**CHARACTERIZATION OF IN-SITU STRESS AND
PERMEABILITY IN FRACTURED RESERVOIRS**

SEMI-ANNUAL TECHNICAL PROGRESS REPORT

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by

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Abstract

Expanded details and additional results are presented on two methods for estimating fracture orientation and density in subsurface reservoirs from scattered seismic wavefield signals. In the first, fracture density is estimated from the wavenumber spectra of the integrated amplitudes of the scattered waves as a function of offset in pre-stack data. Spectral peaks correctly identified the 50m, 35m, and 25m fracture spacings from numerical model data using a 40Hz source wavelet. The second method, referred to as the Transfer Function-Scattering Index Method, is based upon observations from 3D finite difference modeling that regularly spaced, discrete vertical fractures impart a ringing coda-type signature to any seismic energy that is transmitted through or reflected off of them. This coda energy is greatest when the acquisition direction is parallel to the fractures, the seismic wavelengths are tuned to the fracture spacing, and when the fractures have low stiffness. The method uses surface seismic reflection traces to derive a transfer function, which quantifies the change in an apparent source wavelet propagating through a fractured interval. The transfer function for an interval with low scattering will be more spike-like and temporally compact. The transfer function for an interval with high scattering will ring and be less temporally compact. A Scattering Index is developed based on a time lag weighting of the transfer function. When a 3D survey is acquired with a full range of azimuths, the Scattering Index allows the identification of subsurface areas with high fracturing and the orientation (or strike) of those fractures. The method was calibrated with model data and then applied to field data from a fractured reservoir giving results that agree with known field measurements.

As an aid to understanding the scattered wavefield seen in finite difference models, a series of simple point scatterers was used to create synthetic seismic shot records collected over regular, discrete, vertical fracture systems. The model contains a series of point scatterers delineating the top tip and bottom tip of each vertical fracture. When the shot record is located in the middle of the fractured zone and oriented normal to the direction of fracturing, a complicated series of beating is observed in the back scattered

energy. When the shot record is oriented parallel to the fracturing, ringing wavetrains are observed with moveouts similar to reflections from many horizontal layers. These results are consistent with the full 3D elastic modeling results.

An AVOA analysis method was refined and applied to a field data set. An iterative, non-linear least squares inversion that uses the Gauss-Newton method and analyzes the full range of azimuths simultaneously was employed. Resulting fracture location and strike orientation estimates are consistent with other fracture information from the area.

Two modeling approaches for estimating permeability values from seismically derived fracture parameters have been investigated. The first is a statistical method that calculates the permeability tensor for a given distribution of fractures. A possible workflow using this method was tested on fracture distributions obtained from the Transfer Function-Scattering Index analysis method. Fracture aperture and length estimates are needed for this method. The second method is a direct flow model of discrete fractures and fracture networks using a computational fluid dynamics code. This tool provides a means of visualizing flow in fracture networks and comparing expressions for equivalent fracture aperture flow to the actual flow. A series of two dimensional models of fractures and fracture networks, as well as a 3-D model of a single rough fracture, were tested.

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I. Executive Summary

The purpose of this project is to develop and implement large-scale numerical models to quantify the effects of fracture parameter variations on seismic reflection signals and in-situ stress variations on flexural modes in boreholes. These models will be used as the basis of data analysis and inversion routines for estimating the heterogeneous fracture distribution in fractured reservoirs from seismic and borehole field data. Fracture property distributions estimated from seismic data will be used to estimate the permeability tensor in the reservoir for input to reservoir simulators.

During the past six months our efforts have focused on the estimation of fracture orientation, spatial density variations, spacing, and other properties from seismic reflection data, and the conversion of these seismically derived properties into permeability tensor information for use in reservoir simulators. In the area of seismic fracture characterization our results focus on three seismic analysis methods for estimating fracture properties: 1) spectral analysis of prestack scattered wave integrated amplitudes, 2) the transfer function-scattering index method for reservoir interval analysis, and 3) an updated amplitude variations with offset and azimuth (AVOA) method. These approaches are being extensively tested on full wavefield 3-D finite difference modeling data and are also being applied to field data from well-characterized fractured reservoirs. In the area of permeability estimation, two methods have been utilized: 1) the estimation of the permeability tensor from a statistical model of fracture networks and 2) the use of computational fluid dynamics models to model flow in discrete fracture systems.

