Copy

TRANSPORT IN STATISTICAL MEDIA

**Final Report
for the period
5/1/88 - 5/1/90**

LANL Contract No. UC-88-4-H-37

**Principal Investigator
Gerald C. Pomraning
Professor**

July 1990

**School of Engineering and Applied Science
University of California, Los Angeles
Los Angeles, California 90024**

CS
DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

MASTER

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

1 Introduction

This document represents the final report for the research contract UC-88-4-H-37 between the Los Alamos National Laboratory and the University of California. This contract provided support in the amount of \$50,000 for the period May 1, 1988 through May 1, 1990. The principal investigator was Professor Gerald C. Pomraning of the UCLA School of Engineering and Applied Science. The funds were used primarily to support a US citizen graduate student, Fausto Malvagi, during the final stages of his Ph. D. program. Mr. Malvagi successfully defended his Ph. D. dissertation in early July 1990.

This contract led to five submittals to the archived literature. Four of these have appeared, and the fifth has been accepted for publication. The authors of these five publications are Professor Pomraning and Mr. Malvagi, together with colleagues Dr. M. L. Adams (LLNL), Prof. V. C. Boffi (University of Rome), Prof. E. W. Larsen (University of Michigan), and Prof. C. D. Levermore (University of Arizona). These five papers are:

1. M. L. Adams, E. W. Larsen, and G. C. Pomraning, "Benchmark Results for Particle Transport in a Binary Markov Statistical Medium," *Journal of Quantitative Spectroscopy and Radiative Transfer* **42**, 253 (1989).
2. G. C. Pomraning, "Statistics, Renewal Theory, and Particle Transport," *Journal of Quantitative Spectroscopy and Radiative Transfer* **42**, 279 (1989).
3. F. Malvagi, C. D. Levermore, and G. C. Pomraning, "Asymptotic Limits of a Statistical Transport Description," *Transport Theory and Statistical Physics* **18**, 287 (1989).
4. F. Malvagi and G. C. Pomraning, "Renormalized Equations for Linear Transport in Stochastic Media," *Journal of Mathematical Physics* **31**, 892 (1990).
5. V. C. Boffi, F. Malvagi, and G. C. Pomraning, "Solution Methods for Discrete State Markovian Initial Value Problems," *Journal of Statistical Physics*, in press (1990).

The technical content of these five papers is summarized in the next section of this report, and complete copies of the papers are also included in their entirety.

2 Summary of Research

In recent years there has been considerable interest in the problem of describing linear particle transport in a stochastic medium consisting of randomly mixed immiscible fluids. In particular, the Liouville master equation approach has been used to obtain, in the case of a binary mixture, a set of two coupled transport equations in order to describe, in the case of Markovian mixing statistics, the ensemble averaged particle density (angular flux). In the five papers appended, three main objectives have been achieved. Namely: (1) benchmark results for binary Markovian mixtures have been obtained to assess the validity of any proposed model; (2) the previously proposed model has been extended in several ways, to include multi-component mixtures and mixing statistics more general than Markovian; (3) renormalized transport equations are obtained, involving effective sources and interaction cross sections that account, in a very simple way, for the statistical nature of the problem.

In the first paper numerical benchmark results are given for particle transport in randomly mixed binary media, with the mixing statistics described as a homogeneous Markov process. A Monte Carlo procedure is used to generate a physical realization of the statistics. Given this realization, the time independent transport problem with scattering is numerically solved via a standard discrete ordinate method. The expected value for the solution and the variance are obtained by averaging a large number of such calculations. Results are reported for several transmission-reflection problems for finite, source-free systems in both rod and planar geometry. This makes it possible to assess the accuracy of a previously reported model for transport in Markovian binary mixtures. The model appears to give qualitatively as well as semi-quantitatively (with an error of the order of 10%) correct results, with increased accuracy for thin systems.

In a separate development, two coupled transport equations are derived which formally describe transport in a random binary mixture for arbitrary mixing statistics. Closing these equations by approximating their coupling terms in a low order and intuitive way leads to the model for stochastic transport previously obtained via the Liouville master equation approach. The present derivation, based upon approximating exact equations, allows in principle the opportunity to develop more accurate models by making higher order approximations in the coupling terms.

In the second paper, a model involving four coupled integral renewal equations is derived which describes the ensemble averaged intensity for time independent particle transport in a nonscattering medium consisting of two randomly mixed immiscible fluids with arbitrary mixing statistics. It is shown that for a special class of mixing statistics these renewal equations can be recast into integro-differential equations resembling usual transport equations, but with angularly dependent cross sections. An extension of this model is proposed which includes time dependence as well as scattering. This generalizes the earlier description proposed for the special case of Markov mixing statistics.

In the third paper, three asymptotic limits are derived for the two coupled model equations describing transport in a binary Markovian mixture. First it is shown that the "atomic mix" limit is recovered by considering fluid packets that are small compared to the particle mean free path in the packet. The limit of a small amount of large cross section fluid admixed with a large amount of small cross section fluid is then considered. This reduces the underlying model to a single renormalized transport equation of the usual form, with an effective source and effective cross sections that take into account the mixing statistics. The third limit considered corresponds to a nearly isotropic angular dependence of the intensity (angular flux). This limit yields a two equation diffusion approximation. A boundary layer analysis gives the asymptotically consistent boundary conditions for these two coupled diffusion equations.

In the forth paper the master equation approach is used to obtain a model describing linear particle transport in Markovian random mixtures consisting of an arbitrary number of immiscible fluids. This consists of a number of coupled transport equations equal to the total number of fluids. An asymptotic limit corresponding to small amounts of opaque fluids admixed with large amounts of transparent fluids is employed to reduce the complexity of the description to a number of transport equations equal to the number of transparent fluids. The final result depends only on the number of fluids that are considered opaque, and not on the specification of which fluid is opaque and which one is transparent. This provides a simplified description that is robust, in the sense that it gives the qualitatively correct solution even when the physical situation is far from the asymptotic limit considered. In the limit of a single transparent fluid, a renormalized transport equation is obtained, involving an effective source and effective interaction coefficients that

account, in a simple way, for the statistical nature of the problem. Even this very simple renormalized equation preserves the exact deep-in decay rate for the time-independent source-free problem. Appropriate initial and boundary conditions are given for all orders of reduction.

In the fifth paper, a very general class of nonlinear initial value problems for a binary mixture with nonstationary Markovian mixing is considered, and formulated according to the Liouville master equation approach. A numerical method, basically a moment (Galerkin) method, is developed, which enables one to compute the ensemble average for any quantity of interest associated with the problem under consideration. The lowest order numerical approximation produces, for all problems, an analytical result useful for assessing, in a very simple way, the qualitative behaviour of the solution for any problem in this stochastic class, in particular for short times. It is also shown that for a certain subclass of the general class of problems under consideration, one can obtain exact analytical results for the solution of the master equation. Some of these results are used to validate the numerical method and to show that it converges rapidly to the exact answer. An application of the numerical method to a simple problem in the extended kinetic theory of gases (for which no analytical solution is available) is also given.