DOE/ER/54131

GROWTH CURVE ANALYSIS FOR PLASMA PROFILES USING SMOOTHING SPLINES

Final Report

for Period January 1993 to January 1995

Kaya Imre

*City University of New York
College of Staten Island*

July 1995

Prepared for

**THE U. S. DEPARTMENT OF ENERGY
GRANT NO. DE-FG02-91 ER 54131**

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness, of any information, apparatus, product or process disclosed or represents that its use would not infringe privately-owned rights.

BB
DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED *mv*

MASTER

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ABSTRACT

In this project, we parameterize the shape and magnitude of the temperature and density profiles on JET and the temperature profiles on TFTR. The key control variables for the profiles were tabulated and the response functions were estimated. A sophisticated statistical analysis code was developed to fit the plasma profiles. Our analysis indicate that the JET density shape depends primarily on $\bar{n}/B_t$ for Ohmic heating, $\bar{n}$ for L-mode and I_p for H-mode. The temperature profiles for JET are mainly determined by q_{95} for the case of Ohmic heating, and by B_t and $P/\bar{n}$ for the L-mode. For the H-mode the shape depends on the type of auxiliary heating, $Z_{eff} \bar{n}$, q_{95} , and P .

I. INTRODUCTION

The proposal's goal is to parameterize the temperature and density shape in terms of the engineering variables. In fitting the data, we choose from a candidate list of control variables which includes engineering control variables such as the logarithms of the edge safety factor, q_{95} , the plasma current, I_p (in MA), the toroidal magnetic field, B_t (in Tesla), the average density, $\bar{n}_e$ (in $10^{19}/\text{m}^3$), the injected power, P (in MW), the effective ion charge, Z_{eff} , and the plasma elongation, κ . All control variables are normalized to their database averages, thus they are dimensionless.

The temperature is treated as an unknown multidimensional function. An important class of models are the log-additive models of [1,2]:

$$T(\rho, I_p, \bar{n}_e, P, B_t) = \mu(\rho) I_p^{f_I(\rho)} \bar{n}_e^{f_n(\rho)} P^{f_P(\rho)}, \quad (1)$$

where ρ is the normalized flux radius. These models are the analog of confinement expressions: $C_E = C_0 I_p^{C_1} \bar{n}_e^{C_2} P^{C_3}$. The profile parameterization model replaces empirical constants with smooth functions. These unknown functions, $\mu(\rho), f_I(\rho), \dots$, are estimated with smoothing splines [1,2]. One of the most important results of this work is that we determine which control variables need to be included in the model and eliminate those which are insignificant.

There are six main advantages of profile shape scalings:

- 1—The scaling summarizes the characteristic profile shapes over an operating period. We thereby quantify tokamak performance in a concise and easy to understand form.
- 2—Our profile parameterization serves as a benchmark to to compare performance in new classes of discharges with our standard profile shapes.
- 3—By fitting many discharges simultaneously, the signal to noise ratio is enhanced and we average over effects which are not reproducible from discharge to discharge.
- 4—These expressions can be used in transport, stability and heating codes as realistic temperature and density shapes.

- 5—In many cases, physics insight can be gained from examining the profile parameterization. In particular, we are sometimes able to isolate similarity variables in the profile shape dependencies.
- 6—In the near future, we hope to use profile parameterization to extrapolate tokamak performance to ITER, including peaking factor dependencies.

In the next section, the theoretical advances are described. In Section III code development is described. Section IV is the most important section as it presents our physics findings. A list of publications is also given.

II. THEORETICAL DEVELOPMENTS

The basic model [1] was described previously [3]. However, our use of smoothing splines is a significant improvement on [3]. For each choice of control variables we evaluate the predictive average residual (PAR). The PAR value is more effective at model selection than the corrected chi square test because PAR is robust against outliers and correctly counts the degrees of freedom. At each stage of the variable selection, we add the variable which decreases the PAR value the most.

Our smoothing spline package automatically selects the correct level of smoothing which is appropriate for the data. Originally, we based the smoothing on generalized cross-validation. This yielded artificial wiggles, so we developed a new smoothing procedure which suppressed unnecessary wiggles.

In [2], we describe how our methodology may be modified to fit diffusivities and heat fluxes.

III. NUMERICAL IMPLEMENTATION

During this grant, we have developed a code for data analysis. The code uses standard Fortran 77 and can be used by interested members of the fusion community. Because constant additions and modifications are being done, only rudimentary documentation was prepared prior to the end of grant funding. The program can be run in interactive or in batch mode.

Given profile data and a particular model, the code estimates the unknown functions, $\mu(\rho)$, $f_I(\rho)$, $f_P(\rho)$, as well as the corresponding error bars. The predicted values and the residual fit errors are output for future plotting. Various performance measures are calculated. The code routinely runs on a 486 personal computer. We use the Watcom F77 compiler for DOS, Windows and OS/2 applications. Because of the large size of the program, the DOS version requires a protected-mode compiler. The Windows version also runs fine, but because of the large number of input and output, multi-tasking is quite poor. The OS/2 version allows excellent multi-tasking with very efficient execution of the code. A number of preprocessing and post-processing codes have been written. The preparation of a full report including of the code listing and explanation of its usage is planned.