The first method used in seismic fracture characterization uses the wavenumber spectra of the integrated amplitudes of the scattered wavefield as a function of offset in pre-stack data. Spectral peaks corresponding to fracture spacing are clearly identifiable for 100m, 50m, 35m, and 25m fracture spacings from numerical model data using a 40Hz source wavelet. A higher frequency source would allow finer spacings to be resolved. The second method, referred to as the Transfer Function-Scattering Index Method, is based

upon observations from 3D finite difference modeling that regularly spaced, discrete vertical fractures impart a ringing coda-type signature to any seismic energy that is transmitted through or reflected off of them. This coda energy is greatest when the acquisition direction is parallel to the fractures, the seismic wavelengths are tuned to the fracture spacing, and when the fractures have low stiffness. The method derives a transfer function, using a deconvolution-type process, which quantifies the change in an apparent source wavelet propagating through a fractured interval. The transfer function for an interval with low scattering will be more spike-like and temporally compact while for high scattering it will ring and be less temporally compact. We derived a Scattering Index based on a time lag weighting of the transfer function that allows the identification of subsurface areas with high fracturing and the orientation (or strike) of those fractures. The method was calibrated with finite difference model data and then applied to field data from a fractured reservoir giving results that agree with known field measurements. Finally, an AVOA analysis method, originally reported last year, was refined and applied to the same field data set. The inversion method analyzes the full range of azimuths simultaneously and the resulting fracture location and strike orientation estimates are consistent with other fracture information from the area. We have also developed a simple point scatterer model to create synthetic seismic shot records collected over regular, discrete, vertical fracture systems as an aid to understanding the more complex full waveform numerical modeling results. The model represents the top tip and bottom tip of each vertical fracture as point scatterers. The results obtained using this simple travel time model are consistent with the full 3D elastic modeling results and provide insight into P and S wave scattering from fractures.

Two modeling approaches for estimating permeability values from seismically derived fracture parameters have also been investigated. The first is a statistical method that calculates the permeability tensor for a given distribution of fractures. One of the benefits from using this type of approach is that the input to such a model is similar to the format of the data that we can estimate with our seismic methods. Key parameters that are needed for this model include fracture aperture, length, and response to applied stress fields. A possible workflow using this method was tested on fracture distributions

obtained from field data using the Transfer Function-Scattering Index analysis method. The second method is a direct flow model of discrete fractures and fracture networks using a commercial Computational Fluid Dynamics (CFD) code. This tool provides a means of visualizing flow in fracture networks and comparing expressions for equivalent fracture aperture flow to the actual flow. A series of two dimensional models of fractures and fracture networks, as well as a 3-D model of a single rough fracture, were tested.

II. Introduction

The purpose of this project is to develop and implement large-scale numerical models to quantify the effects of fracture parameter variations on seismic reflection signals and in-situ stress variations on flexural modes in boreholes. These forward modeling efforts will be used to develop data analysis and inversion routines for estimating the heterogeneous fracture distribution in fractured reservoir from seismic and borehole field data. These fracture property distributions will be used to estimate the permeability tensor in the reservoir by using statistical and discrete feature network techniques. Permeability estimates will then be used as inputs to reservoir simulators. The objective of the project is schematically displayed in Figure 1.

Our efforts continue to focus on the first two segments of this idealized workflow: first the estimation of fracture orientation, spatial density variations, spacing, and other properties from seismic reflection data; and second, the conversion of these seismically-derived properties into permeability tensor information for use in reservoir simulators.

Figure 1. Schematic display of the project objectives. Surface 3-D seismic data is used to estimate the spatial distribution of fracture parameters (e.g., fracture density, spacing, orientation). A model is used to convert such fracture parameters into permeability values, which are input to reservoir simulators for history matching.

III. Results and Discussion

Our first set of results focuses on three seismic analysis methods for estimating fracture properties: 1) spectral analysis of prestack scattered wave integrated amplitudes, 2) a transfer function method for reservoir interval analysis, and 3) an updated amplitude variations with offset and azimuth (AVOA) method. These approaches are being extensively tested on full wavefield 3-D finite difference modeling data and are also being applied to field data from well-characterized fractured reservoirs.

The second set of results focuses on fluid flow in fracture networks. Through the use of computational fluid dynamics models we are investigating the nature of fluid flow in a wide range of fracture network settings to understand the critical parameters affecting flow and to develop methods for estimating effective permeability values for such

networks. A second approach involves the use of statistical representations of fracture networks and the corresponding permeability tensor.

A. Fracture properties from seismic data

Before providing an update on our results in the three analysis methods for estimating fracture properties, we first look at a qualitative model of scattering from fractures. The model represents fractures as point scatterers and computes the arrival times for P and S scattered energy at a surface receiver array. This model helps us to better understand the scattered wave arrivals that we use for two of our analysis methods.

