Difference between BPM Reading One Bunch and the Average of Multi-Bunches in Booster

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Abstract

Differences caused by BPM reading one bunch and multi-bunch average need to be well understood before the beam parameters, such as the synchrotron tune, betatron tune, and chromaticity, are extracted from those BPM data. It is easy to perform such a study using numerical simulation other than modifying the BPM electronics.

Introduction

The BPM system in Booster can read the average of about 15 bunches due to the electronic limitation.[1] The difference between BPM reading one bunch or the average of fifteen bunches needs to be well understood before the BPM turn-by-turn data are applied for the synchrotron and betatron tune measurement via either the model independent analysis (MIA) or the fast Fourier transform (FFT) of a single BPM reading.[2,3] Situations of BPM reading one bunch and the average of five, ten, and fifteen bunches are simulated using MATLAB when either synchrotron and betatron modes are excited or only betatron modes are excited.

Method

The n^{th} BPM reading at the m^{th} turn ($x_n(m)$) can be written as the summation of three different terms, as shown in eq.1.[2,3]

$$x_n(m) = x_{co} + x_D(m) + x_\beta(m). \quad (1)$$

Eq. 1 was used to generate the turn-by-turn BPM data with the input of the lattice file from MAD.[4] The close orbit (x_{co}) is set to zero due to the consideration of the difference orbit. The displacement due to the non-zero dispersion (D_x) is calculated using eq.2.

$$x_D(m)_n = \frac{1}{L} \times \sum_{k=1}^L D_x(n) \times \delta \times \cos(2\pi Q_s m + 2\pi(k-1) \times \frac{Q_s}{h} + \phi_0(n)). \quad (2)$$

L is the number of bunches to be averaged by BPM. $D_x(n)$ is the dispersion in the horizontal direction (x) at the n^{th} BPM. δ is the momentum offset, and Q_s is the synchrotron tune. h ($=84$) is the harmonic number. $\phi_0(n)$ is the phase of the synchrotron motion at the n^{th} BPM right after the excitation, which can be set to zero, and this is true for all the BPM since the phase advance of the synchrotron motion is small within one Booster turn. The displacement due to the betatron motion:

$$x_\beta(m)_n = \frac{1}{L} \times \sum_{k=1}^L \hat{x}_n \times \cos(2\pi Q_x m + (k-1) \times \frac{2\pi Q_x}{h} + \frac{\xi_x \times \delta \times \sin(2\pi Q_s m + (k-1) \times \frac{2\pi Q_s}{h} + \phi_0(n))}{Q_s} + \phi'_0(n)). \quad (3)$$

$\hat{x}_n$ is the amplitude of the betatron motion at the n^{th} BPM, as shown in eq.4.

$$\hat{x}_n = \frac{\sqrt{\beta_0 \times \beta_n}}{2 \times \sin(\pi Q_x)} \times \theta_0. \quad (4)$$

Q_x is the betatron tune, and ξ_x is the chromaticity. $\phi'_0(n)$ is the phase of the betatron motion at the n^{th} BPM on the 0^{th} turn. It is calculated from the phase advance between the place where the betatron motion is excited and the BPM position using the MAD lattice file. β_0 and β_n are beta functions at places where the angle kick and the n^{th} BPM are, and θ_0 is the amplitude of the angle kick.

Simulation and Results

The operational parameters were used in the calculation. They are the synchrotron tune $Q_s=0.05$, betatron tune $Q_x=6.7$, chromaticity $\xi_x=6$, the number of BPM $N=48$, and the number of tracking turns $M=128$. The beam was excited both transversely and longitudinally at the 0^{th} turn with a 1-mrad angle kick at the 1^{st} BPM and a momentum offset $\delta=0.005$. In the calculation, the number of bunches averaged by each BPM was repeated among 1, 5, 10, and 15. The turn 128^{th} is shown in Fig. 1(a). The red, black, blue, and green curves represent situations of BPM reading one bunch and the average of five, ten, and fifteen bunches respectively. Singular values of all the modes are shown in

Fig. 1(b). There are three significant modes, which are often called modes 1, 2, and 3. And they are caused by the synchrotron motion and betatron motion of the beam. Their temporal eigen vectors are shown in Figs. 1(c), 1(f), and 1(i) for modes 1, 2, and 3 separately, their spatial eigen vectors are shown in Figs. 1(d), 1(g), and 1(j), and their unnormalized power spectrums are shown in Figs. 1(e), 1(h), and 1(k). Singular values of modes 1, 2, and 3 vs. the number of bunched to be averaged by BPM are shown in Fig. 1(l). The red, black, and magenta curves represent modes 1, 2, and 3 separately. It is clear that the synchrotron mode doesn't change with the number of bunches to be averaged by BPM. However, the betatron modes indeed change with the number of bunches to be averaged by BPM. Especially when the number of bunches to be averaged by BPM is 10, spatial eigen vectors of the betatron modes, as shown by the blue curves in Figs 1(g) and 1(i), have a 180° phase shift compared with situations with 1 bunch, the average of 5 and 15 bunches. This can be understood as the following:

$$\Delta\Phi(L) = 2\pi L \times \frac{Q_x}{h}.$$

$$\Delta\Phi(10) = 2\pi \times 10 \times \frac{6.7}{84} \approx 1.6\pi;$$

$$\Delta\Phi(5) \approx 0.8\pi;$$

$$\Delta\Phi(15) \approx 2.4\pi = 2\pi + 0.4\pi.$$

When $(2n-1)\pi \leq \Delta\Phi(L) \leq 2n\pi$, a 180°-phase shift, such as $L=10$, is observed in the spatial eigen vector via MIA, otherwise, the spatial eigen vector is nearly identical with the situation of BPM reading one bunch.

