Anharmonic Vibrator Description of Collective Nuclei

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It is shown that a simple anharmonic vibrator model can account for the properties of yrast states in collective nuclei, as well as the quasi-band states built on the γ -vibration or on the 0_2^+ state. This description extends from nearly harmonic vibrator nuclei to pure rotor nuclei and encompasses both energies and $B(E2)$ values. It survives both "horizontal" and "vertical" perspectives on the data of nuclear physics and offers a challenge to microscopic theories of nuclear structure.

I. INTRODUCTION

The traditional view of the structure of collective nuclei centers on isolated paradigms such as the harmonic vibrator and the symmetric rotor. A few nuclei are envisaged as approximating these limits while others are considered transitional between them. These extended shape transition regions are viewed as undergoing a more or less gradual evolution of structure. Within a given nucleus levels are often grouped into "quasi-bands" patterned on the rotational ideal.

In the last few years we have attempted to take a synoptic view of the vast amount of data that already exists on atomic nuclei to see if a simple perspective on the evolution of structure might emerge. As a result of these studies [1-7] an alternative picture of structure than that described above seems to emerge. Broad sequences of nuclei can be described in terms of an anharmonic vibrator (AHV) model, *with constant anharmonicity*, and the transition to the rotor regime is extremely rapid, obeying the precepts of a true phase transitional region similar phenomenologically to magnetic or condensed matter systems.

An important feature of this viewpoint is a "horizontal" approach to the evolution of structure over sequences of N and Z values. Equally important is the "vertical" view which studies the evolution of structure in a single nucleus as a function of excitation energy and/or spin. In this regard, the AHV model also provides an alternate framework. The quasi-band image of yrast levels, for example, which is inspired by the rotational model, can be phrased equally as successfully in terms of a multi-phonon AHV model. Of course, the phonons involved are not intrinsic excitations such as the γ vibration, but rather describe the successive addition of angular momentum to the mean field configuration. This view, which is empirically successful, is in need of a microscopic rationale.

In the following pages we first outline a systematic interpretation of the low-lying yrast states of nearly all collective *non-rotational* nuclei—that is nuclei with $R_{4/2} \equiv E(4_1^+)/E(2_1^+)$ between ~ 2.0 and 3.15 —in terms of a simple AHV model with *constant* anharmonicity. We will then show that the further evolution into the rotational region occurs very rapidly. A similar phase transitional behavior is observed for the states built on

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Fig. 1. Evolution of empirical $R_{4/2}$ values with $N_p N_n$. Data here and in subsequent figures are taken from Nuclear Data Sheets.

Fig. 2. Correlations $E(4_1^+) - E(2_1^+)$ and $B(E2 : 4_1^+ \rightarrow 2_1^+) - B(E2 : 2_1^+ \rightarrow 0_1^+)$. Based on ref. 8. a,b) Empirical values for all collective, non-rotational (i.e. $2.05 \leq R_{4/2} \leq 3.15$) even-even nuclei with $Z = 38 - 82$. c,d) IBA calculations.

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the quasi- γ band. Following this, we will show that an AHV description is also successful for rotational nuclei and that it extends to the highest (pre-backbending) ground "band" states known.

II. ANHARMONIC VIBRATOR DESCRIPTION OF COLLECTIVE NON-ROTATIONAL NUCLEI

The energy ratio $R_{4/2}$ evolves more or less regularly from shell model values below 2.0 in magic nuclei to rotational values near 3.3 in mid-shell regions. This systematic variation, shown in Fig. 1, seems to suggest a smooth variation of structure. However, another viewpoint suggests a different interpretation.