IV. EXPERIMENTAL RESULTS

We summarize our comparison of temperature and density profiles in JET [4]. This work was done in collaboration with Beatrix Schunke of JET and Kurt Riedel (NYU). For three types of discharge profiles (Ohmic, L-mode, and H-mode), we determine the significant control variables and describe their effects.

The Ohmic density profile shape, analyzed using the JET data obtained during 1990-1991 period, appears to depend predominately on the ratio $\bar{n}_e/B_t$, corresponding to the Murakami parameter. As $\bar{n}_e/B_t$ increases, the profile broadens.

The Ohmic temperature profile shape depends primarily on q_{95} (profile consistency), but other dependencies are discernible [2].

The L-mode density profile depends primarily on $\bar{n}_e$ or $\bar{n}_e/B_t$. A larger data set is required to distinguish between these two parameters. A dependence on κ is seen in the analysis, but we believe that this is artificial, since κ is varied very little, its variation probably correlates with other hidden variables.

The L-mode temperature shape depends primarily on B_t , while the magnitude depends on the power per particle, $P/\bar{n}$, as well as B_t . The temperature peakedness increases with B_t .

The H-mode density shape depends primarily on I_p . As the current increases, the profile

broadens and edge gradient increases.

The H-mode temperature shape depends on the heating type, with ICRH generating more peakedness. The effective charge is the second most important variable with weak dependencies on $\bar{n}$, q_{95} , and power.

A much more detailed description of these results can be found in the paper [4]. An analysis of the TFTR Ohmic profiles is given in [2].

REFERENCES

- 1- K. Riedel and K. Imre, "*Smoothing spline growth curves with covariates.*" Comm. in Statistics, **22**, p.1795-1818, (July 1993).
- 2- K. Imre, K. Riedel and B. Schunke. "*A hierarchy of empirical models for plasma profiles and transport.*" Physics of Plasmas, **2**, p. 1614-1622, (May 1995).
- 3- P.J. McCarthy, K.S. Riedel, O.J.W.F. Kardaun, H. Murmann, K. Lackner, Nuclear Fusion, **31**, 1595 (1991).
- 4- B. Schunke, K. Imre and K. Riedel. "*Log-linear parameterization of JET plasma profiles.*" In progress.

REFEREED ARTICLES

- K. Riedel and K. Imre, "*Smoothing spline growth curves with covariates.*" Comm. in Statistics, **22**, p. 1795-1818, (July 1993).
- K. Imre, K. Riedel and B. Schunke. "*A hierarchy of empirical models for plasma profiles and transport.*" Physics of Plasmas, **2**, p. 1614-1622, (May 1995).
- B. Schunke, K. Imre and K. Riedel. "*Log-linear parameterization of JET plasma profiles.*" In progress.

PROCEEDINGS

- B. Schunke, K. Imre, K.S. Riedel, "*Parametric dependencies of JET electron temperature*

profiles." European Physical Society Proceedings of 21st Conference on Controlled Nuclear Fusion, Montpellier, p. 668-671 (June 1994).

LABORATORY REPORTS

K. Imre and B. Schunke. "*A hierarchy of empirical models for plasma profiles and transport.*" JET Joint Undertaking Report #(94) 59, (Dec. 94).

B. Schunke, K. Imre, K. Riedel, "*Parametric dependencies of JET LIDAR profiles.*" in "*JET posters and papers presented at the 36th APS Plasma Physics Meeting.*" JET Joint Undertaking Report #(94) 61, (Nov. 94).

CONFERENCE PRESENTATIONS

K. Imre and K. Riedel, "*Smoothing Spline of Plasma Profiles*", presented at the 34th APS Plasma Physics Meeting (Nov. 92).

K. Imre and K. Riedel, "*Smoothing Spline Growth Curves with Covariates*", presented at the Biometric Society ENAR Meeting (March 1993)

K. Imre and K. Riedel, "*Empirical Models of Plasma Profiles and Transport: A Hierarchy of Models*", presented at the 35th APS Plasma Physics Meeting (Nov. 93).

K. Imre, K. Riedel and B. Schunke, "*Empirical Models of Plasma Profiles and Transport: A Hierarchy of Models*", presented at the Sherwood 1994 Conference (March 1994).

B. Schunke, K. Imre and K. Riedel, "*Parametric Dependencies of JET LIDAR Profiles*", presented at the 36th APS Plasma Physics Meeting (Nov. 94).

K. Imre, K. Riedel and B. Schunke, "*Log-Additive Parameterization of JET Electron Temperature, Density and Pressure Profiles*", presented at the Sherwood 1995 Conference.