hypothesized correlation with total irradiance gives a value of $\delta L/L \approx -0.15\%$. Based on various measurements of total irradiance during this time, Willson (1984) places $\pm 0.2\%$ as the upper limit of total irradiance change.

As previously noted, the models developed to reproduce the behavior of the total irradiance show significant differences from the observed irradiance during certain phases of the solar cycle. Supplementing these models with the irradiance changes indicated by the differential radius measurements may decrease or even eliminate the differences. Specifically, the model of Foukal and Lean (1988) indicates that from 1978 to 1981 the total irradiance increased by about 0.05% due to the effects of sunspots and faculae. Radiometers aboard the SMM and Nimbus-7 satellites both observed a decrease in the solar total irradiance during the same time period. However, by the correlation proposed above, a decrease of the total irradiance of about 0.1% can be associated with the measured differential radius change during the same time period. The combined effects of solar activity described by the Foukal and Lean model, and the change in the radiation emitted from the quiet photosphere inferred from the differential

radius measurements, result in a net decrease of total irradiance as indicated by satellite observations.

1.2.4. Latitudinal Variations in the Limb-Darkening Function

The SCLERA astrometric telescope allows observations to be obtained in a manner which is useful for determining any latitudinal differences in the solar limb-darkening function. Such observations help ascertain the spatial distribution of radiation from the quiet photosphere and help to uncover mechanisms that effect energy transport through the Sun. Evidence for a spatially non-uniform temperature distribution was reported by Altrock and Canfield (1972) and Kuhn, Libbrecht, and Dicke (1985). Hill *et al.* (1972) reported differences in the polar and equatorial limb-darkening functions. The summary which follows is based on the work of Kroll (1994a,b).

Observations of solar limb darkening were made at 24 limb locations from 1988 through the spring 1990 observing season. Beginning in the fall 1990 observing season, 36 limb locations were observed. The analysis of these data differs from the previous analysis in that daily time series of differential radius values for each slit are used to determine an average value for the season at each limb location observed. In order to compare these averages, corrections must be made for offsets which are present among the six individual detectors. Also apparent in the data from the fall observing runs and from two detectors during the remaining runs are instrumental effects which could not be corrected, so that data is not presented. The results for observing runs 1990a, 1991a, and 1992a are shown in Figure 2. The panel for 1990a shows a rather uniform limb-darkening function in spite of the larger error bars and fewer limb locations than subsequent runs. The 1991a results are of particular interest since there is a strong indication that differential radius values are greater near 20°N and 30°S latitude. The differences between the differential radius at these latitudes and differential radius values at nearby lower latitudes is significant at the 99% level. The 1992a results also show some spatial non-uniformity of the limb darkening, but no smooth patterns are seen. Another important feature is that the average differential radius value over all observed latitudes increases with time.

It is apparent from the differential radius measurements that the solar limb-darkening function was neither uniform over the surface nor constant in time. To elaborate this point, it is useful to focus on the 1991a results and their similarity to the effective temperature measurements of Kuhn and Libbrecht (1991). Their analysis identifies a non-uniform distribution of effective temperature with latitude, distinct from facular enhancements, which they term a "smooth" flux contribution to the total irradiance. The pattern they find in their temperature distribution for observations made in 1990 is like that observed for the 1991a differential radius values. This "smooth" flux contribution is postulated to be the additional mechanism that accounts for the difference between total irradiance models and observations. The origin of the non-uniform temperature distribution proposed by Kuhn (1990,1991) is that the radiation incident on the bottom of the convection zone is modulated by the presence of

Figure 2. Differential radius $\Delta r(8, 24)$ measured at various solar latitudes for three observing seasons. Data have been corrected for offsets among the detectors, and data from slits 3 and 4 are not included.

magnetic fields there. The small temperature variations are magnified to larger temperature variations confined to a narrow layer several hundred kilometers below the base of the photosphere. Such perturbations to the temperature gradient should affect the limb-darkening function and appear in the differential radius measurements.

If, in fact, the two sets of observations are measuring this phenomenon, it should be possible to establish a connection between them. Given the existence of a temperature perturbation in a shallow sub-photospheric layer, Petro, Foukal, and Kurucz (1985) model the change to the limb-darkening function at 4451Å of Pierce and Slaughter (1977). Kroll (1994a) then compares the differential radius value for the perturbed limb-darkening function to the differential radius value for the unperturbed function and finds an increased value for the perturbed function. Thus differential radius values are greater where the shallow-layer temperature enhancements occur, so the distribution pattern of differential radius and effective temperature should be similar. The two sets of observations, then, are qualitatively consistent, based on the temperature perturbation postulated by Kuhn (1990,1991).

Differential radius measurements have revealed that the solar limb-darkening function has been both non-uniform and time variable, which implicates the quiet photosphere as a source of total irradiance variability. The measurements are valuable too for investigating the mechanisms responsible for that variability. Since the solar limb darkening is intimately connected with the apparent solar diameter, differential radius measurements are useful for the interpretation of observed diameter variations (Ribes 1991, Delache and Kroll 1994).

1.2.5. Visual Solar Oblateness

Solar diameter measurements made in 1983 at SCLERA have been analyzed for the Sun's quadrupole and hexadecapole shape terms (Beardsley 1987). These terms represent the shape of an equipotential field at the surface of the Sun that describes the internal mass distribution of the Sun. Variations in the shape of the equipotential surface could lead to a misinterpretation of the effect of a change in the equatorial diameter on the total irradiance.

Expressing the Sun's apparent diameter as a Legendre series, these two terms represent the P_2 and P_4 coefficients, respectively. The theoretical framework used to provide a relationship between the observed shape of the Sun and the multipole moments of the solar gravitational potential field has been improved to include, in general, the effect of both latitudinal and radial variations in the solar rotation. The gravitational potential multipole moments, expressed as the P_2 and P_4 coefficients of a Legendre series, have been found to be $J_2 = (3.4 \pm 1.0)E-6$ and $J_4 = (1.7 \pm 1.1)E-6$, respectively.

The total apparent oblateness ΔR (equatorial-polar radii) found from SCLERA observations is $\Delta R = 13.8 \pm 1.3$ milliarcseconds. The surface rotation contribution $\Delta R'$ to the apparent solar shape is $\Delta R' = 7.9$ milliarcseconds. The quoted uncertainties represent formal statistical 1 σ errors only.

1.2.6. Variability of the Solar Shape

Solar oblateness observations have been made with the solar distortion telescope at Princeton in 1966 (Dicke and Goldenberg 1974), and in 1983, 1984 and 1985 with the telescope operating on Mt. Wilson (Dicke, Kuhn, and Libbrecht; 1985, 1986, and 1987, respectively). SCLERA-type oblateness observations have been made in 1973 (Hill and Stebbins 1975), 1980, 1981, 1983 (Beardsley 1987, for 1980, 1981, and 1983 observations), and 1988. The published results of the Princeton-type observations and the 1973 SCLERA observations led Dicke, Kuhn, and Libbrecht (1987) to suggest that the intrinsic oblateness may be varying with the 22 year Hale double sunspot cycle associated with the reversal of the solar magnetic field. Hill and Beardsley (1986) re-examined the published observational results and demonstrated that, statistically, the limb-darkening function was different along orthogonal diameters for the 1983, 1984, and 1985 data sets of Dicke, Kuhn, and Libbrecht. Also, the form of the variations in the limb-darkening functions changed every year. This result along with several other tests led Hill and Beardsley (1986) to the conclusion that a significant false oblateness due to the limb-darkening differences could remain in the results of Dicke, Kuhn, and Libbrecht. The possibility for systematic error introduced by these differences considerably weakens the basis of the suggestion that the intrinsic oblateness (or surfaces of constant potential) may be varying with the solar magnetic cycle.

Latitudinal changes in the limb-darkening function are well established (see Hill and Stebbins 1975; Beardsley 1987; Hill and Beardsley 1986). However, from recent work at SCLERA it appears that the equator to pole variations in limb-darkening function, which are a major concern in determining solar oblateness and predicting changes in total irradiance, are much smaller than the global secular change that is shown on Figure 1. Also, the latitudinal variations appear to occur on timescales of a few hours or days. Averaged over longer timescales, the latitudinal limb-darkening variations appear to decrease in amplitude with little detectable differences at various solar latitudes.

1.3. Long Term Changes in the Solar Convection Zone

The study of the convection zone and its properties is of particular importance in understanding long-term changes in the total solar irradiance, since all radiation leaving the Sun must pass through the convection zone on its journey to Earth. For instance, it is in this region that changes in the magnetic field alter the efficiency of convective energy transport. As a consequence, the study of modes of oscillation that probe the structure of the convection zone has become an important tool for understanding that region of the Sun and certain types of long-term variability in total irradiance.

When studying the internal solar structure through normal mode oscillations, it is crucial to understand the relationship between the internal structure of the Sun and the individual modes of oscillation. For example, different regions of the solar interior can be studied by making the important observation that g-modes permit one to essentially look from the center of the Sun outward, while the p-modes permit one to look from the surface inward. In particular, the low-order, low-degree acoustic modes and the intermediate-degree f-modes are potentially valuable probes of the solar convection zone. Generally speaking, low-order acoustic modes and f-modes

with lower ℓ values are sensitive to the properties at the bottom of the convection zone, while the modes with higher intermediate values of ℓ are sensitive to the properties near the top of the convection zone.

Not only does the study of the low-order, low-degree acoustic modes and intermediate-degree f-modes open up an avenue for determining the structure of the convection zone, but also allows one to determine how this structure is changing with time. These modes are well suited for this task for two primary reasons. The first is the greater sensitivity of these modes to the convection zone, unlike the low-degree 5-minute modes of oscillation. Second, these modes have line widths ($<0.05 \mu\text{Hz}$) that are intrinsically narrower than the 5-minute acoustic modes. The narrow line widths allow one to detect very small changes in oscillation frequencies, and thus, indirectly to infer changes in the structure of the convection zone. This, of course, is the most interesting aspect of the exercise since it opens up the possibility of predicting future solar events that may have a direct impact upon the Earth.

1.3.1. Acoustic and f-mode Classifications

The low-order, low-degree acoustic modes and intermediate-degree f-modes used in the references cited here were classified in the SCLERA mode classification program. That mode classification program has been described in numerous works (Hill 1984a, 1985a,b, 1986b; Rabaey, Hill, and Barry 1988; Hill and Gu 1990). In general, these classifications are based on the observed temporal and spatial properties of the eigenfunctions obtained from one or more sets of observations.

The 30 low-order, low-degree acoustic mode multiplets used here were classified by Hill (1984a) using the 1979 differential radius observations of Bos and Hill (1983). A typical period for these modes is ≈ 27 min. These mode classifications have been tested and confirmed at a statistically significant level with an independent set of observations by Oglesby (1987a,b,c) and Oglesby, Hill, and Rabaey (1988).

