Technical Report 2nd Quarter 2002

Type of Report: Quarterly Technical
Reporting Start Date: March 17, 2001
Reporting End Date: June 16, 2002
Principal Investigator: Prof. Gary Mavko
Date of Report: May 2002
DOE Award Number: DE-FC26-01BC15354
Submitting Organization: Stanford University, 651 Serra Street, Suite 260, Stanford, CA 94305-4125
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ABSTRACT

As part of our study on “Relationships between seismic properties and rock microstructure”, we have studied

1. How to quantify elastic properties of clay minerals using Atomic Force Acoustic Microscopy. We show how bulk modulus of clay can be measured using atomic force acoustic microscopy (AFAM)

2. We have successfully measured elastic properties of unconsolidated sediments in an effort to quantify attributes for detection of overpressures from seismic

3. We have initiated efforts for velocity upscaling to quantify long-wavelength and short-wavelength velocity behavior and the scale-dependent dispersion caused by sediment variability in different depositional environments.
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INTRODUCTION

In this quarter, we have focused our attention on measurements of clay modulus using Atomic Force Acoustic Microscopy (AFAM) and measurements of seismic properties of unconsolidated sediments at very low effective pressures. Three papers have been published on the work done in this project. This report presents results of the AFAM measurements and the overpressure analyses.

PAPER 1


Presence of clay minerals, hydrous aluminosilicates with grain size smaller than 2 mm, alters the elastic and plastic behavior of materials significantly. Clay in contact zones has significantly lower impedance than the quartz it cements and so ultrasonic velocities are reduced in clay-rich sandstones when the clay is load-bearing. Despite their ubiquitous presence, measurements of the elastic properties of clay minerals remain a challenge. Since clay minerals are an integral part of many formations and seismic measurements are the main tools for identification of subsurface lithologies, knowledge about the elastic properties of clays is essential for interpretation and for modeling the seismic response of clay-bearing formations. However, elastic properties of clay minerals are almost unknown. Until now, estimates of single crystal elastic properties have been either theoretical or based on extrapolations from measurements on clay-epoxy mixtures. We have used Atomic Force Acoustic Microscopy (AFAM) to measure elastic properties of clay minerals. We demonstrate the AFAM technique for measuring elastic properties of soft materials. Using this technique, we present first-ever quantitative measurements of Young’s modulus in clay. The Young’s modulus of dickite was measured as 6.2 GPa.

Paper 1: Total of three figures in paper
Measurement of Young’s modulus of clay minerals using atomic force acoustic microscopy

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Received 7 September 2001; accepted 22 January 2002; published XX Month 2002.

The presence of clay minerals, hydrous aluminosilicates that are smaller than 2 μm can alter the elastic and plastic behavior of materials significantly. We have used Atomic Force Acoustic Microscopy (AFAM) to measure elastic properties of clay minerals. We demonstrate the AFAM technique for measuring elastic properties of soft materials. Using this technique, we present first-ever quantitative measurements of Young’s modulus in clay. The Young’s modulus of dickite was measured as 6.2 GPa. INDEX TERMS: 5102 Physical Properties of Rocks: Acoustic properties; 3909 Mineral Physics: Elasticity and anelasticity; 5112 Physical Properties of Rocks: Microstructure; 3994 Mineral Physics: Instruments and techniques

1. Introduction

[2] Presence of clay minerals, hydrous aluminosilicates with grain size smaller than 2 μm, alters the elastic and plastic behavior of materials significantly. Using acoustic microscopy images of sandstones, Prasad [2001] has shown that the clay in contact zones has significantly lower impedance than the quartz it cements. Ultrasonic velocities are reduced in clay-rich sandstones when the clay is load-bearing [Tosaya, 1982; Han et al., 1986]. As pore filling materials, they block hydraulic pathways and decrease permeability. In the presence of water, clay minerals can swell and cause considerable formation damage. Despite their ubiquitous presence, measurements of the elastic properties of clay minerals remain a challenge.

[3] Since clay minerals are an integral part of many formations and seismic measurements are the main tools for identification of subsurface lithologies, knowledge about the elastic properties of clays is essential for interpretation and for modeling the seismic response of clay-bearing formations. However, elastic properties of clay minerals are almost unknown [Alexandrov and Ryzhova, 2001]. Until now, estimates of single crystal elastic properties have been either theoretical [Katahara, 1996], or based on extrapolations from measurements on clay-epoxy mixtures [Wang et al., 2001].

[4] Recent studies have shown that it is possible to make measurements of dynamic Young’s modulus at nanometer resolution on various materials using Atomic Force Acoustic Microscopy (AFAM) [Kester et al., 2000; Rabe et al., 2001; Amelio et al., 2001]. We have investigated the feasibility of such measurements on clay minerals. We describe the methodology for making quantitative measurements of stiffness and elastic moduli using AFAM and show results of such measurements on clay samples.

2. Atomic Force Acoustic Microscopy

[5] In scanning Atomic Force Microscopy (AFM), the topography of the sample surface is measured from deflections of the cantilever tip from its equilibrium position. AFAM is an enhancement of the AFM technique in which ultrasonic transducers are used to insonify the contact zone between the cantilever tip and the sample surface. To this end, either the sample or the cantilever holder might be excited by an ultrasonic transducer [Rabe et al., 1996; Yamanaka et al., 1999]. The dynamic Young’s modulus is determined by measuring the difference of the cantilever contact-resonance frequencies relative to its free resonances. The resonance frequencies of the flexural modes of the cantilever will depend, amongst other parameters, on the stiffness of the tip-sample contact, which in turn is a function of the Young’s modulus of the sample and the tip, their Poisson’s ratios, the cantilever tip radius and shape, the load exerted on the tip, and the geometry of the sample surface. In an isotropic material, if the elastic modulus of the tip and its radius are known, the Young’s modulus of the sample can be determined if the poisson’s ratio is either known or assumed [Kester et al., 2000; Rabe et al., 2001]. In the anisotropic case, the so-called indentation modulus [Vlassak and Nix, 1993] is determined. For reliable measurements, the sample surface must be of nanometer smoothness over an area of tens of nanometers where the measurements are taken. The Young’s modulus is measured with a spatial resolution of a few tens of nanometers.

[6] In an actual measurement, an area with low topographic variations is first selected with an AFM. Then, the free and contact resonances of the cantilever are measured. The excitation frequency is varied and the cantilever vibration amplitude is measured as a function of frequency. The frequency range used (10 kHz to about 3 MHz) covers typically the first three flexural modes of the cantilever in contact. The procedure is repeated with different normal forces on the tip in order to obtain information on the tip shape and to check whether a Hertzian contact prevails. For a complete set of measurements on an unknown sample, a standard sample is first measured with AFAM to determine the tip radius. The unknown sample is measured next. Finally, the standard sample is measured again to ascertain that the tip remained intact during the measurement. An averaged value between both standard measurements is used as tip radius for the calculation of the Young’s modulus in the unknown sample [Kester et al., 2000; Rabe et al., 2001].

