**CHARACTERIZATION OF IN-SITU STRESS AND
PERMEABILITY IN FRACTURED RESERVOIRS**

SEMI-ANNUAL TECHNICAL PROGRESS REPORT

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by

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ABSTRACT

A 3-D elastic wave propagation finite difference model, including effects of attenuation, has been implemented and compared with other existing modeling codes for validation. Models of seismic scattering from discrete large-scale fractures as well as equivalent anisotropic medium representations of small-scale fractures have been generated and used to develop data analysis methods for applications to seismic field data. An inversion scheme has been developed to estimate fracture orientation and fracture density from amplitude variations with offset and azimuth (AVOA). The method has been tested on synthetic data and field data from an offshore fractured carbonate reservoir with promising results. Spectral characteristics of the numerical model data of the seismic wavefield scattered from aligned fractures with different spacing between fracture zones have been analyzed. Results indicate that the spacing of these large, open fracture zones can be estimated from the wavenumber spectra of the scattered wave amplitude as a function of offset in pre-stack data. Two approaches for converting seismically derived fracture parameters into fluid-flow parameters for use in reservoir simulators have been identified. The first is the numerical modeling of Stoke's flow in fracture networks, and the second uses a statistical model of a fracture distribution that allows for the calculation of the elastic properties and permeability tensor of the resulting equivalent medium. These approaches will be compared in the coming year. Multiple meetings have been held with our industry partner, Shell Oil, to identify a field test site for the project. We are focusing our efforts on a fractured carbonate field. The field application test site selection and data transfer will be completed in the coming year.

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I. EXECUTIVE SUMMARY

The purpose of this project is to develop and implement large-scale numerical models to quantify the effects of fracture parameter variations on seismic reflection signals and in-situ stress variations on flexural modes in boreholes. These models will be used as the basis of data analysis and inversion routines for estimating the heterogeneous fracture distribution in fractured reservoirs from seismic and borehole field data provided by Shell. Fracture property distributions estimated from seismic data will be used to estimate the permeability tensor in the reservoir for input to reservoir simulators.

During the first year of the project we adapted and implemented a 3-D elastic wave propagation finite difference model, including effects of attenuation. Computations have been compared with other existing modeling codes (e.g., from Lawrence Berkeley Laboratory) for validation. Models of seismic scattering from discrete large-scale fractures as well as equivalent anisotropic medium representations of small-scale fractures have been generated and used to develop data analysis methods for applications to seismic field data. Two seismic analysis methods have been investigated for the estimation of fracture parameters. The first is an inversion scheme to estimate fracture orientation and fracture density from amplitude variations with offset and azimuth (AVOA). The method has been tested on synthetic data and field data from a fractured offshore carbonate reservoir with promising results. The second method, which has only been tested on synthetic data sets, looks at the spectral characteristics of seismic wavefield scattered from aligned fractures with different spacing between fracture zones. Results indicate that the spacing of these large, open fracture zones can be estimated from the wavenumber spectra of scattered wave amplitude as a function of offset in pre-stack data. Further testing, especially with field data, will take place in the coming year to test the validity of this method.

We have identified two approaches for converting seismically derived fracture parameters into fluid-flow parameters for use in reservoir simulators. The first is the numerical modeling of Stoke's flow in fracture networks. Initial model development is

underway. The second approach uses a statistical model of a fracture distribution that allows for the calculation of the elastic properties and permeability tensor of the resulting equivalent medium. The model, based on work by Oda (1985) and Brown and Bruhn (1998) and developed by New England Research, will be tested against numerical models in the coming year.

We are continuing to work on methods of estimating the dispersion characteristics of flexural waves from cross dipole acoustic logs in anisotropic and stressed formations. Accurate dispersion information is the critical input to stress estimation techniques such as Huang et al. (1999).

Multiple meetings have been held with our industry partner, Shell Oil, to identify a field test site for the project. We are focusing our efforts on a fractured carbonate field. The field application test site selection and data transfer will be completed in the coming year. We have also been testing our seismic methods on an offshore fractured carbonate field. This field has a very good seismic data set acquired with a four-component ocean bottom cable, and full azimuthal source-receiver coverage, making it an ideal test-bed for our seismic analysis techniques. Data from multiple wells in the field are also available for validation.

II. INTRODUCTION

The purpose of this project is to develop and implement large-scale numerical models (running on parallel Linux clusters) to quantify the effects of fracture parameter variations on seismic reflection signals and in-situ stress variations on flexural modes in boreholes. These forward modeling efforts will be used to develop data analysis and inversion routines for estimating the heterogeneous fracture distribution in fractured reservoir from seismic and borehole field data. Fracture property distributions will be used to estimate the permeability tensor in the reservoir by using statistical and discrete feature network techniques. Such estimates will then be used as inputs to a reservoir simulator (MORES and ECLIPSE) for reservoir performance prediction. History matching will provide a measure of the sensitivity and, along with well data, an estimate of the accuracy of the methods developed. The objective of the project is schematically displayed in Figure 1.

Figure 1 Schematic display of the project objectives. Surface 3-D seismic data is used to estimate the spatial distribution of fracture parameters (e.g., fracture density, spacing, orientation). A model is used to convert such fracture parameters into permeability values, which are input to reservoir simulators for history matching.

During the past six months of the project we have focused on improving our numerical modeling codes by incorporating the effect of attenuation, developed an inversion method for estimating fracture orientation and elastic properties from measured amplitude variations with offset and azimuth (AVOA), and tested methodologies for estimating fracture density variations from measurements of the seismic scattered wave signals related to a fractured reservoir. Our work on fluid flow in fractures and the use of borehole cross dipole acoustic logs for in-situ stress estimation has also continued.

III. RESULTS AND DISCUSSION

A system of aligned fractures effects propagating seismic waves in several ways. If the fractures and fracture spacing are small relative to the seismic wavelength, then the fractures cause the reservoir rock to behave like an equivalent anisotropic medium with a symmetry axis normal to the strike of the 'open' fractures. The resulting seismic anisotropy will therefore be related to the fluid flow directions in the reservoir. We can develop analysis methods to estimate the orientation of these open fractures as well as the fracture density from seismic measurements. In particular, seismic reflections from the top and bottom of a fractured reservoir will display different amplitude variations with offset (AVO) as a function of the orientation of the seismic source-to-receiver direction relative to the orientation of the fractures. Such measurements are referred to as amplitude variations with offset and azimuth (AVOA). We have developed an inversion method for estimating fracture orientation and density from measured AVOA from reservoir reflectors. This method has been tested on synthetic and field seismic data, and the results are promising.

If the fractures and fracture spacing are closer in size to the seismic wavelength, then the fractures will scatter the seismic energy causing a more complex seismic signature. This seismic signature will also vary as a function of the orientation of the seismic acquisition relative to the fracture orientation. This scattered seismic energy can not only provide information about the fracture orientation, but can also be analyzed to provide

information about the fracture spacing and fracture density. One such method looks at the spectral characteristics of reflections in pre-stack data. In this application, the fracture spacing is estimated from the spectral signature of the seismic trace amplitudes versus offset. This method has been tested on synthetic data and the results are promising.

We have also adapted our existing 3D finite difference elastic wave propagation modeling code to include the effects of attenuation using the method of Emmerich and Korn (1987). This development will help us to more accurately model real earth materials and provide better models of the relative amplitudes of different seismic signals.

In the area of fluid flow in fracture systems, we have identified two methods that we will compare for estimating the permeability tensor for a fractured rock. One method is based on numerical approaches for computing Stoke's flow in a fracture network. The second method uses a statistical approach for computing the permeability tensor based on the models of Oda (e.g., 1985) and further developed by Brown and Bruhn (1997). We will compare these methods to help decide the best approach for taking our seismically derived fracture information and converting it into permeability values for input to a dual-porosity reservoir simulator.

Finally, we are continuing to work on an inversion method for estimating velocity anisotropy and dispersion from cross-dipole acoustic logs as an important precursor to the in-situ stress estimation method of Huang et al. (1999). A summary of results for each of these areas is given below.

A Numerical Modeling

We have continued to develop our numerical modeling tools by adapting our 3-D finite difference elastic wave propagation code to include the effects of intrinsic attenuation. We used the method of Emmerich and Korn (1987), which uses a viscoelastic approach to represent a near constant Q value over a frequency range of interest. A paper by

Krasovec et al. (2003) provides a summary of the approach (see Appendix A). We have also continued to use a similar code developed at the Lawrence Berkeley Lab (Nihei et al., 2002) for some of our model computations. Both codes are implemented to run on PC clusters.

B. AVOA – seismic inversion for fracture parameters

Aligned sets of fractures that are small relative to seismic wavelengths (note: for most exploration situations the seismic wavelengths are on the order of 50-100 meters) will cause the reservoir rock to behave like a homogeneous, anisotropic medium. For situations in which the fractures are essentially vertical, this equivalent anisotropic material can be represented as an HTI medium (horizontal transversely isotropic). This HTI medium has a symmetry axis that is normal to the strike of the aligned fractures, and the velocity of the seismic waves will be a function of the propagation direction relative to that symmetry axis. Because fractures are more compliant than the background rock, the seismic velocities are slowest in the direction normal to the fractures and fastest in the direction parallel to the fractures. Such a velocity difference provides several possibilities for the detection of fractures from analysis of seismic data. One detection method would be to look for shear wave splitting of waves that have passed through the reservoir zone (shear waves will be polarized along the symmetry directions, that is parallel and perpendicular to the fracture direction, and the velocities of those waves will be different resulting in two shear wave arrivals at different travel times). Since detection and analysis of shear waves in reflection seismic data is fairly difficult (multicomponent sources and receivers are generally needed), another method can be used to investigate this same velocity anisotropy – amplitude variations with offset and azimuth (AVOA). The AVOA method has received much attention in recent years (e.g., Lynn et al., 1996; Perez et al., 1999; Mallick et al., 1998; Ruger, 1998; Dong and Davidson, 2003). The basic premise is that the amplitude of the P wave reflection as a function of offset will have different behavior at different azimuthal directions because of the aligned fractures in the reservoir. A synthetic example of this behavior is shown in Figure 2 for a case where the symmetry axis (that is the direction normal to the fractures) is oriented at 35°.

Since the seismic velocities are slowest in the direction of the symmetry plane, we see in this figure that the amplitude of the reflection from the top of this ‘reservoir’ drops off most quickly in this direction and least quickly along the fracture strike direction (125°). We developed an inversion method based on this effect, and tested it on data from an offshore fractured carbonate reservoir. The field data was acquired with a 4-component ocean bottom cable and a shot point configuration designed to ensure full azimuthal coverage, providing a good data set for testing the AVOA inversion method. The data is shown in Figure 3, sorted into different azimuthal gathers for the particular CDP location used as input for inversion. The results are shown in Figure 4. In particular, the resulting estimate of the symmetry axis orientation (4th panel in Figure 4) of the fractures shows a constant and consistent orientation of $0-10^\circ$ between 1.7 and 2.2 seconds two-way travel time (the reservoir interval). This result is in agreement with other estimates of the fracture orientation at this field. The inversion method, sensitivity study, and field data example is described in the paper by Minsley et al. (2003) in Appendix B.