The 19 intermediate-degree f-mode multiplets used here were classified by Rabaey, Hill, and Barry (1988). This f-mode work was motivated Hill, Rosenwald and Rabaey (1986) and Hill (1986a). In Hill, Rosenwald, and Rabaey (1986), preliminary evidence of intermediate-degree f-modes was found during a test of the validity of a solar model having a uniformly rotating convection zone. In Hill (1986a), properties of f-mode multiplets of angular degree $\ell \approx 33$ were identified. Rabaey, Hill, and Barry (1988) took these properties as a starting point in their f-mode classification project. These f-mode classifications have been tested and confirmed at a high level of statistical significance with an independent set of observations by Oglesby (1987a,b,c) and Oglesby, Hill, and Rabaey (1988).

1.3.2. Changes in Acoustic and f-mode Eigenfrequencies Over a Six-Year Period

The 1985 disk center intensity observations of Oglesby (1987b) were used initially to test the mode classifications of Hill (1984a) and Rabaey, Hill, and Barry (1988). In addition to testing these mode classifications, the 1985 observations can be used in conjunction with the 1979 differential radius observations of Bos and Hill (1983) to test for shifts of the eigenfrequencies over a six-year period. Since the low-order, low-degree acoustic modes and the intermediate-degree f-modes reside principally in the convection zone, any observed shifts will play an important role in determining how that region of the Sun is changing. Any changes observed in the eigenfrequencies of modes that are sensitive to the properties of the convection zone are necessarily a consequence of changes in, for example, density and temperature in that region of the Sun.

In order to perform these tests, the eigenfrequency spectrum for the low-order, low-degree acoustic modes and the intermediate-degree f-modes were divided into subgroups as shown in Table 2.

TABLE 2
Subgroups of the eigenfrequency spectrum used in this analysis.

	<u>$n = 0$</u>	
Subgroup 0		$18 \leq l \leq 20$
Subgroup 1		$21 \leq l \leq 26$
Subgroup 2		$27 \leq l \leq 31$
Subgroup 3		$32 \leq l \leq 36$
	<u>$n = 1$</u>	
Subgroup 4		$4 \leq l \leq 6$
Subgroup 5		$7 \leq l \leq 12$
Subgroup 6		$13 \leq l \leq 17$
Subgroup 7		$18 \leq l \leq 22$
	<u>$n = 2$</u>	
Subgroup 8		$3 \leq l \leq 10$
	<u>$n = 3$</u>	
Subgroup 9		$2 \leq l \leq 4$

For each subgroup the location of a classified multiplet member was given by a polynomial for the eigenfrequency ν_{nlm} given by

$$\nu_{nlm} = \nu_{nl} + \nu'_{nl} m + \frac{1}{2!} \nu''_{nl} m^2 + \frac{1}{3!} \nu'''_{nl} m^3, \quad (4)$$

where ν_{nl} , ν'_{nl} , ν''_{nl} , and ν'''_{nl} were determined by Hill (1984a) and Rabaey, Hill, and Barry (1987,1988) using the 1979 differential radius observations of Bos and Hill (1983). For the f-modes, Eq. (4) was extended to include fifth order terms. The necessity of retaining fifth order terms for the f-modes is explained in Rabaey, Hill, and Barry (1988). In this analysis for changes in ν_{nl} , the locations of the classified eigenfrequencies determined by Eq. (4) were compared to peaks in the power spectrum of 1985 disk center intensity observations by Oglesby (1987a,b,c) and Oglesby, Hill, and Rabaey (1988). A peak in the 1985 power spectrum was considered to be in coincidence with a corresponding classified eigenfrequency if the separation in their frequencies was $\leq 0.07 \mu\text{Hz}$. For subgroup i, the observed coincidence rate R_i between the 1985 power spectrum and the classified eigenfrequency spectrum is defined as

$$R_i = N_{nl} / N_{\max} \quad , \quad (5)$$

where N_{nl} is the number of peaks in the 1985 power spectrum observed to be in coincidence with the classified spectrum (within subgroup i), and $N_{\max}$ is the maximum number of coincidences possible for subgroup i.

It was found that R_i was maximized if small shifts in the coefficients ν_{nl} and ν'_{nl} in Eq. (4) were allowed. It is very important to note that ν_{nl} and ν'_{nl} were constrained to be constant within the individual subgroups themselves and only allowed to vary from subgroup to subgroup. The observed shifts in ν_{nl} are shown as a function of subgroup in Figure 3, where shifts of $\approx \pm 0.002 \mu\text{Hz}$ were permitted in ν'_{nl} to maximize the coincidence rates. No positive statistically significant result for evidence of group 4 or group 9 was found in the 1985 power spectrum and are therefore, not plotted. The subgroups are ordered in this figure so that l is increasing from left to right. Therefore, roughly speaking, this figure is an indication of the change in the properties of the convection zone as a function of depth, where low l and high l correspond to the bottom and top of the convection zone, respectively.

The determination of the statistical significance of the above results is made through the calculation of the weighted mean difference, $\bar{d}_i$, between the observed coincidence rate for each subgroup R_i , and the coincidence rate p expected from a random distribution, or

$$\bar{d}_i = \langle R_i - p \rangle \quad , \quad (6)$$

where the mean difference is weighted by the standard deviations of the R_i . The coincidence rate expected from a relative random distribution within a window of $\pm \Delta\nu$ of a given set of frequencies was estimated by empirically comparing 10^5 randomly generated frequencies to the actual data. For the subgroups of multiplets, $\bar{d}_i$ is tabulated in Table 3 along with the probability p of obtaining a positive $\bar{d}$ under the null hypothesis. In calculating the probabilities in this table, an order of magnitude was allowed for the extra degree of freedom introduced in shifting ν_{nl} . Additionally, small shifts in ν'_{nl} were permitted ($\approx 0.002 \mu\text{Hz}$), therefore an order of magnitude was also allowed for this degree of freedom. Again, groups 4 and 9 have been omitted due to poor statistics.

TABLE 3
Observed probability p of obtaining a positive mean difference d under the null hypothesis.

Subgroup	d	p
0	0.135 ± 0.043	1×10^{-1}
1	0.105 ± 0.028	5×10^{-5}
2	0.085 ± 0.028	2×10^{-3}
3	0.094 ± 0.027	5×10^{-4}
5	0.219 ± 0.040	2×10^{-6}
6	0.161 ± 0.038	1×10^{-3}
7	0.110 ± 0.034	5×10^{-2}
8	0.212 ± 0.040	3×10^{-5}

1.3.3. Inferred Changes in the Convection Zone

A formal inversion of the results in Figure 3 has not been made to date. However, the systematic monotonic behavior of the observed shifts and a similar systematic behavior of the eigenfunctions makes it possible to estimate the location and magnitude of implied changes in the convection zone.

It is important to make the distinction between the shifts given in Figure 3 and the reported shifts based on the 5-minute oscillations. For the low-degree 5-minute modes, it is not possible to infer the location of changes in the solar interior responsible for the reported shifts. This situation is contrasted to the one presented here which is based on the low-order, low-degree acoustic modes and the intermediate-degree f-modes. Namely, the location of the changes in internal structure responsible for the frequency shifts can be localized to specific regions within the convection zone.

The inversion of the shifts of the low-order, low-degree acoustic mode and intermediate-degree f-mode eigenfrequencies must take into account the main differences between acoustic and f-modes. The f-modes are generally characterized by $\vec{V} \cdot \vec{\xi} \approx 0$ which is not the case for the acoustic modes ($\vec{\xi}$ is the displacement eigenfunction). An immediate consequence of this is that it is relatively easy to estimate the sensitivity of the acoustic modes to changes in the velocity of sound, but this is not true for the f-modes. From a sensitivity analysis of the $n=1$ acoustic modes and f-modes, we have the order of magnitude expressions (Rosenwald, Hill, and Czarnowski 1988)

$$\frac{\Delta v}{v} \sim \begin{cases} -0.05 \Delta c / c \\ 0.02 \Delta(\delta s / \xi_r) / (\delta s / \xi_r) \end{cases} ; \text{ f-modes } , \quad (7)$$

$$\frac{\Delta v}{v} \sim \begin{cases} -0.50 \Delta c / c \\ 0.03 \Delta(\delta s / \xi_r) / (\delta s / \xi_r) \end{cases} ; \text{ p}_1\text{-modes } ,$$

where c is the sound speed, δs is the entropy eigenfunction, v is the eigenfrequency and ξ_r is the radial part of the displacement eigenfunction. Over the six year period from 1979 to 1985, one is able to make estimates of changes in the internal structure of the convection zone given the relationships in Eq. (7) and the observed shifts in Figure 2. Namely, from the f-mode shifts of $0.05 \mu\text{Hz}$, it is estimated that there was an increase in the ratio of the entropy eigenfunction δs to the radial displacement eigenfunction ξ_r , i.e., $\Delta(\delta s / \xi_r) / (\delta s / \xi_r) \sim 0.5\%$, in the upper half of the convection zone. From the observed p_1 -mode shifts, taking into account the implied change in $(\delta s / \xi_r)$, it is inferred that $\Delta c / c \sim 0.01\%$ in the upper half of the convection zone and $\Delta c / c \sim 0.1\%$ in the bottom half of the convection zone. The change in $(\delta s / \xi_r)$ for example, could be due to changes in the opacity within the convection zone. Work on the formal inversion is currently in progress.

1.4. Evidence of Gravity-Mode Signals in Total Irradiance Observations

The work at SCLERA on the identification of gravity-mode signals in total irradiance observations prior to 1987 was initiated as a means to quantify the relationship between changes in total irradiance to changes in the apparent solar radius. The initial investigation of Hill and Kroll (1986) consisted of analyzing the coincidence of the frequencies and amplitudes of power density peaks in a $6 \mu\text{Hz}$ region of a power spectrum of ACRIM observations determined by Woodard and Hudson (1983a,b) with the frequencies and amplitudes of classified gravity modes given by Hill (1985b). From their findings it was inferred for gravity modes with eigenfrequencies near $100 \mu\text{Hz}$ that

$$\eta' \approx 5 , \quad (8)$$

where

$$\eta' \equiv \frac{(\Delta L / L)}{(\Delta R / R)} , \quad (9)$$

$\Delta L / L$ is the fractional change in luminosity and $\Delta R / R$ is the fractional change in apparent radius associated with the gravity modes. The result given by Eq. (8) indicated that the observable $\Delta R / R$ has a relatively high sensitivity to changes in total irradiance.

Work continued at SCLERA during 1987 and 1988 on the detection and study of gravity-mode signals in total irradiance observations. The motivation was to obtain improved values of η' for use in interpretation of observed changes in the apparent solar diameter and/or radius in terms of changes in total irradiance as well as to evaluate how gravity-mode oscillations

themselves were affecting changes in total irradiance. This work entailed the extension of the work by Hill (1985b) on gravity-mode classification from 31 multiplets to 53 multiplets (Hill and Gu 1990), tests of the extended set of classifications by Oglesby (1987a) using 1985 observations and by Hill (1988) using 1978 observations and an extensive series of analyses by Hill, Huang, and Kroll (1988); Kroll, Huang, and Hill (1987); Kroll, Hill, and Chen (1988); and Kroll, Chen, and Hill (1988) of the 1980 and 1984 total irradiance observations.

