Conf-951013--2

UCRL-JC-120969
PREPRINT

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This paper was prepared for submittal to
Society of Exploration Geophysicists International Expo and 65th Annual Meeting
Houston, Texas, October 8-13, 1995

June 2, 1995

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Constraints on Elastic Parameters and Implications for Lithology in VTI Media

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SUMMARY

Energy considerations provide constraints on elastic stiffnesses in media exhibiting transverse isotropy with a vertical axis of symmetry. If the anisotropy is due to thin layers, additional constraints hold. The constraints can be used to provide insight into the mechanisms causing the anisotropy, which in turn gives information about the lithology. These ideas are illustrated by some examples of anisotropic sedimentary rocks and sediments from the literature.

INTRODUCTION

The elastic properties of a medium exhibiting transverse isotropy with a vertical axis of symmetry (VTI medium) are described by five independent elastic stiffnesses. The stiffnesses C_{11} , C_{13} , C_{33} , and C_{44} describe P - and SV -wave propagation, while C_{44} and C_{66} describe SH -waves (Auld, 1973). Requiring positive strain energy in a VTI medium provides the following rigorous constraints on the elastic stiffnesses:

$$C_{11} \geq C_{66} \geq 0, \quad (1)$$

$$C_{33} \geq 0, \quad (2)$$

$$C_{44} \geq 0, \quad (3)$$

and

$$C_{13}^2 \leq C_{33}(C_{11} - C_{66}) \quad (4)$$

(Auld, 1973). When the anisotropy is caused by thin layering, additional constraints apply:

$$C_{33} > C_{13}, \quad (5)$$

$$C_{33} > \frac{4}{3}C_{44}, \quad (6)$$

and

$$C_{66} \geq C_{44} \quad (7)$$

(Backus, 1962). The VTI medium can also be described using vertical ray velocities $V_P = \sqrt{C_{33}/\rho}$ and $V_{SV} = \sqrt{C_{44}/\rho}$ (where ρ is the density) together with the parameters ϵ , δ , and γ defined by Thomsen (1986):

$$\epsilon \equiv \frac{C_{11} - C_{33}}{2C_{33}}, \quad (8)$$

$$\delta \equiv \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})}, \quad (9)$$

and

$$\gamma \equiv \frac{C_{66} - C_{44}}{2C_{44}}. \quad (10)$$

The Thomsen parameters were originally defined for the purpose of quantifying weak anisotropy, but provide convenient notation for describing any VTI medium since the algebraic expressions in (8) through (10) appear in the equations for dip-moveout corrections, normal-moveout velocities, and variations of amplitude with offset for seismic reflection data (e.g., Thomsen, 1986; Berge, 1991; Tsvankin, 1994; Alkhalifah and Tsvankin, 1994; Tsvankin and Thomsen, 1994; Blangy, 1994). The factor $\epsilon - \delta$ is particularly important. The above constraints on elastic stiffnesses translate into constraints on the velocities and the Thomsen parameters. For anisotropy due to thin layers, $V_P/V_{SV} > 4/3$ from (6), and also

$$\epsilon - \delta \geq 0 \quad (11)$$

(Backus, 1962; Berryman, 1979; Thomsen, 1986). The P -waves are elliptical and the SV -waves are spherical if $\epsilon = \delta$ (Helbig, 1983; Thomsen, 1986). Thin isotropic layers with constant isotropic Poisson's ratio give $\delta = 0$ (Thomsen, 1986). Schwartz et al. (1994) have shown that for models of granular materials and models containing randomly-oriented cracks, applied uniaxial stress induces anisotropy with VTI symmetry and $\epsilon \approx \delta$. These constraints and special cases highlight the physical meaning of the elastic stiffnesses and Thomsen parameters, and can be used to explore constraints on lithology in VTI media.

ESTIMATING ELASTIC STIFFNESSES AND ϵ , δ , γ

Values of ϵ and δ can be estimated from velocity analysis of seismic reflection data (e.g., Thomsen, 1986; Alkhalifah and Tsvankin, 1994; Tsvankin and Thomsen, 1994). V_P can be found from check shots or well log data. Then C_{11} and C_{33} can be determined from ϵ and V_P . Additional information would be needed to determine both C_{13} and C_{44} from δ and C_{33} .