[7] Kester et al. [2000] give an accuracy of the Young’s modulus from the average of the scatter and considerations of the simplifications mentioned above to be about 40%. By comparing indentation moduli of silicon, Rabe et al. [2001] have shown that it is possible to differentiate between indentation moduli of Si (100) and Si (111) crystals, that differ only by 4%. However, our
measurements on a softer material, polystyrene, showed that an accuracy of at least 40% can be expected for such materials. We present here initial results of AFM and AFAM measurements for clay minerals and the Young’s modulus derived from them.

3. Samples and Sample Preparation

[8] Clay mineral powders were used in this study. Fused silica, mica and polystyrene served as the standard samples with known Young’s modulus [Amelio, 2000]. The Young’s modulus of polystyrene measured with ultrasonic pulse-echo was 3.5 GPa.

[9] Kaolinite, dickite, and mica powders were immersed in distilled water and insonified for 30 minutes. After allowing the larger particles (mainly silts) to settle, a dilute water-clay mixture was extracted from the top of the container. This suspension was diluted further, mixed, and allowed to settle for 5 minutes. The water-clay mixture taken from the top of this solution was put on a glass slide heated by a glass lamp. In this manner thin layers of kaolinite and dickite samples were obtained. Mica particles were co-mingled with the clay. With this method, the clay minerals were aligned with their C-axis perpendicular to the glass slide.

4. AFM Imaging

[10] AFM images of the kaolinite and dickite samples are shown in Figures 1a and 1b, respectively. In these images, as in all AFM images, the topographic differences are originally color coded and then converted to a gray scale (black = dips and white = highs). The kaolinite image shows grain stacks with a typical rosette structure of the kaolinite. Each mineral stack is less than 1 µm wide. Although the topography varies only by a few tens of nanometers within each stack, the image shows numerous disturbances caused mainly by movement of the sample with respect to the cantilever. The dickite sample with slightly larger grain size shows the typical clay booklets observed in SEM studies. This mode of operation was found to create disturbances in the imaging mode due to sample movement and grain deformation. The so-called tapping mode [Zong et al., 1993] gave better resolution and imaging results. Using this mode, we ascertained that the texture shown in Figure 1b is repeated at random locations throughout the dickite sample.

5. AFAM Quantitative Analyses

[11] Despite the good quality AFM images, quantitative AFAM of clay measurements were more difficult to obtain. The AFAM image (Figure 2) often had disturbances in the scanning mode. However, good repeatability was observed in the single-location contact-resonance spectra obtained at random clay locations with a rectangular cantilever (stiffness = 1.5 N/m). Quantitative analyses were made at 10s different clay locations. A typical spectra sequence is shown in Figure 3 for different materials. The contact-resonance spectra for polystyrene are at a lower frequency than mica and fused silica. The second contact-resonance spectra...
Table 1. Calculated Effective Stiffness ($k^*$) and Young’s Modulus (E) for Various Positions on the Dickite Sample

<table>
<thead>
<tr>
<th>Sample</th>
<th>$k^*$ (loadd in nN)</th>
<th>$E_1$ (GPa)</th>
<th>$E_2$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polystyrene</td>
<td>56 58 3.2 3.7</td>
<td>30 45</td>
<td></td>
</tr>
<tr>
<td>Mica</td>
<td>301 300 56.9 60.2</td>
<td>30 45</td>
<td></td>
</tr>
<tr>
<td>Position 1</td>
<td>81 76 5.99 5.69</td>
<td>6.7 6.13</td>
<td></td>
</tr>
<tr>
<td>Position 2</td>
<td>72 100 4.99 8.75</td>
<td>5.58 9.45</td>
<td></td>
</tr>
<tr>
<td>Position 3</td>
<td>81 83 5.99 6.52</td>
<td>6.7 7.04</td>
<td></td>
</tr>
<tr>
<td>Position 4</td>
<td>80 83 5.88 6.52</td>
<td>6.57 7.04</td>
<td></td>
</tr>
<tr>
<td>Position 5</td>
<td>78 75 5.65 5.57</td>
<td>6.32 6.0</td>
<td></td>
</tr>
<tr>
<td>Position 6</td>
<td>76 77 6.08 6.26</td>
<td>5.16 5.78</td>
<td></td>
</tr>
<tr>
<td>Position 7</td>
<td>77 78 6.19 6.38</td>
<td>5.26 5.9</td>
<td></td>
</tr>
<tr>
<td>Position 8</td>
<td>80 81 6.57 6.77</td>
<td>5.56 6.25</td>
<td></td>
</tr>
<tr>
<td>Position 9</td>
<td>78 59 6.32 4.14</td>
<td>5.37 3.87</td>
<td></td>
</tr>
<tr>
<td>Position 10</td>
<td>70 85 6.19 7.3</td>
<td>5.26 6.74</td>
<td></td>
</tr>
</tbody>
</table>

$E_1$ corresponds to the tip radius obtained from measurements on the standard sample before measurements on clay and $E_2$ corresponds to the tip radius obtained from measurements on the standard sample after the measurements on clay. The mean value of $E$ is 5.9 GPa at 30 nN and 6.4 GPa at 45 nN static load.

are as much as 500 kHz apart. Note that the relative frequency difference of the contact resonances on different materials depends on the mode of the cantilever vibrations [Turner et al., 1997]. The contact-resonance spectra in polystyrene and clay also appear to be broader, possibly due to internal friction losses in the sample.

Young’s modulus for dickite was calculated from the spectra shown in Figure 3 by assuming a Poisson’s ratio of 0.3. The values obtained for all 10 locations are given in Table 1 along with the standard deviation for the calculations at different locations (0.54 GPa and 1.27 GPa for 30 nN and 45 nN static load, respectively). The mean value for Young’s modulus from our measurements is 5.9 GPa at 30 nN and 6.4 GPa at 45 nN static load.

[12] The Young’s modulus was calculated by assuming a spherical tip and Poisson ratio = 0.3 for the dickite. The average value according to Table 1 is 6.2 ± 1.0 GPa. Although the values show a high reproducibility, a larger error of about 40% (± 2.5 GPa) needs to be assessed for the absolute values at present. This error arises from uncertainties in the theories required to model the cantilever vibration modes and the contact area between tip and sample. The input parameters for these calculations, such as beam geometry and tip shape, are only known to a certain approximation. The Young’s modulus is calculated by comparing the vibration modes of the AFM cantilever on a known reference sample with those on the unknown sample. The error can be relatively small if the elastic constants of the calibration sample are close to those of the unknown sample. A future set of measurements will improve the precision of our measurement by using new reference samples. Furthermore, the complex clay-tip interactions will be modeled to account for deviations from the assumed model. For example, surface tension forces of clay might lead to higher lateral forces than assumed. Elastic moduli in the c11 and c22 directions in sheet silicates can differ by a factor of 3 to 5.5 [Alexandrov and Ryzhova, 1961]. Future work will improve accuracy and resolve the issue of anisotropy in these samples.

6. Conclusions

Our analyses of the AFAM measurements and comparisons with measurements on similar materials give Young’s modulus values of 6.2 GPa (+ 2.5 GPa) in dickite in the c11 direction. This value is in agreement with the theoretically derived bulk modulus value of 10 – 12 GPa by Berge and Berryman [1995]. Our study resolves the controversy surrounding elastic modulus values of clay minerals. Various values have been suggested for bulk modulus of clay minerals. For example, extrapolation from shale measurements [Tosaya, 1982; Castagna et al., 1985; Han et al., 1986], theoretical models using measured values on other sheet silicates [Katahara, 1996], and extrapolation from measurements on clay-epoxy mixtures [Wang et al., 2001] gives bulk modulus values of 6.2 GPa. We also demonstrate for the first time how the AFAM technique can be used for soft materials. Implications of this study go beyond the applications for earth sciences.