Instead of looking at the dependence of collective observables on "external" variables, such as N or Z , or even $N_p N_n$, it is interesting to look at the correlation of one collective observable with another. To this end we show in Fig. 2a the 4_1^+ energies of all collective even-even nuclei from $Z = 28 - 82$ as a function of $E(2_1^+)$. The empirical phenomenology is startling. The data fall along a compact smooth straight line. Even more striking, this line has a slope of 2.00. In fact, the equation of the best linear least squares fit to the data for $R_{4/2} \lesssim 3.15$ is

$$E(4_1^+) = 2.00E(2_1^+) + 150 \text{ keV} \quad (1)$$

This is, however, exactly the equation of an anharmonic vibrator [8] with *constant* anharmonicity, ϵ_4 . The implication seems to be that the 4_1^+ level can be interpreted as a 2-phonon excitation within the ground "band" intrinsic configuration. The remarkable result is that, although the mean field structure is varying enormously across this plot, the anharmonicity, or phonon-phonon interaction, remains nearly constant. How this can be and what it implies is one of the challenges for a microscopic interpretation that we referred to above.

The AHV interpretation extends to transition rates as well. Figure 2b shows $B(E2 : 4_1^+ \rightarrow 2_1^+)$ values as a function of $B(E2 : 2_1^+ \rightarrow 0_1^+)$ values. They too lie on a straight line and satisfy an AHV expression identical in form to eq. 1.

It is interesting to ask if there is a simple model that can account for this phenomenology. A rather general collective model applicable to a wide range of structures is the IBA. We have carried out IBA calculations for a wide variety of parameters and found that for essentially all parameter combinations that describe collective non-rotational nuclei, the IBA reproduces the observed behavior [9]. This is illustrated for $E(4_1^+)$ and for $B(E2 : 4_1^+ \rightarrow 2_1^+)$ values in Figs. 2c,d. An explanation of the success of the IBA in this regard [10] may provide clues to the microscopic understanding of the constant-anharmonicity behavior observed empirically.

The extension of the energy level data into the rotational region discloses further surprises. Figure 3a extends the data of Fig. 2a into the rotor region. One notes in Fig. 3a the sudden rollover of the linear trajectory as the $R_{4/2} = 3.33$ line (a practical limit) is approached. The suddenness of this transition, *as a function of $E(2_1^+)$* , suggests that it might even be useful to view this observable as a kind of critical parameter. The distinction between the vibrator and rotor regimes is seen most clearly in the sudden slope change (Fig. 3b). The slope is constant at 2.00 until a certain critical value of $E(2_1^+)$ and then suddenly increases towards the limiting value 3.33.

The usefulness of $E(2_1^+)$ as a critical marker of structure extends to other data as well. In the AHV description the 4_1^+ level is a 3-phonon state that involves two anharmonicities, ϵ_4 and ϵ_2 [which is the deviation of the 2_1^+

Fig. 3. a) Correlation of $E(4_1^+)$ with $E(2_1^+)$ for all collective even-even nuclei with $Z = 38 - 82$. (Based on ref. 3.) The solid curve is a linear least squares fit to the data. b) The derivative $dE(4_1^+)/dE(2_1^+)$ as a function of $E(2_1^+)$ obtained from the fit in Fig. 3a.

Fig. 4. a) Correlation of $E(4_γ^+)$ with $E(2_1^+)$ for all collective even-even nuclei with $Z = 30 - 82$. See eq. 3 for the definition of $E'(4_γ^+)$. (Based on ref. 11.) The solid curve is the IBA calculation discussed in ref. 11. b) The slope of the IBA calculations as a function of $E(2_1^+)$. c) Empirical values of $E(2_γ^+)$ for some rare earth nuclei as a function of $E(2_1^+)$.

level from $2E(2_1^+)$.]

$$E(4_\gamma^+) = 3.0E(2_1^+) + \frac{10}{7}\epsilon_2 + \frac{10}{7}\epsilon_4 \quad (2)$$

To test if the 4_γ^+ state can be viewed as a 3-phonon excitation, it is useful to re-write eq. 2 as

$$E'(4_\gamma^+) \equiv E(4_\gamma^+) - \frac{11}{7}\epsilon_2 - \frac{10}{7}\epsilon_4 = 3.0E(2_1^+) \quad (3)$$

Taking ϵ_2 and ϵ_4 from the 2_γ^+ and 4_1^+ levels of each nucleus, we can evaluate $E(4_\gamma^+)$ and plot this against $E(2_1^+)$. This is done in Fig. 4a. Again the data lie close to a straight line whose slope of 3.12 is surprisingly close to the predicted value.