1. A qualitative model of scattering from fractures

Finite difference modeling of regular, vertical discrete fracture systems shows many complicated patterns of scattering. It is difficult to know from these full 3D elastic modeling results whether the energy observed is P wave energy scattered back to P wave energy or back to S wave energy. One would think that travel times alone should be sufficient to determine something as seemingly simple as this; however, the complicated wave trains are difficult to decipher. This can be seen in the full wavefield models shown in Figures 2 and 3 and Table 1, which have five layers with the middle layer having vertical fractures. This center layer is 200-m thick and contains parallel, vertical fractures that are as tall as the layer, one grid cell thick (5m) and run the entire width of the model. We modeled five different fracture spacings, including 50m, but we limit our discussion here to only the case with vertical fractures spaced 50m apart. We used the 3-D elastic finite difference code, developed by Lawrence Berkeley National Laboratory (Nihei et al., 2002), to synthesize all reflections and scattered energy. Figure 3 shows the shot records oriented (a) normal and (b) parallel to the fractures. It is clear on the shot record normal to the fracture direction that the scattered wavefield is quite complex and mostly incoherent in or below the reservoir level (300 ms). Parallel to the fractures, there are many more coherent events that can be seen primarily below the base reservoir reflection. In this direction the seismic energy seems to be guided by the aligned fractures and the resulting scattered energy is similar to the direct P wave reflection.

Table 1. Parameters for model

Layer	Thickness (m)	V _p (m/s)	V _s (m/s)	Density (g/cc)
1	200	3000	1765	2.2
2	200	3500	2060	2.25
3	200	4000	2353	2.3
4	200	3500	2060	2.25
5	200	4000	2353	2.3

Figure 2. Diagram showing the geometry of the 3D finite difference model. The layer velocities and densities are shown in Table 1. The source is located in the left front corner (red symbol) and the receivers are spread out in a rectangular area 1000 m in the x direction and 1000m in the y direction. The receiver spacing is 5m in each direction.

Figure 3. Vertical component of the 3D Finite difference modeling for 50m fracture spacing from Willis et al. (2004a, b): a) shows the shot record acquired normal to the fractures, b) shows the shot record acquired parallel to the fracture direction.

In order to more fully understand the mechanics and gain insight into the processes that create these patterns of scattered energy, we use a much simpler model and modeling method consisting of a constant velocity medium and we make no corrections for spreading or attenuation. The P wave velocity used is 4000 m/s and the S wave velocity used is 2353 m/s. Figure 4a shows the diagram of an array of receivers on the surface of the ground, spaced 5m apart in both x and y directions and shown in black triangles. Below the surface are located two sets of identical fracture systems, represented by a series of point scatterers. The first set is located a depth of 400m. The second set is located at a depth of 600m. These sets of scatterers are intended to mimic the top tip and bottom tip of each fracture in Figure 2. The extension of the fracture in the y direction is modeled by closely spaced point scatters to simulate the continuous edges of the fracture in that direction. In the x direction, the scatterers are spaced 50m apart, representing the distance between the fractures.

To create each seismogram, we utilize a Huygens-type approach in which each scatterer acts as a point source that is time delayed by the one way travel time from the shot location to the point scatterer. Each synthetic trace is simply the summation of the contributions from each of the scatterers. This amounts to repeatedly summing together a 40Hz Ricker wavelet delayed in time by the total time from the source down to each scatterer and then up to the receiver. The model itself contains no reflectors other than the point scatterers and we assume no secondary scattering.

P-P Scattering

We first examine P-P scattering, that is, downward propagating P wave energy that is scattered as P wave energy back to the surface by each point scatterer. The seismograms for P-P scattering recorded at the front edge ($y=0$) of the fracture model, acquired normal to the fractures, are shown in Figure 4b. The source is located at the origin ($x=0, y=0$), which is shown as a yellow symbol. In Figure 4c the red line corresponds to acquiring a shot record in the middle of an area with fractures. The corresponding P-P shot record is shown in Figure 4d. Since this model does not contain any continuous reflectors or layering, the constructive interference of the scatterers themselves forms what seem to be strong primary and secondary arrivals in both the edge and centered acquisitions. Note how on both records, most of the ringing, scattered energy is seen later in time than what appears to be the second event. Also evident on the centered seismogram (Figure 4d) is a strong lateral beating phenomenon corresponding to the back scattering from fractures located symmetrically on both sides of the source.

Figure 4. Vertical component of simple P-P diffractor model with a receiver line (red) normal to the fractures. a) model geometry of shot at edge of fractured zone, b) the simulated waveforms acquired at edge of fractured zone, c) model geometry of shot in center of fractured zone, d) the simulated waveforms acquired in the center of the fractured zone. Each trace is spaced 5m apart.

We next examine the case of acquiring shot records parallel to the fracture orientation. The seismograms are shown in Figure 5b for P-P scattering observed from the left edge ($x=0$) of fracture model, acquired parallel to the fractures (Figure 5a). The source is located at the origin ($x=0, y=0$), which is shown as a yellow symbol. The red line in Figure 5c corresponds to acquiring a shot record in the middle of an area with fractures. The corresponding P-P shot record is shown in Figure 5d. Notice how in both of these cases the shot records show long, laterally coherent, ringing wavetrains. In fact there seems to be very little difference in the kinematic aspects of the two records. They show time varying amplitudes, which seem to bring out two distinct secondary zones with

higher amplitudes. As before these general observations are very similar to the results seen in the finite difference modeling.