Acknowledgments. M.P.’s work was performed under the auspices of the National Science Foundation (Grant No. EAR 0074330) and Department of Energy (Award No. DE-FG26-01BC15354). M.P. also thanks the German Academic Exchange Program for support of an exchange visit. The IZFP authors thank the German Science Foundation for support within the SFB277.

References

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We have investigated the sensitivity of seismic methods for detection of shallow water flows and over-pressured zones. Using P-wave information alone can be ambiguous, because a drop in P-wave velocity (Vp) can be caused both by overpressure and by presence of gas. The ratio of P-wave velocity to S-wave velocity (Vp/Vs), which increases with overpressure and decreases with gas saturation, can help differentiate between the two cases. Since P-wave velocity in a suspension is slightly below that of the suspending fluid and Vs = 0, Vp/Vs and Poisson’s ratio must increase exponentially as a load-bearing sediment approaches a state of suspension. On the other hand, presence of gas will also decrease Vp but Vs will remain unaffected and Vp/Vs will decrease. Our analyses of ultrasonic P- and S-wave velocities in sands show that the Vp/Vs ratio, especially at low effective pressures, decreases rapidly with pressure.
Elevated pore pressures, commonly encountered in the shallow, unconsolidated section of the sedimentary column, present a significant hazard during the drilling and completion of offshore wells. The porosity of the oceanfloor sediments is high and a cover of low-permeability clay can prevent the underlying sediments from draining, even at very shallow depths below the seafloor. As further deposition loads the sediment, the entrapped fluids impede normal compaction by becoming pressurized. The lowered effective stress that results from the higher pore fluid pressure produces a proportional drop in the strength of noncohesive sediments, resulting in very weak shallow sands. This weakness can result in washouts or fracturing in the shallow strata, potentially leading to the loss of the well and of neighboring wells.

These problems are exacerbated in deepwater wells where the long column of drill mud makes it difficult to control the pressure placed on the formation. In addition, slight overpressures result in a very narrow mud-weight window. A mud-weight below the pore pressure can result in shallow water flow, analogous to large scale washouts of unconsolidated sand layers. On the contrary, high mud weights can fracture these weak, shallow strata, with fractures potentially propagating all the way up to the surface and out to other wells in the same template (Ostermeier et al., 2000). The detection of high pore-pressure regions prior to drilling and the remote measurement of in-situ pressures has the potential to prevent the loss of many deepwater wells and of the millions of dollars that would be spent on them.

For a number of years, pore pressure evaluation has mainly relied on the use of empirical relationships between pressure and seismic velocities or interval transit times. Recently, Huffman and Castagna (2001) and Prasad (2002) have demonstrated the use of the $V_p/V_S$ ratio as an indicator of pressure, as extracted from AVO analysis or multicomponent reflection data. One weak link in these analyses is the paucity of laboratory data available to calibrate the measured velocities or $V_p/V_S$ to the in-situ pressure, especially for unconsolidated samples at the low effective pressures appropriate to this environment. This is in a large part due to the difficulties in propagating an ultrasonic signal through unconsolidated sediments at such low pressures. In an effort to constrain the velocity-pressure trends and to investigate the influence of porosity reduction on these trends, we have undertaken a systematic set of experiments to measure the ultrasonic velocities through loose sediments at effective pressures from below 100 kPa up to 20 MPa. Here we present measurements of $P$- and $S$-wave velocities through sand and glass bead samples at a range of porosities. We also discuss the effects of pressure, sorting, and compaction on the velocities and porosities, and implications for the evaluation of pore pressures using $V_p/V_S$ in unconsolidated sands. This study did not include a number other factors that could also influence the velocity-pressure relationship, including the age of the sediment, clay content, packing (or depositional environment), or stress-induced or inherent anisotropy, though they should be considered when attempting to predict in-situ pressure from velocity or $V_p/V_S$ measurements.

Experimental methods and samples. We adapted a hydrostatic pressure apparatus to be able to make $P$- and $S$-wave velocity measurements on unconsolidated samples at low effective pressures. One adaptation was the use of lower frequency (200 kHz) transducers to produce and receive the ultrasonic signals. We also made the transducer face plates out of a glass-filled polycarbonate (shatterproof glass) in order to improve the acoustic impedance matching between the samples and the transducers. With this arrangement, we were able to get interpretable shear signals at pressures as low as 0.05 MPa, with errors of approximately 2% for $P$-wave velocities and 4% for $S$-wave velocities.

The data presented are from samples of a fine grained,
well sorted, quartz sand called the Santa Cruz Aggregate, and from synthetic bead samples. Four samples of the Santa Cruz Aggregate were run, two dry (Dry 1, Dry 2) and two water-saturated (Wet 3, Wet 4). A fifth sample (Big Sand) was also prepared by sieving the 0.295-0.350 mm grain sizes out of this sand, and running this fraction dry. A total of seven glass bead samples were run, all dry. Three samples (GB Big, GB Small, and GB Tiny) consisted of different narrow size ranges of beads. Three samples (GB 35% Small, GB 35% Tiny 1 and 2) were made with a “bimodal” mixture of grain sizes, with 35% of the mass made up of smaller grains and 65% of larger grains. Finally, one sample (GB Broad) was made up of a broad range of particle sizes. Table 1 summarizes the grain sizes and initial porosities of all samples.

Special attention was given to preparing the samples in such a way as to ensure complete mixing of the different grain sizes, and to maintain full saturation of the water-saturated samples. However, the sample preparation varied based on whether the sample was a single grain size and dry (air pluviated) or water-saturated (water pluviated), or was a mixture of grain sizes (some samples mixed dry, some mixed while moist and allowed to dry once in the sample holder). These differences in sample preparation produced variations in the packing of the grains and some scatter in the data.

The porosity was calculated from the grain density, dry sample mass, and sample volume. The changes in the sample volume and porosity with pressure were then monitored by measuring changes in the length and circumference of the samples. The error in the porosity is estimated to be about 0.02, or about a 5% relative error.

The pressure path followed generally included a number of pressure cycles of increasing peak pressure for each cycle, up to 20 MPa. The velocities and porosity were measured at the same set of pressures during each cycle (e.g., 0.1, 0.2, 0.5.. MPa). This allowed us to compare the velocities and porosities measured at the same pressure for a sample that had been preconsolidated to a range of higher pressures. Four samples were cycled through 8 cycles, while the rest were cycled through between 1 and 5 cycles.

### Ultrasonic results

Figure 1 shows the P- and S-wave velocities plotted against pressure for all our samples, and for measurements on clean sands from Prasad and Meissner (1992), Domenico (1977), Yin (1992), Estes et al. (1994), and Robertson et al. (1995). This figure demonstrates similar trends between the data from all sources. The scatter is presumably a result of textural differences between the samples such as their sorting, angularity, packing (sample preparation), and mineralogy.