Instead of using MIA, the calculated turn-by-turn data of BPM #1 and #4 are shown in Figs. 2(a) and 2(c), and their unnormalized power spectrums are shown in Figs. 2(b) and 2(d). In Figs. 2(a) and 2(c), the slow sin-waves are caused by the synchrotron motion of the beam since their period is 20 turns ($1/Q_s$), and they stay the same no matter when the BPM reads one bunch, the average of 5, 10, and 15 bunches; however, the fast oscillations due to the betatron motion get smaller and smaller with the increase of the number of bunches to be averaged by BPM.

The synchrotron motion and chromatic modulation on the betatron motion are turned off in the calculation by setting the momentum offset to zero. The turn-by-turn data of BPM #1 and BPM #4 are shown in Figs. 3(a) and 3(c) and their corresponding unnormalized

power spectrums are shown in Figs. 3(b) and 3(d). The red, black, blue, and green curves represent the number of bunches to be averaged by BPM to be 1, 5, 10, and 15. It is clear from Figs. 3(a) and 3(c) that the betatron motion is smoothed out by an increase in the number of bunches to be averaged by BPM. The red curve has a 180° -phase shift compared to the blue curve in Fig. 3(a), and this is also true for Fig. 3(c). This 180° -phase shift between BPM reading one bunch and ten-bunch average is consistent with the observation from Figs. 1(g) and 1(i). Also, the relative phase advance between BPM #1 and BPM #4 is still observable even when the number of bunches to be averaged by BPM increases. The difference in betatron amplitudes of BPM #1 and BPM #4 is caused by the different in their beta functions.[4]

Conclusion

The synchrotron-mode analysis via either MIA or fast Fourier transform (FFT) of a single BPM turn-by-turn data doesn't change with the number of bunches to be averaged by BPM. The result is predictable since the synchrotron phase advance at one Booster turn is comparable small. However, the betatron mode is more sensitive to the number of bunches to be averaged by BPM. With the number of bunches to be averaged increasing, the betatron amplitude decreases. However, the betatron tune still can be measured via MIA and FFT. Furthermore, the betatron phase advance between two BPM is observable via MIA and FFT, and special attention should be paid to the 180° -phase shift in the situation when the betatron phase covered by the bunch train (L) is in between $(2n-1)\pi$ and $2n\pi$. The analytical expressions of eqs.1, 2, 3 and their FFT will be derived in the future publication.

Acknowledgement

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References:

[1] FERMILAB-TM-405.

- [2] X. Yang, “Chromaticity Measurement Via the Fourier Spectrum of Transverse Oscillations”, FERMILAB-TM-2262-AD, submitted.
- [3] X. Yang, “Beam Diagnostics Via Model Independent Analysis of the Turn-by-Turn BPM Data”, FERMILAB-TM-2263-AD, submitted.
- [4] <http://www-ap.fnl.gov/~drozhdin/prdriver/>

Fig. 1(a)

Fig. 1(b)

Fig. 1(c)

Fig. 1(d)

Fig. 1(e)

Fig. 1(f)

Fig. 1(g)

Fig. 1(h)

Fig. 1(i)

Fig. 1(j)

Fig. 1(k)

Fig. 1(l)

Fig. 1(a) the turn 128th calculated at the following conditions: the synchrotron tune $Q_s=0.05$, horizontal betatron tune $Q_x=6.7$, chromaticity $\xi=6$, after the beam was excited both transversely and longitudinally at the 0th turn with a 1-mrad angle kick at the 1st BPM and momentum offset $\delta=0.005$. The red, black, blue, and green curves represent situations of BPM reading one bunch, the average of five, ten, and fifteen bunches respectively for Figs. 1(a)-(k).

Fig. 1(b) singular values of all the modes.

Fig. 1(c) the temporal eigen vector of mode 1.

Fig. 1(d) the spatial eigen vector of mode 1.

Fig. 1(e) the unnormalized power spectrum of the temporal vector of mode 1.

Fig. 1(f) the temporal eigen vector of mode 2.

Fig. 1(g) the spatial eigen vector of mode 2.

Fig. 1(h) the unnormalized power spectrum of the temporal vector of mode 2.

Fig. 1(i) the temporal eigen vector of mode 3.

Fig. 1(j) the spatial eigen vector of mode 3.

Fig. 1(k) the unnormalized power spectrum of the temporal vector of mode 3.

Fig. 1(l) singular values vs. the number of bunched to be averaged by BPM. The red, black, and magenta curves represent modes 1, 2, and 3 separately.

Fig. 2(a)

Fig. 2(b)

Fig. 2(c)

Fig. 2(d)

Fig. 2(a) the calculated turn-by-turn data of BPM #1 at the same conditions with Fig. 1.

The red, black, blue, and green curves represent situations of BPM reading one bunch, the average of five, ten, and fifteen bunches respectively for Figs. 2(a)-(d).

Fig. 2(b) the unnormalized power spectrum of the data in Fig.2(a).

Fig. 2(c) the calculated turn-by-turn data of BPM #4.

Fig. 2(d) the unnormalized power spectrum of the data in Fig.2(c).

Fig. 3(a)

Fig. 3(b)

Fig. 3(c)

Fig. 3(d)

Fig. 3(a) the turn-by-turn data at BPM #1 calculated under the same conditions with those in Fig. 1 except with a zero momentum offset. The red, black, blue, and green curves represent the number of bunches to be average to be 1, 5, 10, and 15.

Fig. 3(b) the unnormalized power spectrums of the data in Fig. 3(a).

Fig. 3(c) the turn-by-turn data at BPM #4 under the same conditions with Fig. 3(a). The red, black, blue, and green curves represent the number of bunches to be average to be 1, 5, 10, and 15.

Fig. 3(d) the unnormalized power spectrums of the data in Fig. 3(c).