Figure 2 P wave reflection amplitude versus angle of incidence (which is equivalent to offset) for 5 different azimuthal directions given by the colored lines. Calculations based on the Ruger (1998) model for a fracture system whose normal is oriented at 35° . Note that the azimuth closest to the fracture normal has the largest variation with incidence angle and the azimuth closest to the fracture strike has the smallest variation. (from Minsley et al., 2003)

Figure 3 Seismic data over a fractured carbonate reservoir. Each panel represents the CDP gather along a particular azimuth (two way travel time along the vertical axis, source-receiver offset along the horizontal). The reservoir interval is below the high amplitude reflector at 1.8 seconds. (from Minsley et al., 2003)

Figure 4 AVOA inversion results from field data in Figure 3. Each panel shows the resulting parameter value as a function of two way travel time (vertical axis). The first 4 panels are the inversion results for the parameters in the model, panel 5 is a ratio of two 'B' parameters, and panel 6 is the RMS error in the data fit. Note the constant symmetry axis orientation (panel 4) in the time interval of 1.7 – 2.2 seconds, which contains the fractured reservoir. The direction (0-10°) is consistent with other fracture indicators in the field. (from Minsley et al., 2003)

C. Scattered Wave Analysis –new seismic measures for fractured reservoirs

The way in which fractures affect seismic waves depends on the fracture compliance (due to roughness, saturating fluid, etc) and geometric parameters, such as dimensions and spacing. If fractures have characteristic lengths on the order of the seismic wavelength then there will be scattering of energy due to the presence of the fractures. If fractures are spaced much closer than the wavelength of the seismic waves, then the medium will behave like a homogeneous, anisotropic equivalent medium, as described in the previous section. When fractures are spaced further away (about a quarter of a wavelength or more) the seismic waves will interact with the fractures and scatter appreciable energy from the direct wave (Pearce, 2003). Observations by previous researchers have demonstrated that scattering effects are sensitive to fracture density in real data as well as numerical and laboratory experiments (e.g., Ata and Michelena, 1995).

Using the 3-D finite difference code developed by Lawrence Berkeley National Laboratory (Nihei et al., 2002), we modeled a simple reservoir geometry consisting of three horizontal layers. The first and third layers bound the reservoir and are homogeneous and isotropic with the same material properties. The second layer is 100-m thick and contains the fractures. Figure 5 shows a schematic of the model with the source and receiver line oriented normal to the fractures, which are parallel and spaced at 50m apart (note: the P wavelength in the model is 100m). Snapshots of the propagating wavefield normal and parallel to the fracture direction are shown in Figure 6. It is easy to see that normal to the fractures the scattered energy from each of the fractures interferes in a way that produces a fairly complex wavefield. In the direction parallel to the fractures, the seismic energy seems to be guided by the aligned fractures and the resulting scattered energy is more coherent and similar to the direct P wave reflection. By generating models with different fracture spacing (10m, 25m, 35m, 50m, and 100m) we have been able to study the effect of spacing on the scattered wavefield. Figure 7 shows the resulting seismograms oriented normal to the fractures for each of the cases (including no fractures), together with a measure of the scattered wave amplitude (sum of

the rectified trace amplitude) as a function of offset. If we look at the wavenumber spectrum of each of those trace amplitude figures, we get the results shown in Figure 8. These spectra (normalized by the case with no fractures) show peaks corresponding to the fracture spacing for the 50m and 100m models. The 35m model shows no peaks because of inadequate spatial sampling in the model. These results indicate that a simple spectral analysis of time windows in the seismic data may provide a measure of the spacing of major fluid-conducting fractures in a reservoir. Details of this method and the results are given in Pearce et al. (2003), which can be found in Appendix C. We will further test this method in the coming year.

Figure 5 Geometry used in the 3-D finite difference modeling. Fractures are represented by discrete features with elastic properties different from the background properties (see Nihei et al., 2003). The fractures are at a constant spacing (50 m in this example).

Figure 6 Snapshots of the propagating wavefield (vertical component at $t=0.45$ seconds) for the model shown in Figure 5. The snapshot on the top is oriented perpendicular to the fractures, while the one on the bottom is parallel to the fractures. The vertical axis in both cases is depth, and the horizontal axis is distance (from Pearce et al., 2003)

Figure 7 Plots of the seismic traces as a function of source-receiver offset (vertical axis is two way travel time) for each of the fracture models for the receiver line oriented normal to the fractures. The models are (top, left to right): no fractures, 100m spacing (0.01 fractures/m), and 50m spacing (0.02 fractures/m); and (third row, left to right): 35m spacing (0.0286 fractures/m), 25m spacing (0.04 fractures/m), and 10m spacing (0.1 fractures/m). The panel below each trace figure is the corresponding plot of integrated, rectified trace amplitude as a function of offset (note: the amplitudes are summed from the blue line down, in order to focus on the scattered wave energy). Note that the variance in the curves is greater for the case of fracture spacing close to the seismic wavelength (i.e., 50m and 100m spacing). (from Pearce et al., 2003).

Figure 8 Wavenumber spectra of the integrated amplitude plots from Figure 7. The top plot is the spectra for the 100m spacing (0.01 fractures/m) case, the middle plot is for 50m spacing (0.02 fractures/m), and the lower plot is for 35m spacing (0.0286 fractures/m). Each spectrum has been normalized by the spectrum for no fractures. Spectral peaks are seen corresponding to the fracture density in the top two plots, while the lower plot has inadequate receiver spacing to see the peak at 0.0286 fractures/m. (from Pearce et al., 2003)

D. Fracture permeability estimation – flow modeling

An important, and challenging, task in the project is to convert fracture information derived from the seismic data into fluid flow properties for use in reservoir simulators. As we reported previously, we have been developing some numerical models of flow in simple fracture networks in 2-D using a finite difference representation of Stoke's flow. An example of the fluid velocity field for a simple case is shown in Figure 9. This type

of numerical approach could be used to estimate the permeability tensor in 3-D for a given fracture network. Another approach described in Brown and Bruhn (1998) uses a statistical modeling approach based on work by Oda (1985). In this model, the fractures are represented as penny shaped features with a known distribution of orientations, sizes, and apertures (Figure 10). The anisotropic elastic properties and permeability tensor of the resulting equivalent medium can be calculated quite easily. The effect of an applied stress field can also be considered. This model has a number of desirable features that make it attractive for this project: it relates the elastic and flow properties of a known distribution of fractures, it includes a way to incorporate stress effects, and it is easy to compute. We have had discussions with Dr. Steve Brown of New England Research about testing this model against other numerical flow models at MIT and at Shell Oil Company to see if the estimated permeability tensor is accurate enough for our needs. This test will take place in the coming year.

E. Borehole dispersion analysis and stress estimation

We are continuing to work on methods of estimating the dispersion characteristics of flexural waves from cross dipole acoustic logs in anisotropic and stressed formations. Accurate dispersion information is the critical input to stress estimation techniques such as Huang et al. (1999).

Figure 9. Flow velocity plot for Stokes' flow through a simple fracture network. The top plot is the pressure field (color) with velocity vectors superimposed. The lower left is the x-component of velocity, the lower right is the y-component (hot colors are positive, cool colors are negative). (from Sarkar et al., 2003 in project mid-year report).

Figure 10. Schematic representation of the Oda (1985) fractured rock model. (from Steve Brown, personal communication, 2003).

IV. PLANNED ACTIVITY FOR THE COMING YEAR

In the coming year we will continue extensive testing of our seismic analysis methods on field data, with particular attention to validation tests of the seismically-derived fracture properties against well and production data. We will also complete a comparison of permeability estimates from the equivalent medium approach (e.g., Oda model) and discrete feature models. The field application test site selection and data transfer will be completed with our industry partner Shell Oil Company.

V. CONCLUSIONS

During the past six months of the project we have focused on improving our numerical modeling codes by incorporating the effect of attenuation, developed an inversion method for estimating fracture orientation and elastic properties from measured amplitude variations with offset and azimuth (AVOA), and tested a method for estimating fracture density variations from measurements of the seismic scattered wave signals related to a fractured reservoir. Following further testing and validation, these methods may provide us with techniques to estimate important fluid-flow related fracture parameters, including fracture orientation, spacing between major, open fracture zones, and fracture density variations within the reservoir.

The first method is an inversion scheme to estimate fracture orientation and fracture density from amplitude variations with offset and azimuth (AVOA). The method has been tested on synthetic data and field data from a fractured offshore carbonate reservoir with promising results. The second method, which has only been tested on synthetic data sets, looks at the spectral characteristics of seismic wavefield scattered from aligned fractures with different spacing between fracture zones. Results indicate that the spacing of these large, open fracture zones can be estimated from the wavenumber spectra of scattered wave amplitude as a function of offset in pre-stack data.

We have identified two approaches for converting seismically derived fracture parameters into fluid-flow parameters for use in reservoir simulators. The first is the numerical modeling of Stoke's flow in fracture networks, and the second uses a statistical model of a fracture distribution. We will compare these two approaches in the coming year. We are also continuing to work on methods of estimating the dispersion characteristics of flexural waves from cross dipole acoustic logs in anisotropic and stressed formations.

Multiple meetings have been held with our industry partner, Shell Oil, to identify a field test site for the project. We are focusing our efforts on a fractured carbonate field. The field application test site selection and data transfer will be completed in the coming year. We have also been testing our methods on seismic data from an offshore fractured carbonate field. Data from multiple wells in the field are also available for validation.

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Finite Difference Modeling of Attenuation and Anisotropy

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Abstract

A finite difference scheme which includes the effects of attenuation and anisotropy is tested for seismic reflection and borehole acoustic models. The validity of the scheme is established using a 3D homogenous isotropic model to compare results to the discrete wavenumber method. Three models are then investigated. First, reflections from a 3D flat layered model are analyzed for offset and azimuthal dependence of attenuation. Second, discrete fractures are included in a 2D flat layered model to examine their effect on reservoir top and bottom reflections. Third, a 3D borehole in both hard and soft formations is modeled to test the effect of attenuation on guided waves.

1 Introduction

Since attenuation is an important property of the subsurface, there is a need for models of seismic wave propagation in heterogenous attenuating media. Although previously available methods, such as the discrete wavenumber method (Bouchon, 1981) can handle attenuation, they are limited in the types of models that can be represented.

For one topic of interest, the characterization of fractured media, common indicators of the presence of fractures include velocity anisotropy, shear wave splitting, and AVO effects. It is less common to use attenuation because it can be difficult to measure, and the relationship between fracture set properties and attenuation is complicated. However, it is clear that fractures have an effect on the attenuation of a medium (Walsh, 1966), suggesting that attenuation information could aid in delineating fractured reservoirs.

Borehole guided waves can be used to estimate formation properties and stress regime (Huang, 2003), but such guided waves are affected by attenuation in the formation and the borehole fluid.

Physical models of attenuation are derived in Walsh (1966) and O'Connell and Budiansky (1977), among others. There have been many papers presenting laboratory attenuation measurements, including Toksöz et al. (1979), Winkler and Nur (1979), and Winkler et al. (1979). Numerical viscoelastic finite difference schemes have been developed, such as Day and Minster (1984) which used a Padé approximation to model the attenuation. The methods of Emmerich and Korn (1987) and Blanch et al. (1995) improve on the accuracy and computational efficiency of the Padé approximation. Carcione (1993) simulates the response of linear isotropic-anelastic media, and Robertsson et al. (1994) contains a detailed study of the stability, accuracy, numerical dispersion, physical dispersion, and computational efficiency of their viscoelastic finite difference scheme.

In an effort to expand the modeling tools available at ERL, we have updated a finite difference code written by Cheng (1994) to model seismic waves in anisotropic, viscoelastic media. Originally written in Fortran, the code has been converted to MPI C to allow it to run large models on a PC cluster. A front end graphic user interface has also been developed to make the code easier to use.

Our focus in this paper is on establishing the validity of the finite difference forward modeling scheme by testing it on various models. The paper has two sections: the first discusses briefly the modeling method and the data processing steps. The second section goes through three applications of the finite difference scheme: a 3D flat reflector model, a 2D flat reflector model with discrete fractures, and a borehole model.

2 Method

2.1 The Forward Modeling method

We use a finite difference algorithm originally developed at ERL (Cheng, 1994). It is 2nd order in time and 4th order in space, and models anisotropy with up to nine elastic constants. The wave equation is formulated in velocity and stress, and discretized on a staggered grid.