Figure 2a shows all dry velocity results plotted against porosity, and color-coded according to the effective pressure. The lower porosity data (below 0.35) come from the glass bead samples where two size fractions of beads were mixed, with the smaller grains partially filling the pore space between the larger grains. The lines in Figure 2a represent velocities calculated with the Reuss (isostress) average between the pure mineral moduli (quartz) and the high-porosity dry frame moduli for a given pressure. The high porosity end-member moduli come from the highest porosity data for each pressure, taken from the virgin compaction curve of sample Dry 2, which are highlighted with black circles in Figure 2a. The Reuss average, the weighted harmonic average between the two end member moduli, simulates the weakest possible way to combine two materials. Here the Reuss average is used to represent the minimum possible effect on the velocities of adding solid grain material (quartz) to the high porosity sample. The Reuss average was calculated as follows:

$$\frac{1}{M} = \frac{1}{M_0} + \frac{1}{M_D}$$

where $M$ is the resulting average modulus, $M_0$ is the modulus of the dry frame at the pressure of interest, and $M_D$ is the modulus of pure quartz. The fraction of dry frame, $f_D$, is simply given by $f_D = \phi / \phi_0$, where $\phi$ is the porosity, and $\phi_0$ is the initial porosity at that pressure from sample Dry 2. The fraction of pure quartz, $f_Q$, is then just $1-f_D$ or (1-$\phi / \phi_0$).

The data from a single sample (the streaks of points that rise more or less vertically in Figure 2a—e.g., the black or blue highlighted points) demonstrate the effect of the pressure on the velocity-porosity trend: a significant velocity increase with relatively little change in the porosity. Data collected at the same pressure, shown in the same color, illustrate the effects of sorting. Poorer sorting produces large decreases in the porosity, but relatively small increases

### Table 1.

<table>
<thead>
<tr>
<th>Sample Name</th>
<th>Initial Porosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sands:</td>
<td></td>
</tr>
<tr>
<td>Dry 1</td>
<td>0.414</td>
</tr>
<tr>
<td>Dry 2</td>
<td>0.432</td>
</tr>
<tr>
<td>Wet 3</td>
<td>0.400</td>
</tr>
<tr>
<td>Wet 4</td>
<td>0.411</td>
</tr>
<tr>
<td>Big Sand</td>
<td>0.409</td>
</tr>
<tr>
<td>Glass Beads:</td>
<td></td>
</tr>
<tr>
<td>GB Big</td>
<td>0.381</td>
</tr>
<tr>
<td>GB Small</td>
<td>0.411</td>
</tr>
<tr>
<td>GB 35% Small</td>
<td>0.315</td>
</tr>
<tr>
<td>GB Tiny</td>
<td>0.421</td>
</tr>
<tr>
<td>GB 35% Tiny 1</td>
<td>0.296</td>
</tr>
<tr>
<td>GB 35% Tiny 2</td>
<td>0.258</td>
</tr>
<tr>
<td>GB Broad</td>
<td>0.338</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fraction Size, µm (φ scale)</th>
<th>Fraction Size, µm (φ scale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>295-350 (1.5-1.75)</td>
<td>74-88 (3.5-3.75)</td>
</tr>
<tr>
<td>37-44 (4.5-4.75)</td>
<td>37-710 (0.5-4.75)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample Name</th>
<th>Initial Porosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big Sand</td>
<td>0.409</td>
</tr>
</tbody>
</table>

1 $D_v$ and $D_o$ are the grain diameters below which 10%, or 60% respectively, of the mass of the sample is found.

2 $\phi = -\log_d$, where $d$ is the grain diameter in mm.
in the velocities. While there is some scatter in the data above and below the Reuss average, likely because these data come from glass bead and sand data prepared in slightly different ways, all in all the data collected at similar pressures do tend to lie along this trend. This result confirms the supposition of Dvorkin and Nur (1996) that the porosity variation produced by variations in sorting has a minimal (Reuss average) effect on the velocities. The implication is that the poorer sorting results in small grains lying passively in the pores between larger grains. Most small grains reduce the porosity but do not add significantly to the stiffness of the sediment, and so the velocity is only slightly increased.

Figure 2b illustrates the effect on the velocities of pre-consolidation or compaction. This figure shows a velocity-porosity plot of the data from only the large size fraction sand, Big Sand, again color-coded by the pressure of the measurement. The data from this sample are highlighted with blue circles in Figure 2a. Also shown in Figure 2b are the same Reuss average lines as in Figure 2a, based on the virgin compaction curve of sample Dry 2. Sample Big Sand was run through eight pressure cycles, during which measurements were made at the same eight pressures for each cycle, once the peak pressure had exceeded that measurement pressure. Velocity measurements taken at the same pressure but compacted under higher preconsolidation pressures are seen to rise slightly as the porosity is reduced by compaction. This effect is more pronounced and more consistent for the higher measurement pressures. The rise is also slightly more pronounced for the $P$-wave velocities than for the $S$-wave velocities. For the higher measurement pressures, this effect is noticeably greater than the basically flat trend predicted by the Reuss averages based on the Dry 2 data. The data follow the Reuss average trend fairly well, although the $S$-wave velocities rise slightly, then fall again with greater consolidation pressures. Nevertheless, as for the case of sorting, the total effect on
the velocities of this compaction-driven porosity reduction is relatively minor for these cohesionless sediments.

Figure 3 shows the results of Gassmann fluid substitution (with water) of the velocities of all dry glass bead samples, plotted against pressure and color-coded according to porosity. As seen in this figure, higher porosities produce lower $P$-wave velocities for the water-saturated samples. The shear-wave velocities, which require only a density substitution, show similar porosity dependence at the higher pressures, with the low-porosity data resulting in higher velocities. At lower pressure, however, the porosity effect on the $S$-wave velocities is relatively small, as shown in the reduced scatter in Figure 3b and the rather flat Reuss average lines for the lower pressures in Figure 2a.

**Pressure prediction.** $V_p/V_s$ ratios for all the Gassmann fluid-substituted glass bead data are shown in Figure 4, plotted against pressure and color-coded by porosity. As discussed by Prasad (2002) and Huffman and Castagna (2001), the water-saturated $V_p/V_s$ shows a dramatic rise as the effective pressure decreases. The $V_p/V_s$ ratio increases from below 3 at 10 MPa to a mean value of about 7 at 0.5 MPa. Unfortunately, at the lower pressures there is a considerable amount of scatter in the $V_p/V_s$ ratio that would generate a significant uncertainty in the in-situ effective pressure determined from $V_p/V_s$ measurements. For example, if the measured $V_p/V_s$ is 5, the in-situ pressure (based on our fluid-substituted glass bead data) could vary between 0.15 and 2 MPa.

In this study, we considered the effect of two factors...
on the relationship between $V_P/V_S$ and pressure: preconsolidation and sorting. Both factors influence the porosity of the sediment: preconsolidation by compaction of the grain matrix primarily through rearrangement of grains, and sorting by smaller grains filling pore spaces between larger grains.