If no additional information were available, the constraints above at least give an upper bound on C_{13} and a lower bound on C_{44} , plus an upper bound on C_{44} if the anisotropy is due to thin layers. For consolidated sedimentary rocks, the usual range for V_P/V_{SV} is about 1.5 to 2. This could be used with C_{33} to approximate C_{44} . C_{44} can be

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estimated if V_{SV} is known (e.g., from well log data), or if SV or $P - SV$ stacking velocities are available.

The SH -wave stacking velocity provides an estimate of γ , which can be used to find C_{66} . Although SH data are not usually available, the constraints above give upper and lower bounds for C_{66} . In the case of elliptical anisotropy, with $\epsilon - \delta = 0$, (4) and (8)–(10) can be used to show that

$$\epsilon - \gamma \geq \frac{C_{33} - 2C_{13}}{2C_{33}}. \quad (12)$$

IMPORTANCE OF C_{13} AND δ

The value of C_{13} controls the P - and SV -wave velocities at ray angles intermediate between the horizontal and vertical directions. In an isotropic medium, $C_{13} = C_{33} - 2C_{44}$. But in a VTI medium, it is possible to have a large C_{13} with

$$C_{13} > C_{33} - 2C_{44}, \quad (13)$$

or alternatively a small C_{13} with

$$C_{13} < C_{33} - 2C_{44}. \quad (14)$$

A VTI medium with a small C_{13} value has slower P -wave velocities and faster SV -wave velocities at intermediate ray angles than an isotropic medium would have. A VTI medium with a large C_{13} value has faster P -wave velocities and slower SV -wave velocities at intermediate ray angles than an isotropic medium would have. C_{13} affects the amplitudes as well as the velocities since it controls the partitioning of energy.

It is always true that $\delta = 0$ when $C_{13} = C_{33} - 2C_{44}$. These conditions describe isotropic media, and also VTI media where the anisotropy is due to thin layers with constant Poisson's ratio.

For the case of large C_{13} in (13), $\delta > 0$ unless $C_{33} < C_{44}$ and $C_{33} > -C_{13}$. From (6), $\delta > 0$ for large C_{13} if thin layers cause the anisotropy. Since most rocks have $V_P > V_{SV}$ and $C_{33} > C_{44}$, it is usually true that $\delta > 0$ when C_{13} is large.

If C_{13} is small as in (14), $\delta < 0$ unless $C_{33} > C_{44}$ and $-C_{33} > C_{13}$. Negative values of C_{13} may not be very common, which implies that usually $\delta < 0$ when C_{13} is small.

EXAMPLES

Table 1 gives the values of the elastic stiffnesses and the Thomsen parameters for some typical VTI sedimentary rocks and sediments. Velocity surfaces showing wavefront shapes in the sagittal plane are plotted in Figure 1 for these VTI media. The sedimentary rocks in Table 1 were described by Thomsen (1986). The marine silty clay properties were determined in an ocean-bottom refraction experiment in unconsolidated sediments, as described by Berge et al. (1991).

Note that the only rocks and sediments having non-negative values of $\epsilon - \delta$ in Table 1 are the Dog Creek shale, Taylor sandstone, and the marine silty clay. The anisotropy in these rocks and sediments may be due to thin layers, but none of the other rocks can have anisotropy due to thin layers. If thin isotropic layers cause the anisotropy in the marine silty clay and the Taylor sandstone, the Poisson's ratios of the isotropic components do not vary greatly, since δ is close to 0 in these materials.

The Berea sandstone, Pierre shale, and marine silty clay all have $\epsilon - \delta \approx 0$, indicating that it is possible the anisotropy in these rocks and sediments is caused by application of vertical stress. That cannot be the case for any of the other rocks in Table 1.

The Cotton Valley shale and Mesaverde mudshale have large values of C_{13} as described by the inequality in (13). The wavefronts for these rocks are similar, showing a decrease in the SV -wave velocities as the ray angle increases from 0 to intermediate angles (as measured from the Z -axis). As expected, $\delta > 0$ for these rocks having large C_{13} values.

The rest of the rocks and sediments in Table 1 have C_{13} values that are fairly close to the isotropic case. The Taylor sandstone and the marine silty clay have slightly small values for C_{13} as described by inequality (14). Note δ is negative, although very close to 0. The SV -wave velocity has a maximum at intermediate ray angles, as shown in Figure 1. The Pierre shale and the Berea sandstone in Table 1 have C_{13} values that are very close to the value for the isotropic case. These rocks have $\delta \approx 0$, and the SV -wave surfaces in Figure 1 are nearly circular in cross-section. The Dog Creek shale has a slightly large value for C_{13} , and δ is positive.