The code was originally developed in Fortran, and has been converted to MPI C so that large models can be run on computer clusters. It has also been updated to allow a choice between the boundary conditions of Cerjan et al. (1987) or the original boundary conditions (Higdon, 1986, 1987).

The method of Emmerich and Korn (1987) is used to include attenuation in the calculations, with three relaxation frequencies. The approximation of the attenuation value is shown in Figure 1 for Q values of 100 and 20. The approximation is best in the range from a few Hertz to about 70 Hertz. The reflector finite difference models in this paper use wavelets centered at 30 and 40 Hertz, which are both in the range of good approximation. The borehole model uses a 7 kHz wavelet, with the reference frequencies shifted appropriately to achieve a good approximation of the attenuation value within that frequency band.

Figure 2 shows the wavelet shape after it has traveled 5 wavelengths. Results from two methods are compared: the finite difference result and the solution from the discrete wavenumber method of Bouchon (1981). The approximation for $Q=100$ is quite good; for $Q=20$ it shows more error, which is expected since the attenuation estimation fails at very low and very high frequencies.

The example presented in Section 3.1 was originally run for comparison to discrete wavenumber results, and so uses a Ricker wavelet at 30 Hertz. The example in Section 3.2 uses a 40 Hertz Kelly wavelet. The borehole example in Section 3.3 uses a 7 kHz Kelly wavelet.

2.2 Data Processing

In analyzing the synthetic data from seismic reflection models, the following steps are used:

1. Calculate NMO times, where the NMO velocities take into account dispersion (using the velocity at the source peak frequency) and effective medium properties, if necessary.
2. Time shift to align the wavelets.
3. Window appropriately. We use a rectangular window, with a Blackman window to taper the edges.
4. Zero pad to increase the sampling of the Fourier transform.
5. Calculate the spectrum of the wavelet as the absolute value of the fast Fourier transform.
6. Attenuation estimates are calculated from the interval top and bottom spectra, as discussed in Section 2.3.

2.3 Q Estimation

We use the spectral ratio method to estimate Q , following the notation of Alshammery (1998), with the exception that we use frequency f rather than angular frequency ω . $A_n(f)$ is the amplitude spectrum of a direct arrival at some distance d_n from the source, and $A_1(f)$ is the spectrum of the arrival at some nearer reference distance d_1 . The log of their ratio

$$\delta(f) = \log A_1(f)/A_n(f) \quad (1)$$

can be fit to a straight line

$$\delta_{fit}(f) = mf + \delta_0. \quad (2)$$

The slope m of this line relates to the Q value of the medium, the distance traveled $d_n - d_1$, and the background velocity c as follows:

$$Q = \frac{\pi(d_n - d_1)}{mc} \quad (3)$$

In order to estimate interval Q from reflection data, the compared spectra are those of the reflection events from the top and bottom of the interval. In this case, Alshammery (1998) replaces $(d_n - d_1)/c$ in Equation 3 by Δt , the time spent in the interval by the bottom reflected wave (White, 1992).

$$Q = \frac{\pi \Delta t}{m}. \quad (4)$$

Like Alshammery (1998), we use a ray tracing algorithm to estimate Δt .

3 Synthetic Data Examples

Three different model types are investigated, as listed in Table 1.

3.1 Interval Q estimation

We test the effects of interval Q using a three layer reservoir model, shown in Figure 3. The top and bottom layers are a background material, the middle layer is the reservoir.

We run three variations on the model by adjusting the reservoir Q value Q_{res} , the background Q value Q_o , and also by adding vertically aligned fractures to the reservoir layer. The parameters of the models are listed in Table 1.

To model the presence of fractures, we use an effective velocity model based on the crack compliances of Schoenberg and Douma (1988), as discussed in Krasovec et al. (1998). The model allows for one set of aligned fractures, which we assume to be horizontal and aligned parallel to the y axis. The fracture set is defined by a crack density value, a crack aspect ratio, and the compressibility of the pore fluid, which for this study we assume to be 0.

The effect of the fractures is shown in Figure 4. We use an aspect ratio of .1, which results in a fracture set porosity of 4.2% and a high degree of anisotropy: 20% for the P-wave velocity. Note that since the density is slightly lowered by the fracture set, the effective shear wave velocity in the crack normal direction (angle 0) actually increases slightly.

Figures 5, 6, and 7 show the synthetic data for the three models, as well as the NMO corrected events and their spectra. The trace scales on the wiggle plots and the colorlimits on the spectra plots are the same for each of three figures. Only the receivers located on the x axis (crack normal) are shown. The top reflector shows a polarity change because of the extremity of the change in the elastic constants caused by the fracture set.

The effect of the attenuation can be seen in the amplitudes of the reflected events in Figures 6 and 7. The bottom shear reflection, which is at traveltime 650 msec for the furthest offset trace, is barely visible in Figure 7. The amplitude of the P reflections can be compared in the spectra plots: the top reflector shows a slight decrease in energy due to the attenuation, while the decrease in the bottom reflector energy is more pronounced.

The attenuation also broadens the reflection events. The peak frequency of the spectrum at zero offset for the elastic model is 38.2 Hertz, but 32.9 Hertz for the attenuated model.

Figure 8 shows the interval Q value estimated from the top and bottom reflector spectra. The top plot shows the Q estimation for model B1, where the interval has $Q=20$ and the overburden has $Q=100$; The Q value at zero offset is underestimated, which agrees with the findings of Alshammery (1998), who estimated interval Q values from discrete wavenumber reflection data. The misfit is likely due to the limited bandwidth of the source.

The Q estimation for model B2 is not plotted, as there is no attenuation in the model.

In the bottom plot of Figure 8, results for Model B3 are shown. The values are similar to those found in the isotropic case, except at far offsets where there is a polarity change in the interval top reflector in the crack normal direction. This polarity change causes the spectrum of the wavelet to go to zero, and the Q estimation to be unstable.

3.2 Effective attenuation from discrete fractures

The next example puts discrete fractures into a 2D layered model. Pearce (2003) uses similar models, but runs them in 3D and looks at the dependence of the scattered wavefield on the fracture set characteristics. Our approach is to investigate the effects which discrete fractures have on reflection events.

The discrete fractures are modeled as columns of grid elements which have altered elastic constants (Daley et al., 2002). These columns could be thought of as highly fractured zones.

The model is run in 2D so that the grid can be very finely sampled (.5 meters) and the discrete fractures can have spacing much smaller than the seismic wavelength. We compare the results of regularly spaced to randomly spaced fractures, as well as adding background attenuation to the model. The background material properties are listed in Table 1.

The geometry of the model with irregularly spaced fractures is shown in Figure 9. The bottom plot shows a histogram of the fracture spacings. The spacing has a mean of 2 meters, matching the fracture spacing in the regularly spaced model so that the total number of fractures in the two cases is the same. The fracture spacing was generated by sampling a random process, using a correlation length of 1 so that the space between any two fractures was independent of the neighboring spacings. The spacing values are rounded to the nearest multiple of the grid spacing. This results in a large number of .5 spaced fractures, because anything less than .5 had to be rounded up.

Figure 10 shows the data in the case of no discrete fractures, with and without attenuation. The bright events with zero offset arrival times of 330 ms and 410 ms are the P wave reflections from the top and bottom of the reservoir. The two later events which increase amplitude with offset are the converted wave reflections off the top and bottom of the reservoir. They have times of about 550 and 620 for the furthest offset traces. Other visible events are interbed multiples.

Figure 11 shows the data from the model with regularly spaced discrete fractures. Compared to Figure 10, all events show slightly different amplitudes. In addition, the traveltimes of the reservoir bottom reflections are later because the waves are traveling through the fractures. The reservoir bottom P reflection and the reservoir top S reflector overlap at far offsets.

Figure 12 shows the data in the case of irregularly spacing discrete fractures, with and without attenuation. The reflection events in these gathers are similar to those in Figure 11, but the irregular spacing of the fractures causes low amplitude scattering at later times.

So what we can see in these examples is that the discrete fractures at such a close spacing have two effects. First, both the regular and irregular fracture distributions act as an effective medium, lowering the material velocity in the fractured layer. The second effect happens only for the case with irregular fractures: regions of more sparsely or densely spaced fractures cause scattering.

3.3 Attenuation in an empty borehole

Figure 13 shows the model geometry for the empty borehole models. The borehole radius is .1 meters. The source is centered in the borehole.

Figure 14 shows the synthetic data for the hard formation, with and without attenuation. The P, pseudo-Rayleigh, and Stoneley waves are visible. All three arrivals are still present in the traces from the attenuating model, but high frequencies and the Stoneley wave have been greatly reduced by the attenuation.

The effect of attenuation is more drastic in the soft formation, as the lower velocity means more cycles per unit length. Figure 15 shows the synthetic data for the soft formation, with and without attenuation. The bottom plot has the amplitude scaled by a factor of 4 so that the events are visible. Again the Stoneley wave is greatly reduced by the attenuation.

4 Conclusions

The formulation of Emmerich and Korn (1987) for modeling attenuation in 3D finite difference models works reasonably well for a wide range of models. This will provide an additional tool for fracture characterization, in the form of interval attenuation estimates. It will also allow for more realistic simulations of borehole guided wave applications.

5 Acknowledgments

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A. 3D Layered Models (Figure 3)									
Model	α_{bk}	β_{bk}	ρ_{bk}	α_{res}	β_{res}	ρ_{res}	Q_{bk}	Q_{res}	fracture density
A1	5	2.78	2.7	4.5	2.5	2.6	100	20	0
A2	5	2.78	2.7	4.5	2.5	2.6	10000	10000	0.1
A3	5	2.78	2.7	4.5	2.5	2.6	100	20	0.1

B. 2D Discrete fracture models (Figure 9)									
Model	α_{bk}	β_{bk}	ρ_{bk}	α_{res}	β_{res}	ρ_{res}	Q^*	fracture spacing	
B1	4	2.353	2.3	3	1.765	2.2	10000	no fractures	
B2	4	2.353	2.3	3	1.765	2.2	100	no fractures	
B3	4	2.353	2.3	3	1.765	2.2	10000	regular	
B4	4	2.353	2.3	3	1.765	2.2	100	irregular	
B5	4	2.353	2.3	3	1.765	2.2	10000	regular	
B6	4	2.353	2.3	3	1.765	2.2	100	irregular	

*background and reservoir have the same Q.

C. Empty borehole models (Figure 13)								
Model	α_{fm}	β_{fm}	ρ_{fm}	Q_{fm}	α_{fl}	ρ_{fl}	Q_{fl}	
C1 (hard)	4	2.3	2.3	-	1.5	1.0	-	
C2 (hard)	4	2.3	2.3	40	1.5	1.0	10	
C3 (soft)	2	1.15	1.6	-	1.5	1.0	-	
C4 (soft)	2	1.15	1.6	40	1.5	1.0	10	

Table 1: Model parameters. The indicated figures show the model geometries. Unspecified attenuation values (-) mean a non-attenuating version of the code was used.

Figure 1: Estimation of Q for the finite difference code, used for the flat layered models A1-A3 and B1-B6.

Figure 2: Comparison of finite difference and discrete wavenumber results. The top plot is for a Q value of 100, the bottom for $Q=20$.

Figure 3: Geometry of 3D reservoir models A1-A3. The material properties are listed in Table 1.

Figure 4: Effective velocities due to the presence of aligned vertical fractures in the reservoir layer in Models A2 and A3.

Figure 5: Synthetic data and spectra for Model A1. Model parameters are listed in Table 1. The traces shown are for receivers on the x axis, which is the crack normal direction.

Figure 6: Synthetic data and spectra for Model A2. Model parameters are listed in Table 1. The traces shown are for receivers on the x axis, which is the crack normal direction. The black asterisk on the bottom spectrum plot is at the maximum of the zero offset spectrum, which is at 38.2 Hertz.