**Preconsolidation:** Because these noncohesive sediments demonstrate only relatively small changes in both the velocity and porosity with compaction, we do not expect preconsolidation to produce a significant $V_P/V_S$ signature. Preconsolidation does have a slightly larger effect on the dry $P$-wave velocities than on the $S$-wave velocities, as demonstrated in Figure 2b. At a given pressure, this would result in a slightly higher $V_P/V_S$ in overconsolidated sediments than in normally consolidated ones. This effect does not persist once the data is fluid-substituted, which, except for some scatter at the very lowest pressures, shows no significant $V_P/V_S$ signature from preconsolidation. So for water-saturated sands our results demonstrate a much larger reduction in the porosity of clay-rich sediments (Bowers, 1995; Sayers et al., 2001), this effect could potentially be important for cohesive sediments.

**Sorting:** At low pressures, the fluid-substituted $V_P/V_S$ ratio is a function of both the pressure and the porosity, with low porosities corresponding to high $V_P/V_S$ ratios (see Figure 4b). The porosity variation comes primarily from differences in the initial porosities of the samples due to differences in their sorting. The spread in the $V_P/V_S$ ratio comes from the different effects that the porosity has on the velocities, as demonstrated in Figure 3 and discussed above. As the relative porosity effect at high pressures is similar for the $P$- and $S$-wave velocities, there is relatively little scatter in $V_P/V_S$ above about 10 MPa. The diminishing porosity-dependence of the $S$-wave velocities at lower pressures allows the porosity dependence of the $P$-wave velocity to show through. The implication is that for robust pressure estimates to be made from measured $V_P/V_S$, it will be necessary to correct for this porosity effect at pressures below about 10 MPa. Laboratory measurements on natural sands could potentially produce a relatively general porosity correction, but at present we would recommend cautious use of $V_P/V_S$ as a quantitative pressure predictor.


**Acknowledgments:** This work was supported by the Stanford Rock Physics and Borehole Geophysics Project and by the Department of Energy (under Award No. DE-FC26-01BC15354). The opinions, findings, conclusions, or recommendations expressed herein are those of the authors and do not necessarily reflect the views of the DOE. Gilbert Palafox provided invaluable assistance in designing the experimental apparatus.

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PAPER 3


The pressure dependence of compressional (P) and shear (S) wave velocities in unconsolidated sediments is an important consideration in a number of engineering applications. This pressure dependence is often used to correct or project velocities to depths or locations where measurements have not been made. It also allows velocity changes to be used to monitor pressure changes in weak reservoirs or aquifers. The hazards posed to offshore drilling by unknown overpressures at shallow depths have also prompted attempts to use the seismic velocities or Vp-Vs ratio to quantitatively predict in situ effective pressures.

Our measurements of compressional (P) and shear (S) wave velocity in unconsolidated sand and glass bead samples at pressures from 100 kPa to 20 MPa show that the S-wave velocity to varies with approximately the fourth root of the effective pressure over this entire range of pressures, while the P-wave velocity shows a slightly lower pressure dependence.

Paper 3: Total of seven figures in paper
VELOCITY-PRESSURE TRENDS IN SANDS FROM ULTRASONIC P AND S-WAVE VELOCITY MEASUREMENTS

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Abstract

We present both compressional (P) and shear (S) wave velocity measurements from unconsolidated sand and glass bead samples at pressures from 100 kPa to 20 MPa. We found the S-wave velocity to vary with approximately the fourth root of the effective pressure over this entire range of pressures, while the P-wave velocity shows a slightly lower pressure dependence. The pressure dependence of both velocities was influenced by preconsolidation, though the effect was greater for the P-wave velocities. We prepared samples with a range of initial porosities by varying the sorting of the samples, generally by mixing two different sizes of sand grains or glass beads. The samples with lower sorting qualities had significantly lower porosities, but the effect on the P and S-wave velocities was minimal. The Reuss average between the moduli of the highest porosity sample and of solid quartz described the effect of sorting on the velocities and porosity relatively well. The variation in initial porosities due to the sorting did not produce a systematic variation in the pressure dependences. Water-saturated P and S-wave velocities were modeled with Gassmann fluid substitution. The water-saturated P-wave velocities show a large porosity dependence, which results in a significant porosity dependence in the V_P-V_S ratio, especially at low pressures.

Introduction

The pressure dependence of compressional (P) and shear (S) wave velocities in unconsolidated sediments is an important consideration in a number of engineering applications. This pressure dependence is often used to correct or project velocities to depths or locations where measurements have not been made, especially for site amplification predictions or liquefaction susceptibility analyses (Youd and Idriss, 1996). It also allows velocity changes to be used to monitor pressure changes in weak reservoirs or aquifers. The hazards posed to offshore drilling by unknown overpressures at shallow depths have also prompted attempts to use the seismic velocities or V_P-V_S ratio to quantitatively predict in situ effective pressures.

Because of the frequent use of the dynamic shear modulus in geotechnical applications, a great deal of research has been conducted on the pressure dependence of the shear modulus and of the shear wave velocity in soils. Hardin and Blandford (1989) developed a semi-empirical expression for the small-strain shear modulus. The form for an isotropic stress state is as follows:

\[ G^e_s = \frac{OCR^k S_{ij}}{F(e)} \left( \frac{p_a}{2(1+\nu) \sigma_{av}^{\nu-n}} \right)^{\frac{1}{\nu-n}} \]

where \( G^e_s \) is the shear modulus in the plane of propagation, which experiences the mean stress \( \sigma_{av}^\nu \) in all directions, \( p_a \) is the atmospheric pressure, and \( \nu \) is the Poisson’s ratio of the grain material. Equation (1) includes two free parameters: \( S_{ij} \), a multiplier to account for textural factors and structural anisotropy, and \( n \), which dictates the pressure dependence of the modulus. The void ratio function, \( F(e)=0.3 + 0.7e^2 \), is meant to account for the effect of porosities differences, whether the result of textural differences between samples or of the compaction of a given sample. The \( OCR^k \) term corrects the pressure dependence for the effects of compaction or preconsolidation of the sample, where \( OCR \) is the...
overconsolidation ratio, and $k$ is a function of the plasticity index usually assumed to be zero for sands. Since the overconsolidation ratio is defined as the preconsolidation pressure the current pressure, the pressure dependence of the modulus for unloading or reloading paths is simply the effective pressure, $p$, to the $n$-$k$. A large body of work has demonstrated that the value of $n$ for sands is generally near 0.5 (Hardin, 1980; Yu and Richart, 1984; Hryciw and Thomann, 1993). Most of this work has been conducted at pressures below a few hundred kPa. Hryciw and Thomann (1993) measured the pressure dependence of a number of texturally different sands at pressures up to 300 kPa and found $n$ to vary between values of 0.39 and 0.72, and to correlate to the compressibility of the sand. Their measurements showed $S_{ij}/[2(1+\nu)]$ to vary from 478 to 734, and to be inversely related to $n$. They also recognized that $k$ can be greater than zero for loose, compressible sands.

For offshore applications compressional waves are more commonly measured than shear waves. In these applications pressures are generally assessed by developing a normal P-wave interval travel-time vs. depth trend. This trend is usually assumed to be linear when depth is plotted against the log of the interval travel-time. Deviations above this trend are assumed to be the result of high pore pressures and low effective pressures (Hottman and Johnson, 1965; Pennebaker, 1970; Pilkington, 1988).