The evidence for the detection of gravity-mode signals in the total irradiance observations has been considerably expanded by the work of SCLERA during 1987 and 1988 and the results expressed by Eq. (8) confirmed. In addition, it was found that the rotating nonuniform solar surface introduces sidebands to the gravity-mode multiplets, a discovery which may prove valuable in resolving some of the highly discrepant findings that have been reported for the internal rotation of the Sun.

1.5. Work on Solar Gravity-Mode Classifications

Five significant projects on gravity-mode detection, classification, and subsequent testing of detection-classification results were completed during the period. One project by Hill and Gu (1990) was concerned with the detection and classification of low-degree gravity modes. The projects by Cornuelle and Hill (1990) and Hill (1992) were concerned with testing of the Hill and Gu (1990) work using independent sets of observations. The fourth project concerned the detection and classification of intermediate-degree gravity modes. The fifth project by Hill *et al.* (1991) was an exhaustive review of all the work done to date by the different research groups on the detection and classification of low- and intermediate-degree gravity modes.

The work of Hill (1992) referenced in the first paragraph of this section is a major addition to the series of independent tests made of the Hill (1985b), Gu and Hill (1987), and Hill and Gu (1990) gravity-mode classifications. The first series by Hill and Kroll (1986); Kroll, Huang, and Hill (1987); and Kroll, Hill, and Chen (1988) is based on an analysis of the total irradiance observations obtained from the Solar Maximum Mission. The second series is based on the power spectrum of the 1985 radiation intensity observations made by Oglesby (1987a,b). The third series by Hill (1992) is based on an analysis of the 1978 solar diameter observations made by Caudell *et al.* (1980). These series of tests fall into several categories. Some test for detection of gravity modes, some test for identification of multiplets, and others test l and m assignments that have been made. Highly statistically significant positive results have been obtained in each of the series of tests and, in particular, in the work of Hill (1992).

1.5.1. Low-Degree Gravity-Mode Detection and Classifications

The work of Hill and Gu (1990) reported detection and classification of series of low-degree gravity-mode multiplets of angular order $m=0$ with eigenfrequencies between 60 and 220 μHz . That work on gravity-mode classifications is part of an extensive mode classification program at SCLERA that was started in 1982. This program has been described in numerous works (Hill 1984a, 1985a,b, 1986a; Rabaey, Hill, and Barry 1988) and is based on the physical properties of the eigenfunctions, the use of combinations of observations that possess different

spatial filter functions, and the theoretical eigenfrequency spectrum of a standard solar model for determining the radial order n .

The gravity-mode spectrum, in general, is much more complex than the low-order, low-degree, acoustic-mode spectrum. The complexity of the spectrum was first reduced to a manageable level in the work of Hill (1985b) by the successful combination of differential-velocity observations from the Crimean Astrophysical Observatory (Kotov *et al.* 1983) and of the 1979 differential-radius observations made at SCLERA (Bos and Hill 1983). Because of the different spatial filter functions for these two sets of observations, it was possible to decouple ℓ and m in the mode classification program, where ℓ is the angular degree of a mode. The result was the classification of 31 gravity-mode multiplets with $\ell = 1, \dots, 5$. In a subsequent analysis, Gu and Hill (1987) and Hill and Gu (1990) extended the Hill (1985b) work using the mode classification techniques developed at SCLERA. The result was 53 classified gravity-mode multiplets. The 53 multiplets reported by Hill and Gu (1990) include the original 31 classified by Hill (1985b) with several reclassifications.

1.5.2. Gravity-Mode Classification Tests Based on 1978 Solar Diameter Observations

In order to test the correctness of the gravity-mode classification, solar diameter observations made at SCLERA in 1978 (Caudell *et al.* 1980) were examined. The tests are for correlations of power densities, mean power densities and mean square deviations of power densities between the 1978 power spectrum and the 1979 power spectrum. Positive results were obtained for each of the four correlation tests made. Taken together the probability for obtaining the observed correlations is 9.6×10^{-7} if it is assumed that either there are no gravity-mode signals present or the gravity-mode classifications are incorrect. Two of the tests indicate that if the m classifications are incorrect the probability of the observed correlations is 7.6×10^{-3} . It was also observed that the correlations were maximized if the 1978 power spectrum was shifted by $0.02 \pm 0.07 \mu\text{Hz}$ with respect to the 1979 power spectrum based on the $\ell = 2$ and 4 even m modes. The findings indicate that the 1978 observations contain evidence of low-degree gravity modes and that the observed gravity modes are, on the whole, correctly classified.

1.5.3. Gravity-Mode Classification Test Based on 1985 Radiation Intensity Measurements

A method to measure variations in the radiation intensity in a 200×200 arcsec region at the center of the solar disk was developed and observations were made during the spring of 1985 (Oglesby 1987b). To determine if evidence of gravity modes appears in this data, a test for a correlation of classified eigenfrequencies with peaks in the power spectrum of the observations was made. The classified modes were divided into two categories - the symmetry allowed ($\ell + m$ even) modes and the symmetry forbidden ($\ell + m$ odd) modes. To be considered coincident, the classified g-mode eigenfrequency Hill and Gu (1990) and the 1985 power density peak were required to be within a specified interval $\pm \Delta\nu$. For the symmetry forbidden modes no statistically significant correlation was observed for a range of values of $\Delta\nu$, confirming the null hypothesis that either the modes are incorrectly classified or the gravity-mode signals are not present in the power spectrum. The latter is what one would expect for symmetry forbidden modes. For the symmetry allowed modes, however, a statistically significant correlation was

found which maximized at $\Delta v = 0.063$. The correlation indicated that the probability of the null hypothesis being correct is 5.0×10^{-4} . These results indicate again that it is quite unlikely that the gravity modes are incorrectly classified as well as providing evidence that the 1985 radiation intensity measurements are sensitive to the low-degree gravity modes.

1.5.4. Test for Gravity-Mode Signals in the 1980 and 1984 Total Irradiance Observations

The total irradiance data used in the following tests was obtained with ACRIM during 1980 and 1984. Four non-overlapping time-series of approximately 135 days each were used to obtain four independent power spectra. This approach provides for multiple values of the power density while preserving a good frequency resolution. The eigenfrequencies included in the analysis of Hill, Huang, and Kroll (1988) are between 75 and 135 μHz . A total of 68 classified modes were included. This set of 68 modes is a subset of that used by Kroll, Huang, and Hill (1987) where dual mode classifications of some peaks in the 1979 power spectra were retained.

The tests made by Hill, Huang, and Kroll (1988) are based on the correlation of both the mean power densities and mean square deviation of power densities found in the total irradiance power spectra with the mean power density found in the power spectra of the 1979 differential radius observations at the frequencies of the classified modes. With the assumption of the null hypothesis, that no gravity-mode signals are present in the total irradiance and/or the gravity-mode classifications are incorrect, the probability of the observed correlation of the mean power densities is 4.8×10^{-2} and the correlation of the mean square deviation of the power densities is 5.6×10^{-4} .

Subsequent to this work the analysis was again expanded by Kroll, Huang, and Hill (1987) to include a larger set of gravity modes and a wider range of frequencies. This analysis employs the power densities of 90 classified gravity modes with frequencies between 75 and 165 μHz and l values from 2 to 5. For the work reported here, the set of 90 classified modes are divided into groups with frequencies from 75 to 135 μHz and 135 to 165 μHz in order to obtain more detailed information from the correlation studies.

The results, again assuming the null hypothesis, show that the probability for obtaining the observed correlations of both mean power density and mean square deviation of power density is 0.014, indicating that the likelihood of the null hypothesis being correct is indeed quite small. Thus, a correlation between total irradiance power and power attributed to classified gravity modes can be deemed significant. It is evident that the total irradiance observations are sensitive to gravity-mode oscillations and that the gravity-mode classification is predominately correct.

Since strong evidence exists for gravity-mode signals in solar total irradiance, certain other observable properties of the Sun may be manifest in the ACRIM data. In particular, if the amplitude of a gravity mode were modulated by regions of solar activity which are moving across the solar surface, additional patterns in the total irradiance power spectrum may be apparent. Kroll, Hill, and Chen (1988) and Kroll, Chen, and Hill (1988) have examined the ACRIM observations for evidence of deviations of the observed eigenfunctions from that predicted for an axisymmetric system, deviations that might be expected because of the nonuniformity of the solar surface. Examples of the manifestations of such deviations are well

documented in the literature (Hill and Kroll 1986, Anderson and Maltby 1983, Edmunds and Gough 1983, Hill and Czarnowski 1986, Jeffries *et al.* 1988). In general, the rotating nonuniform surface of the Sun will lead to gravity-mode power density at sidebands of the eigenfrequencies $\nu_{n\ell m}$ of the normal modes where n , ℓ , and m specify the radial order, angular degree and angular order, respectively, of the modes. Differences between the mean power density at frequencies where gravity mode associated sideband power density is predicted and the mean power density at frequencies where only background power density is expected are calculated. Similar differences are calculated for the mean square deviations of the power density.

The analysis is designed to test the hypothesis that there is no more statistically significant power in the sidebands of the eigenfrequencies than there is in regions outside the sidebands. The tests of Kroll, Hill, and Chen (1988) are restricted to modes with angular order $m=0$. Their results give the probability of the null hypothesis being correct is $p = 5.8 \times 10^{-4}$ for all of the data taken together. If the data from 1980 and 1984 are considered separately the probability for the same hypotheses are $p = 0.014$ for 1980 and $p = 0.008$ for 1984. Kroll, Chen, and Hill (1988) have extended these tests to modes with $|m|=1$. Their result for this case, that the probability that the null hypothesis is valid, is $p = 1.7 \times 10^{-3}$. Again considering 1980 and 1984 observations separately the probabilities are $p = 0.015$ for 1980 and $p = 0.025$ for 1984 that the null hypothesis is correct for the $|m| = 1$ modes.

It is interesting to note that the strengths of the sidebands for the $\ell = 1$ and 3, $m = 0$ classified modes are similar to that observed for the sidebands of the $\ell = 2$ and 4, $m = 0$ classified modes. Comparable strengths for what is usually considered the symmetry allowed even ($\ell + m$) signals and the symmetry forbidden odd ($\ell + m$) signals is consistent with the sidebands being generated by a rotating nonuniform surface.

The strength of the sidebands in 1984 relative to 1980 are also different by a factor of 0.54 ± 0.33 . The mean background power density also differs by a similar factor. This correlation is consistent with the sideband power density being generated by a rotating nonuniform surface whose characteristics vary from one year to another.

It was further found that the total gravity mode associated power density in the sidebands is an order of magnitude larger than the gravity mode power density located at the $\nu_{n\ell m}$. This finding may be important in understanding the difficulty experienced in detecting gravity-mode signals in total irradiance observations (Woodard and Hudson 1983a,b; Fröhlich 1988)

In summary, statistically significant evidence of gravity mode associated sideband power density has been found in both the 1980 and 1984 total irradiance power spectra for both $m = 0$ and $|m| = 1$ modes. Furthermore, the sideband structure is interpreted as the effects of rotation of a nonuniform surface and not the signature of rotational splitting, since the strength of the sidebands for 1980 and 1984 are different by a factor of ≈ 2 , the sideband strength is proportional to the mean background power density, and the strength of the even ($\ell + m$) and odd ($\ell + m$) sidebands are comparable.