Figure 7: Synthetic data and spectra for Model A3. Model parameters are listed in Table 1. The traces shown are for receivers on the x axis, which is the crack normal direction. The black asterisk on the bottom spectrum plot is at the maximum of the zero offset spectrum, which is at 32.9 Hertz, a reduction by 13% from the nonattenuating case.

Figure 8: Interval Q estimations from the data in Figures 5 and 7. The Q estimation is cut off near where the polarity of the reflection reverses, at about offset 425 meters in the azimuth 0 and 30 directions.

Figure 9: Geometry of 2D discrete fracture models B1-B6. Attenuating models have the same value of Q for all materials, as listed in Table 1. The random distribution of discrete fractures (vertical gray lines in the middle layer) is for models B5 and B6.

Figure 10: Synthetic data for models B1 and B2: the 2D reservoir model with no discrete fractures, with and without attenuation.

Figure 11: Synthetic data for models B3 and B4: the 2D reservoir model with regularly spaced discrete fractures, with and without attenuation. The grid spacing is .5 meters, the fracture spacing is 2 meters, and the seismic wavelength is 75 meters. Compared to Figure 10, the regularly spaced fractures act as an effective medium, delaying the reflections from the reservoir bottom. There is no visible scattered wavefield.

Figure 12: Synthetic data for models B5 and B6: the 2D reservoir model with irregularly spaced discrete fractures; the fracture spacing has a mean value of 2 meters. In addition to the reflection events seen in Figures 10 and 11, there is also a scattered field visible at later times. These are waves scattered off regions of tightly or sparsely spaced fractures.

Figure 13: Empty borehole model geometry for models C1 through C4. The white is fluid, the black is formation. The borehole radius is .1 m, and the material properties are given in Table 1. The blue asterisk is the source position, the triangles are the receivers.

Figure 14: Results for models C1 and C2, the empty borehole in the hard formation. The amplitude scale is the same for the two plots. The high frequency guided waves are diminished by the attenuation.

Figure 15: Results for models C3 and C4, the empty borehole in the soft formation. The high frequency guided waves have been attenuated out. The bottom plot has the amplitudes scaled by a factor of four relative to the top plot so that the events will be visible.

Fractured Reservoir Characterization using Azimuthal AVO

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Abstract

Ordinary least squares is used to investigate the ability to detect changes in physical properties using Amplitude Versus Offset (AVO) information collected from seismic data. In order to characterize vertically aligned fractures within a reservoir, this method is extended to Azimuthal AVO (AVOA) analysis. Azimuthal AVO has the potential not only to detect fractured zones, but to spatially describe the fracture strike orientation and changes in fracture or fluid properties. Depending on the data acquisition geometry, signal-to-noise ratio, and extent of fracturing, AVOA analysis can be marginally successful. A study of the robustness and limitations of AVOA analysis is therefore first classified with synthetic data. These methods are then applied to seismic data collected during an Ocean Bottom Cable (OBC) survey over a known fractured reservoir.

1 Introduction

1.1 Objectives

A common problem with geophysical data interpretation involves finding a robust method for clearly extracting subtle features from large, and possibly complicated, datasets. Amplitude versus offset analysis is a fairly well-used technique in the field of exploration seismology, as it has the ability to numerically describe features within the data that are not otherwise apparent. The methods for using AVO as an analysis tool are relatively straightforward, though results can be obscured by the many complications involved with analysis of field data. This case study will investigate the capabilities and limitations of azimuthal AVO as a tool for the detection and characterization of fractured reservoirs.

The fracture model is limited to vertical features that are aligned at a preferred orientation, which is commonly the situation in reservoir settings controlled by regional tectonic stresses. This fracture geometry is referred to as Horizontal Transverse Isotropy (HTI), as the fractures are transversely isotropic with a horizontal axis of symmetry. Extension to AVOA is very appropriate, as it allows for the simultaneous analysis of seismic data collected at different azimuths relative to the fractures. Inversion for AVO parameters related to the fracture orientation and elastic properties will be better constrained through this holistic approach, though a calibration of this method is necessary to understand the ability to resolve individual model parameters in the presence of imperfect data.

As a first order detection tool, AVOA analysis should be able to find the primary orientation of the fractures, as well as some quantification as to the degree of fracturing, which is measured essentially as an anisotropic AVO behavior. A secondary level of analysis should describe spatial differences in the physical properties of the fractures and saturating fluid such as fracture density, aperture, and elastic moduli. The ability to determine these properties is not straightforward, as they are coupled together within the AVO model parameters and can be easily biased.

The promise of using AVO as a reconnaissance/analysis tool with regards to fracture properties is quite attractive, as it is a relatively inexpensive addition to the typical acquisition and processing of seismic data.

Analysis of a 3D dataset also allows for a spatially continuous model of the fractures, which might otherwise only come from sparsely sampled well logs. Because AVO parameters can be related to fracture geometry and rock/fluid elastic properties, this analysis has the potential to infer some information about fluid saturation and flow.

1.2 Background

The underlying concept behind AVO was first generally developed by Knott and Zoeppritz near the beginning of the 20th century. They described the reflection coefficient at an interface as a function of angle of incidence and the elastic properties of upper and lower materials. Simplifications of the general Zoeppritz equations have since been made that are applicable to the relatively subtle changes found in Earth materials at angles of incidence commonly used in exploration seismology. The linear approximations made by [Aki and Richards \(1980\)](#) and [Shuey \(1985\)](#) provide the basis for the AVO model used here. An extension of the Shuey method which describes AVOA for the HTI case was made by [Rüger \(1997\)](#).

There are many examples of techniques used for measuring response of fractured reservoirs using AVO, as well as some other velocity-analysis oriented methods. While much of the work is based on theoretical models, there is also a large amount of AVO analysis performed on field data, particularly in the exploration industry. [Mallick et al. \(1998\)](#) used a method similar to Rüger for describing the azimuthal variations of AVO over an area known to exhibit vertically aligned fractures. A more recent work by [Dong and Davidson \(2003\)](#) addresses the issue of 3D survey geometry and its implications for the adequacy of AVOA analysis.

1.3 Approach

To begin, a synthetic two-layer model is studied in order to understand the estimates and assumptions commonly made with simple least squares inversion methods applied to AVO analysis. The effects of these estimations, as well as geometry and noise considerations, will be taken into account. Building on this knowledge, the synthetic model will be extended to an HTI regime in order to use a similar analysis of the Rüger approximation. These techniques can then be applied to seismic datasets with a better understanding of their limitations. A particular example from an OBC dataset will be used to explore the reliability of this approach. A discussion of the synthetic results compared with the actual data analysis should help to capture the success of these methods, as well as focus the needs for future improvements.

2 Development

2.1 AVO approximation

It is useful to begin with a description of Shuey's form of Aki and Richard's approximation to the Zoeppritz equations.

$$R_{PP}(\Theta) \approx R_P + [A_0 R_P + \frac{\Delta\sigma}{(1-\sigma)^2}] \sin^2 \Theta + \frac{1}{2} \frac{\Delta V_P}{V_P} (\tan^2 \Theta - \sin^2 \Theta) \quad (1)$$

where,

$$R_P = \frac{\rho_2 V_{P2} - \rho_1 V_{P1}}{\rho_2 V_{P2} + \rho_1 V_{P1}} \quad (2)$$

$$A_0 = B_0 - 2(1 + B_0) \frac{1 - 2\sigma}{1 - \sigma} \quad (3)$$

$$B_0 = \frac{\frac{\Delta V_P}{V_P}}{\frac{\Delta V_P}{V_P} + \frac{\Delta \rho}{\rho}} \quad (4)$$

and,

$R_{PP}(\Theta)$ is the angle of incidence-dependent reflectivity

Θ is the incidence angle of an incoming plane wave

$$\sigma = \frac{(\sigma_2 + \sigma_1)}{2} \quad (\sigma \text{ is Poisson's ratio, and can be written in terms of } V_P \text{ and } V_S)$$

$$\Delta\sigma = \sigma_2 - \sigma_1$$

$$\Delta V_P = V_{P2} - V_{P1}$$

$$V_P = \frac{(V_{P2} + V_{P1})}{2}$$

$$\Delta\rho = \rho_2 - \rho_1$$

$$\rho = \frac{(\rho_2 + \rho_1)}{2}$$

Equation 1 can be written as a linear equation in three parts, commonly given in terms of AVO coefficients A, B, and C.

$$R_{PP} \approx A + B \sin^2 \Theta + C(\tan^2 \Theta - \sin^2 \Theta) \quad (5)$$

The A term, equal to R_P , describes the normal incidence ($\Theta = 0$) reflectivity, while the $\sin^2$ term applies to angles of incidence up to about 30 degrees, and the $(\tan^2 - \sin^2)$ term dominates for angles mostly above 30 degrees. This last term is sometimes disregarded in AVO analysis because these angles of incidence are larger than those used in most exploration surveys, though it still can play an important role.

Each term involves a combination of elastic parameters, a fact that will later define the ability to classify material changes through AVO model parameters. Also, note that R_{PP} is meant to represent the actual value of the reflection coefficient in equation 1. This is a difficult quantity to obtain with reflection seismic data; actual reflection amplitudes are often used instead as they are directly proportional to the reflection coefficient. This approximation further degrades the ability to use AVO as a quantification tool with regards to material properties, though careful calibration can be used to give some estimation of them.

2.2 Extension to AVOA in HTI media

Equation 5 describes the reflection amplitude dependence as a function of offset (or angle of incidence) and elastic parameters for an isotropic medium. For the HTI model, a set of vertically aligned fractures embedded in a homogeneous background material, shown in figure 1, is used.

Figure 1: Sketch of HTI model with aligned vertical fractures

These fractures typically have small aspect ratios due to the orientation of local stresses. In the direction of the isotropy axis, these apparently thin fractures have little effect on the elastic properties, and thus reflection amplitudes, compared with the homogeneous case. For ray-paths along the symmetry-axis however, fracture compliance can play a large role in the behavior of reflection amplitudes. This is a manifestation of the rock

and fluid compressibility, density, and fracture geometry. These two directions represent the extremes of the azimuthal AVO behavior, where seismic waves typically propagate more slowly in the fracture normal (symmetry-axis) direction.

An analysis of seismic waves propagating over a 180 degree range of azimuths should therefore sample one complete 'cycle' of this transverse isotropy. The goal of AVOA analysis is to capture the azimuthal variation of reflection amplitudes, which requires data collected at an absolute minimum of three independent azimuths. In practice, wide-azimuth 3D survey methods are required to adequately sample the offset-azimuth space. Reflection amplitudes at a given offset will have an azimuthal dependence that is periodic with respect to the symmetry axis.

Rüger's addition to Shuey's approximation is meant to capture this azimuthal variation in amplitudes. The AVO gradient ('B') term is split into a constant, isotropic term, and an azimuthally varying anisotropic term.

$$B(\phi_k) = B^{iso} + B^{ani} \cos^2(\phi_k - \phi_{sym}) \quad (6)$$

In this equation, ϕ_k represents an arbitrary *known* source to receiver azimuth within the dataset, and ϕ_{sym} represents the *unknown* symmetry-axis orientation. This is a non-linear equation with several terms coupled together. The result of this non-linearity is that there are two possible solutions for B^{ani} . Often, *a-priori* knowledge about the general fracture orientation (or regional stress) can be used to select one of the solutions.

