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MARTIN MARIETTA

**New Insights into Input Relegation
Control for Inverse Kinematics of a
Redundant Manipulator
Part 2: The Optimization of a
Secondary Criteria Involving Self
Motion of the Joints**

M. A. Unseren

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Computer Science and Mathematics Division

**NEW INSIGHTS INTO INPUT RELEGATION CONTROL FOR INVERSE
KINEMATICS OF A REDUNDANT MANIPULATOR
PART 2: THE OPTIMIZATION OF A SECONDARY CRITERIA
INVOLVING SELF MOTION OF THE JOINTS**

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Abstract

The input relegation control (IRC) technique for redundancy resolution [1] is extended to solve the problem of optimizing a scalar performance criteria representing a secondary objective to be accomplished via self motion of the joints. The criteria is defined to be the error between the vector of joint velocities and a new vector of "corrective" joint velocities, which is minimized in a Euclidean norm sense. The corrective velocities represent a "corrective" action to be applied to the system and are projected into the null space of the Jacobian in the solution for the joint velocities. The report demonstrates that there exists a component in the solution for the joint velocities that induces self motion of the joints but is not a function of the "corrective action". A technique for eliminating this undesired component is presented. The method is compared to the well known gradient projection technique [4] and its advantages are discussed.

1 Introduction

The purpose of Part 2 of this report is to investigate how a designer could utilize the input relegation control (IRC) redundancy resolution method to optimize a secondary objective involving self motion of the joints while satisfying the primary objective of end effector trajectory tracking. The key to accomplishing this is the determination of the redundant degree of freedom (DOF) variable ϵ , which is defined in eq. (2)^{†1}.

In our previous work [1], it was demonstrated that in principle it is feasible to control the quantities $\{\dot{x}, \epsilon\}$ independently to track reference trajectories. Please note that with this approach the Cartesian degrees of freedom (DOF) and the redundant DOF have equal priorities. Thus it was suggested in [1] to determine the value of ϵ by optimization techniques. Using the latter way, the designer can prioritize the objectives. Unfortunately, the only example provided in [1] was determining ϵ to minimize the square of the Euclidean norm of the joint velocities.

In this report we extend the idea prioritizing our objectives by defining a scalar performance criteria representing the secondary objective as the square of the Euclidean norm of the error between the vector of joint velocities and a vector of "corrective" joint velocities. The corrective joint velocities represent a "corrective action" to be applied to the system via a self motion component in the final solution for the joint velocities. The value of ϵ is determined during the optimization of the performance criteria.

Our motivation for going the optimization route is understood by the following scenario. Each joint in the manipulator has upper and lower physical hardware limits which define its range of motion. The aforementioned "corrective action" is to be applied to the system only when it is detected, by sensing, that one or more joints are in close proximity to their upper or lower hardware limits. Indeed, the intent of the corrective action is to move those joints in a direction away from their respective limits. On the other hand, when all of the joints are sufficiently away from their limits, no corrective action is needed and there is no corrective action component (and thus no self motion component) in the solution for the joint velocities.

The report demonstrates that there exists a component in the final solution for the joint velocities that contributes to self motion of the manipulator that is undesired because it is not a function of the "corrective action". The usefulness of choosing B orthogonal to J that was espoused in Part 1 of this report is advanced by showing that it results in the elimination of this unwanted component.

Our approach is motivated by a desire to overcome certain drawbacks and deficiencies associated with the well known gradient projection technique [4], which are discussed in section 2. A general framework for a single performance criteria representing a secondary objective to be accomplished by self motion of the joints is presented in section 3, that, when optimized using IRC, results in a solution for the joint velocities which satisfies the end effector trajectory tracking requirement. The vector of corrective velocities is treated as a known exogenous quantity here, but becomes an endogenous quantity in Part 3 of this report [3] where it is calculated to yield a joint limit avoidance capability.

^{†1}Superscript $\dagger$ denotes that the referenced equation is in Part 1 of this report [2]

2 Review of Gradient Projection Technique

To date the most popular method for accomplishing a secondary objective by utilizing self motion of the joints while simultaneously satisfying the end effector trajectory tracking requirement is the gradient projection technique originally proposed in [4] and thoroughly investigated by multiple researchers [6, 7, 8, 9, 10, 11, 12]. In [4], an arbitrary scalar function $g(q)$ representing a secondary performance criteria was optimized by determining its gradient and mapping the gradient into the null space of the Jacobian. This was accomplished by augmenting Whitney's minimum Euclidean norm (pseudoinverse) solution [5] to eq. (1)[†] with an additional term [4]:

$$\dot{q} = J^T (J J^T)^{-1} \dot{x} + \left(I_{N \times N} - J^T (J J^T)^{-1} J \right) \left(\frac{dg}{dq} \right)^T \quad (1)$$

where, here again, $I_{k \times k}$ denotes a $(k \times k)$ identity matrix and the $(1 \times N)$ vector dg/dq is the gradient of g .