1.5.5. Phase-Coherence and Lifetimes of Solar Gravity-Mode Signals in 1986-1987 Observations

The continued analysis of solar differential radius data collected at the SCLERA Mt. Bigelow site in 1986 and 1987 has yielded substantial evidence for solar gravity-mode signals. Our current effort focuses on the search for these signals by using their temporal properties. This will extend the previous work on oscillation power (Cornuelle and Hill 1990) to search for more evidence of low-order, low-degree gravity-mode signals. Standard solar theory envisions gravity modes surviving within the solar interior for 10^5 to 10^6 years. This contrasts with the well-known "5-minute" global acoustic modes, which have measured lifetimes of the order of a few days. We are using this predicted, lengthy coherence of gravity modes to aid our search for their signal in the data set. This effort has provided new evidence regarding the existence of these oscillations.

One technique of analysis, which has yielded good results when applied to past observations, is calculation of the $D(i,j)$ parameter values. This quantity reflects the spatial properties of a mode and has previously been used to determine the value of the index m for individual gravity modes. To calculate these values, the data strings from 1986 to 1987 were first divided into five "seasons," each about 41 days in length. This was done for ease of handling. Second, the differential radius data from six different axisymmetric locations on the solar limb near the equator were linearly combined into four data strings, S_i . These S_i reflect the four possible parities of the spherical harmonic indices ℓ and m , which characterize each gravity mode in a spherically symmetric Sun. The quantity $D(i,j)$ is defined, then, as the ratio of the imaginary parts of the Fourier transforms of S_i and S_j at a particular frequency location.

The frequencies at which the $D(i,j)$ were calculated were those found by Hill and Gu (1990) to contain gravity-mode signals in the 1979 SCLERA data. By determining the $D(i,j)$ at these frequencies, we hoped to improve our chance of finding measurable oscillation power in the more recent observations, assuming long-term coherency. Because of the small signal-to-noise ratio for the individual gravity-mode signals, ensembles of the $D(i,j)$ were calculated. These $\langle D(i,j) \rangle$ were determined by averaging over modes of appropriate ℓ and m parity, and over seasons.

The result was that the ensemble of these parameter values was non-zero at the three-standard-deviation level for each ℓ and m parity. As shown in Hill, Alexander, and Caudell (1985), the $\langle D(i,j) \rangle$ are antisymmetric in m -value, and the $\langle D(i,j) \rangle$ exhibit this behavior as well. This result indicates that, when examined statistically, the gravity-mode frequency classifications based on the 1979 data exhibit significant signal power in the 1986-87 observations, thus implying the overall coherence of the signals for at least eight years, since correlation is between modes that are excited. This is far longer than has been previously observed in solar acoustic modes and is consistent with solar oscillation theory.

Another method which shows promise is an examination of phase properties in Fourier transforms of the S_i at the above-mentioned classified gravity-mode frequencies. The phases at these frequencies for consecutive pairs of seasons are compared statistically. The seasons are separated by as much as six months, thus giving a direct method for showing signal coherence on

a time scale longer than that of the 5-minute oscillations. Results are consistent with ensemble phase stability between each season and its successor.

A much more extensive analysis of the power levels at the classified gravity-mode eigenfrequencies via comparison to those of the 1979 data have been made, expanding on earlier work (Cornuelle and Hill 1991). In a nonparametric rank-order analysis, the Spearman correlation coefficient r_s for the spectra at those frequencies was found for $\ell+m$ even parity eigenmodes, with the results given in Table 4.

Overall, the values for r_s and the associated probabilities of null hypothesis correctness P_r are consistent with gravity-mode signal power being present in the 1986-87 data seasons. Another, simpler, comparative power analysis was also completed. A linear fit between the same powers used in forming Table 4 seek to reveal evidence of gravity-mode signal power in the later data sets by a significant positive first-order term. Results are shown in Table 5. Every slope value obtained from the analysis was positive, with the ensemble result for all $\ell+m$ even modes and all 1986-87 data seasons of 16.129 ± 2.317 . Not only is this value significant and positive, the magnitude of the slope implies interesting physics lay in the visibility of these gravity-mode oscillations.

1.6. Stochastic Background Gravitational Radiation

A strong source of stochastic gravity wave background (Thorne 1987) is binary stars in our galaxy which radiate in the frequency range $\nu \leq 0.03$ Hz. Another potentially important background for this frequency range is primordial gravity waves. The oscillations of massive astronomical bodies, such as the Earth, can be used as detectors of such gravity waves (Forward *et al.* 1961, Weber 1967, Dyson 1969, Burke 1973, Ashby and Dreitline 1975, Gilbert 1980, Zimmermann and Hellings 1980, Boughn and Kuhn 1984).

Boughn and Kuhn (1984) carried out an analysis of the gravity wave implications of solar oscillations. For frequencies of $\nu \approx 3 \times 10^{-3}$ Hz, the work is based on the observed velocity amplitude (Isaak 1981) of the much-studied 5-minute oscillations. For frequencies of $\nu \leq 1 \times 10^{-3}$ Hz, the conclusions of Boughn and Kuhn (1984) are based on unpublished estimates by Van der Raay and Isaak of velocity noise level in resonance scattering velocity spectrometers. The reported 1σ confidence level limit on the gravitational flux has a minimum at 4×10^{-4} Hz with an energy density per unit logarithmic frequency interval that is ≈ 100 larger than the critical energy density required to close the universe.

TABLE 4
Correlation statistics for 1979 data vs. 1986-87 data

Season	Symmetry	r_s	P_r
1	1	0.205	0.323
	2	0.0504	0.380
2	1	-9.49×10^{-3}	0.885
	2	0.336	5.13×10^{-4}
3	1	0.215	0.0122
	2	0.357	2.71×10^{-4}
4	1	0.164	0.144
	2	0.319	8.31×10^{-4}
5	1	0.450	5.87×10^{-5}
	2	0.245	4.42×10^{-3}

TABLE 5
Intercepts, slopes, and uncertainties for first-order polynomial least-squares fits of 1986-87 spectral power at mode locations plotted against similarly determined 1979 values of spectral power.

Season	Symmetry	Fit intercept	Fit slope
1	1	9.507 ± 2.879	13.655 ± 21.439
	2	11.470 ± 3.120	6.488 ± 10.394
2	1	6.853 ± 1.363	3.710 ± 10.145
	2	4.121 ± 2.601	32.361 ± 8.664
3	1	3.733 ± 1.612	31.916 ± 12.002
	2	4.895 ± 1.538	16.929 ± 5.122
4	1	6.961 ± 1.496	9.736 ± 11.138
	2	1.704 ± 0.996	13.211 ± 3.317
5	1	5.600 ± 3.857	95.853 ± 28.719
	2	8.905 ± 3.688	27.833 ± 12.286

Mean of all intercepts = 4.768 ± 0.544

Mean of all slopes = 16.129 ± 2.317

It should be possible during the nineties to improve on the solar results of Boughn and Kuhn (1984) in several different ways: 1) use measurements of the temperature eigenfunction in lieu of the velocity eigenfunction, 2) work with observed gravity modes and longer-period acoustic modes of the Sun, and 3) take advantage of the rather unique dependence on radial order of the gravitational quadrupole moment for the $l=2$ modes (Hill, Oglesby, and Gu 1991, Hill and Gu 1991). The disadvantage of working with velocity measurements arises because the energy in a mode excited by gravitational waves is proportional to the square of gravitational quadrupole moment, $(J'_{n,2,m})^2$, and the detection power sensitivity is proportional to the square of the mode amplitude (i.e., $\langle v^2 \rangle / (2\pi v)^2$), where $\langle v^2 \rangle$ is the mean-square average of the observed velocity signal and v is the eigenfrequency. The $1/v^2$ factor is not present for analyses based on the temperature eigenfunction.

There is considerable evidence available now in support of the detection of gravity modes (Hill *et al.* 1991). Progress made at SCLERA in this area is described in Section 1.5. This situation should only improve during the nineties. The most important new development concerns the radial order n dependence of $J'_{n,2,m}$. Any contribution of gravity waves to mode excitation should be proportional to $|J'_{n,2,m}|^2$, which has a rather unique dependence on n . The theoretical properties of $|J'_{n,2,m}|^2$ have been calculated using a standard solar model (Saio 1982). The results are shown in Figure 4 as $|J'_{n,2,0}|^2 / E_{n,2,0}$ plotted as a function of n , where $E_{n,2,0}$ is the kinetic energy of the mode. The double peak feature in Figure 4 can be used specifically to look for the contribution to an observed level of excitation for an ensemble of gravity and long-period acoustic modes due to gravity waves.

Work is underway at SCLERA (Hill and Gu 1991; Hill, Oglesby, and Gu 1991) that is specifically designed to utilize this unique signature of gravity wave excitation. Prospects are good for significant improvement in the upper limit on the stochastic gravity wave flux referenced in the work of Boughn and Kuhn (1984).

Figure 4. The values of $|J'_{n,2,0}|^2 / E_{n,2,0}$ plotted in relative scale, where $E_{n,2,0}$ is the kinetic energy of the corresponding mode of oscillation. Only the $m = 0$ mode is plotted. The negative value of radial order n corresponds to gravity modes.

1.7. Evidence of Gravity-Mode Coupling

The intermediate-degree gravity modes ($\ell \approx 30$) which reside in the region $0.05 < r/R_{\odot} < 0.45$ of the Solar radius provide a useful observational tool for probing the solar interior. The first evidence of the excitation of the intermediate-degree solar gravity modes was obtained in the late 1970's by Hill and Caudell (1979). They found evidence which indicated the excitation of a number of gravity modes with frequencies between ~ 250 and ~ 400 μHz during an examination of the 1973 solar diameter observation taken at SCLERA. However, it was not until 1986 that Hill (1986b) identified and preliminarily classified a set of four intermediate-degree gravity-mode multiplets using the 1979 differential radius observations of Bos (1982). These four multiplets were identified to have $m = 0$ eigenfrequencies of $\nu_{n\ell} \approx 350$ μHz with a classified angular degree of $\ell \approx 30$. These modes were also observed to be mode-coupled (phase locked) which implies that these modes have large temperature eigenfunctions in the core. During 1987 and 1988, the number of detected and classified mode coupled gravity-mode multiplets was increased from 4 to 20 by Rabaey and Hill (1988).

The presence of the relatively large temperature eigenfunctions of these mode-coupled gravity modes and the associated energy transport of these modes are both expected to alter the effective temperature profile in the core of the Sun. This modification of the effective temperature profile is expected to change the neutrino production rates as predicted by the standard solar model (thus providing a possible resolution to the solar neutrino paradox) and may also have an impact on the long term solar variability of the Sun.