While there will also be an azimuthal variation of the higher angle ('C') term, it will not be developed here. This may be a partial limitation of results shown for larger angles of incidence. Equation 5 can then be written for the azimuthally varying case as:

$$R_{PP} \approx A + [B^{iso} + B^{ani} \cos^2(\phi_k - \phi_{sym})] \sin^2 \Theta \quad (7)$$

2.2.1 Linearization of azimuthal AVO approximation

Equation 7 must be linearized before it can be used in a simple least squares inversion. Taking advantage of the two trigonometric identities,

$$\cos^2(\phi_k - \phi_{sym}) = \frac{1}{2}(1 + \cos(2\phi_k - 2\phi_{sym})) \quad (8)$$

$$\cos(2\phi_k - 2\phi_{sym}) = \cos 2\phi_k \cos 2\phi_{sym} + \sin 2\phi_k \sin 2\phi_{sym} \quad (9)$$

this equation can be put into a linear form.

$$R_{PP} \approx A + (B^{iso} + \frac{1}{2}B^{ani}) \sin^2 \Theta + (\frac{1}{2}B^{ani} \cos 2\phi_{sym}) \cos 2\phi_k \sin^2 \Theta + (\frac{1}{2}B^{ani} \sin 2\phi_{sym}) \sin 2\phi_k \sin^2 \Theta \quad (10)$$

Or, it can be written as a linear equation in four unknowns,

$$R_{PP} \approx C_1 + C_2 \sin^2 \Theta + C_3 \cos 2\phi_k \sin^2 \Theta + C_4 \sin 2\phi_k \sin^2 \Theta \quad (11)$$

which is similar to the approach used by Xu et al. (2001), where

$$\begin{aligned} A &= C_1 \\ B^{ani} &= \pm 2\sqrt{C_3^2 + C_4^2} \\ B^{iso} &= C_2 - \frac{1}{2}B^{ani} \\ \phi_{sym} &= \frac{1}{2} \arctan \frac{C_4}{C_3} \end{aligned}$$

2.3 Application of Least Squares to AVO

An ordinary least squares inversion can now be used with either equations 5 or 11. These equations are put into a simple linear form,

$$\underline{\underline{E}} \tilde{x} + \underline{n} = \underline{y} \quad (12)$$

where the solution for the overdetermined problem that minimizes the data residual norm, $||\underline{n}||^2$, is

$$\tilde{x} = (\underline{\underline{E}}^T \underline{\underline{E}})^{-1} \underline{\underline{E}}^T \underline{y} \quad (13)$$

In these linear equations, $\underline{y}$ represents the data, or reflection amplitudes, $\tilde{x}$ is the estimate of the AVO model, $\underline{n}$ is the data residual, and the matrix $\underline{\underline{E}}$ is made up of the terms that multiply the AVO parameters. For example, $\underline{\underline{E}}$ matrices for the AVO and AVOA inversions have the following form:

$$\underline{\underline{E}}_{AVO} = \begin{pmatrix} 1 & \sin^2 \Theta & (\tan^2 \Theta - \sin^2 \Theta) \\ 1 & \sin^2 \Theta & (\tan^2 \Theta - \sin^2 \Theta) \\ 1 & \sin^2 \Theta & (\tan^2 \Theta - \sin^2 \Theta) \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ 1 & \sin^2 \Theta & (\tan^2 \Theta - \sin^2 \Theta) \end{pmatrix} \quad \underline{\underline{E}}_{AVOA} = \begin{pmatrix} 1 & \sin^2 \Theta & \cos 2\phi_k \sin^2 \Theta & \sin 2\phi_k \sin^2 \Theta \\ 1 & \sin^2 \Theta & \cos 2\phi_k \sin^2 \Theta & \sin 2\phi_k \sin^2 \Theta \\ 1 & \sin^2 \Theta & \cos 2\phi_k \sin^2 \Theta & \sin 2\phi_k \sin^2 \Theta \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \sin^2 \Theta & \cos 2\phi_k \sin^2 \Theta & \sin 2\phi_k \sin^2 \Theta \end{pmatrix}$$

There is one row for every measured reflection amplitude and, typically, the AVO inversion is performed on a single common mid-point (CMP) gather. This results in a linear equation for each trace, where the angle of incidence, Θ , will be different for each offset in the CMP gather at a given depth.

3 Synthetic Analysis

3.1 Ordinary (non-azimuthal) AVO

3.1.1 Description of the model and data

In order to understand the behavior of the least squares analysis, a simple model for the interface between two layers with different elastic parameters is used to create AVO coefficients based on equation 5. This example uses the parameters summarized in table 1. The synthetic AVO coefficients are calculated directly from equations 2 - 4.

From these coefficients, synthetic reflectivity data (R_{PP}) can be calculated over a range of incidence angles commonly found in reflection seismology. The synthetic data can then be contaminated with noise, or partially removed over certain offset ranges, and the ability of the inversion to recover the original coefficients is studied. The modeled behavior for the offset dependant reflectivity is shown in figure 2, where the angles are based on fixed source-to-receiver offsets and a depth to the interface of 800m.

In this figure, the effect of modeling only the A and B terms is clear; there is little deviation below 20°, around 10% at 30°, and then they diverge significantly at larger angles. The three term reflectivity curve will be treated as the 'true' value for the following inversion sensitivity discussions. The effects of changing source-to-receiver geometry is studied using both 2- and 3-term AVO models, as well as their behavior in the presence of noise. For this experiment, random noise with a Gaussian distribution and standard deviation of 0.02 is added to the 'true' reflectivity data. This standard deviation corresponds to about 13% of the mean value of the original model.

	Layer 1	Layer 2
V_P	3000 m/s	4000 m/s
V_S	1732 m/s	2309 m/s
ρ	2300 kg/m ³	2600 kg/m ³
σ	0.25	0.25
AVO-A	0.202	
AVO-B	-0.316	
AVO-C	0.143	

Table 1: Elastic parameters for two-layer AVO model

Figure 2: Modeled AVO for a simple two-layer interface using equation 5 and the parameters in table 1

3.1.2 Synthetic AVO tests

The first case, shown in figure 3, uses data sampled from the 'true' reflectivity curve in one degree intervals from 0° to 45° . Using the high angle of incidence data, it is clear that a 2-term AVO model is incapable of recovering the correct values, even for the noise-free case. The fact that the 2-term model is inappropriate can also be seen in the residual, which shows structure. In the presence of noise, the AVO gradient term has an error of 31% for the 2-term model, compared with only 8% for the 3-term model.

Typically, the source-to-receiver offsets and target depths in a seismic survey do not extend all the way to 45° , and this same experiment is repeated using only data from 0° to 30° . These results are shown in figure 4. With noise-free data, the 2-term AVO model is more accurate than before, but is still unable to recover the true solution. When some noise is added to the reflectivity measurements, however, the 2-term and 3-term results are much more similar. By eliminating the far offset data and adding noise, the 3-term solution becomes much less accurate. Repeating this experiment several times with different noise realizations, one finds that the estimate of the 3-term model is unbiased about the correct answer, but has a fairly large standard deviation. The 2-term model estimate does not have the correct mean, but has much sharper resolution.

Finally, for a more representative approximation of real acquisition geometries, information about the near offsets is also removed, so that the data lies between 10° and 30° . An example of the least squares solution for a single noise realization is shown in figure 5. Repeating this experiment several times, one again finds that the 2- and 3-term AVO solution estimates involve sacrifices between resolution and the accuracy.

3.1.3 Discussion of results

Another useful way to compare the effects of these geometries is through the use of a resolution matrix. Resolution matrices for three of the examples are shown in figure 6. It is clear that as information is removed from the data, the model parameters become more dependent on an broader set of measurements. In the 3-term case, it seems that there is a more dramatic linear dependence of measurements, which is likely the reason why a little noise in the data can produce more spurious results.

These three simulations are useful in understanding the relationship between the AVO model being used, survey geometry, and the presence of noise. Without performing a more rigorous statistical analysis of the results, one might conclude that the 2-term AVO model is robust for use with typical seismic datasets. This reliability does not come for free however, as there is some sacrifice in resolution.

Least Squares Estimates of AVO Parameters for 0° to 45° Data

Figure 3: AVO inversion results for the two-layer model using data from 0° to 45°

Least Squares Estimates of AVO Parameters for 0° to 30° Data

Figure 4: AVO inversion results for the two-layer model using data from 0° to 30°

Figure 5: AVO inversion results for the two-layer model using data from 10° to 30°

Figure 6: Resolution matrices for the least squares method using 2- and 3- term AVO models and three different cases of source-receiver geometry. N.B. Color scales are all the same

3.2 Azimuthal AVO

3.2.1 Description of the model and data

A similar synthetic analysis can be preformed using azimuthal AVO, where equation 7 is used to create the 'true' R_{PP} reflectivity model. Synthetic data is created from this model by decimating the offset-azimuth space to represent a realistic survey geometry, and by adding random noise. The data is then used with equation 11 in a least squares setting to find the azimuthal model parameters A, B^{iso}, B^{ani} , and ϕ_{sym} . Construction of the $\underline{\underline{E}}$ matrix is similar to the previous section, though there are now four columns, with dependence on incidence angle *and* azimuth.

Two synthetic models are investigated using evenly distributed datasets with slightly different geometries (an example of which is shown in figure 7), and a third model will simulate the true 3D survey geometry associated with the OBC data discussed at the end of the report. This analysis will be a good calibration tool for the results obtained when working with the actual field data. In each case, Gaussian noise with standard deviation of 0.05 is added to the 'true' R_{PP} measurements. This standard deviation corresponds to about 33% of the mean value of the noise-free data.

Figure 7: Azimuth-offset geometry used for synthetic AVOA analysis

For consistency with the non-azimuthal AVO analysis, the same elastic model parameters are used, with some slight modifications for the azimuthal case. The AVO-A term is unchanged, as it does not vary with azimuth. The AVO-B term is split into two parts, where 80% of the value used in the previous section is assigned to B^{iso} , and 20% to B^{ani} . An arbitrary fracture symmetry-axis orientation of 35° is used. The 'true' model parameters for the azimuthal analysis are summarized in table 2

A	0.202
B^{iso}	-0.2528
B^{ani}	-0.0632
ϕ_{sym}	35°

Table 2: True AVOA model parameters for two-layer azimuthal AVO model

The 'true' reflectivity behavior, based on equation 7 is shown in figure 8 for a few selected azimuths. In this figure, the azimuthal behavior of the AVO gradient term is apparent. At azimuths closest to the fracture symmetry axis (35°), the greatest curvature is seen (blue and green curves) due to the contribution of the B^{ani} term. The least squares inversion aims to capture this behavior in order to uniquely and accurately determine the four model parameters.

Figure 8: Synthetic AVOA model curves for several azimuths based on the parameters in table 2

3.2.2 Synthetic AVOA tests

The first test of the four-term least squares inversion method is really a validation, as the input data comes from an even distribution of offsets and azimuths. The synthetic reflection amplitudes are sampled at a one degree spacing between 0° and 45° in incidence angle, and at every four degrees between 0° and 180° in azimuth. This dataset is inverted for the four AVOA model parameters using equation 11 and the subsequent relationships between coupled terms. The inversion is repeated with 2000 realizations of the random noise, and the average results are summarized in table 3. Note that the noise-free example has perfectly resolved the model, which should be expected since it is using a linearized form of the same model that created the data. The inversion of the noisy data has been distorted somewhat, but the AVOA parameters have been resolved within reason.

Next, only the data with angles of incidence between 10° and 30° , and azimuths separated by 16° (figure 7), are used in the same inversion process to simulate a more realistic survey geometry. Again, the average results are summarized in table 3. There is clearly a significant loss of accuracy in the inverted model when the offset-azimuth space is this sparse.