Hardin and Blandford (1989) also propose a semi-empirical relationship for the constrained (P-wave) modulus, $M_i$, similar in form to that of the shear modulus:

$$M_i = \frac{OCR^k}{F(e)} \frac{S_i(1-\nu)}{(1+\nu)(1-2\nu)} P_{eff}^{\nu-n} \sigma_i^{\nu-n}$$

where $\sigma_i^{\nu-n}$ is the stress in the direction of wave propagation, and all the other symbols are the same as for equation (1). Little experimental data has been collected on unconsolidated sediments to constrain the free parameters for the P-wave modulus.

Recently Huffman and Castagna (2001) and Prasad (2002) have demonstrated the potential of the $V_P$-$V_S$ ratio as an indicator of pressure, as extracted from AVO analysis or multi-component reflection data. The $V_P$-$V_S$ ratio is especially sensitive to pressure for water saturated sediments. At low pressures the shear velocity drops to near zero. The water-saturated compressional wave velocity is limited by the compressional resistance of water to values above 1500 m/s, and so the $V_P$-$V_S$ ratio rises dramatically at low pressures. The appeal to using the $V_P$-$V_S$ ratio as a pressure indicator is that it can be easily extracted from AVO data, whereas pure shear wave velocities are difficult to measure in situ in offshore environments. It is also thought that this parameter might reduce the ambiguity in the compressional wave velocity-pressure trend that results from the presence of gas in the sediment, and at the same time might reduce the effect of textural effects inherent in both of the velocities.

A number of theoretical models (Digby, 1981; Walton, 1987; Goddard, 1990) have been proposed to describe the elastic moduli of granular materials. These models generally assume that the material is made up of an assemblage of perfect spheres, with the stiffness of the contacts described by Hertz and Mindlin solutions (Mindlin, 1949) to the displacements of two spheres subject to normal and shear forces. The behavior at the contacts is then used to predict the shear or compressional moduli either by assuming a regular packing (Santamarina and Cascante, 1996) or by assuming a random arrangement of contacts (Walton, 1987). In either case these models predict both the bulk and shear moduli to have a pressure dependence of $p^{1/3}$. If the contacts are assumed to be a cone in contact with a plane, or if the number of inter-particle contacts are allowed to vary by buckling of the particle chains, the pressure dependences are predicted to be $p^{1/2}$ at low pressures, transitioning to $p^{1/3}$ at high pressures (Goddard, 1990).

We have run a series of experiments on sand and glass bead samples designed to measure the effects of porosity changes due to sorting and compaction on the velocities and on the velocity-pressure relationship. We measured the P and S-wave velocities at pressures from below 100 kPa up to 20 MPa in order to test the pressure dependence over this entire pressure range. In this paper we report our results
and discuss the porosity-velocity and pressure-velocity trends observed. We also discuss the implications of our results for quantitative pressure prediction over the range of pressures measured.

Figure 1: The experimental apparatus. A) The pressure vessel, about 80 cm long and 10 cm in diameter, with the sample holder. B) The sample holder.

Experimental Apparatus and Samples

The experimental apparatus consists of a sample holder that is inserted into a hydrostatic pressure vessel (see Fig. 1). The two end caps of the sample holder contain both P and S-wave transducers made with 200 kHz piezoelectric (PZT) crystals. In order to improve the impedance matching between the transducers and the sample, the faces of the transducers are made of 30% glass filled polycarbonate (shatter proof glass). Velocities were calculated by picking first arrivals from pulse-transmission signals. With this arrangement we were able to get interpretable shear signals at pressures as low as 50 kPa, with errors of approximately 2% for the P-wave velocities and 4% for the S-wave velocities.

The data presented here are from samples of a fine grained, well sorted, quartz sand, called the Santa Cruz Aggregate, as well as from synthetic samples made from sieved fractions of this sand and of glass beads. Four samples of the Santa Cruz Aggregate were run, two dry (Sa Dry 1, Sa Dry 2) and two water-saturated (Sa Wet 3, Sa Wet 4). Two other samples were made of sieved fractions of this sand, both of which were run dry. One sample (Sa Big) was made of 0.295-0.350 mm grain sizes. A second sample (Sa 35% Small) was made up of 65% by mass of the 0.295-0.350 mm size fraction, and the other 35% of 0.053-0.088 mm size grains. A total of seven glass bead samples were run, all dry. Three samples (GB Big, GB Small, and GB Tiny) consisted of different narrow size ranges of beads. Three samples (GB 35% Small, GB 35% Tiny 1 and 2) were made with a “bimodal” mixture of grain sizes, with 35% of the mass made up of smaller grains and 65% of larger grains. Finally, one sample (GB Broad) was made up of a broad range of particle sizes. Table 1 summarizes the grain sizes used and the initial porosities of all the samples.

Special attention was given to preparing the samples in such a way as to insure complete mixing of the different grain sizes, and to maintain full saturation of the water-saturated samples. However, the sample preparation varied based on whether the sample was a single grain size and dry (air pluviated) or water-saturated (water pluviated), or was a mixture of grain sizes (some samples mixed dry, some mixed...
while moist and allowed to dry once in the sample holder). These differences in sample preparation produced variations in the packing of the grains and led to some scatter in the data.

The initial porosity of the samples was calculated from the grain density, dry sample mass, and sample volume. The sample dimensions were determined initially by measuring the diameter of the jacket around the sample and the distance between the two end caps once the sample had been prepared in the holder. The samples, 3.8 cm (1.5 in.) in diameter, were generally prepared to be about 3 cm long. The changes in the sample volume and porosity with pressure were then monitored by measuring changes in the length and circumference of the samples. An error analysis of our porosity measurements estimates the error at 0.012 to 0.016, or 3 to 4% of the total porosity.

The pressure paths followed for the different samples generally included a number of pressure cycles of increasing peak pressure for each cycle. The velocities and porosity were measured at the same set of pressures during each cycle (e.g. 0.1, 0.2, 0.5... MPa). This allowed us to compare the velocities and porosities measured at the same pressure for a sample that had been preconsolidated to a range of higher pressures. Five of the samples were cycled through 8 or 9 cycles, while the rest were cycled through between 1 and 5 cycles.

**Pressure Trends**

The shear and compressional velocity results from all of our samples are shown plotted against effective pressure in a log-log plot in Figure 2. Also shown are other measurements made on clean sands over similar pressure ranges from Domenico (1977), Prasad and Meissner (1992), Yin (1992), Estes et al. (1994), and Robertson et al. (1995). Figure 2 also includes lines showing pressure dependences of \( p^{1/6} \), \( p^{1/4} \), and \( p^{1/3} \). This figure demonstrates that the measurements are fairly consistent, and are comparable to other data collected on loose sands over the same pressure range. It also illustrates that in general the pressure dependence of these samples tends to remain close to \( p^{1/4} \) for the shear wave velocities, but with a slightly lower pressure dependence for the P-wave velocities.

Figure 3 shows the data from sample Sa 35% Small: plotted against effective pressure in (A) and (B), and plotted against porosity and color coded according to the measurement pressure in (C) and (D). These figures demonstrate that there is a slight preconsolidation effect to the samples, especially visible in (C) and (D) where the velocities measured at the same pressure rise as the porosity decreases with compaction from higher preconsolidation pressures.

