Plasma Inverse Transition Acceleration

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Abstract
The world has not heard of thee, until now...

1 INTRODUCTION

It can be proved fundamentally from the reciprocity theorem with which the electromagnetism is endowed that corresponding to each spontaneous process of radiation by a charged particle there is an inverse process which defines a unique acceleration mechanism, from Cherenkov radiation to inverse Cherenkov acceleration (ICA) [1], from Smith-Purcell radiation to inverse Smith-Purcell acceleration (ISPA) [2], and from undulator radiation to inverse undulator acceleration (IUA) [3]. There is no exception. Yet, for nearly 30 years after each of the aforementioned inverse processes has been clarified for laser acceleration, inverse transition acceleration (ITA), despite speculation [4], has remained the least understood, and above all, no practical implementation of ITA has been found, until now.

Unlike all its counterparts in which phase synchronism is established one way or the other such that a particle can continuously gain energy from an acceleration wave, the ITA to be discussed here, termed plasma inverse transition acceleration (PITA), operates under fundamentally different principle. As a result, the discovery of PITA has been delayed for decades, waiting for a conceptual breakthrough in accelerator physics: the principle of alternating gradient acceleration [5, 6, 7, 8, 9, 10]. In fact, PITA was invented [7, 8] as one of several realizations of the new principle.

Transition radiation occurs when a charged particle transits an inhomogeneous medium, often in the form of a metal foil. To discover its inverse process for practical acceleration, one has to make two critical steps through a conceptual evolution. First, replace the metal foil by an underdense plasma layer such that a focused TM mode with longitudinal electric field of such a mode. The medium of interest is specified by \( \mu = \mu_0 \) and \( \epsilon = \epsilon_0 \epsilon_r(z) \), where \( \epsilon_r(z) = \nu^2(z) \) and \( \nu(z) = 1 + \delta \nu(z) \). Assuming \( |\delta \nu| < 1 \) and \( \left| \frac{d\epsilon_r}{dz} \right| \ll k_0 \epsilon_r \), vector potential is shown to be governed under Lorentz gauge by a wave equation of the same usual form \( \nabla^2 \mathbf{A} + k^2 \mathbf{A} = 0 \), except where \( k = k_0 \nu(z) \), \( k_0 = \omega/c \), with time dependence \( e^{-i\omega t} \) understood.

It has been shown [12] that TM mode in vacuum can be derived by assuming \( \mathbf{A} = A_z(z) \mathbf{z} \). Following the same recipe, the vector wave equation is converted into a scalar one \( \nabla^2 A_z + k^2 A_z = 0 \), which is much easier to solve. Seeking slowly varying envelope solution of the form

\[
A_z(r, z) = \Psi(r, z)e^{i\int k(s)ds}, \tag{1}
\]

we then obtain the usual equation for the envelope

\[
\nabla_+^2 \Psi + 2ik \frac{\partial \Psi}{\partial z} = 0, \tag{2}
\]

which admits the well-known fundamental mode

\[
\Psi = \frac{\Psi_0}{1 + iq} e^{-\frac{q}{z_r}}, \tag{3}
\]

where \( \rho = r/w_0, q = z/z_r \), and \( z_r = k_0 w_0^2/2 \) is the Rayleigh range. The difference between \( k \) and \( k_0 \) is neglected in Eqs.(2,3), since it contributes only to higher order corrections compared to the dominant one preserved in the phase factor of Eq.(1).
Given solution for $A$, electromagnetic fields can be determined from $B = \nabla \times A$, $E = i\omega A + \frac{\partial A}{\partial t} \nabla (\nabla \cdot A)$, yielding for TM$_{01}$ mode under paraxial approximation

$$B_z = B_r = E_\phi = 0, \quad B_\phi = \frac{vE_r}{c},$$

$$E_r = -\frac{i\gamma_g E_\alpha P}{(1 + iq)^2} e^{-\frac{\rho^2}{1 + iq}} e^{\frac{\rho^2}{1 + iq}} i \int k(s) ds,$$

$$E_z = \frac{E_\alpha}{(1 + iq)^2} \left( 1 - \frac{\rho^2}{1 + iq} \right) e^{-\frac{\rho^2}{1 + iq}} i \int k(s) ds,$$

where $\gamma_g = z_r/w_0 \gg 1$ is the condition used when making the paraxial approximation.

Acceleration by the TM$_{01}$ mode of a charged particle traveling along the axis, $\rho = 0$, is determined by

$$\frac{dW}{dq} = \frac{eE_\alpha z_r \cos \psi(q)}{1 + q^2}, \quad (4)$$

where $W = \gamma mc^2$ is the particle energy,

$$\psi = \omega t(z) - \int k(s) ds + 2 \tan^{-1} \left( \frac{z}{z_r} \right), \quad (5)$$

and $t(z) = t_0 + (1 + 1/2\gamma^2)(z/c)$ is the particle orbit.