The coefficient matrix of $(dg/dq)^T$ in eq. (1) lies in the null space of the Jacobian matrix and thus "projects" the gradient of g into the null space. This matrix, however, was not determined by the optimization process itself. No explanation on how this matrix was determined was provided, either. Furthermore, the second term to the right of eq. (1) sacrifices Whitney's minimum Euclidean norm solution to eq. (1)[†], which is acknowledged in [6]. An additional criticism of the gradient projection technique is that the gradient of g is a function of the joint positions but not of the joint velocities. The units of the second term to the right of eq. (1) are not of a velocity and thus are inconsistent.

The coefficient matrix of $(dg/dq)^T$ in eq. (1) is idempotent and therefore singular because the only nonsingular idempotent matrix is the identity matrix [13]. Because the column vectors comprising the coefficient matrix form a linearly dependent set, there always exists the possibility that the components $(dg/dq)^T$ can have values, not all zero, that result in linear combinations of the column vectors which equate to a vector of zeros [13, pg. 113]. Indeed, this condition occurs when $(dg/dq)^T$ can be expressed as a linear combination of the first M columns of our joint space basis G defined in eq. (11)[†], i.e.:

$$\left(\frac{dg}{dq} \right)^T = \sum_{i=1}^M (J_i^T \alpha_i) \quad (2)$$

where $\{\alpha_1, \alpha_2, \dots, \alpha_M\}$ are scalars, not all zero. When eq. (2) is satisfied, $(dg/dq)^T$ is an eigenvector corresponding to a zero valued eigenvalue of its coefficient matrix in eq. (1). In this case it is obvious that the secondary criteria is not optimized.

In this report we take issue with the fundamental essence of gradient projection whereby the particular and homogeneous components of solution for $\dot{q}$ are determined by separate yet conflicting optimization procedures. Our philosophy is that the complete solution (particular and homogeneous) for $\dot{q}$ should be obtained by a optimizing one scalar performance criteria. The performance criteria is presented in a general framework in section 3 and the proposed optimization procedure yields a general form for the solution.

Another criticism of the gradient projection technique is that the gradient of $g(q)$ is not determined by a conditional algorithm. That is to say, the gradient of g and the second term to the right of eq. (1) are always computed, regardless of whether or not

the secondary objective needs to be enforced or not. In the IRC method presented here, the vector of "corrective" joint velocities can be calculated by a conditional algorithm, and the self motion component of the joint velocity solution (which is a function of the corrective velocities) is calculated only when conditions necessitate it be done. This point will be made clear when an application to joint limit avoidance is considered in Part 3.

3 The Proposed Technique

Let $\dot{q}^*$ denote an $(N \times 1)$ vector of "corrective" joint velocities whose values are determined by the state of the manipulator as well as the specific self motion objective selected by the designer. Here $\dot{q}^*$ is assumed to be a known exogenous quantity. An example of making $\dot{q}^*$ an endogenous quantity and calculating its value to establish a joint limit avoidance capability are presented in Part 3.

Our objective is to minimize the square of the Euclidean norm of the error between the vector of joint velocities and $\dot{q}^*$ while simultaneously satisfying the end effector trajectory tracking requirement. This will be accomplished through application of the input relegation control (IRC) redundancy resolution method [1]. The performance criteria representing the secondary objective is proposed:

$$P = (\dot{q} - \dot{q}^*)^T (\dot{q} - \dot{q}^*) . \quad (3)$$

Substituting for $\dot{q}$ in eq. (3) using the right hand side of eq. (10)[†] gives:

$$P = (E \dot{x} + F \epsilon - \dot{q}^*)^T (E \dot{x} + F \epsilon - \dot{q}^*) . \quad (4)$$

Please note that eqs. (1)[†] and (2)[†] have been implicitly satisfied by the substitution leading to eq. (4), and this is why P is not a Lagrangian.

The only unknown variable on the right of eq. (4) is ϵ . Therefore the necessary optimality condition is obtained by differentiating P with respect to ϵ , and equating the result to zero:

$$F^T F \epsilon + F^T (E \dot{x} - \dot{q}^*) = 0_{L \times 1} \quad (5)$$

where, here again, $L = N - M$. Since F has full rank L , then $(F^T F)$ is positive definite and therefore nonsingular. Thus eq. (5) can be solved for ϵ :

$$\epsilon = (F^T F)^{-1} F^T (\dot{q}^* - E \dot{x}) . \quad (6)$$

Backsubstituting the right side of eq. (6) into eq. (10)[†] yields the symbolic solution for the joint velocities:

$$\dot{q} = E \dot{x} + F (F^T F)^{-1} F^T (\dot{q}^* - E \dot{x}) . \quad (7)$$

Please note that $\dot{q}^*$ has been projected into the null space of J by its coefficient matrix in eq. (7). Therefore the corrective action induced by $\dot{q}^*$ does not contribute to end effector motion. Furthermore, the coefficient matrix of $\dot{q}^*$ in eq. (7) is determined as a result of the optimization process, unlike the coefficient matrix of $(dg/dq)^T$ in eq. (1).