Figure 5. Vertical component of simple P-P diffractor model with a receiver line (red) parallel to the fractures. a) model geometry of shot at edge of fractured zone, b) the simulated waveforms acquired at edge of fractured zone, c) model geometry of shot in center of fractured zone, d) the simulated waveforms acquired in the center of the fractured zone. Each trace is spaced 5m apart.

P-S Scattering

We next examine P-S scattering representing a downgoing energy from a P wave source, which is converted to S wave energy by each point scatterer. The seismograms for P-S scattering observed from the front edge ($y=0$) of the fracture model, acquired normal to the fractures, are shown in Figure 6b. The source is located at the origin ($x=0, y=0$)

(yellow symbol). The red line in Figure 6c corresponds to acquiring a shot record in the middle of an area with fractures. The corresponding P-S shot record is shown in Figure 6d. In contrast to the P-P scattering, notice how on both of the P-S records, significant scattered energy is seen above (i.e. earlier in time) than the secondary event. As before with the P-P scattered shot records (Figure 4d), the centered P-S scattered energy acquired normal to the fractures also shows the prominent lateral beating.

Figure 6. Vertical component of simple P-S diffractor model with a receiver line (red) normal to the fractures. a) model geometry of shot at edge of fractured zone, b) the simulated waveforms acquired at edge of fractured zone, c) model geometry of shot in center of fractured zone, d) the simulated waveforms acquired in the center of the fractured zone. Each trace is spaced 5m apart.

Finally we look at P-S scattering for the case of acquiring shot records parallel to the fracture orientation. The seismograms are shown in Figure 7b for P-S scattering observed

from the left edge ($x=0$) of the fracture model, acquired parallel to the fractures (Figure 7a). As before, the source is located at the origin ($x=0, y=0$), which is shown as a yellow symbol. The red line in Figure 7c corresponds to acquiring a shot record in the middle of an area with fractures. The corresponding P-S shot record is shown in Figure 7d. As with the PP scattering acquired parallel to the fractures, these shot records show long, laterally coherent, ringing wavetrains.

Figure 7. Vertical component of simple P-S diffractor model with a receiver line (red) parallel to the fractures. a) model geometry of shot at edge of fractured zone, b) the simulated waveforms acquired at edge of fractured zone, c) model geometry of shot in center of fractured zone, d) the simulated waveforms acquired in the center of the fractured zone. Each trace is spaced 5m apart.

Summary

These models, using a simple series of point scatterers, are able to match the overall trends in synthetic shot records generated from the more complete full elastic finite difference method (Figure 3). For shot records acquired parallel to the fracture systems, the long ringing wavetrains generated for both P-P and P-S scattering are coherent and have positive moveout with offset. The most interesting effects take place when the shot records are acquired normal to the fractures for both P-P and P-S scattering. In these cases, when the shot record is acquired near the edge of the fractured area, strong scattered events are visible having a generally negative moveout (i.e. moving back into the spread toward the source) with offset. When the shot record is acquired in the middle of the fractured zone, normal to the fractures, a strong beating pattern is generated. This pattern is caused by the constructive and destructive interference of the scattered energy moving back toward the source location. This energy is from fractures located in the positive and negative offset directions. These simple model results help us to understand the nature of the scattered wave arrivals we see in the finite difference models and in field data.

2. Full waveform analysis – model and field data

The previous section provides us with an understanding of the scattered wavefield generated by a regularly spaced set of fractures, including the effect of back scattering and forward scattering, as well as P and S wave scattering. We have developed and applied three methods to the full wavefield from finite difference models and field data. The first two, integrated amplitude analysis and the transfer function method, focus on the direct analysis of the scattered wavefield. The third method, amplitude variations with offset and azimuth (AVOA), measures the degree of azimuthal anisotropy in the subsurface and relates that anisotropy to the presence of subwavelength oriented fractures.

a. Integrated Amplitude Analysis of Scattered Waves

Our first method of analyzing the scattered wavefield signatures involves pre-stack data. Close inspection of the shot records acquired normal to the fracture direction reveals a beating or resonance phenomenon (see Figure 3). The pattern of beating is different for each of the fracture spacing cases, and is most prominent on the near to middle offset range of traces. To study this, we computed the integrated absolute value of the amplitudes for each trace in a given shot record in an offset and velocity controlled time window through the scattered energy. The far offset traces were omitted from the analysis. The integrated amplitudes will be affected by the AVO response of the top and bottom reflection of the reservoir. However, this effect will be a predominantly long wavelength term on the integrated amplitudes versus offset. The approximate effect of the AVO term is removed by subtracting the best-fit linear trend with offset from the integrated amplitudes prior to spectral analysis. A series of initial results was included in our previous report and a full description of the method is given in Appendix D.