Finally, the same method is used with an offset-azimuth distribution that comes from data collected during an OBC survey. This distribution is shown in figure 10 and, though it is dense, it is not as uniformly sampled as the synthetic cases. As can be seen in table 3, this survey geometry has performed somewhere in-between the previous two synthetic models.

parameter	true value	without noise	all offsets/azimuths	sparse offsets/azimuths	OBC geometry
A	0.202	0.202	0.2019	0.2022	0.205
B^{iso}	-0.2528	-0.2528	-0.2517	-0.2378	-0.2609
B^{ani}	-0.0632	-0.0632	-0.0645	-0.0962	-0.0621
ϕ_{sym}	35°	35°	34.8°	57.1°	40.1°

Table 3: Average results of azimuthal AVO inversions using noisy synthetic data for different geometries and 2000 noise realizations

3.2.3 Discussion of results

The four-term azimuthal AVO inversion method seems to have the fundamental ability to resolve azimuthally varying features in seismic data. The combination of survey geometry and noise content are two of the major controlling factors in the success of this approach, and can clearly lead to suprious results. A more rigorous statistical analysis of various geometries and levels of noise would be a very useful tool with regards to the adequacy of AVOA analysis for a specific dataset, data preparation, and interpretation of inversion solutions. For example, an optimal scheme of offset-azimuth selection and trace stacking to suppress noise can be chosen for a specific dataset based on these statistical conclusions. It is also possible that a weighted least squares inversion could be developed based on these results that would be more appropriate for use with AVOA.

4 Field data analysis

In an attempt to test the lessons learned throughout this study, some field seismic data collected during an OBC survey will be analyzed. This survey is known to cover regions of oriented vertically aligned fractures, which will help to test the argument for using AVOA in the detection of HTI features. A comparison of the AVOA inversion results with known fracture parameters will provide a useful calibration for this method.

4.1 Data overview

As already discussed, one of the fundamental requirements for use in AVOA analysis is an acquisition geometry that covers a wide range of offsets and azimuths. A more detailed discussion of the geometry adequacy is addressed in [Dong and Davidson \(2003\)](#). The reason for choosing this OBC survey is that it contains data over a full range of azimuths, with offsets ranging from a few hundred meters to over 4000 meters. The offset-azimuth distribution for an 8-by-8 block of CMPs is shown in figure 9. This block of CMP data will be used in the following AVOA analysis.

Figure 9: Offset-azimuth distribution for all traces that fall within an 8 x 8 super-CMP bin

One of the major problems commonly found with analyzing field data is noise. Because AVO analysis is performed on pre-stack data, it is often difficult to obtain a good signal-to-noise ratio; therefore, the pre-processing steps meant to enhance the signal must also be careful not to distort the natural AVO behavior. For this reason, it is common to analyze more than one CMP at a time, and to stack traces with common offset-azimuth values. While this will have the effect of blurring true amplitudes across offset-azimuth space, it is a necessary step in overcoming the noise problem. Before performing the azimuthal AVO analysis on this super-CMP gather, a few steps are taken to enhance the signal, while regularizing the data space used in the analysis.

4.2 AVOA analysis of field data

There is an obvious clustering of the traces in the block of CMPs shown in figure 9, which is more dense near zero and 180 degrees. The traces are first smoothly discretized in this offset-azimuth space so that the solution is not biased towards azimuths with more data points, and to reduce the overall volume of data. The criteria for discretizing the traces is such that only one trace is accepted in any offset-azimuth bin with dimensions of $1m$ by 30° . The resulting CMP dataset is shown in figure 10. The CMP gather corresponding to this set of traces is shown in figure 11. The reason for the odd appearance at the bottom of the record is due to the Normal Moveout function applied to the data, and is not included in the AVO analysis.

The traces in the super-CMP are now sorted into common-azimuth gathers, which are shown in figure 12. The azimuth bins are each 30 degrees wide, and are annotated with the orientation of the center of each bin. At this point, common offset stacking is applied within the azimuth gathers to enhance the signal. This involves stacking all of the traces within 100m offset groups for each azimuth gather. Besides helping to improve the signal-to-noise ratio, common offset stacking has the benefit of regularizing the offsets that are input into the inversion.

Figure 10: Regularized offset-azimuth distribution for traces in $1m \times 30^\circ$ bins

These azimuth gathers now contain all of the information used as input for the AVOA analysis. Using local velocity information, the time axis can be converted into approximate depths. Along with the known offsets for each trace, approximate angles of incidence can be calculated, though ray-tracing would be required for a more accurate representation. With known angles of incidence and source-to-receiver azimuths, the $\underline{E}$ matrix can be computed with the same procedure as with the synthetic example. For a single time horizon in the gather, amplitudes are collected from all of the traces and are used as the observations in equation 11. The associated resolution matrix for this setup is shown in figure 13, and it reveals that there is a strong interdependence of measurements over several different azimuths.

The least squares inversion will produce a set of values for the four AVOA model parameters at any given time step. Sliding down the trace in incremental steps of 20ms, this analysis is repeated at each time, and the results are collected for the gather between 1 and 3 seconds. A log-type display of the four parameters versus time is shown in figure 14.

Figure 11: CMP display of the traces to be used in AVOA analysis

Figure 12: CMP traces sorted into 30 degree azimuth gathers and stacked on common offsets

Figure 13: Resolution matrix for the AVOA inversion used on data collected in azimuth gathers

4.3 Discussion of the field data inversion

While this solution represents only a tiny subset of the OBC data, and uses a simple criteria for data selection and stacking, there are some promising results. Probably the most outstanding feature is the alignment of the symmetry-axis orientation near zero degrees between 1.7 and 2.2 seconds. This solution corresponds fairly well with the time and orientation of a known fractured zone. The fact that the solution for ϕ_{sym} is much more focused in this window than it is for earlier or later times suggests that the inversion is capable of resolving features within the data. It is difficult to comment on the behavior of the other parameters however, as there are no other similarly striking trends. This may be related to several factors; including the method of data selection/stacking, the non-uniqueness of the B^{ani} solution (which was always forced to be positive for this example), and estimation of the incidence angle based on average velocities.

Further effort towards improving the inversion method is certainly justified, and should help to reveal more information about the other model parameters. The B^{ani} term should also be well correlated with fracture zones, as it captures the difference in AVO behavior between the fracture-normal and fracture-parallel orientations. Isotropic zones should therefore have a small B^{ani} , while fractured regions should have proportionally larger values. B^{iso} will be key in describing the actual differences in material properties at an interface, but is more subtle, and has the draw-back of being coupled to the non-unique solution of the anisotropic term. Finally, one would like to see a decrease in the RMS of the data residual in areas where strong fracturing produces a good fit to the inversion. The reason for the general increase in this RMS curve after 2.2 seconds is likely due to the increased noise in the data at depth. This does not necessarily imply a poor solution, but the probability of a good fit is certainly decreased.

Figure 14: Azimuthal AVO parameter estimates at multiple depths for the OBC super-CMP dataset

5 Conclusions

5.1 AVOA as a tool for fracture characterization

This study has attempted to show the limiting factors that play a role in the ability to resolve fracture information using azimuthal AVO with typical datasets. The synthetic and field data analysis presented here supports the use of AVOA for fracture characterization, though this tool should be considered on two levels. The first of these levels is the ability to detect and spatially map oriented fractures, and to describe basic features such as strike angle and degree of anisotropy. A second, more involved, level is to use the AVOA information to conclude something about the physical properties of the fractures and saturating fluid.

It is important to take into account the limitations and assumptions made with this method however, as the results can be biased significantly due to geometric and noise-related factors. This approach was directed entirely towards describing HTI-like features, which may not be appropriate in all cases. Azimuthal analysis of field data without sufficient constraining information can result in a non-unique solution, and should be interpreted as such.

5.2 Future improvements

There are a few areas of improvement to be addressed that may significantly improve this azimuthal AVO inversion method, providing a much stronger fracture characterization tool. A few of these include: better estimates of the incidence angle with depth, *a-priori* constraints on the values of ϕ_{sym} and sign of B^{ani} , statistical analysis of the inversion behavior with various geometries and levels of noise (possibly leading to a weighted inversion), optimization of the data discretization/stacking method, and extension of the model beyond HTI. Analysis of numerical model output with controlled fracture properties should also prove useful as a calibration tool for this technique.

6 Acknowledgements

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APPENDIX C

Fracture Density Estimation Using Spectral Analysis of Reservoir Reflections: A Numerical Modeling Approach

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Abstract

We use a 3-D finite difference numerical model to generate synthetic seismograms from a simple fractured reservoir containing evenly spaced, discrete, vertical fractures. The fractures are represented using a single column of anisotropic grid points. Analysis of seismic amplitudes and spectral characteristics were carried out on the top and base reservoir reflections as well as scattered wave coda for models with fracture spacing ranging from 0.01 to 0.1 fractures/m. Results show that the bulk scattered wave energy contained in a common shot gather increases greatly when the fracture spacing is greater than about $1/4$ wavelength. Wavenumber spectra for integrated amplitude versus offset from a time window containing the base reservoir reflector show spectral peaks corresponding to the fracture spacing.

1. Introduction

Fractures are common within the subsurface and play a critical role in the mechanical and fluid flow properties of earth materials. In regions where the maximum compressive stress is vertical, open fractures will tend to be oriented vertically. The ability to interpret properties such as fracture spacing, orientation, and fluid content associated with such fracture systems is vital to the effective extraction and management of reservoir resources and the monitoring of contaminant migration. In particular, the ability to delineate zones of high fracture density is a key component in developing reservoir-scale fluid flow models.

The way in which fractures affect seismic waves depends on mechanical parameters, such as compliance and saturating fluid, and on their geometric properties, such as dimensions and spacing. If fractures are small relative to the wavelength of the seismic waves, the waves will be only weakly affected by the fractures. If fractures have characteristic lengths on the order of the wavelength then there will be scattering of energy due to the presence of the fractures. If fractures are spaced much closer than the wavelength of the seismic waves, then the medium will behave like a homogeneous, anisotropic equivalent medium. When fractures are spaced further away (about a quarter of a wavelength or more) the seismic waves will interact with the fractures and scatter appreciable energy from the wavefront.

Observations by previous researchers have demonstrated that scattering effects are sensitive to fracture density in real data as well as numerical and laboratory experiments (Ata and Michelena, 1995 and Rathore et al., 1994). In addition, numerical modeling of fractured media has proven to be an efficient way to explore the feasibility of using seismic waves to estimate fracture parameters. In particular, the applicability of finite difference methods to simulate wave propagation in a fractured elastic medium has been well documented in the literature (Shen and Toksoz, 2000, Daley et al. 2002, Nihei et al., 2002, Wu et al. 2002, Groenenboom and Falk, 1999, and Nakagawa et al. 2002).

Most previous work in estimating fracture parameters from seismic reflection data has focused on using an equivalent medium approach where the fractures themselves or the spacing between the fractures is assumed to be small relative to the wavelength of the seismic wave. In this paper, we explore the characteristics of reflected and scattered elastic waves that may be used to develop seismic attributes sensitive to fracture spacing. A 3D finite difference method is used to simulate wave propagation through a simple fractured reservoir which is modeled as a single horizontal layer containing parallel, evenly-spaced fractures. Previous work has shown that the finite difference method is accurate over a wide range of scattering regimes and can account for all converted waves and

their interactions (Shen et al., 2002). Synthetic seismograms are generated along transects perpendicular and parallel to the fracture orientation and spectral characteristics of the wavefield scattered from the fracture zones are analyzed. Our objective is to investigate how the intensity of seismic scattering changes with the spacing of discrete fractures, and develop potentially diagnostic seismic attributes sensitive to the fracture spacing. Our analysis focuses on the primary P-wave reflections from the top and bottom of the reservoir, as well as the scattered wave coda.