For simplicity, $\gamma$ is taken as a constant when evaluating the right hand side of Eq.(4), which is valid, strictly speaking, only if the relative change of $\gamma$ remains small during interaction. For laser propagation through an underdense plasma layer, we have $\delta v(z) = -\hat{f}_p(z)/2\gamma_p^2$, where $\gamma_p = \omega/\omega_p \gg 1$, $\omega_p = c/\sqrt{4\pi n_e n_0}$ is the plasma frequency corresponding to the peak electron density $n_0$ of a profile $n_e(z) = n_0 f_p(z)$ with $f_p(z) \leq 1$ and $r_e$ being the classical radius of electron. It is convenient to introduce the following profile function for a model for an ionized gas jet

$$f_p(z) = \frac{1}{\cosh^2 \left( \frac{z - z_p}{l_p} \right)},$$

since it is analytically integrable as

$$\int_{-\infty}^{\infty} f_p(s) ds = l_p \left[ 1 + \tanh \left( \frac{z - z_p}{l_p} \right) \right],$$

where $l_p$ is the half thickness and $z_p$ is the location of the plasma layer. The phase from Eq.(5) is then reduced to

$$\psi = \alpha_\gamma q + \alpha_p \left[ 1 + \tanh \left( \frac{q - q_p}{\beta_p} \right) \right] + 2 \tan^{-1} q + \psi_i,$$

where the scaled parameters are defined by

$$\alpha_\gamma = \frac{\gamma_g^2}{\gamma^2}, \quad \alpha_p = \beta_p \gamma_p^2, \quad \beta_p = \frac{l_p}{z_r}, \quad q_p = \frac{z_p}{z_r}.$$

The net energy change of the particle traveling from $-\infty$ to $\infty$ along the trajectory is given by

$$\Delta W = eE_\alpha z_r \Lambda, \quad (6)$$

where $\Lambda = \Lambda(\alpha_\gamma, \alpha_p, \beta_p, q_p, \psi_i) = \int_{-\infty}^{\infty} dq F(q), \quad (7)$

$$F(q) = \frac{\cos \psi(q)}{1 + q^2}, \quad (8)$$

When expressed in practical units, Eq.(6) becomes

$$\Delta W[\text{MeV}] = 31 \sqrt{P[\text{TW}]} \Lambda,$$

where $P = \pi E_\alpha^2 \gamma^2 / 8 Z_0$ is the laser power, and $Z_0$ is the vacuum impedance.

![Figure 1: $\Lambda$ as a function of $\{\alpha_p, \beta_p\}$ with $\{\alpha_\gamma = 0, q_p = 0\}$ and $\psi_i$ optimized everywhere to maximize $\Lambda$.](image1)

![Figure 2: $\Lambda$ as a function of $\{\alpha_\gamma, \alpha_p\}$ with $\{\beta_p = 0.5, q_p = 0\}$ and $\psi_i$ optimized everywhere to maximize $\Lambda$.](image2)

### 3 PERFORMANCE

Due to phase slippage, a particle moving on the axis sees a varying field, Eq.(8). It is well understood that the integral effect or the net energy change of the particle is identically zero, $\Lambda = 0$, in vacuum where $\alpha_\gamma = 0$, regardless of
ψ_i. This result is often known as Lawson-Woodward theorem [13, 14, 15]. We will see in this section how much net acceleration can be achieved with θ_p > 0. Let’s first examine the dependence of Λ on the five scaled parameters through Eq.(7) and then look at a specific example given in terms of real parameters.

Shown in Fig.1 is a contour plot of Λ as a function of \{α_p, β_p\} with \{α_q = 0, q_p = 0\} and ψ_i optimized everywhere to maximize absolute maximum. It is noted that (1): Λ = 0 for α_p = 0, as expected; (2): ∂Λ/∂β_p < 0; (3): optimal value of α_p decreases from \pi/2 as β_p increases from 0; (4): the absolute maximum, Λ_{max} = 2, verified over the entire parameter space, is reached at \{α_q = 0, α_p = π/2, β_p = 0, q_p = 0, ψ_i = π/2\}. Shown in Fig.2 is a contour plot of Λ as a function of \{α_q, α_p\} with \{β_p = 0.5, q_p = 0\} and ψ_i optimized. It is noted that (1): ∂Λ/∂α_q < 0; (2): the reduction of Λ due to finite particle energy can be neglected if α_q < 1 or γ^2 ≫ \gamma_p^2, a condition equivalent to requiring that the phase slippage caused by finite particle energy is negligibly small compared with Guoy phase shift. In addition, we note without showing the plot due to page limit, that ∂Λ/∂q_p < 0, and the most effective location to place a plasma layer is at the waist q_p = 0.

A specific example is given in Table 1 for electron acceleration, and the integrand F(q) of Λ from Eq.(7) is plotted in Fig.3. For comparison, F(q) is also plotted in Fig.4 for the same case, but without a plasma layer. It is noted that Λ = 1 with the plasma layer and Λ = 0 without. The effect of the plasma layer is to decrease the region over which the particle experiences a deceleration field by enhancing phase slippage over that region. In the extreme case when the thickness of the plasma layer is reduced to 0 while still maintaining an overall phase shift of \pi by increasing plasma density, the area of the negative field region in Fig.3 would be reduced to 0, leading to Λ_{max} = 2.

In conclusion, a new acceleration mechanism is made transparent and established decisively from now on. Stimulating discussions with Max Zolotorev about the intimate relationship between radiation and acceleration are acknowledged. This work was supported by the U.S. Department of Energy under contract No.DE-AC03-76SF00098.

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**Table 1: A PITA Example**

### 4 REFERENCES