The steps used in the method are illustrated in Figure 8. The top panel of Figure 8 shows the synthetic shot record normal to the fractures for the 50m fracture spacing case. The scattered wavefield is windowed and the absolute value of the amplitude for each trace is summed within that window. The integrated trace amplitude versus offset is shown for the near and mid-offset traces. The analysis window followed the NMO velocity of the reservoir. The bottom panel of Figure 8 shows the wavenumber amplitude spectrum computed from the fast Fourier transform of these de-trended amplitudes. Clearly visible in the wavenumber spectrum is a peak at 0.02/m, which corresponds to the fracture spacing of 50m. The same analysis was done on the other fracture spacing cases. Spectral peaks correctly identified the 50m, 35m, and 25m fracture spacings. However, due to the combination of source frequency content, receiver spacing and reservoir velocity, the 10m spacing case was not resolvable (see Figure 9). For the 100m spacing case the spectral peak is shifted to a smaller wavenumber, representing a fracture spacing of 130m. This effect is being investigated. For the model of a Gaussian distribution of

fractures with a mean spacing of 35m, a broader Gaussian spectral peak was present in approximately the correct wavenumber region.

Figure 8. Top panel shows the synthetic shot record normal to the fractures at 50m spacing. The highlighted zone is the window for the integrated amplitude analysis. The middle panel shows the integrated amplitude versus offset and lower panel the corresponding wavenumber spectrum with a peak at 0.02/m (50m spacing).

Figure 9. Wavenumber spectra computed from integrated amplitude data for models with different fracture spacing. The blue vertical lines correspond to the actual fracture spacing in the each model. The ‘control’ spectrum is for a model with no fractures. For this particular set of model parameters, fracture spacings of 25m, 35m, and 50m are well resolved. The 10m spacing is too small to be resolved. The disagreement seen in the 100m spacing case is still being investigated.

b. Reservoir scattering analysis – the Transfer Function-Scattering Index method

Figure 10 shows the shot records for the 50m fracture spacing case acquired in directions normal and parallel to the fractures as generated by the 3-D finite difference model. To the right of each shot record is its velocity analysis. Normal to the fracture direction there is no coherent stackable energy in or below the reservoir level (300 ms), while for the parallel case, there are many coherent events that can be seen primarily below the base reservoir.

Figure 10. Left two plots show the seismic shot records for the model with 50m fracture spacing. The top left plot show the shot record normal to fractures, left bottom plot shows the shot record parallel to the fractures. The right two plots show the velocity spectra for the corresponding shot records on the left.

In this finite difference model data it is reasonably straight forward to separate and analyze the scattered wavefield from the isolated reflectors in the reservoir zone. However, in field data we expect to have a more difficult time observing these scattered wavetrains due to the nearly continuous nature of the subsurface reflectivity. In addition, if there are fractured zones or other scatterers in the overburden, those scattered waves will contaminate, or overprint, the scattered energy from our zone of interest in the reservoir. So our analysis approach starts with the concepts from existing methods of seismic wavelet estimation to obtain two apparent (or temporally local) source wavelets from the reflection time series – one from above the fractured zone (the “input” wavelet) and one below it (the “output” wavelet). These wavelets are represented by their autocorrelations obtained from windowed portions of the reflection time series above and below the fractured zone of interest.

We then compute the time domain transfer function between the autocorrelations of the two extracted wavelets. The transfer function is computed by deconvolving the autocorrelation of the input wavelet from the autocorrelation of the output wavelet. This transfer function characterizes the effect of scattering in the interval of interest, between the two windowed portions of the trace (see previous report). A simple pulse shaped transfer function indicates no scattering, while a long ringing transfer function captures the scattering within the reservoir interval. It is important to note that any contamination from scattering above the interval will be present in both the input and output extracted wavelets and thus will be excluded from the transfer function.

This method can be used on both pre-stack and post-stack data. If it is applied to data stacked in different azimuthal directions, the interval transfer function should exhibit greater ringing in the direction parallel to fracturing. The left panel of Figure 11 shows the results of applying NMO and stacking the model traces in different azimuthal directions. The trace labeled “normal” corresponds to the stack of the traces in the top left plot of Figure 11. The trace labeled “parallel” corresponds to the stack of traces in the bottom left plot of Figure 11. In between these two traces are those stacked traces corresponding to 10 degree increments between normal and parallel directions to the fractures. (For comparison, the bottom trace labeled “control” is the trace from the model with no fractures.) The right panel of Figure 11 shows the transfer functions derived for the stacked traces in the left panel of the same figure.

We can see that the transfer functions are all compact and similar to the non-fractured case (labeled “control”) for orientations that are not parallel to the fractures. However, for azimuthal stacks oriented parallel to the fractures, the transfer function contains significant energy at non-zero lag values.