2. Background

The importance of fractures in the production and stimulation of reservoirs has resulted in considerable efforts to develop methods for fracture detection and characterization using seismic waves. In particular, offset dependent attributes are useful in characterizing vertical fractures because of the variation in incidence angle of waves impinging on the fractures. Much effort has focused on measuring amplitude variations with offset (AVO) for seismic reflections from a fractured reservoir (Gray et al., 2002, Perez et al., 1999, and Shen et al., 2002). Amplitude variation with offset and azimuth has been widely used to identify the orientation of vertical fractures with variable levels of success (Ruger, 1998; Mallick et al., 1998; Perez et al., 1999; Shen and Toksoz, 2000; Minsley et al., 2003). In these applications, the fractures are assumed to be small relative to the wavelength of the seismic waves. This allows the fractured medium to be modeled using an equivalent anisotropic medium (Schoenberg and Douma, 1988). Others have looked at shear wave data for fracture analysis (Lynn et al., 1995; Gaiser et al., 2002).

Shen et al. (2002) used numerical experiments and field data from a fractured reservoir in Venezuela to demonstrate the advantages of using attributes based on frequency variations with offset and azimuth (FVOA) in addition to AVOA based attributes to further constrain the interpretation of parameters related to the presence of vertically aligned fractures.

Recently, researchers have begun to focus on relaxing the assumption of relatively small fracture spacing by incorporating the effects of discrete fractures. Shen and Toksoz (2000) used stochastic models to generate heterogeneous fracture density distributions. They examined the effect of such fracture density variations using both time-domain and frequency-domain attributes and found that the presence of the fractures was more easily identified in the attributes associated with the base reflector. They concluded that first-order effects of fractures on FVO and AVO were controlled by the mean fracture density used in their stochastic fracture representation; however, they did not investigate systematic variations in fracture spacing. Daley et al. (2002) conducted 2D finite difference modeling of discrete fracture zones aimed at identifying scattering mechanisms and resolvability issues. They concluded that fracture tip diffractions and P-to-S conversions are the dominant scattered events from discrete fracture zones observable in surface seismic data, and identified fracture stiffness, spacing, and spatial scale as important parameters in controlling the resolvability of discrete fracture zones. Although their FD modeling and fracture representation are the same as what we use here, they do not specifically address the effect of fracture density on the development of the scattering wavefield.

Nakagawa et al. (2002) developed a hybrid numerical method that combines the plane wave method and the finite element method in order to examine the characteristics of the scattered wavefield from a layer containing discrete, vertical, evenly spaced fractures. However, their paper is more concerned with the presentation of the model and doesn't address variations in background velocities between fractured and unfractured layers, and doesn't vary the fracture spacing with the intention of developing seismic attributes sensitive to fracture spacing.

3. Modeling

As discussed in the previous section, a great deal of research has focused on the applicability of finite difference methods to modeling seismic scattering phenomena. In particular, Nihei et al. (2002) test the performance of the Coates and Schoenberg (1995) equivalent medium anisotropic cell approach for FD modeling of discrete fractures. The finite difference method is advantageous when modeling the scattering effect of discrete fractures because of its stability over a wide range of material property contrasts and its ability to model all wave types with minimal numerical dispersion and anisotropy. In addition, synthetic seismograms can be generated at any point within the model domain facilitating the simulation of surface seismic acquisition geometries.

We use the time domain, staggered grid finite difference code developed by Lawrence Berkeley National Laboratory (Nihei et al., 2002), which solves the 3-D, anisotropic stress-velocity formulation of the elastic wave

equation (Levander, 1988). This formulation does not include intrinsic attenuation. The finite difference operator is 4th order accurate in space and 2nd order accurate in time.

Our numerical experiments use a simple reservoir geometry consisting of three horizontal layers. The first and third layer bound the reservoir and are homogeneous and isotropic with the same material properties. The second layer is 100-m thick and is where the fractures are placed. Figure 1 shows the model geometry for a fracture density of .02-frac/m and the background velocities used for each layer. Although absorbing boundary conditions are incorporated into the model, the boundaries are also placed far enough away so that any boundary reflections are not recorded in the seismograms during the time interval of interest.

The model uses a standard surface seismic reflection acquisition geometry, however, the source and receivers are embedded within the first layer to eliminate free surface effects. Receivers are placed in the crack normal and crack parallel directions with the first receiver fixed at the source. The receiver spacing is 20-m with the maximum source-receiver offset at 800-m which corresponds to a maximum angle of incidence of 45° relative to the top of the reservoir. A pressure source is used with a Ricker wavelet having a center frequency of 40-Hz. The model uses a uniform grid spacing of 5-m and a time step of .5-msec.

The fractures begin at a distance in the x-direction of 5-m from the edge of the grid and are then placed in even increments across the reservoir depending on the fracture spacing value. The source is also fixed at a point above the reservoir corresponding to a distance in the x-direction of 550-m from the edge of the model domain. As a result, the actual locations of fractures relative to receivers are different for models with different fracture densities.

In practice, we incorporate discrete vertical fracture zones into the reservoir layer using a single column of anisotropic grid points. The elastic properties of the discrete fractures are calculated using the Coates and Schoenberg (1995) thin anisotropic grid cell approach. The fractures are assumed to have negligible mass and thickness relative to the seismic wavelength in accordance with the linear slip model of Schoenberg (1980) for imperfect interfaces. The fractures are given equal normal and tangential stiffness of 8×10^8 -Pa/m. We assume fracture zones are bed terminated and have constant fracture density across the reservoir for each of our simulations.

4. Model Results

We generated synthetic data using the finite difference code and model geometry presented in the previous section. Synthetic seismograms were produced for the no fracture case as well as for five different fracture density values: 0.01, 0.02, 0.0286, 0.04, and 0.1-frac/m. For the purposes of describing the model output, we will focus on two time windows of the time series. The first travel time window is bounded by the two coherent P-wave reflections and the second window contains all data after the P-wave reflection from the base of the reservoir.

4.1 Receiver Array Normal to Fractures

Figure 2 shows the vertical component of velocity recorded at each receiver position normal to the fractures for each fracture density case. The no fracture case is useful for identifying the main P- and S-wave reflections from the top and bottom of the reservoir. The first time window contains only the coherent P-wave reflections. However, careful observation shows a third event separates from the 2nd P-wave reflection at large offsets. The event is associated with the downgoing P-wave reflected as an S-wave at the base of the reservoir, and then converted back to a P-wave upon leaving the reservoir. Because of this conversion, the event has a slightly slower move-out velocity than the primary P-wave reflection from the base of the reservoir.

The dominant events in the second time window are the S-wave reflections, which show rapid increases in amplitude as offset increases because we are looking at only vertical particle velocities from our receivers. The S-wave reflection from the top of the reservoir has a relatively large amplitude while the S-wave from the base of the reservoir has a much smaller amplitude at least partially due to the additional conversions to P-waves as seen in the first time domain.

As we begin to incorporate fractures into the reservoir, we see a drastic change in the character of the time series. For the .01-fractures/meter case, the first time window shows the P-wave reflection from the top of the reservoir remains relatively unaffected while the P-wave reflection from the base of the reservoir shows significant interference from scattered events starting at about 400-m offset. The curvature of the second P-wave reflection increases owing to a reduced effective velocity in the reservoir due to the presence of the fractures.

In the second time window, we see that both S-wave reflections have become difficult to follow. The S-wave reflection from the top of the reservoir is affected by interference from scattered waves and shows better

coherence at larger offsets where its amplitude is large. However, the S-wave reflection from the base of the reservoir is completely drowned out by scattered events due to the presence of the fractures. Overall, the second time window is dominated by many different events derived from interactions with the fractures. Close observation shows these events seem to focus at offsets between 200-m and 600-m and at travel times after the S-wave reflection from the top of the reservoir.

As we increase the fracture density to .02-fractures/meter, the same trends are observed. In the first time window, the P-wave reflection from the top of the reservoir shows an appreciable decrease in amplitude with offset. The P-wave reflection from the base of the reservoir becomes more incoherent at smaller offsets and is more difficult to follow from receiver to receiver. The decrease in amplitude of both P-wave reflections can be attributed to a decrease in the impedance contrast between the two layers and to interference from scattered waves with shorter travel paths.

In the second time window, the S-wave reflection from the top of the reservoir is only observable at very large offsets and the S-wave reflection from the base of the reservoir is absent. The scattered events have a very similar appearance to those for the .01-fractures/meter case except there doesn't seem to be a focus of scattered energy in the mid-offset receivers.

For the .0283-fractures/meter case, the first time window shows the P-wave reflection from the top of the reservoir is being interfered with at much smaller offsets (~400-m). In addition, we observe a large increase in the amplitude of events at large offsets within the first time window. These observations suggest shorter travel paths and larger amplitudes for these scattered events.

The second time window contains a relatively large amplitude S-wave reflection from the top of the reservoir whose amplitude reaches a maximum between 400-m and 500-m offsets. In fact, scattered events tend to have large amplitudes and arrive at earlier times than the previous cases. Again, these observations support shorter travel paths and larger amplitudes for scattered events.

The .04-fractures/meter case shows a sudden change in the character of the time series. In the first time window, the P-wave reflection from the top of the reservoir is still interfered with at larger offsets but the magnitude of the interference is smaller. The P-wave reflection from the base of the reservoir becomes much more coherent; however, it is now interfered with by an event that arrives at almost the same time even at zero offset and has a very similar move-out behavior. In general, we can now clearly observe that the apparent velocity of the P-wave reflection from the base of the reservoir has decreased substantially from the case with no fractures.

For the second time window, we see that the S-wave reflection from the top of the reservoir is clear and its amplitude increases with offset. The amplitude of the scattered events has decreased substantially and there are patches within the time series where no noticeable scattered energy is present. However, the weak scattered events that remain still show the linear move-out trend observed in the previous fracture density cases.

The .1-fractures/meter case, we see the almost the same behavior as the no fracture case. In the first time window, we only observe the P-wave reflections from the top and bottom of the reservoir. The amplitude of the P-wave from the top of the reservoir decreases more rapidly with offset than the no fracture case. The move-out velocity of the P-wave reflection from the bottom of the reservoir is much smaller. In the second time window, the S-wave reflection from the top of the reservoir is larger in amplitude. Almost no scattered energy is present within either time window. These observations are consistent with the equivalent medium representation that is applicable when fracture spacing is much smaller than the wavelength of the seismic waves.

4.2 Receiver Array Parallel to Fractures

The vertical component of velocity for receivers parallel to the fractures can be observed for each fracture density case in Figure 3. As we expect, the no fracture case has identical time series data as obtained from receivers in the normal direction. For the .01-fractures/meter case, we observe very little change in the first time window. Both P-wave reflections are coherent and the apparent velocity of the P-wave reflection from the base of the reservoir is slightly slower than the no fracture case.

For the second time window, the S-wave reflection from the top of the reservoir is observable at only large offsets. The scattered events are much more coherent from receiver to receiver and seem to have move-out trends similar to the P-wave reflections. In addition, the scattered events have smaller amplitudes than those obtained from the receivers normal to the fractures.

The .02-fracture/meter case shows similar trends to the .01-fractures-meter case. The P-wave reflection from the top of the reservoir is coherent across all offsets. However, the P-wave reflection from the base of the reservoir is difficult to follow at large offsets and has significant interference from scattered events at small offsets.

The S-wave reflection from the top of the reservoir can only be observed at large offsets. Again, we see that the scattered events have move-out trends similar to the P-wave reflections.

For the .0283-fractures/meter case, we again see that the P-wave reflection from the top of the reservoir is unaffected. However, the P-wave reflection from the base of the reservoir loses coherence at an offset of about 400-m. The S-wave reflections can not be distinguished; however, there is a focusing of large amplitude scattered energy in around the time interval of the S-wave reflections. In general this scattered energy has a similar move-out trend but is more concentrated particularly at small offsets.

In the .04-fractures/meter case, both P-wave reflections are very coherent and there is little scattered energy present. In general the .04-fractures/meter case is similar to the .1-fractures/meter case with a couple of exceptions. The .04-fractures/meter case has some small interference of the P-wave reflection from the base of the reservoir as compared to the .1-fractures/meter case. Also, the .04-fractures/meter case has enough residual scattered energy to blur the S-wave reflection from the base of the reservoir while this event is clear in the .1-fractures/meter case.