We treated \( n, k \), and \( S/F(\nu) \) (lumping the Poisson’s ratio terms, \( F(\nu)=2(1+\nu) \) for the S-waves and \( F(\nu)=(1+\nu)/(1-2\nu)/(1-\nu) \) for the P-waves, in with \( S \)) as free parameters and fit equations (1) and (2) to the moduli calculated from our velocity measurements. We show the empirical fit for these two equations to the data from sample Sa 35% Small in Figure 3. The coefficients for the fits for each of the samples are given in Table 1, while the coefficients for the fits to the dataset as a whole are given in Table 2. The coefficients are plotted against the initial porosity of the samples in Figure 4. For the shear modulus we found the value of \( n \) to vary from between 0.352 to 0.612, with most of the values in the range from 0.45 to 0.5. This implies that the pressure dependence of the S-wave velocity would range from \( p \) to the 0.176 and the 0.306. These values are within the range of the results of Hryciw and Thomann (1993). The value of \( k \) for the shear modulus varied between \(-0.073\) and \(0.101\) and averaged \(-0.003\) (effectively zero), generally slightly lower than most of the values measured by Hryciw and Thomann (1993). These values of \( k \) are still relatively small compared to the values predicted for clays, which can be as large as 0.5 for high plasticity clays (PI=100) and would be about 0.1 for a plasticity index of 10 (Hardin and Drnevich, 1972). Some of the samples, especially a couple of the glass bead samples, actually demonstrates a velocity drop with consolidation, indicated by a negative \( k \) value.
Figure 2: P and S-wave velocities plotted against effective pressure for all of our sand and glass bead samples, and for similar measurements on clean sands from Prasad and Meissner (1992) (water sat.-▼, ▲), Domenico (1977) (dry-•, water sat.-♦), Yin (1992) (dry-▲, water sat.-▼), Estes et al. (1994) (dry-■, water sat.-★), and Robertson et al. (1995) (water sat.-+, x).

Figure 3: The velocity results from sample Sa 35% Small: A) $V_P$ vs. pressure, B) $V_S$ vs. pressure, C) $V_P$ vs. porosity and color-coded by the pressure, and D) $V_S$ vs. porosity. The black lines in parts (A) and (B) and the black dots in parts (C) and (D) represent fits to the Hardin and Blandford (1989) equations. The lines, the same as shown in Figure 5, are the Reuss average and Hardin and Blandford predictions for the velocity-porosity trend based on the moduli of the high porosity end-member (Sa Dry 2).
The pressure dependence of the P-wave modulus is consistently lower than that of the shear modulus, with $n$ ranging from 0.304 to 0.478 and averaging 0.07 less than the $n$ for the shear modulus for the same sample. While the value of $n$ was lower for the P-wave modulus of each sample, the value of $k$ was almost always higher for the P-wave modulus than for the shear modulus, averaging 0.026 and ranging from $-0.033$ to $0.104$.

Figure 4 demonstrates that there is not a significant systematic initial porosity effect on the pressure dependence ($n$) or on the preconsolidation dependence ($k$). Thus the variation in initial porosity that comes from the differences in sorting does not significantly affect these dependences over the porosity range tested.

For one of the samples, GB Big, the pressure dependence of the P-wave modulus is actually below $p^{1/3}$, the dependence predicted by Hertz-Mindlin contact models. For all of the rest of the samples the pressure dependences of both moduli are higher. This might indicate that the pressure dependence is influenced by the grain angularity, as suggested by Goddard (1990), and grain size, though there are a number of other factors that we also did not systematically investigate that might influence the pressure dependence as well, including the packing, age, and mineralogy.

**Sorting Effects**

Figure 5 shows the velocity data from all of the samples plotted together against the porosity and color coded according to the pressure. The lower porosity data (below 0.35) come from the glass bead samples where two size fractions of beads were mixed, with the smaller grains partially filling the pore space between the larger grains. These plots illustrate the relative effects of the pressure and the sorting on the velocity and porosity. While for these coarse sediments, increased pressures result in large increases in the velocity, the porosity is only slightly reduced. On the contrary, the porosity reductions that result from a decrease in the sorting quality produce only minor increases in the velocity.

The lines shown in Figure 5 demonstrate the velocity effect predicted for the sorting-induced porosity changes in two ways: 1) by the Reuss average between the dry frame moduli and the solid quartz moduli (solid lines), and 2) by the Hardin and Blandford void ratio equation, $F(e)$ (dashed lines). The Reuss (isostress) average, the weighted harmonic average between two end member moduli, simulates the weakest possible way to combine two materials. Here the Reuss average is used to represent the theoretically minimal possible change in the velocities produced by adding solid grain material (quartz) to the high porosity sample (assuming constant packing) (Dvorkin and Nur, 1996). The high porosity end-member moduli come from the highest porosity data for each pressure, taken from the virgin compaction curve of sample Sa Dry 2. The Reuss average was calculated as follows:
where $M$ is the resulting average modulus, $M_{df}$ is the modulus of the dry frame at the pressure of interest, and $M_{qtz}$ is the modulus of pure quartz. The fraction of dry frame is given by $\frac{\phi}{\phi_{df}}$, where $\phi$ is the current porosity, and $\phi_{df}$ is the initial dry frame porosity at that pressure from sample Sa Dry 2. The fraction of quartz is then just $1 - \frac{\phi}{\phi_{df}}$. The Hardin and Blandford porosity effect on the moduli was calculated as follows:

$$M = \frac{F(e_{df})}{F(e)} M_{df} = \frac{0.3 + 0.7 e_{df}^2}{0.3 + 0.7 e^2} M_{df}$$

where $M$ is the resulting porosity-corrected modulus, $M_{df}$ is the modulus of the high porosity dry frame at the pressure of interest (calculated from the velocities and density of sample Sa Dry 2 at that pressure), $e_{df}$ is the void ratio of the dry frame, and $e$ is the current void ratio. Either relationship predicts the sorting effect relatively well, given the scatter in the data, though the Reuss average is likely to fit lower porosity data better than Hardin and Blandford's expression. The fact that the trend is so well described by the Reuss average implies that the small grains added tend to sit more or less passively in the pores, reducing the porosity but not adding significantly to the stiffness of the sediment.

Figure 6 shows the results of Gassmann fluid substitution with water of the velocities of all the dry samples, plotted against pressure and color-coded for porosity. As seen in this figure, the water saturated P-wave velocities have a significant porosity dependence, with lower porosities correlating to higher P-wave velocities. The presence of water in the pores stiffens a low porosity sediment more than a high porosity one, though the trend is still described reasonably well by the Reuss average between the moduli from the water-saturated, high porosity end-member and solid quartz. The shear wave velocities, which only require a density substitution, show a slight porosity dependence at the higher pressures, with the low porosity data resulting in higher velocities. At lower pressures, however, the porosity effect on the S-wave velocities is smaller than the scatter in the data.