Figure 11. Left Panel: a plot showing the azimuthal stacks of traces from the 50m fracture spacing model. The traces represent azimuth stacks starting in the direction parallel to fracturing (top), and then increasing in 10 degree increments until normal to the fractures. The bottom trace shows the stack for the model without a fractured layer. Right panel: a plot showing the transfer functions corresponding to the azimuthal stacks in the left panel.

Non-regular spacing and multiple fracture sets

One of the first questions to be answered is whether this methodology is applicable for fracture systems that are not perfectly regular or have multiple sets of fractures. We generated two models to address these questions. Figure 12 shows the azimuthal stacks for the case of uniform 35m fracture spacing and the corresponding transfer functions. Figure 13 shows the same plots for a model with the spacing of the fractures having a Gaussian distribution with a mean of 35m and standard deviation of 10m. In both cases the strong ringing of the stack and the transfer function are still present in the direction parallel to the fractures.

Figure 12. Left panel: azimuthal stacks for a model with uniform 35m spacing of fractures. Right panel: the transfer functions corresponding to the same stacks.

Figure 13. Left panel: azimuthal stacks for a model with a Gaussian distribution of fracture spacings with a mean of 35m and a standard deviation of 10m. Right panel: the transfer functions corresponding to the same stacks.

We generated one model to test the applicability of the methodology to the presence of multiple fractures sets. A more thorough analysis will be needed to test the method against the full range of critical variables, including fracture orientation, spacing, and stiffness. We chose to build a model with two fracture sets. The primary set of fractures is uniformly spaced at 35m, while the secondary set of fractures is oriented at 45 degrees to the primary set and spaced 100m apart. The stiffness of the secondary set of fractures was chosen to be 5 times greater than the primary set. Figure 14 shows the associated azimuthal stacks and transfer functions. For this particular model the secondary set of fractures has little effect on the stacks and transfer functions.

Figure 14. Left panel: azimuthal stacks for a model with two sets of fractures, the primary set spaced at 35m and the secondary set spaced at 100m and oriented at 45 degrees to the first set. Right panel: the transfer functions corresponding to the same stacks. Note that the secondary set of fractures has a stiffness value five times greater than the primary set.

These results show that stacks made in the direction parallel to the primary fractures enhance the ringing scattered energy on the traces. Stacks made in other directions tend to diminish the scattered energy. This same trend is evident on the corresponding transfer functions. The transfer functions for the orientation parallel to fractures have significant energy away from the zero lag, while in the normal direction the transfer functions are comparatively compact.

Scattering Index

In order to quantify the amount of transfer function ringiness or non-compactness, we created a Scattering Index, SI, defined by:

$$SI = \sum_{i=0}^m |t_i| i^n$$

where i is the time lag, t_i is the transfer function (time domain) amplitude at lag i , n is an exponent, typically equal to unity, and m is a lag at which there is no more significant energy in the transfer function. (It is also possible to normalize the scattering index based upon its energy and interval time sample or other such criteria.) The more the transfer function rings, the larger the value of the Scattering Index. If the transfer function is a simple spike (i.e. representing no scattering) then the Scattering Index attains a value of zero.

Figure 15. Polar plot of the azimuthal variation of Scattering Indices derived from the transfer functions of the 25, 35, 50 and 100m fracture spacing models. The Scattering Index is largest in the direction parallel to the fracture orientation. The largest scattering index is for the 35m fracture spacing, while the smallest shown is for the 25m spacing. (Quadrants 2 through 4 were created from the measurements in quadrant 1.)

Figure 15 shows the scattering index values for models run with 25, 35, 50 and 100m fracture spacings. These results show that there is a clear maximum of the scattering index in the parallel direction. It is also clear that in the non-parallel directions the scattering index is not zero but fluctuates about a smaller, but fairly consistent, value. For this set of models, the 35m spacing produces the largest value due to a tuning effect from the particular values of P and S wave velocity and source wavelet frequency.

Application to field data

We applied the method to data from a 3D/4C seismic survey collected over an offshore fractured carbonate reservoir. Borehole breakouts, FMS logs, and oriented cores suggest the presence of two orthogonal fracture sets oriented ENE and NNW (Angerer et al, 2002). Recent studies have investigated this 3D seismic data using PP and PS wave anisotropy to identify fracture characteristics of the reservoir level (Vetri et al, 2003; Gaiser et al, 2002).

The near to mid range offsets of the preprocessed PP data (Vetri et al, 2003) were stacked in eighteen different azimuthal orientations from East to West using 20 degree wide overlapping ranges. This process created eighteen 3D stacked volumes. The transfer functions and Scattering Indices for the formation zone were computed for each of these stacked volumes. The Scattering Indices were sorted and directions for those with the highest ranges of values are plotted in Figure 16, giving a map view of the location and direction of possible fractures determined by this method. Figure 17 shows histogrammed rose plots of the fracture distributions corresponding to the sixteen boxes outlined in green in Figure 16. The ENE and NNW fracture directions are strongly evident in much of the field.