4.3 Snapshots

Snapshots of the wavefield in the presence of discrete fracture zones highlight the different scattering events and their interactions. Figure 4 shows the vertical component of velocity after .3-seconds for a fracture density of .02-fractures/meter. We see that the P-wave reflection from the top of the reservoir remains relatively coherent, while the P-wave reflection from the base of the reservoir is much more effected by the discrete fractures. This figure also illustrates the difference in coherence of the scattered wavefield normal and parallel to the fractures. Normal to the fractures (Figure 4a), interference of scattered waves from the fractures results in a complex wavefield. Fractures act as secondary sources and multiple scattering effects can be seen. Parallel to the fractures the wavefield is much more coherent. The fractures appear to act more as a waveguide.

5. Analysis

The purpose of this section is to present methods developed to quantify the effect of variations in fracture density in our models.

5.1 Velocity Anisotropy

In general, aligned fractures will result in velocity anisotropy, which is often used as a measure of fracture intensity. Therefore, we examined the effect of variable fracture density on the normal move-out (NMO) velocity estimated by following the P-wave reflections as a function of offset (note: although our models are generated for a single source position with multiple receivers, and therefore are common shot gathers, we treat them as if they were a CMP gather). In our simple model, the normal move-out (NMO) velocity of the upper layer will simply be the P-wave velocity specified for the upper layer, 3000-m/s. However, energy reflecting from the base of the reservoir will pass through both layers and potentially fractures as well. Therefore, the NMO velocity of the P-wave reflection from the base of the reservoir should depend on the fracture density and this effect should be strongest when measured normal to the fractures.

To estimate the NMO velocity, we pick the closest peak in amplitude associated with the P-wave reflection from the base of the reservoir at every offset and use the hyperbolic move-out equation to fit the time values of these peaks using a linear least squares regression. Figure 5a shows the best-fit velocity curve for the .02-fractures/meter case and shows appreciable variation in the peak amplitudes about the best-fit model.

Using this approach, we calculated NMO velocities parallel and perpendicular to the fractures for each fracture density case (Figure 5b). In the parallel direction, the NMO velocities remain relatively constant although slight variations exist at fracture density values between .01-fractures/meter and .0283-fractures/meter. Conversely, the NMO velocities in the normal direction decrease as the fracture density increases and again we see irregular fluctuations in NMO velocity for the fracture density values between .01-fractures/meter and .0283-fractures/meter.

As we pointed out in the previous section, the presence of discrete fractures in our model affects the coherence of the P-wave reflections because of interference from scattered events. Therefore, as our observations indicate, we expect to see the greatest variability in NMO velocity for fracture density values associated with the greatest amount of scattering. This is important because using velocity anisotropy alone to estimate the fracture density associated with large discrete fractures may lead to erroneous results.

5.2 Bulk Scattering

In addition to velocity anisotropy, it is also important to quantify the amount of scattering present in the different fracture density cases. To calculate the amount of scattered energy in each time series output, we select a time window that incorporates as much of the data as possible (Figure 5c). We then take the absolute value of the windowed amplitudes and sum them to get a single integrated amplitude value. This procedure is done for the normal and parallel directions and for each fracture density value.

Figure 5d shows the resulting integrated amplitude values as a function of fracture density and orientation. The shapes of the integrated amplitude curves are nearly identical in the normal and parallel case. However, the normal case has much larger integrated amplitude values in the fracture density range between .01-fractures/meter and .0283-fractures/meter. The integrated amplitude values rise sharply as you move away from the no fracture case, reach a peak at the .0283-fractures/meter case, and decline more gradually towards the .1-fractures/meter case. The fracture density of .0283-fractures/meter obtains a value almost three times that of the case with no fractures in the normal direction. The .1-fractures/meter case falls to the no fracture case in the normal direction and actually falls below the no fracture case in the parallel direction, due to a change in AVO response.

5.3 Spectral Analysis of Integrated Amplitude vs. Offset

We have demonstrated that the strongest scattering of seismic energy occurs for the fracture cases between .01-fractures/meter and .0283-fractures/meter. Now we will extend the integrated amplitude approach to develop an offset dependent attribute sensitive to fracture spacing within this strong scattering regime. We will focus on the P-wave reflection from the base of the reservoir and construct a robust window to capture this event and scattered events with similar arrival times. As we have shown, the P-wave reflection from the base of the reservoir is difficult to follow because of interference from scattered events. For this reason, we use a relatively large window based upon the NMO velocity of the P-wave reflection from the top of the reservoir. However, the P-wave reflection from the top of the reservoir does not sample the fractures directly and so will be excluded from the window. The same window is used for all fracture density cases making it robust in its application. Amplitude values within the window are rectified and then summed to give a single integrated amplitude value for each offset.

Figure 6 shows the plots of integrated amplitude vs. offset for each of the fracture density values with receivers normal to the fractures. In the absence of strong scattering, as in the no fracture case, we see that the curves show a smooth line dominated by the AVO response of the P-wave reflection from the base of the reservoir along with secondary effects from P to S conversions as described in the Data section. For the fracture density range associated with strong scattering, we also observe an AVO trend in the integrated amplitudes with offset; however, there are also periodic fluctuations about the AVO trend. These fluctuations are the greatest for the .01-fractures/meter and .02-fractures/meter case and are substantially less in the .0283-fractures/meter case.

Figure 7 displays the same integrated amplitude vs. offset plots parallel to the fractures. We see the same trends in these plots but they lack the oscillations about the AVO trend. The difference between the normal and parallel cases is consistent with our earlier observation that scattered events in the parallel case tend to have move-out trends similar to the primary reflections. Therefore, we conclude that the interference patterns observed in the integrated amplitude vs. offset from the normal direction may have a direct correlation with fracture density.

To test this hypothesis, we transform the integrated amplitude vs. offset data in Figure 6 to the amplitude vs. wavenumber domain using the Fast Fourier Transform (FFT). Since the resulting spectra are dominated by the AVO trend in the data, we remove this dominant trend by normalize the spectra. If the method is to be robust, the fracture density estimate must be insensitive to the spectra used to normalize. To address this issue, we normalize the spectrum obtained from each of the strong scattering fracture density values (i.e. .01-, .02-, and .0286-fractures/meter) with each of the spectra obtained from the weak scattering regime (Figure 8). Several important observations can be made from the resulting normalized spectra:

- 1) Peaks in the normalized spectra correspond to the input fracture density values for the .01-fractures/meter and .02-fractures/meter cases.
- 2) The ability to resolve the .0283-fractures/meter spacing is beyond the resolution of the analysis, which is determined by the receiver spacing.

- 3) The method is robust with respect to the spectrum used to remove the dominant trend and gives accurate estimates even when the .04-fracture/meter spectrum is used.

6. Discussion and Conclusions

We used a finite difference code to model scattering effects from discrete, vertical fractures with variable fracture spacing. Observations from vertical velocity time series show that scattering along receiver transects normal to fractures contain more complex wave patterns while receiver transects parallel to fractures tend to only have hyperbolic move-outs similar to the primary P-wave reflections. This implies that stacking data obtained normal to fractures may remove valuable information regarding fracture parameters if those fractures behave as scatterers.

Analysis of NMO velocities associated with the P-wave reflection from the base of the reservoir showed that using velocity anisotropy to estimate fracture intensity might lead to erroneous results when fractures strongly scatter seismic waves. For our numerical experiment, this strong scattering regime occurs between fracture densities of .01-fractures/meter and .0283 fractures/meter. Given the background velocities in our reservoir, this strong scattering regime corresponds to spacing between the fractures of about a P-wavelength to about half an S-wavelength. This observation agrees with well-established theory on scattering of waves by heterogeneities.

We use a robust window about the P-wave reflection from the base of the reservoir to calculate integrated amplitude values as a function of offset. When receivers are normal to the fractures, we observe periodic fluctuations about the general AVO trend within the strong scattering regime. Transforming this information to the wavenumber domain yield peaks in the spectra that correspond to the fracture density of the model, which can be easily observed when the general trend is removed. The peaks in the spectra are not sensitive to the model used to remove the general trend.

However, two important issues in using this method were identified when attempting to resolve the fracture density of .0283-fractures/meter. The spatial resolution of this method is controlled by the spacing of the receivers. For a specified maximum fracture density to be resolved, the receiver spacing must be at least half of that maximum fracture density as governed by the Nyquist criterion.

Another consideration is that the amplitudes of the fluctuations in the .0283-fractures/meter case are much smaller than in the other two cases that were successful. This may be due to the window of the data not capturing enough of the scattered energy responsible for the interference patterns that lead to the fracture density estimation. In fact, this observation can be somewhat validated by examining the time series for the .0283-fractures/meter case, which show a lack of almost any events within the time window used for this analysis. Instead, this case shows strong scattered energy just below the window used in this analysis. Therefore, the optimal window used for this method may depend on the fracture density and may require additional constraints.

We have focused our more detailed spectral analysis on the time window around the P-wave reflection from the base of the reservoir. However, we have shown that a great deal of energy is scattered into events that arrive significantly later than the primary P-wave reflections. As we have stated, the time series data normal to the fractures have many different scattered events with different apparent NMO velocities while the time series parallel to the fractures have move-out trends that follow the primary reflections. Therefore, examining scattering derived coda using the two-dimensional FFT may show some relationship between apparent velocity and peak amplitude wavenumber or frequency that relate to fracture spacing. In addition, time shifting the windowed data used in the two-dimensional FFT using an NMO velocity can separate the spectra of individual events with different move-out trends. An example of two-dimensional FFT output for the scattered wave coda from the four fracture cases with the strongest scattering are shown in Figure 9.

In this paper we have only investigated the effect of variations in the spacing between discrete fractures. The effect of fracture stiffness, fracture zone thickness (in this study the fracture zones were set to the grid size of 5 meters), heterogeneous fracture spacing, and reservoir thickness on the scattered wavefield should also be investigated. In addition, we have only focused on one component of the wavefield. It would be beneficial to look at horizontal components as well in order to investigate the relative strength of P and S wave scattering.

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Model Geometry

Figure 1 – The model geometry for the .02-fractures/meter case showing the source and receiver locations, fracture locations, and velocity and density parameters

Figure 2 – Vertical component of velocity normal to fractures for each fracture density case

Figure 3 – Vertical component of velocity parallel to fractures for each fracture density case

Figure 4 – Snapshots of the vertical component of velocity in the normal (a) and parallel directions (b) at a time of .3-seconds

Figure 5 – a) shows the traces for the .02-fractures/meter case where the bold line represents the best fit NMO velocity, b) plot with the best fit NMO velocity for each fracture density in the normal and parallel directions, c) shows the windowing used to calculate integrated amplitudes summed across all offsets in d) for the normal and parallel case

Figure 6 – Integrated amplitude as a function of offset for receivers normal to fractures. Blue lines show window used in calculations.

Figure 7 – Integrated amplitude as a function of offset for receivers parallel to fractures. Blue lines show window used in calculations.

Figure 8 – Spectra of integrated amplitude vs. offset curves in Figure 6. a) Spectra used for normalizing (from left to right) - no fracture spectrum, .1-fractures/meter spectrum, and .04-fractures/meter spectrum. b) Normalized spectra for the .01-fractures/meter case, c) Normalized spectra for the .02-fractures/meter case, and d) Normalized spectra for the .0283-fractures/meter case

Figure 9 – Two-dimensional FFT output generated using NMO corrected time series data from the window enclosed by blue lines for a) fracture density of .01-fractures/meter, b) fracture density of .02-fractures/meter, c) fracture density of .0283-fractures/meter, and d) fracture density of .04-fractures/meter.