Table 1. Parameters for VTI rocks and sediments.*

Material	C_{11}/ρ (km/s) ²	C_{13}/ρ (km/s) ²	C_{33}/ρ (km/s) ²	C_{44}/ρ (km/s) ²	C_{66}/ρ (km/s) ²	V_P (km/s)	V_{SV} (km/s)	ϵ	δ	γ	ρ (g/cm ³)
Cotton Valley shale	28.31	9.58	22.29	8.352	11.359	4.72	2.89	0.135	0.205	0.180	2.14
Mesaverde mudshale	21.91	9.68	20.51	7.306	7.978	4.53	2.70	0.034	0.211	0.046	2.52
Pierre shale	4.99	3.25	4.85	0.939	0.995	2.20	0.97	0.015	0.060	0.030	2.25
Berea sandstone	17.76	3.84	17.69	7.097	7.168	4.21	2.66	0.002	0.020	0.005	2.14
Taylor sandstone	13.84	4.25	11.34	3.345	5.051	3.37	1.83	0.110	-0.035	0.255	2.50
Dog Creek shale	5.10	2.48	3.52	0.682	1.153	1.88	0.83	0.225	0.100	0.345	2.00
Marine silty clay	2.64	2.56	2.64	0.025	0.033	1.62	0.16	0.000	-0.011	0.160	2.10

*Thomsen (1986); Berge et al. (1991).

All of the rocks and sediments in Table 1 have $C_{11} \approx C_{33}$, except for the Cotton Valley shale, Taylor sandstone, and the Dog Creek shale. $C_{44} \approx C_{66}$ for all but the Cotton Valley shale, Taylor sandstone, Dog Creek shale, and the marine silty clay. The shear wave velocity anisotropy is more significant than the compressional wave velocity anisotropy, for all the rocks and sediments in Table 1. Note that V_P/V_{SV} is about 1.6–2.3 for all the rocks, but is much larger for the unconsolidated marine silty clay. For unconsolidated VTI sediments, it may be typical to have $C_{13} \approx C_{11}$ or C_{33} and $C_{44} \ll C_{33}$ or C_{11} .

CONCLUSIONS

The constraints on the elastic stiffnesses provide constraints on the sign of the quantity $\epsilon - \delta$ and the value of δ that are linked to possible mechanisms causing the anisotropy. This information in turn can guide lithology interpretation for VTI media.

In the examples shown, the anisotropy in the Taylor sandstone and the marine silty clay could be caused by thin layers of isotropic materials with similar Poisson's ratios, such as sand and lithics in the case of the Taylor sandstone and interbedded sand, silt, and clay particles for the marine sediments. The Dog Creek shale anisotropy may also be caused by some subhorizontal layering, but the components have contrasting Poisson's ratios. Possibly this indicates subhorizontal cracks rather than layers.

For the Pierre shale and the Berea sandstone, uniaxial stress such as overburden pressure could cause the anisotropy. The Berea sandstone may behave like a granular model that is isotropic under hydrostatic stress and VTI under uniaxial stress, and the Pierre shale may contain randomly oriented cracks that exhibit anisotropy under uniaxial stress as the horizontal cracks close and vertical cracks dilate.

The Cotton Valley shale and the Mesaverde mudshale have some form of intrinsic anisotropy. Of particular interest in these materials are the large values of C_{13} and δ and their effects on wavefronts. Note that the physical meaning of δ can be described in terms of the vertical ray velocity V_P and the approximate hyperbolic P -wave normal moveout velocity V_{NMO} :

$$\delta = \frac{V_{NMO}^2 - V_P^2}{2V_P^2}. \quad (15)$$

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Figure 1. Ray velocity surfaces for the transversely isotropic rocks and sediments listed in Table 1. Cross sections of ray velocity surfaces in the sagittal plane show wavefront shapes since ray velocity is proportional to distance a wave travels in unit time. Solid lines indicate the P ray velocity curves; dashed lines show SV and dotted lines show SH . (a) Cotton Valley shale. (b) Mesaverde mudshale. (c) Pierre shale. (d) Berea sandstone. (e) Taylor sandstone. (f) Dog Creek shale. (g) Marine silty clay.

(b) Mesaverde mudshale

(e) Taylor sandstone

(c) Pierre shale

(f) Dog Creek shale

(d) Berea sandstone

(g) Marine silty clay