Figure 16. The spatial orientation and distribution of fractures over the field as indicated from Scattering Indices of multiple azimuthal stacks. The black and red bars show the orientations for the top two bins of highest Scattering Indices. The green grid shows the location of the histogram plots in figure 17.

Figure 17. Angular histogram plots of the fracture distributions shown in Figure 16. Each plot represents the corresponding subportion of the field cut up by the green dashed lines in Figure 16. The top of each plot represents fractures aligned North.

c AVOA

Amplitude Variations with Offset and Azimuth (AVOA) analysis was completed on the same field data set. We use a surface-fitting algorithm for the AVOA analysis that is similar to the approach of Hall and Kendall (2003). Many azimuthal AVO studies separate the data into a few discrete azimuths and compare the individual AVO results in those directions. However, by analyzing the full range of azimuths simultaneously the problem is better constrained. An iterative, non-linear least squares inversion that uses the Gauss-Newton method is employed (Minsley et al., 2004). Figure 18 shows the results of this analysis at four well locations in the field and compares the fracture orientation to that obtained by Gaiser et al. (2002) from shear wave birefringence. The details of the inversion method are given in Appendix C.

Figure 18. Comparison of the AVOA isotropy axis direction (blue) found in this study with fast S-wave direction (red) reported by Gaiser et al. (2002) for the Gessoso horizon at four locations. The magnitude of the arrows represents the amount of anisotropy for each method at each location. Two fast S-wave arrows are used to show the 20° uncertainty of the direction.

B. Fracture permeability estimation – flow modeling

We are exploring two different modeling approaches to estimating permeability values from seismically-derived fracture parameters. The first is a statistically-based method that calculates the permeability tensor for a given distribution of fractures. The second is a direct flow model of discrete fractures and fracture networks.

1. Statistical model approach – the Oda model

We are continuing to investigate the potential of using the method of Brown and Bruhn (1998), based on the Oda model (Oda, 1985) of fracture networks to estimate the permeability tensor from seismically derived fracture parameters. One of the benefits from using this type of approach is that the input to such a model is similar to the format of the data that we can estimate with our seismic methods. Figure 19 shows an example of this. In the left panel we see the rose diagrams obtained from the Transfer Function-Scattering Index analysis method applied to the field data set as given in Figure 17. The right panel shows the permeability tensor estimated from the Oda model (S. Brown, personal communication). In this figure the radial distance represents the permeability value. Key parameters that are needed for this model include fracture aperture, length, and response to applied stress fields. For this example, a uniform fracture aperture of 70 microns was used (based on core measurements from a well in the field) and a constant fracture length of 7m was assumed. This modeling approach looks quite promising.

Figure 19. Left panel: angular histograms plots of the fracture distributions estimated from the field data (same as Figure 17). The top of each plot represents fractures aligned North. Right panel: estimated permeability tensor computed from the Oda model as adapted by Brown (personal communication). Fracture apertures and lengths were assumed to be 70 microns and 7 meters respectively. The aperture value was based on mean fracture apertures measured in cores from the field site.

2 Fluid flow modeling – computational fluid dynamics

For the numerical solution of the steady-state Navier-Stokes equation, we use a commercial Computational Fluid Dynamics (CFD) code (FLUENT). The code uses a finite volume based technique (Versteeg and Malalasekera, 1995). The code is supplemented by a CAD based geometry construction and meshing engine, which allows users to build and mesh complex flow models to be used by the solver. Fracture flow modeling with this tool provides a means of visualizing flow in fracture networks and comparing expressions for equivalent fracture aperture flow to the actual flow. We constructed a series of two dimensional models of fractures and fracture networks. These tests provided the following results:

1) For fractures connected in series, the equivalent hydraulic aperture is a weighted harmonic mean of cubed apertures of all fractures. 2) For fractures connected in parallel, the equivalent flow is simply the sum of all flows through individual fractures. 3) If a fracture is inclined with respect to the axis of pressure gradient, then the amount of flow will be reduced by the cosine of the inclination angle. 4) Any network of randomly intersecting fractures can be replaced by a single fracture to give flow equivalence; the aperture of that equivalent fracture will roughly be close to the aperture of the fracture in the network that cuts across the boundaries (inlet and outlet) in the most continuous fashion and has the smallest inclination (with respect to the pressure gradient axis).

Simple 3-D models of a single rough fracture have also been constructed, and more realistic networks of such fractures will be tested in the future. A full description of these modeling results is given in Appendix E.

C. Borehole dispersion analysis and stress estimation

There are no new results to report in this area. Development continues on a method to jointly invert for azimuthal angle and dispersion relations from cross dipole data, which are needed for in-situ stress estimation.