1.7.1. Classification Program for the Intermediate-Degree Gravity Modes

The work of Hill (1986b) provided the initial starting point for the intermediate-degree gravity-mode study done by Rabaey and Hill (1988) which culminated in the classification of 20 intermediate-degree gravity-mode multiplets. The mode classification program used by Rabaey and Hill (1988) to classify the intermediate-degree g-modes was similar to the classification schemes used by Hill (1984a, 1986b), and Rabaey, Hill, and Barry (1988). This mode classification program provides a framework that enables one to determine if a set (or family) of peaks in the power spectra defines a rotationally split intermediate-degree g-mode multiplet. There are several important features to note about the classification procedure used by Rabaey and Hill (1988).

The first feature is the technique used to classify the $m = 0$ mode and to determine the ℓ classification for the multiplet. This technique is based upon an analysis done on the properties of $\overline{Y}_{\ell}^m$, where $\overline{Y}_{\ell}^m$ is the average of the spherical harmonic $Y_{\ell}^m(\theta, \phi)$ over the detector geometry used by Bos (1982) (see Bos and Hill 1983). This analysis revealed that (for a given value of ℓ) as one incremented $|m|$ from $m = 0$ to a maximum of $m = \ell$, the value of $\overline{Y}_{\ell}^m$ is observed to pass through zero. The locations of these zeros for ℓ values between 27 and 34 are presented in Table 6. At these zeros of $\overline{Y}_{\ell}^m$ it was found that there was an overall sign change in the $\overline{Y}_{\ell}^m$. This

overall sign change introduces a π phase shift in the plots of mode phase versus m . A search for this unique signature during the mode classification procedure allowed the identification of the $m = 0$ mode and the classification of the angular degree ℓ . This property of the detector geometry was initially introduced by Hill (1986b) to make the preliminary classifications of four intermediate-degree gravity-mode multiplets.

TABLE 6

Values of m where $\overline{Y}_\ell^m = 0$. The quantity $\overline{Y}_\ell^m$ is the spherical harmonic, $\overline{Y}_\ell^m(\theta, 0)$, averaged over the detector geometry. There are two cases of interest; $(\ell + m)$ even and $(\ell + m)$ odd.

ℓ	m_0 for $(\ell + m)$ even	m_0 for $(\ell + m)$ odd
27	22	11
28	23	14
29	24	15
30	25	18
31	26	19
32	27	20
33	30	23
34	9, 31	24

The second feature also deals with the properties of the detector geometry. It was discovered during the analysis of $\overline{Y}_\ell^m$ that the relative detector sensitivity as measured by $\overline{Y}_\ell^m$ was reduced for certain values of m (given that ℓ is fixed). In order to maintain a proficient mode classification procedure these reduced or low detector sensitive mode locations were excluded from the mode classification procedure. This property of the detector geometry was initially introduced and used successfully by Rabaey, Hill, and Barry (1988) to classify a set of 19 intermediate-degree f-mode multiplets.

The final feature deals not with the inclusion of a criterion for mode classification but with the exclusion of a fundamental criterion used in the SCLERA mode classification program. A fundamental part of the mode classification program used to classify modes in the works of Hill (1984a), Hill (1985a,b; 1986b,c), Rabaey, Hill, and Barry (1988) and Hill and Gu (1990) was the use of the functional D_{ij} . Formally, D_{ij} is essentially the ratio of the finite Fourier transform of the $\sin(m\phi)$ component of the eigenfunction to the finite Fourier transform of the $\cos(m\phi)$ component of the eigenfunction [see Hill (1984a) and Hill, Alexander, and Caudell (1985) for a complete discussion of D_{ij}]. From a qualitative viewpoint, if this quasi-periodic function exists for a given set of modes, it establishes a relationship between a family of modes ordered according to frequency and this same family ordered according to m (i.e., according to D_{ij}). In the six mode-classification works cited above, the D_{ij} was instrumental in the determination of the $m = 0$ member of the multiplet. The use of this important part of the SCLERA classification

program was superceded due to a reduced effectiveness caused by the closeness of the first order rotational splitting, ν'_{nl} , to the frequency of $1/(4 \text{ days})$. However, the D_{ij} was used to test the effectiveness of the mode classification procedure after the initial classification program was completed and results of this test will be discussed below (see section 1.7.2).

The two criteria listed above along with the following two requirements: (1) that the observed modes symmetry properties must be internally consistent and (2) that the observed properties of the multiplets as a set must be self consistent form the main components of the intermediate-degree gravity-mode classification program. With this mode classification criteria and the properties of the $g_{15,30}$ multiplet tentatively classified by Hill (1986b) as a starting point the mode classification program was initiated.

1.7.2. The Observed Mode-Coupling and the Results of the Classification Program

As was mentioned in section 1.7.1, the mode classification program resulted in the identification and classification of a set of 20 intermediate-degree gravity-mode multiplets. This set of 20 multiplets is composed of a total of 639 classified normal modes of oscillation which belong to the following 3 subsets; 8 multiplets with radial order $n = 14$ and angular degree $\ell = 27$ to 34, 7 multiplets with radial order $n = 15$ and angular degree $\ell = 28$ to 34, and 5 multiplets with radial order $n = 16$ and angular degree $\ell = 30$ to 34. Each one of the multiplets exhibited 3 mode-coupled sections. These mode coupled sections were clearly shown by a nearly linear relationship between the mode frequencies and the azimuthal eigenvalue m and also by a corresponding nearly linear relationship between the mode phases and the azimuthal eigenvalue m . Shown in Figure 5 is a plot of the mode phases versus azimuthal eigenvalue m for the multiplet tentatively classified by Hill (1986b) and subsequently confirmed by Rabaey and Hill (1988) to be a $g_{15,30}$ gravity-mode multiplet. In this plot, those modes affected by the π phase shift discussed above in Section 1.7.1 were shifted by $-\pi$. Clearly shown in this figure is the linear relationship between the mode phases and the azimuthal eigenvalue m . Shown in Figure 6 is the corresponding plot of mode frequencies versus azimuthal eigenvalue m . The vertical lines separate each mode-coupled section and the line drawn through the frequencies belonging to each mode coupled section is the result of a linear least-squares analysis done on that section. Similar results were obtained for the 19 other multiplets identified and classified by Rabaey and Hill (1988).

In section 1.7.1 it was noted that the mode classification program for the intermediate-degree gravity modes did not use the functional D_{ij} . This was a significant departure from the SCLERA mode classification programs of Hill (1984a) and Rabaey, Hill, and Barry (1988). However, the D_{ij} was used to test the effectiveness of the classification procedure.

Recognizing that the observed D_{ij} will be contaminated by aliasing of peaks in the power spectrum due to the observing window and using the antisymmetry of D_{ij} about $m = 0$ we examine an average of D_{ij} over all modes with the same $|m|$. This average, $\langle D_{ij} \rangle$, is presented in Figure 7. In the works of Hill (1984a,b); Hill, Alexander, and Caudell (1985); and Rabaey, Hill, and Barry (1988); the $\langle D_{ij} \rangle$ was found to be a quasi-periodic function of the azimuthal eigenvalue m with zeros at $|m| = 0$, $|m| \approx 6.3$, $|m| \approx 15.6$, $|m| \approx 22.5$, $|m| \approx 27.5$ and $|m| \approx 34$. Comparing this to the $\langle D_{ij} \rangle$ obtained from the intermediate-degree gravity modes shown in

Figure 7, one can see a very good agreement. The positive outcome of this test is strong evidence that a significant number of these families of peaks are indeed multiplets and also that a significant number of the modes belonging to these multiplets have the correct m classification.

Figure 5. Observed phase of modes in the $g_{15,30}$ multiplet as a function of m . The solid lines are linear least-squares fits to the three different sections.

Figure 6. The observed fine structure for the multiplet $g_{15,30}$. In order to better display the details of the structure, a constant plus linear term in m has been subtracted from the observed ν_{nlm} .

1.7.3. The Implications of Observed Mode-Coupled Gravity Modes

The work of Rabaey and Hill (1988) provides strong evidence of phase locking (mode-coupling) observed in a set of 20 intermediate-degree gravity modes. The evidence of a single mode-coupled section leads to a modification of the neutrino production rates as predicted by a standard solar model with no solar oscillations present. The evidence of two or more mode-locked sections should lead to a periodic modulation in time of the neutrino production rates. The periods of modulation will depend upon the properties of the mode-locked sections, while the amplitude of the modulation will depend upon both the properties of the mode-coupled sections as well as the details of the neutrino production.

Both the presence of modes in the mode-coupled sections with their relatively large temperature eigenfunctions and the associated energy transport of these modes in the region of $0.05 \leq r/R_{\odot} \leq 0.45$ are expected to alter the neutrino production rates. These two mechanisms lead to a modification of the effective temperature profile as defined for a given process in the Sun. Of these two mechanisms, it is anticipated that the former will be the most important process for the modification of the ^8B neutrino production rate.

Figure 7. The azimuthal dependence of the eigenfunction as represented by the function $\langle D_{ij} \rangle$. D_{ij} is defined in terms of the finite Fourier transform of the $\sin m\phi$ and $\cos m\phi$ dependent terms of the oscillatory component of the radiation intensity.

The modulation of the neutrino production rates occurs in a slightly different manner. For a set of phase-locked modes belonging to a the multiplet $g_{n,l}$, there exists a rotating coordinate system with a rotational speed of $\Omega = -2\pi\nu'_{n,l}$ where each mode will have the same frequency of oscillation. Thus, there will be regions of constructive interference where the temperature eigenfunctions will be all in phase. These regions of constructive interference will also be rotating at $-2\pi\nu'_{n,l}$. For a different set of mode-coupled modes, there will be another rotating coordinate system, with a slightly different rotation rate, having identical properties. It is this difference in rotation rates between the respective regions of constructive interference that are important. Periodically these two regions of constructive interference belonging to different mode-coupled sets will come into phase. Because of the importance of the higher-order terms in the process as indicated by the mode coupling, this passage of the two different mode-coupled regions will lead to enhanced neutrino production rates. The periodicity of the enhanced neutrino production rates is obtained directly by taking differences between the observed $\nu'_{n,l}$ found from each of the mode-coupled sections. The results of a preliminary analysis done by Rabaey and Hill (1988) indicate a typical period for this periodicity of ≈ 2 years.

The ability to predict both a change in amplitude of solar neutrino production and a modulation of the solar neutrino production places solar seismology in a unique position. Unlike most, if not all of the other hypotheses put forward to resolve the solar neutrino paradox, the ability to predict both a change in production rates and a modulation of these production rates is natural consequence of mode coupling. Furthermore, this hypothesis can be tested directly by studying neutrino capture rates and periodicities found in these capture rates as observed by neutrino detectors (see Rowley, Cleveland, and Davis 1985).

