A numerical analysis of 3D EM imaging from a single borehole
David L. Alumbaugh*, Sandia National Laboratories, Albuquerque, NM; Currently at the University of Wisconsin-Madison, and Michael J. Wilt, ElectroMagnetic Instruments, Inc., Richmond, CA.

Summary
In this study we analyze the feasibility of three dimensional (3D) electromagnetic (EM) imaging from a single borehole. The proposed logging tool consists of three mutually orthogonal magnetic dipole sources and multiple three component magnetic field receivers. A sensitivity analysis indicates that the most important sensor configuration for providing 3D geological information about the borehole consists of a transmitter with moment aligned parallel to the axis of the borehole, and receivers aligned perpendicular to the axis. The standard coaxial logging configuration provides the greatest depth of sensitivity compared to other configurations, but offers no information regarding 3D structure. Two other tool configurations in which both the source and receiver are aligned perpendicular to the borehole axis provide some directional information and therefore better image resolution, but not true 3D information. A 3D inversion algorithm has been employed to demonstrate the plausibility of 3D inversion using data collected with the proposed logging tool. This study demonstrates that an increase in image resolution results when three orthogonal sources are incorporated into the logging tool rather than a single axially aligned source.

Introduction
Often the drilling of an oil well is followed by a logging process to characterize the region immediately surrounding the well bore. The electromagnetic (EM) induction tool, which provides the formation resistivity, is among the most frequently run logs. Commercial EM induction logging tools use a magnetic dipole source aligned with the axis of the borehole, and measure the axial field at offsets of 2m or less. This allows for one and/or two dimensional, interpretation of the formation. However, in regions of complex geology it is desirable to have sensitivity to off-axis structures such as fractures or formation heterogeneities. For such an application it is necessary to use three component magnetic field sensors. At present there are no commercial tools offering this configuration, and only one prototype tool has been described (e.g. Wilt and Alumbaugh,1998).

In this paper we demonstrate how three dimensional (3D) information can be obtained by measuring three components of the field generated by a 3D magnetic field source, and the degree of resolution that is available from these measurements. We begin with a sensitivity analysis, and follow this with a brief imaging analysis using a 3D inversion scheme. For simplicity, all results we will be presented for an 'induction number' (l=σωμL²) of 1. Here σ is the average electrical conductivity of the medium, ω is the angular frequency (2πf where f has units of Hertz), μ is the magnetic permeability (assumed to be that of free space) and L the distance between the source and receiver (see Figure 1). The advantage to presenting the results in terms of the induction number is that a single figure can provide valuable information for a wide variety of different scenarios.

Figure 1 - The four basic source-receiver configurations that can be provided in a logging tool. Here the direction of the arrows indicate the polarization with the black arrows representing sources, and the white arrows receivers (note the circle represents an arrow oriented normal to the page). CA=coaxial configuration; CANC=coaxial-null coupled configuration; CP= coplanar configuration; CPNC=coplanar-null coupled configuration.

Sensitivity Analysis
We wish to analyze the region of the earth surrounding the borehole that each different coil configuration is sensitive to, how that sensitivity varies spatially, and what source-receiver combinations are most important for 3D analysis. To derive the sensitivities we use the Born Approximation method in the same manner as utilized by Spies and Habashy (1995) for crosswell EM applications. Here a set of equations are developed that relate the secondary or scattered magnetic fields measured by a receiver to a unit perturbation in conductivity at a certain point within the medium. These are the sensitivity functions, or Frechet derivatives for the EM inversion problem, and they can be analyzed to yield valuable information about the resolution and sensing capability of different source-receiver-frequency combinations. For more information on the derivation see Spies and Habashy (1995).

The four primary source-receiver configurations that can be housed within a logging tool are shown in Figure 1. Note that the coaxial (CA) configuration is the only configuration that is used in well logging operations. Although there are other possible configurations of source and receivers, in a homogenous medium all other combinations are either reciprocals or rotations of these basic four. Also, we envision an 'extended offset' logging configuration that is comprised of multiple sondes interconnected with logging cable within the
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.
borehole. As it will be demonstrated this allows for greater source-receiver offset, and thus greater depth sensitivity away from the borehole.