Once again, the evolution into the rotor regime is revealing. We show in Fig. 4b the slope as a function of $E(2_1^+)$. This slope changes rapidly in exactly in the same range of $E(2_1^+)$ values as the slope change for the yrast states. It leaps from just above 3.0 to almost 50 within a range of a few keV. The phase transition corresponds to an interesting structural change. In the AHV regime the 2_γ^+ level is a 2-phonon excitation of the ground state. In the rotor region, it becomes a 1-phonon γ -vibrational mode. Correspondingly, the 4_γ^+ level changes from a 3-phonon state to a rotational excitation built on the γ vibration. Figure 4c shows this evolution of the structure of the 2_γ^+ state for a couple of rare earth elements. The 2_γ^+ energies drop smoothly until the critical 2_1^+ energy and then exhibit wildly fluctuating behavior.

The AHV can also be extended to the quasi-band built on the lowest excited 0^+ state (which is a two phonon excitation in the vibrator scheme). We use the label " β " for those states for historical continuity but do not imply thereby any specific structure. The AHV eigenvalue expressions for the quasi- β band are:

$$E(0_\beta) = 2.0E(2_1^+) + \epsilon_0 \quad (4)$$

$$E(2_\beta) = 3.0E(2_1^+) + \frac{7}{5}\epsilon_0 + \frac{4}{7}\epsilon_2 + \frac{36}{35}\epsilon_4 \quad (5)$$

$$E(4_\beta) = 4.0E(2_1^+) + \frac{9}{5}\epsilon_0 + \frac{8}{7}\epsilon_2 + \frac{107}{35}\epsilon_4 \quad (6)$$

Defining the relative energies in the quasi- β band as $E^{rel}(2_\beta) \equiv E(2_\beta) - E(0_\beta)$ and $E^{rel}(4_\beta) \equiv E(4_\beta) - E(0_\beta)$ we have

$$E^{rel}(4_\beta) = 2.0E^{rel}(2_\beta) + \epsilon_4 \quad (7)$$

This relation is identical in form to eq. 1. The data are presented in Fig. 5 and clearly satisfy eq. 1 very well for $R_{4/2} \geq 3.15$. Moreover, the evolution to good rotors ($R_{4/2} \sim 3.33$) is again very sharp.

Fig. 5. Correlation of $E^{rel}(4_{\beta})$ with $E^{rel}(2_{\beta})$ for all collective nuclei ($2.00 \leq R_{4/2} \leq 3.33$). The solid curve is a linear least squares fit to the non-rotational nuclei $2.00 \leq R_{4/2} \leq 3.15$.

III. EXTENSION OF THE AHV DESCRIPTION TO ROTATIONAL NUCLEI AND HIGHER SPIN YRAST STATES

These results encourage further studies of the AHV. Following the “horizontal” perspective of Section II we now follow a “vertical” perspective and ask if the AHV can describe higher spin states of the yrast bands. We also ask if the AHV might be able to account for nuclei normally considered as ideal examples of rotational behavior as well. Certainly the AHV can describe a pure $I(I+1)$ spectrum, provided ϵ_4 is taken as $\epsilon_4 = 1.33 E(2_1^+)$. But, from this, it is hardly obvious that a vibrational model can explain the subtle deviations from the pure rotational limit that have prompted the development of sophisticated energy expressions such as the VMI over many years. In any case, this is the idea that we now assess. To do so, we write more general AHV expressions for yrast energies [12] and $B(E2)$ values extending to the next anharmonic term. We have:

$$E(n) = nE(2_1^+) + \frac{n(n-1)}{2}\epsilon_4 + \frac{n(n-1)(n-2)}{6}\epsilon_6 \quad (8)$$

and

$$B(E2: I \rightarrow I-2) = \frac{1}{n} \left[nb + \frac{n(n-1)}{2}\tau_4 + \frac{n(n-1)(n-2)}{6}\tau_6 \right]^2 \quad (9)$$

where $n = I/2$ is the number of phonons. We now ask if these expressions can account for the yrast data of *all types of collective nuclei* with $R_{4/2}$ values ranging from ~ 2.0 all the way to 3.33.

We have fitted all even-even nuclei from $Z=50-100$. We show some completely typical results in Fig. 6. The fits for other nuclei are comparable to these results. To compare the 3-parameter AHV fits (in the fitting we take $E(2_1^+)$ in eq. 4 as a fitting parameter but its value is always almost exactly the same as the empirical 2_1^+ energy) with competitive models, we also show in Fig. 6 the results for the generalized VMI which is perhaps the best existing phenomenological description of yrast spectra.

The nuclei included in Fig. 6 include near harmonic vibrators, intermediate nuclei and nuclei that would normally be described as excellent rotors. In some cases the data extends to $I \sim 30^+$. The AHV fits are excellent, competitive to any other approach.

For $B(E2 : I \rightarrow I-2)$ values, similar results are obtained using eq. 9. This is shown in Fig. 7 for a variety of nuclei. Though the patterns vary considerably, from linear to parabolic to S-shaped, a single AHV expression successfully reproduces the observed phenomenology.

IV. SUMMARY

We have seen that the AHV model with constant anharmonicity fits the low-lying yrast levels of nearly all collective nuclei from $Z=28-82$ with a single eigenvalue expression. Yrast $B(E2)$ values are also well fit. These results suggest an alternative view of the evolution of nuclear structure. Instead of a smoothly varying character, there is an underlying constancy which persists [as a function of $E(2_1^+)$] virtually to the edge of the rotor region, at which point a sudden phase transition occurs. That is, the persistence of an AHV description squeezes the structural changes that occur when the rotor regime is entered and leads to a transitional behavior that is phenomenologically very similar to that of true phase transitions in other physical systems. When the AHV expressions are slightly generalized, they are seen to account for the "vertical" as well as the "horizontal" phenomenology of nuclei. The AHV accounts for yrast energies and $B(E2)$ values not only of vibrational and intermediate nuclei but even of the best "rotor" nuclei as well.

These results raise all sorts of questions concerning the meaning of a microscopic phonon description of rotational motion and of the apparent success of such a multi-phonon description of nuclear structural evolution (both in N and Z and in spin) in the face of the expected fragmentation due to Pauli effects. For years the work of Soloviev and his colleagues [13] has imaginatively probed the interrelationship of quasi-particle and phonon excitations (albeit "intrinsic" phonon modes of a different sort). They have dealt with the subtle issues of the interplay of collective and single particle degrees of freedom in fermionic systems. Lately, the model has undergone rapid, substantial, and promising improvements [14] in the face of new data on intrinsic phonon excitations in nuclei. We offer the above alternative phenomenological description of collective yrast states in nuclei in the hope that an approach and a technology such as that already developed by Soloviev and his co-workers might be applied more generally to such issues and to encourage the study of the evolution of nuclear structure not only vertically within a given nucleus but horizontal over major swaths through the nuclear chart.

Fig. 6. Yrast energies vs. spin for typical nuclei spanning the collective range of $R_{4/2}$ values. The solid lines are fits using eq. (8). The bottom panels show the deviations of predicted and experimental energies on an expanded scale (crosses for AHV and circles for GVMI).

Fig. 7. Similar to Fig. 6 for yrast $B(E2)$ values. Fits using eq. (9).

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