1.7.4. Intermediate-Degree Gravity Modes, Phase-Locking, and the Solar Neutrino Paradox

Work continued during 1989-1991 on the observed phase-locking of the intermediate-degree gravity modes. Numerous test were conducted on the observed properties of these multiplets. From these statistical tests, several important results have been obtained (cf. Rabaey 1989, Rabaey and Hill 1990). The probability that these multiplets were obtained from a set of peaks randomly distributed in frequency was estimated to be very small, possibly as small as $< 10^{-15}$. This test result implies that the multiplets constitute order within the power spectra under study. The observed horizontal spatial properties of the multiplets were tested and found to be consistent with what one would expect if the peaks were modes of oscillation. The outcome of this test implies that a significant number of the peaks that comprise these multiplets are due to modes of oscillations and that these peaks classified as modes have the correct m classification. The l classification of these multiplets was tested by searching for evidence of modes at locations where the detector sensitivity was low. The outcome of this test confirms the l classification of the multiplets and supports the conclusion that the multiplets are comprised of modes of global oscillation which have eigenfunctions that are well-represented by spherical harmonics. The symmetry properties of the modes within the multiplets were tested and found to be consistent with the symmetry properties that one would expect from normal modes of oscillation. Furthermore, the number of correctly classified modes was estimated to be greater than $67\% \pm 4\%$. The significance of the observed phase-locking was examined and the probability of obtaining a set of multiplets with the observed linear phase relationship was estimated to be within three or four orders of magnitude of 10^{-12} . Based upon this body of

evidence and the range of peak frequencies, one can conclude that the observed multiplets consist of intermediate-degree g-modes.

The discovery of phase-locked gravity modes offers a possible resolution to the discrepancy between the theoretically predicted ${}^8\text{B}$ neutrino flux and the observed ${}^8\text{B}$ neutrino flux. The evidence of a single mode-coupled section leads to a modification of the time-averaged theoretical neutrino production rates as determined from a standard solar model with no solar oscillations. Because these modes reside deep in the interior of the Sun, the very fact that these mode-coupled sections are observed allows one to make two important statements about the properties of these modes. The first property is that the relative amplitudes of the temperature eigenfunctions in the core are large. The second property is that the mode-coupling implies that nonlinear effects are important. These observed properties provide two mechanisms which may modify the effective temperature profile for a given process in the Sun. The first of these mechanisms is the energy transport associated with the modes of oscillation. This mechanism could decrease the effective temperature in the solar interior because of the efficiency of the energy transport by the gravity modes. However, compared to the second mechanism, it is expected to have a lesser impact. The observed large temperature eigenfunctions, in conjunction with the nonlinear effects, provide the second mechanism. These nonlinear effects (and the large temperature eigenfunction) could directly modify the effective temperature profile for a given nucleosynthesis reaction. Because the ${}^8\text{B}$ neutrino production rate is highly temperature sensitive, one can estimate the required relative temperature change to be $\approx 3\%$. In either case (increased energy transport or nonlinear effects), the end result is a reduction in the effective temperature profile for the ${}^8\text{B}$ neutrino production.

In the work of Czarnowski (1989), a detailed theoretical analysis of the nonlinear multi-mode coupling of solar gravity modes was carried out in third order. From the results of his analysis and the properties of the multiplets presented here, Czarnowski (1989) was able to estimate the relative Lagrangian temperature perturbation, $\delta T/T$, for an individual mode to be about 7×10^{-5} . While this amplitude in itself is not large, when one sums over 20 or more phase-locked modes, the resulting amplitude may have an appreciable effect. This theoretical work lends credibility to the above hypothesis.

1.8. Convective Flux and Nonadiabatic Solar Oscillation

In this work, the role of convective flux in the solar nonradial oscillation is investigated by Lee (1994). Although the adiabatic treatment is a good first-order approximation of some physical process in the solar nonradial oscillations, a nonadiabatic analysis is needed to explain or compute other important aspects, especially the excitation and damping mechanisms. Since the nonadiabatic effects may rise to a significant level in the region that ranges from the outer part of the convection zone to the photosphere, an accurate description of the convective flux is required. Due to a lack of accurate and reliable theory of time-dependent convection, previous works on nonadiabatic analysis simply neglected the perturbation of convective flux, or adopted the mixing length theory (MLT). We find the theory of time-dependent turbulent convection developed by Xiong (1989) has many of the desired properties. We tried and tested many alternative ways of mathematical formulation and numerical computation to find a practical and reliable way to incorporate Xiong's work (1989) with our nonadiabatic analysis. While the

treatment in this work may not be completely satisfactory, some interesting results are obtained. The curves of intensity amplitude vs. frequency display unique patterns from which we may suggest that the long period modes can be detected. The positions of minima in plots of temperature amplitude vs. frequency for $\ell = 2, 3, 4$ are consistent with the results of SCLERA observations. This agreement speaks to the credibility of both the low-order g-mode observations obtained at SCLERA and the way we treat the nonadiabatic oscillations coupled with the convective flux. The curves of heat input vs. radius suggest that the hydrogen ionization zone does play an important role for the coupling of convective flux and the low-order g-mode oscillations.

Our set of eigenfrequencies calculated from the adiabatic equations and YALE solar model data is consistent with the results of Guenther *et al.* (1992), the authors of the YALE model, within an acceptable difference (0.3% $\sim$ 0.1). Those findings do not differ much from the previous results calculated using different solar models, while the accuracy of each solar model should be further investigated. This consistency justifies our method of calculation and the choice of solar model.

Xiong's formulae for turbulent convection, which are obtained from basic fluid dynamic equations, could be incorporated with our set of six differential equations based on the first law of thermodynamics and the conservation laws. This new set of equations may be regarded as a more generalized version of Unno *et al.* (1989) and Saio-Cox (1980), since our set converges to their expressions when the convective terms are neglected.

Because Saio-Cox's (1980) numerical calculations were performed for more massive stars, our results cannot be directly compared with their results. However, the nonadiabatic effects are also only appreciable in the region near the surface of the Sun. The frequencies calculated from the nonadiabatic analysis do not differ significantly from the adiabatic results. Though the difference increases as the frequency increases, it remains slight (within 0.1%).

One of the most distinct features in our result is that we could reproduce frequency of minimum in the plot of the intensity eigenfunction vs. n for each ℓ as found in SCLERA observations. This is the first accurate computational evidence of the observed pattern. We believe this agreement supports or enhances the credibility of our effort.

The second, probably the most important, feature is that we could obtain significant temperature or associated intensity perturbations at $\tau = 1$ for low- ℓ g-mode oscillations. The validity of g-mode observations has been disputed by theorists since the fast radiative damping at the surface has been believed to keep the amplitudes of low frequency oscillations undetectable. Unlike their prediction that this damping mechanism monotonically decreases the amplitude of intensity perturbation as the frequency of oscillation decreases, our results show a quite different amplitude vs. frequency curve (see Figure 8). The patterns of those figures suggest that g-modes of longer periods (of an order of 100 minutes) may have detectable amplitudes of temperature or intensity perturbations at the surface.

Figure 8. Temperature perturbations at the surface for $\ell = 2, 3, 4$; all amplitudes are normalized by y_1 at $\tau = 1$.

In summary, we find that the coupling of convection with nonradial oscillations cannot be ignored, and a time-dependent theory of solar convection like Xiong's (1989) treatment shows that the intensity eigenfunctions do indeed remain large at the surface relative to y_1 . Our results strongly support SCLERA observations and Xiong's (1989) theoretical analysis.

1.9. Development on Inverse Techniques and Inversion Results

The inverse problem in solar seismology has two facets. The first is to obtain high-accuracy observational data. Here at SCLERA the observation and mode identification programs have been successful for over a decade, and more than 70 sets of low- and intermediate-degree gravity-mode multiplets have been identified and classified (Hill and Gu 1990, Rabaey and Hill 1990). The second facet is to infer from the observed data the internal structure parameters of the Sun.

Work on the development and application of an inverse algorithm which uses the continuous orthonormalization integration technique and the adjoint sensitivity technique in parallel was undertaken at SCLERA. The continuous orthonormalization technique allows for the determination of eigenfrequencies with both fidelity and efficiency, while the adjoint sensitivity technique allows one to determine the sensitivity of these eigenfrequencies to the model parameters. This inversion technique will allow one to invert observational data sets to obtain information about adjustments required for the standard solar model. Once a baseline, standard solar model has been obtained, one could use the inversion technique to determine secular or quasi-periodic changes in the baseline model.

As a first step toward determining the baseline solar model, a set of three intermediate-degree f-modes was used in an initial inversion exercise. From this exercise, the theoretical eigenfrequencies were moved 0~10 μHz in order to match the observed frequencies by using the adjoint sensitivity technique for determining the necessary adjustments to the given solar model (cf. Rosenwald 1989).

Some of the basic difficulties encountered in the numerical inversion exercise arise from the use of discrete zoning. First, it is difficult to implement physical constraints on the model parameters. For example, when two model parameters are related by a differential equation, one can only use some constraint equations to approximate the relationship. In order to achieve higher accuracy in an inversion, the usual procedure is to increase the number of zones used. The trade-off is that the larger the number of zones used, the larger the number of parameters required in the inversion process. At some point, the inversion exercise will become an underdetermined problem when the number of parameters exceeds the size of the data set available.

This problem could be avoided if the solar internal structure could be characterized by some set of analytical functions. In this case, the inversion problem transforms to one of inferring the parameters in these analytical functions. Now one can increase the number of discrete zones to achieve higher accuracy in the numerical computation without increasing the number of free parameters. Physically this is more reasonable since the solar internal structure can not be perturbed freely and there are some physical constraints that must be satisfied. For

example, below the bottom of the convection zone one would not expect any discontinuity in density. Another advantage of using these continuous parameters is that non-independency of different structure parameters involved in the inversion can be easily built into the formulation.

Efforts have been made at SCLERA to develop a new inversion algorithm using continuous parameters. Also, during 1990 several new features have been added to the inversion algorithm to adopt the generalized inverse method and singular value decomposition (SVD) technique. The generalized inverse method which utilizes the SVD technique has been developed in geophysics data analysis for solving the linearized inverse problem. This method combines the characteristics of both the least-square method and the minimum-length method and provides a better understanding of the inverse process and its results. For the work of SCLERA, the solar interior was subdivided into a relatively small number of zones, and polynomial fits were applied to the solar structure parameters in each zone. The eigenfrequency sensitivities to these polynomial coefficients - the continuous parameters - are computed. These sensitivities can be used to predict the change in solar eigenfrequencies when the structure parameters are perturbed (with the necessary physical constraints satisfied).

The inverse problem for the solar internal structure is formulated by using these sensitivities. The generalized inverse technique is used to solve the nonlinear inverse problem in an iterative process. Observed frequencies of low-degree g-modes have been used for the first application of these new inversion techniques. The nonlinearity of the solar seismic inverse problem is clearly demonstrated. The results from the nonlinear inversion have been analyzed and compared with results from previous inversions. It is found that the inversion results from low-degree g-modes are roughly consistent with those from p-modes in the range of 0.25 to 0.7 solar radius. The inversion results indicate that the standard solar model is a good approximation of the real Sun in a few percent level (Gu, 1992,1993; Gu and Hill 1993).