Figure 5: The velocity data from all of the samples plotted against porosity and color-coded according to the pressure at the time of measurement: A) $V_p$ B) $V_s$. The solid lines represent the Reuss average between the moduli of the high-porosity dry frame end-member and solid quartz. The dashed lines represent the porosity correction to the high porosity moduli proposed by Hardin and Blandford (1989).
Implications for Overpressure Prediction

The $V_P$-$V_S$ ratios for all of the Gassmann fluid-substituted data are shown in Figure 7, plotted against pressure and color-coded by porosity. As discussed by Prasad (2002) and Huffman and Castagna (2001), the water saturated $V_P$-$V_S$ ratio shows a dramatic rise as the effective pressure decreases. The $V_P$-$V_S$ ratio increases from below 3 at 10 MPa to a mean value of about 7 at 0.5 MPa. Unfortunately there is a considerable amount of scatter in the $V_P$-$V_S$ ratio, especially at low pressures, that would generate a significant amount of uncertainty in the in situ effective pressure determined from $V_P$-$V_S$ ratio measurements. For example, if the measured $V_P$-$V_S$ ratio is 5, the in situ pressure (based on our fluid-substituted data) could vary from between 0.2 and 2 MPa.

Figure 7(B) demonstrates that there is a porosity dependence to much of this scatter in the $V_P$-$V_S$ ratio at low pressures. Since these non-cohesive sediments demonstrate only relatively small changes in both the velocity and porosity with compaction, preconsolidation has only minor influences on the $V_P$-$V_S$ ratio. This implies that for water-saturated sands there should be only very little difference between the $V_P$-$V_S$ ratio signatures of overpressures produced through undercompaction or through repressurization mechanisms such as fluid expansion. As preconsolidation can cause a much larger reduction in the porosity of clay-rich sediments (Bowers, 1995; Sayers et al., 2001), this effect could potentially be more important for cohesive sediments.

The porosity variation between the samples comes primarily from the differences in the initial porosities of the samples due to differences in their sorting. The scatter in the $V_P$-$V_S$ ratio is therefore a product of the different effects that the porosity has on the water-saturated P- and S-wave velocities, as demonstrated in Figures 5 and 6 and discussed above. As the relative porosity effect at high pressures is similar for the P and S-wave velocities, there is relatively little scatter in the $V_P$-$V_S$ ratio above about 10 MPa. The small porosity-dependence of the S-wave velocities at lower pressures allows the porosity dependence of the water-saturated P-wave velocity to show through. The implication is that for robust pressure estimates to be made from measured $V_P$-$V_S$ ratios it will be necessary to correct for this porosity effect at pressures below about 10 MPa. Such a correction could potentially be developed through a suite of laboratory measurements on natural sands, and would need to be developed in order to permit the use of the $V_P$-$V_S$ ratio as a quantitative pressure indicator.
Figure 7: The $V_p/V_S$ ratio plotted against pressure and color coded according to the porosity: A) linear plot, and B) log-log plot.

Conclusions

We have measured the pressure dependence of the P and S-wave velocities through sand and glass bead samples with a variety of initial porosities produced by varying their sorting. We also ran the samples through a series of pressure cycles of increasing peak pressures in order to observe the effects of preconsolidation on the samples. We have observed the following results:

1) The pressure dependence of the S-wave velocities is around the fourth root of the effective pressure ($p$). The pressure dependence of the shear modulus ranges from $p$ to the 0.352 to $p$ to the 0.612. For the P-wave velocities the pressure dependence is slightly lower. The P-modulus dependence varies between $p$ to the 0.304 and $p$ to the 0.478, though generally still higher than the $p^{1/3}$ dependence predicted by Hertz-Mindlin contact models.

2) These non-cohesive sediments do exhibit a small preconsolidation effect on the velocities, though the effect is generally smaller for the S-wave velocities than for the P-wave velocities.

3) While poorer sorting can result in large decreases in the porosity, it has a relatively small effect on the S-wave velocities and dry P-wave velocities. The porosity reduction does produce a larger effect on the water-saturated P-wave velocities, as calculated from Gassmann fluid-substitution of our dry velocity results. This porosity dependence of the water-saturated P-wave velocities, and the limited porosity dependence of the S-wave velocities, results in a porosity dependence in the $V_p/V_S$ ratio. Higher porosities correspond to a lower $V_p/V_S$ ratio.

4) The sorting appears to have no significant effect on the pressure dependence of the velocities or on the effect of preconsolidation. The variation in the values of $n$ and $k$ for these non-cohesive sediments could be largely a result of differences in sample preparation that lead to inconsistency in the packing from sample to sample, or of other textural parameters that were not systematically investigated here.

References


Table 1: Sample summary and fit parameters for individual samples

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<tr>
<th>Sample Name</th>
<th>Porosity</th>
<th># of cycles</th>
<th>Vs: $S/F(\nu)^n k$</th>
<th>$R^2$</th>
<th>Vp: $S/F(\nu)^n k$</th>
<th>$R^2$</th>
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<td>clean, quartz sand; well sorted</td>
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<td>0.459</td>
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<td>0.409</td>
<td>9</td>
<td>1</td>
<td>295-350 (1.5-1.75)</td>
<td>782</td>
<td>0.449</td>
</tr>
<tr>
<td></td>
<td>0.378</td>
<td>9</td>
<td>0.65</td>
<td>295-350 (1.5-1.75)</td>
<td>539</td>
<td>0.507</td>
</tr>
<tr>
<td>Glass Beads: GB Big</td>
<td>0.381</td>
<td>8</td>
<td>1</td>
<td>295-350 (1.5-1.75)</td>
<td>1075</td>
<td>0.352</td>
</tr>
<tr>
<td></td>
<td>0.411</td>
<td>8</td>
<td>1</td>
<td>74-88 (3.5-3.75)</td>
<td>671</td>
<td>0.448</td>
</tr>
<tr>
<td></td>
<td>0.315</td>
<td>8</td>
<td>0.65</td>
<td>295-350 (1.5-1.75)</td>
<td>526</td>
<td>0.490</td>
</tr>
<tr>
<td></td>
<td>0.421</td>
<td>3</td>
<td>1</td>
<td>37-44 (4.5-4.75)</td>
<td>704</td>
<td>0.448</td>
</tr>
<tr>
<td>GB 35% Tiny 1</td>
<td>0.296</td>
<td>3</td>
<td>0.65</td>
<td>295-350 (1.5-1.75)</td>
<td>564</td>
<td>0.503</td>
</tr>
<tr>
<td>GB 35% Tiny 2</td>
<td>0.258</td>
<td>3</td>
<td>0.65</td>
<td>295-350 (1.5-1.75)</td>
<td>287</td>
<td>0.612</td>
</tr>
<tr>
<td>GB Broad</td>
<td>0.338</td>
<td>3</td>
<td></td>
<td>295-710 (0.5-4.75)</td>
<td>393</td>
<td>0.494</td>
</tr>
</tbody>
</table>

Table 2: Fit parameters to whole dataset

<table>
<thead>
<tr>
<th>$S/F(\nu)^n k$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_S$</td>
<td>0.38</td>
</tr>
<tr>
<td>$V_P$ (dry)</td>
<td>0.41</td>
</tr>
</tbody>
</table>

GB: Glass Beads; CB: Clay Beads; SB: Sand Beads; BB: Bivium (shape); PS: Perforated cycles; I: Initial # of
BIBLIOGRAPHY