In Figure 2 we plot the sensitivity as a function of position for the CA configuration shown in Figure 1 for an induction number of 1. The values displayed in these figures have been processed as suggested by Spies and Habashy (1995), where the real and quadrature sensitivities are calculated at a constant interval and plotted in a logarithmic scale with red representing positive sensitivity and blue negative values. The maximum on the grid is assigned a value of 60dB's, and all other values scaled accordingly, with amplitudes less than 1dB set to zero. Thus in Figure 2a and all other subsequent 2D plots we are showing those values which are within three orders of magnitude of the maximum sensitivity. For the 3D renderings, for example Figure 2b, the ±20dB isosurface has been plotted. Thus the 3D volume (or volumes) encloses sensitivities that are within two orders of magnitude of the maximum value. Note that all spatial axes in these plots have been normalized by the coil separation.

Figure 2 — Logarithmically normalized coaxial sensitivity as a function of position for an induction number of 1. a) Two dimensional sensitivity distribution in the y=0 plane. b) Three dimensional isosurface corresponding to the −20dB level of the coaxial sensitivity.

Figure 2 closely resembles the geometrical factor plots originally introduced by Doll (1949; Figure 6) for simple two-coil induction tool analysis. Note that the sensitivity peaks near the source and receiver and is symmetric about the borehole. This symmetry is due to the fact that we have assumed a homogenous whole space for these calculations. If we were to apply this approach to a more general 3D model, then the resulting sensitivity plots would be asymmetric. Also note that due to the cylindrical symmetry of the tool we can not delineate a scattering target on one side of the borehole from one on the opposite side. Rather all that can be determined is the position along the borehole axis, and the radial position away from the borehole of the body.

In Figure 3 we plot the sensitivity versus position for the CANC configuration. The utility of this arrangement for delineating 3D structures becomes apparent as both the real and quadrature sensitivity change sign but are equal in amplitude on opposite sides of the borehole. This indicates that the scattered fields generated by inhomogeneities located the same distance away from the tool, but in different directions, will be of the same amplitude but differ in phase by 180°. Also note from Figure 3b that there is no sensitivity to regions located along the x=0 plane. Sensitivity to these regions is provided by measuring the y component of the magnetic field in conjunction with the x. The CANC configuration is the only tool configuration of the four shown in Figure 1 that provides true 3D information of the region surrounding the borehole, and thus it is essential for the success of a 3D tool. Unfortunately, the sensitivity does not extend as far away from the borehole as the CA tool, and thus the CANC configuration will not sample as great a volume around the borehole as the former.

In Figure 4 we plot the sensitivity distributions for the coplanar and coplanar-null coupled configurations (Figure 1) in 3D isosurface format. Notice that neither one of these configurations offer true 3D information as offered by the CANC configuration. That is, a target on one side of the borehole will produce the same response as a target 180° on the
other side of the well. However, unlike the CA configuration, these two systems do offer some directional information. This characteristic is especially true of the CPNC system (Figure 4b) for which the sensitivity changes sign from one quadrant of the xy plane to the next. Also note that the sensitivity for the CPNC coupled configuration is maximum where the CANC is minimum, and vice versa. Thus these two null coupled configurations when combined should offer increased resolution over the CANC system alone.

Figure 4 - Logarithmically normalized sensitivity as a function of position for an induction number of 1 for two different configurations employing a x directed dipole source. A) Three dimensional isosurfaces corresponding to the ±20dB level of the sensitivity for the coplanar configuration. b) Three dimensional isosurfaces corresponding to the ±20dB level of the sensitivity for the coplanar-null coupled configuration.

In this section we have demonstrated how the different source-receiver combinations on a three component logging tool can sense the medium around the borehole. In the next section we demonstrate using an inversion code that 3D imaging is indeed possible, and how additional resolution is provided by employing 3D field measurement.