1.10. References to Section 1

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2. SCLERA PUBLICATIONS OF WORK RELATED TO THE GRANT (1987 - 1993)

Refereed Papers

1. P. H. Oglesby, "Confirmation of the Detection and Classification of the Low-Order, Low-Degree Acoustic Modes with 1985 Observations," in Stellar Pulsation: Lecture Notes in Physics No. 274, eds. A. N. Cox, W. M. Sparks and S. G. Starrfield (Springer-Verlag, Berlin, 1987), pp. 314-317.
2. G. F. Rabaey, H. A. Hill, and C. T. Barry, "The Observed Intermediate-Degree f-Mode Eigenfrequency Spectrum of the Sun and Tests of the Mode Classification Proficiency," Astrophysics and Space Science **143**, 81-97 (1988).
3. H. A. Hill and Y. Gu, "Extension of the Range in Radial Order in the Detection and Mode Classification of Solar Low-Degree Gravity Modes," Science in China A **33**, No. 4, 854-866 (1990).
4. G. F. Rabaey and H. A. Hill, "The Observed Properties of the Intermediate-Degree Gravity Modes," Astrophysical Journal **362**, 734-744 (1990).
5. T. P. Caudell and H. A. Hill, "Sun," in Encyclopedia of Physics, Second Edition, eds. R. G. Lerner and G. L. Trigg (VCH Publishers, New York, 1991), pp. 1189-1190.
6. R. D. Rosenwald and G. F. Rabaey, "Application of the Continuous Orthonormalization and Adjoint Methods to the Computation of Solar Eigenfrequencies and Eigenfrequency Sensitivities," Astrophysical Journal Supplement **77**, 97-117 (1991) (work done while at SCLERA, UA).
7. H. A. Hill, "Tests of the Detection and Mode Classification of Low-Degree Solar Gravity Modes with 1978 Solar Diameter Observations," Astrophysical Journal Supplement **78**, 283-300 (1992).
8. Yeming Gu, "Nonlinear Inversion of Solar Oscillation Frequencies," Astrophysical Journal **413**, 422 (1993).
9. Ronald J. Kroll, "Latitudinal Variation of the Solar Limb-Darkening Function," Solar Physics **152**, 105-110, 1994.

Invited Papers

1. H. A. Hill and Paul H. Oglesby, "New Solar Seismograph for Use in a Global Network," Ninth Annual Fairborn-Smithsonian IAPP Symposium, Tucson, AZ, Feb. 18-21, 1988; in Automatic Small Telescopes, ed. D. S. Hayes and R. M. Genet (Fairborn Observatory, 1988), pp. 131-142.

2. H. A. Hill, C. Fröhlich, M. Gabriel, and W. A. Kotov, "Solar Gravity Modes," in The Solar Interior and Atmosphere, eds. A. N. Cox, W. C. Livingston, and M. S. Matthews (University of Arizona Press, Tucson, 1991), pp. 562-617.
3. H. A. Hill and R. J. Kroll, "Long-Term Solar Variability and Solar Seismology: I," paper given at the UN/EAS Workshop on Basic Space Science for the Benefit of Developing Countries, Bangalore, India, April 30-May 5, 1991; in Basic Space Science, AIP Conference Proceedings, Vol. 245, (AIP, New York, 1992).
4. H. A. Hill, P. Oglesby, and Y.-M. Gu, "Long-Term Solar Variability and Solar Seismology: II," United Nations/European Space Agency Workshop on Basic Space Science for the Benefit of Developing Countries, Bangalore, India, April 30-May 5, 1991, in Basic Space Science, AIP Conference Proceedings, Vol. 245, (AIP, New York, 1992).
5. Henry Hill, "The SCLERA Astrometric Solar Seismograph," SCLERA Symposium: Selected Topics in Science & Technology, Nov. 19-22, 1991, Tucson, Arizona. To be published in the Conference Proceedings.
6. Henry Hill and Yeming Gu, "The Sun as a Gravitational Radiation Detector," SCLERA Symposium: Selected Topics in Science & Technology, Nov. 19-22, 1991, Tucson, Arizona. To be published in the Conference Proceedings.
7. Henry Hill and Paul Oglesby, "Background Compensation," SCLERA Symposium: Selected Topics in Science & Technology, Nov. 19-22, 1991, Tucson, Arizona. To be published in the Conference Proceedings.
8. Henry Hill, "Internal Rotation Problem," SCLERA Symposium: Selected Topics in Science & Technology, Nov. 19-22, 1991, Tucson, Arizona. To be published in the Conference Proceedings.
9. Yeming Gu, Ross D. Rosenwald, Greg F. Rabaey, and H. A. Hill, "The Forward and Inverse Problems in Solar Seismology: Algorithms and Numerical Results," SCLERA Symposium: Selected Topics in Science & Technology, Nov. 19-22, 1991, Tucson, Arizona. To be published in the Conference Proceedings.
10. Ronald Kroll, "Monitoring Long-Term Solar Limb-Darkening Changes with Differential Radius Measurements," SCLERA Symposium: Selected Topics in Science & Technology, Nov. 19-22, 1991, Tucson, Arizona. To be published in the Conference Proceedings.
11. Paul Oglesby, "Photometric Telescopes used in the International Network of SCLERA," SCLERA Symposium: Selected Topics in Science & Technology, Nov. 19-22, 1991, Tucson, Arizona. To be published in the Conference Proceedings.

12. Paul Oglesby, "The Objectives of the International Network of SCLERA," SCLERA Symposium: Selected Topics in Science & Technology, Nov. 19-22, 1991, Tucson, Arizona. To be published in the Conference Proceedings.
13. Ronald J. Kroll, "The Observed Spatial Non-Uniformity and Temporal Variability of the Solar Limb-Darkening Function," in The Solar Engine and Its Influence on Terrestrial Atmosphere and Climate, Proceedings of the NATO Advanced Research Workshop, Paris Observatory, October 25-29, 1993, ed. Elizabeth Nesme-Ribes (NATO ASI Series I: Global Environmental Change, Springer-Verlag, Berlin, 1994), pp. 185-193.
14. Philippe Delache and Ronald J. Kroll, "Panel Discussion on Solar Diameter Variations," in The Solar Engine and Its Influence on Terrestrial Atmosphere and Climate, Proceedings of the NATO Advanced Research Workshop, Paris Observatory, October 25-29, 1993, ed. Elizabeth Nesme-Ribes (NATO ASI Series I: Global Environmental Change, Springer-Verlag, Berlin, 1994), pp. 194-202.

Contributed Papers

1. G. F. Rabaey, H. A. Hill and C. T. Barry, "The Intermediate-Degree f-Mode Multiplets of the Sun," in Stellar Pulsation: Lecture Notes in Physics, No. 125, A. N. Cox, W. M. Sparks and S. G. Starrfield eds. (Springer-Verlag, Berlin, 1987), pp. 307-310.
2. R. J. Kroll, Huang You-ran, and H. A. Hill "Evidence of Solar Gravity Modes Found in Total Irradiance Observations," 171st AAS Meeting, Austin, TX, Jan. 10-14, 1988; abstract in Bull. Am. Astron. Soc. **19**, 1119 (1987).
3. H. A. Hill, "Tests of Solar Gravity Mode Classifications," 171st AAS Meeting, Austin, TX, Jan. 10-14, 1988; abstract in Bull. Am. Astron. Soc. **19**, 1119 (1987).
4. Y. Gu and H. A. Hill "Extension of the Range in Radial Order in the Detection and Classification of Low-Degree Gravity Modes," 171st AAS Meeting, Austin, TX, Jan. 10-14, 1988; abstract in Bull. Am. Astron. Soc. **19**, 1119 (1987).
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6. M. E. Germain and H. A. Hill "Gravity Mode Detection Sensitivity of Resonant-Scattering Spectrometers," 171st AAS Meeting, Austin, TX, Jan. 10-14, 1988; abstract in Bull. Am. Astron. Soc. **19**, 1120 (1987).
7. B. J. Beardsley and H. A. Hill "The Visual Shape and Multipole Moments of the Sun," 171st AAS Meeting, Austin, TX, Jan. 10-14, 1988; abstract in Bull. Am. Astron. Soc. **19**, 1120 (1987).

8. R. D. Rosenwald, H. A. Hill, and Y. Gu "Observed Quasi-periodic Deviations from Second Order Asymptotic Theory of Low-Degree Gravity Mode Eigenfrequencies," 171st AAS Meeting, Austin, TX, Jan. 10-14, 1988; abstract in Bull. Am. Astron. Soc. **19**, 1120 (1987).
9. C. T. Barry, R. D. Rosenwald, Y. Gu, and H. A. Hill "Test of Second Order Asymptotic Theory with Low-Degree Solar Gravity Modes," 171st AAS Meeting, Austin, TX, Jan. 10-14, 1988; abstract in Bull. Am. Astron. Soc. **19**, 1120 (1987).
10. H. A. Hill, "Perturbations of the Sun's Gravitational Field Due to Solar Oscillations," in International Symposium of Experimental Gravitational Physics, eds. P. F. Michelson, Hu En-ke, and G. Pizzella (World Scientific, Singapore, 1988), pp. 510-514.
11. H. A. Hill, Y. Huang, R. J. Kroll "The Detection of Gravity Mode Signals in Differential Radius and Total Irradiance Observations," in Solar Radiative Output Variations: Proceedings of a Workshop, P. Foukal ed., pp. 221-223 (1988).
12. R. J. Kroll, H. A. Hill, and Jian Chen, "Test for the Properties of Solar Gravity Mode Signals in Total Irradiance Observations," COSPAR (Committee on Space Research) XXVII Plenary Meeting, Helsinki, Finland, July 18-29, 1988; Adv. Space Res. **8**, No. 7, (7)39-(7)42 (1988).
13. P. H. Oglesby, H. A. Hill, and G. F. Rabaey, "Evidence for Temporal Variations in the Acoustic and f-Mode Eigenfrequency Spectrum," COSPAR (Committee on Space Research) XXVII Plenary Meeting, Helsinki, Finland, July 18-29, 1988; Adv. Space Res. **8**, No. 7, (7)115-(7)118 (1988).
14. B. Beardsley, C. Cornuelle, R. Kroll and H. A. Hill, "Long Term Solar Variability Studies Using 1986 and 1987 Differential Radius Observations," Bull. Am. Astron. Soc. **20**, 678 (1988).
15. Y. Gu, H. A. Hill and R. D. Rosenwald, "Observed Asymptotic Properties of Low-Degree Solar Gravity Mode Eigenfrequencies," in Proc. Symp. Seismology of the Sun and Sun-like Stars, Tenerife, Spain, September 26-30, 1988; ESA SP-286, pp. 399-401.
16. H. A. Hill, Q. Gao and R. D. Rosenwald, "Inversion of Quasi-Periodic Deviations Between Low-Degree Solar Gravity Mode Eigenfrequencies and Asymptotic Theory Eigenfrequencies," in Proc. Symp. Seismology of the Sun and Sun-like Stars, Tenerife, Spain, September 26-30, 1988; ESA SP-286, pp. 403-406.
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18. B. J. Beardsley, W. M. Czarowski, R. J. Kroll, C. S. Cornuelle, P. H. Oglesby, L. Yi, and H. A. Hill, "Variations in Solar Luminosity Since 1957 Using Differential Radius