It should be noticed that there are two distinct terms in eq. (7) that are explicit functions of $(E \dot{x})$, which is the end effector motion inducing component in eq. (10)[†]. The second occurrence of $(E \dot{x})$, however, has been projected into the null space of J by the same coefficient matrix that projects $\dot{q}^*$ into the null space.

The aforementioned term $\{-F(F^T F)^{-1} F^T E \dot{x}\}$ in eq. (7) has nothing to do with the "corrective action" $\dot{q}^*$ we desire to apply to the system via self motion. The self motion inducing joint velocities $\{-F(F^T F)^{-1} F^T E \dot{x}\}$ and $\{F(F^T F)^{-1} F^T \dot{q}^*\}$ might conflict with one another when determining a solution for the joint velocities that satisfies a secondary criteria such as joint limit avoidance, the topic of Part 3.

To prevent such a potential conflict, it is observed that the unwanted velocity term vanishes when the designer choose B to be orthogonal to J . Indeed, when eq. (25)[†] applies, a solution for E is obtained by postmultiplying eq. (9)[†] by J^T :

$$E = J^T (J J^T)^{-1}. \quad (8)$$

When E is defined by eq. (8), it is easy to verify that its columns are orthogonal to those of F :

$$E^T F = 0_{M \times L} \quad (9)$$

where eq. (6)[†] has been used. Eq. (7) now simplifies to:

$$\dot{q} = G \begin{bmatrix} (J J^T)^{-1} \dot{x} \\ (F^T F)^{-1} F^T \dot{q}^* \end{bmatrix} \quad (10)$$

where G is the joint space basis defined by eq. (11)[†]. It should be noticed that the solution for $\dot{q}$ has been obtained in eqs. (7) and (10) by minimizing just one scalar criteria function in eq. (4). This is in marked contrast to the gradient projection scheme [4] that first obtains a minimum Euclidean norm solution to eq. (1)[†], then sacrifices that solution by introducing a homogeneous solution that projects the gradient of a scalar function into the null space of J .

4 Conclusion

The report has addressed the problem of optimizing a secondary performance criteria by self motion of the joints of a serial link manipulator through application of the input relegation control (IRC) redundancy resolution algorithm. The approach has sought to overcome a fundamental problem with the gradient projection technique. Namely, that it involves two successive optimizations, where the second optimization (gradient of a scalar secondary criteria $g(q)$) sacrifices the minimum Euclidean norm solution for end effector trajectory tracking obtained by the first optimization. Using input relegation control, it has been shown how a single scalar performance criteria can be optimized to obtain the complete solution (particular and homogeneous components) for the joint velocities $\dot{q}$, where the particular solution satisfies the end effector trajectory tracking requirement. The performance criteria proposed is the square of the Euclidean norm of the error between the vector of joint velocities and a vector of "corrective" joint velocities $\dot{q}^*$. Vector $\dot{q}^*$ is projected into the null space of the Jacobian matrix in the final solution for the joint velocities. Also, the coefficient matrix of $\dot{q}^*$ in the solution for $\dot{q}$ is a result of the optimization process, whereas the coefficient matrix of $(dg/dq)^T$ in eq. (1) was not calculated by optimization techniques, nor was it explained how it was selected using gradient projection.

A contribution of the report has been to identify a component in the general solution for the joint velocities given by eq. (7) that contributes to self motion of the joints but is not a function of the corrective action $\dot{q}^*$. It was shown that selecting B to be orthogonal to the rows of J results in this unwanted component vanishing.

It must be acknowledged that the proposed method has a deficiency which is in common with one associated with the gradient projection technique that was pointed out in section 2. The coefficient matrix of $\dot{q}^*$ in eqs. (7) and (10) is idempotent and therefore singular. Because the column vectors comprising the coefficient matrix form a linearly dependent set, there always exists the possibility that the components $\dot{q}^*$ can have values, not all zero, that result in linear combinations of the column vectors which equate to a vector of zeros. This occurs when $\dot{q}^*$ can be expressed as linear combinations of the first M vectors in the joint space basis defined in eq. (11)[†]. Should this happen, the solution reduces to a pure minimum norm solution for $\dot{q}$ and the performance criteria is not optimized.

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