Inversion Examples
To test the feasibility of generating a 3D image from single borehole data in a somewhat realistic scenario, we have employed the model shown in Figure 5. In this model a horizontal well has been drilled into an oil bearing sandstone (blue: $\sigma=0.15S/m$) that is interbedded with fresh water bearing sandstones (green: $\sigma=0.2S/m$), shales (yellow: $\sigma=0.55S/m$), and a brine filled sandstone (orange: $\sigma=0.8S/m$). Four main structural features within this model are used to define the imaging capability of a multi-component logging tool: 1) the horizontal and vertical extent of the various formations; 2) the dip of the beds; 3) the fault located at x=0m; and 4) the 'hidden' oil zone located below the main oil reservoir in the -x half of the model. Seven frequencies spaced logarithmically from 1kHz to 100kHz were employed, and the source-receiver separations were chosen such that at each frequency an induction number ($\mathcal{I}$) of either 1 was approximately maintained.

We used the 3D finite difference scheme described in Alumbaugh et al. (1996) to generate the synthetic data. A measurement interval of 5m was used along the borehole for source positions starting at -100m, and receiver positions extending to +100m. The modeling domain consisted of 88 cells in $x$, and 42 cells in both $y$ and $z$, and random noise with a standard deviation equal to 1% of the primary field at each offset were added to the data. The resulting synthetic data set was inverted using the scheme of Newman and Alumbaugh (1997) where a slightly coarser inversion (54 by 38 by 38) forward modeling mesh was employed along with an inversion domain of 48 by 32 by 32 cells. Thus we were inverting for approximately 50,000 unknown conductivity values.

The image that results from inverting these synthetic data using a single polarization source oriented parallel to the axis of the borehole is shown in Figure 6. The data for this configuration consisted of 219 individual measurements. The run time for this inversion was approximately 4 hours on 512 Pentium 200 MHz P2 processors with convergence occurring in 8 iterations. In general we have recovered a smoothed version of the 3D structure immediately surrounding the well. Specifically the fault has been imaged as has a portion of the primary oil zone. However, the oil zone does not extend outward away from the well more than 30m. Also, it would be difficult from the image to recognize the dip in the strata, as the only place it is discernable is in the conductive portion underlying the oil zone in the $x=+10m$ cross section. Finally, it would be very difficult to deduce that the hidden oil zone exists from this image.

The image that results from inverting synthetic data generated from three separate sources that are orthogonally polarized is shown in Figure 7. Because the number of sources has been tripled, the total number of data points is 657. The run time for this inversion was approximately 13 hours on 512 Pentium 200 MHz P2 processors with convergence occurring in 11 iterations. Notice that we have recovered a better estimate of the 3D geology than we did for the single polarization source (Figure 6). Specifically the oil zone is shown to extend across the entire region of interest, and the dip is much more apparent. The fault has been imaged and the conductive layer above the oil zone is better represented, although its continuity in the $y$ direction is still poor. Finally, notice that comparing the $x=+10m$ and $x=-10m$ cross sections that the oil zone is thicker on the right hand side of the $x=-10m$ cross section. Thus the extra data generated with the three sources are more sensitive to the second or hidden oil when compared to the single source data. However, the resolution does not exist to image the sequence as a conductor sandwiched between two resistors, and thus is manifested as a slightly thicker resistive section both below and to the sides of the borehole.
Discussion and Conclusions

In this paper we demonstrate the plausibility of generating a 3D image of the region surrounding a borehole. If a source aligned parallel to the borehole axis is employed, the phase of the perpendicular components contain the necessary information to reconstruct 3D geology. All other combinations of sources and receivers will not offer true 3D information, but incorporating them into the inversion will improve resolution.

The inversions presented utilized the massively parallel computer systems at Sandia National Laboratories, and thus at present powerful computers will be needed to process this type of data. However as the cost and speed of computers change in the near future, we believe that this problem will become less of an issue.

Acknowledgments

This work was performed at Sandia National Laboratories with funding provided by ElectroMagnetic Instruments, Inc., and the Department of Energy's Office of Geothermal Technology. Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under Contract DE-AC04-94AL85000.

References