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19. R. D. Rosenwald, H. A. Hill and W. M. Czarowski, "Eigenfrequency Sensitivities of Solar p_1 -Modes and f-Modes," 173rd AAS Meeting, Boston, MA, January 8-12, 1989; abstract in Bull. Am. Astron. Soc. **20**, 1011 (1988).
 20. P. H. Oglesby and H. A. Hill, "Temporal Changes in the Internal Structure of the Sun Inferred From Shifts in Eigenfrequency of the Low-Degree p_1 -Modes and Intermediate-Degree f-Modes," 173rd AAS Meeting, Boston, MA, January 8-12, 1989; abstract in Bull. Am. Astron. Soc. **20**, 1011 (1988).
 21. L. Yi and H. A. Hill, "Evidence of Gravity Mode Signals in 1983 Differential Radius Observations of the Sun," 173rd AAS Meeting, Boston, MA, January 8-12, 1989; abstract in Bull. Am. Astron. Soc. **20**, 1011 (1988).
 22. H. A. Hill, C. S. Cornuelle and M. E. Germain, "Properties of Solar Gravity-Mode Eigenfunction in Photosphere," 173rd AAS Meeting, Boston, MA, January 8-12, 1989; abstract in Bull. Am. Astron. Soc. **20**, 1012 (1988).
 23. G. F. Rabaey and H. A. Hill, "The Intermediate-Degree g-mode Spectrum of the Sun: Its Properties and Their Implications to the Solar Neutrino Paradox," in Fourteenth Texas Symposium on Relativistic Astrophysics, ed. E. J. Fenyves, (Ann. NY Acad. Sci.) **571**, 594-600 (1988).
 24. E. Ribes, B. J. Beardsley, T. M. Brown, Ph. Delache, J. Kuhn, F. Laclare and N. Leister, "The Variability of the Solar Diameter," in Proceedings of the Sun in Time Conference, Tucson, AZ, March 6-10 1989, in press.
 25. W. M. Czarowski, "Nonlinear Multi-mode Coupling of the Solar Gravity Modes in a Rotationally Split Multiplet," 174th Meeting of the American Astronomical Society, Ann Arbor, MI, June 11-15, 1989, Bull. Am. Astron. Soc. **21**, 752 (1989).
 26. G. F. Rabaey, "The Observed Properties of the Intermediate-Degree Gravity Modes and their Relevance to the Solar Neutrino Paradox," 174th Meeting of the American Astronomical Society, Ann Arbor, MI, June 11-15, 1989, Bull. Am. Astron. Soc. **21**, 752 (1989).
 27. C. S. Cornuelle and H. A. Hill, "Evidence of Solar Gravity-Mode Oscillations in 1986-87 SCLERA Data," poster paper presented at the 175th AAS Meeting, Washington, DC, Jan. 9-13, 1990; Bull. Am. Astron. Soc. **22**(1), 742 (1990).
 28. R. J. Kroll, B. J. Beardsley, and H. A. Hill, "Recent Differential Radius Results at SCLERA," poster paper presented at the 175th AAS Meeting, Washington, DC, Jan. 9-13, 1990; Bull. Am. Astron. Soc. **22**(1), 742 (1990).

29. R. J. Kroll, H. A. Hill, and B. J. Beardsley, "A Correlation between Changes in Solar Luminosity and Differential Radius Measurements," presented at the NASA-WCRP International Conference on the Climatic Impact of Solar Variability, Goddard Space Flight Center, April 24-27, 1990; in Climate Impact of Solar Variability, NASA Conference Publication 3086, 65-72 (1990).
30. P. H. Oglesby, H. A. Hill, D. Fraser, Y.-M. Gu, J. Chen, J. Hanson, and R. C. Harlow, "The SCLERA International Network: Update," poster paper presented at the 177th AAS Meeting, Philadelphia, Pennsylvania, Jan. 13-17, 1991; Bull. Am. Astron. Soc. 22(4), 1197 (1990).
31. G. F. Rabaey, "A Comparison of Observed Linewidths for Low-Angular-Degree ($l \leq 20$) Solar Acoustic Modes," poster paper presented at the 176th AAS Meeting, Albuquerque, NM, June 10-14, 1990; abstract in Bull. Am. Astron. Soc. 22, No. 2, 856 (1990).
32. J. A. Hanson, H. A. Hill, and G. F. Rabaey, "The Sun's Rotational Angular Velocity, as Determined from the Observed Fine Structure of Intermediate Degree f-modes," poster paper presented at the 177th AAS Meeting, Philadelphia, Pennsylvania, Jan. 13-17, 1991; Bull. Am. Astron. Soc. 22(4), 1198 (1990).
33. R. J. Kroll and H. A. Hill, "The SCLERA Program for Monitoring Solar Limb Darkening and Diameter," Poster paper presented at the NSF/NCAR Workshop on Solar-Terrestrial Impacts on Global Change, Boulder, CO, May 8-10, 1991.
34. C. S. Cornuelle and H. A. Hill, "Observed Stability of Solar Gravity Mode Signals," abstract submitted for the 67th Annual Meeting, SWARM Division, AAAS, Lubbock, Texas, May 15-18, 1991.
35. Yeming Gu, G. F. Rabaey, and H. A. Hill, "Eigenfrequency Sensitivities to Structure Parameters for Solar Normal Mode Oscillations and the Inversion Problem," abstract submitted for the 67th Annual Meeting, SWARM Division, AAAS, Lubbock, Texas, May 15-18, 1991.
36. P. H. Oglesby and H. A. Hill, "The Study of Long-Term Solar Variability and Global Solar Oscillations Through an International Network," IAU 21st General Assembly meeting, Buenos Aires, Argentina, July 23-August 1, 1991.
37. R. Kroll, H. Hill, and K. Beeson, "Variability of the Solar Limb-Darkening Function with Latitude," abstract published in Bull. Am. Astron. Soc. 24, No. 4, 1253 (1992).
38. Li Rufeng, Ye Binxun, Chen Hailin, Liu Shaohus, Deng Bailian, Ma Jiagu, H. A. Hill, and P. H. Oglesby, "Solar Oscillations Instrument at an Infrared Wavelength of 1.6 μm at Yunnan Observatory," Proceedings of IAU Symposium 154, Tucson, AZ, March 2-6, 1992. In Infrared Solar Physics, eds. D. Rabin, J. T. Jefferies, and C. Lindsey (Kluwer Academic Publishers, Dordrecht, 1994), p. 283.

39. Yeming Gu and H. A. Hill, "Nonlinear Inversion for Solar Oscillations," abstract in Bull. Am. Astron. Soc. **25**, 741 (1993). AAA 181st Meeting, Phoenix, AZ, January 3-7, 1993.
40. C. S. Cornuelle and H. A. Hill, "Solar g-Modes: Observed Signal-Amplitude Minima and Adiabatic Theory," abstract in Bull. Am. Astron. Soc. **25**, 741 (1993). AAA 181st Meeting, Phoenix, AZ, January 3-7, 1993.
41. Ronald J. Kroll, "Latitudinal Variation of the Solar Limb-Darkening Function," poster paper presented at IAU Colloquium No. 143, June 20-25, 1993; in Solar Physics **152**, 105-110 (1994).

SCLERA Monograph Series in Astrophysics

1. "Properties of the Solar Gravity Mode Eigenfunctions in the Photosphere," H. A. Hill, C. S. Cornuelle, and M. E. Germain, SCLERA Monograph Series in Astrophysics, No. 8 (1989).
2. "Visual Solar Oblateness Observations and Changes in the Shape of the Limb Darkening Function," Henry A. Hill and Burt Beardsley, SCLERA Monograph Series in Astrophysics, No. 9 (1986).
3. "Gravity Mode Detection Sensitivity of Resonant-Scattering Spectrometers," M. E. Germain and H. A. Hill, SCLERA Monograph Series in Astrophysics, No. 10 (1988).
4. "Observed Asymptotic Properties of Low-Degree Solar Gravity Mode Eigenfrequencies," Ye-ming Gu, Henry A. Hill and Ross D. Rosenwald, SCLERA Monograph Series in Astrophysics, No. 11 (1988).
5. "Eigenfrequency Sensitivities of Solar p_1 -Modes and f-Modes," R. D. Rosenwald, H. A. Hill and W. M. Czarowski, SCLERA Monograph Series in Astrophysics, No. 12 (1989).
6. "Temporal Changes in the Internal Structure of the Sun Inferred From Shifts in Eigenfrequency of the Low-Degree p_1 -Modes and Intermediate-Degree f-Modes," P. H. Oglesby and H. A. Hill, SCLERA Monograph Series in Astrophysics, No. 13 (1989).
7. "Evidence of Gravity Mode Signals in 1983 Differential Radius Observations of the Sun," L. Yi and H. A. Hill, SCLERA Monograph Series in Astrophysics, No. 14 (1989).
8. "A Comparison of Observed Linewidths for Low-Angular-Degree ($\ell \leq 20$) Solar Acoustic Modes," G. F. Rabaey, SCLERA Monograph Series in Astrophysics, No. 15 (1990).
9. "Solar g-Modes: Observed Signal-Amplitude Minima and Adiabatic Theory," C. S. Cornuelle and H. A. Hill, SCLERA Monograph Series in Astrophysics, No. 16 (1993).

Theses and Dissertations

1. P. H. Oglesby, "Global Solar Oscillations Observed in the Visible to Near Infrared Continuum," Ph.D. Dissertation, University of Arizona, Tucson, Arizona (1987).
2. B. J. Beardsley, "The Visual Oblateness and Multipole Moments of the Sun," Ph.D. Dissertation, University of Arizona, Tucson, Arizona (1987).
3. W. M. Czarowski, "Nonlinear Multi-mode Coupling of the Solar Gravity Modes in a Rotationally Split Multiplet," Ph.D. Dissertation, University of Arizona, Tucson, Arizona (1989).
4. G. F. Rabaey, "The Observed Properties of the Intermediate-Degree Gravity Modes and Their Relevance to the Solar Neutrino Paradox," Ph.D. Dissertation, University of Arizona, Tucson, Arizona (1989).
5. R. D. Rosenwald, "The Development of the Continuous Orthonormalization and Adjoint Methods for Solar Seismology: Eigenfrequency Computation and Sensitivity Analysis for Direct and Inverse Problems," Ph.D. Dissertation, University of Arizona, Tucson, Arizona (1990).
6. Yeming Gu, "Inverse Problem with Continuous Parameters for Solar Oscillations," Ph.D. Dissertation, University of Arizona, Tucson, Arizona (1992).
7. C. S. Cornuelle, "The Detection of Low-Degree Solar Gravity Mode Oscillations by Differential Radius Observations," Ph.D. Dissertation, University of Arizona, Tucson, Arizona (1992).
8. M. Germain "Evidence of Solar Oscillations in Rayleigh-Scattered Light," Ph.D. Dissertation, University of Arizona, Tucson, Arizona (1993).
9. Jaeshin Lee, "Convective Flux and Nonadiabatic Solar Oscillation," P.D. Dissertation, University of Arizona, Tucson, Arizona (1994).