D. Field Data

We are using two field data sets for analysis and testing of methods. Agip provided a 3-D Ocean Bottom Cable (OBC) multicomponent seismic data set from an offshore fractured carbonate reservoir in the Adriatic Sea. Well log and production information has also been provided. Shell Oil Company provided the second data set. This data set includes a 3-D seismic data set from an onshore field in the Middle East. The field is a fractured carbonate field with a well-characterized fracture network. The availability of data from two different fields provides us with an opportunity for more thorough testing of our methods. The Agip

data has been used for most of our seismic method tests (as shown in this report). Analysis of the Shell data will get underway in the coming quarter.

IV. Conclusions

Two methods for estimating fracture orientation and density in subsurface reservoirs from scattered seismic wavefield signals have given promising results. Peaks in the wavenumber spectra of the integrated amplitudes of the scattered waves as a function of offset in pre-stack data correspond to fracture spacing in numerical model data. This method has not yet been tested on field data. The Transfer Function-Scattering Index Method has been extensively tested on numerical model data as well as field data. The method appears to be quite stable and robust and the results to date suggest that the approach is sensitive to fracture orientation and fracture density. Further tests with more complex fracture network models and with a second field data set are planned in the coming months. An AVOA analysis method was refined and applied to the same field data set. An iterative, non-linear least squares inversion that uses the Gauss-Newton method and analyzes the full range of azimuths simultaneously was employed. Resulting fracture location and strike orientation estimates are consistent with other fracture information from the area. Fracture distribution estimates from field data using the Transfer Function-Scattering Index method were used as input to a statistical fracture network model to estimate the permeability tensor. Although there are additional parameters needed for such a model (specifically, fracture aperture and length), this approach shows promise. We have also investigated flow modeling of discrete fractures and fracture networks using a commercial Computational Fluid Dynamics (CFD) code. This tool provides a means of visualizing flow in fracture networks and comparing expressions for equivalent fracture aperture flow to the actual flow.

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Appendix A

Reservoir Fracture Orientation and Density from Reflected and Scattered Seismic Energy

by

M. E. Willis, F. Pearce, D .R. Burns, J. Byun, and B. Minsley
(presented at the 2004 EAGE Meeting)

Z-99 Reservoir Fracture Orientation and Density from Reflected and Scattered Seismic Energy

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Abstract

Synthetic seismic shot records were generated using a 3D elastic finite difference modeling code simulating the seismic scattering from sets of discrete, large-scale vertical fractures in a subsurface reservoir. We used these shot records to develop data analysis methods applicable to reflection seismic field data in order to determine the orientation and density of the fractures. For shot records acquired parallel to the fractures, coherent, ringing scattered events are seen across the near to mid offset ranges. These traces stack together constructively when a reasonable NMO velocity correction is applied. In contrast, for shot records acquired normal to the fracture direction, the observed scattered wavefield is greatly disruptive and does not stack together well with any NMO velocity. From these observations, the strike of the fracturing may be determined by identifying the azimuthal acquisition direction with shot records containing coherent, ringing energy which are enhanced the most when stacked. Two methods were developed to estimate fracture density variations from the scattered wavefield signals. In the first, fracture density is estimated from the wavenumber spectra of the integrated amplitudes of the scattered waves as a function of offset in pre-stack data. The second method uses a time variant deconvolution process to extract interval estimates of scattering, which provides a means of separating the effects of fracture scattering from other forms of non-reservoir, overburden scattering. The method was applied to OBC field data from a fractured reservoir and the results were consistent with known fracture trends in the field.

Introduction

The way in which fractures in rock formations affect the seismic waves propagating through them depends on the fractures' mechanical parameters, such as compliance and saturating fluid, and on their geometric properties, such as dimensions and spacing. If the fractures and fracture spacing are small relative to the seismic wavelength, then the fractures cause the reservoir rock to behave like an equivalent anisotropic medium with a symmetry axis normal to the strike of the 'open' fractures. Resulting seismic reflections from the top and bottom of a fractured reservoir will display amplitude variations with offset and azimuth (AVOA) (e.g., Lynn et al., 1996). If the fractures and fracture spacing are close in size to the seismic wavelength, then the fractures will scatter the seismic energy causing a complex seismic signature or coda. This seismic signature will vary as a function of the orientation of the seismic acquisition relative to the fracture orientation. The scattered seismic energy not only provides information about the fracture orientation, but can also be analyzed to provide information about the fracture spacing and fracture density. Observations by previous researchers have demonstrated that scattering effects are sensitive to fracture density in field data as well as numerical and laboratory experiments (e.g., Ata and Michelena, 1995).

Modeling Results

We used the 3-D elastic finite difference code developed by Lawrence Berkeley National Laboratory (Nihei et al., 2002) to model a simple reservoir geometry consisting of five horizontal layers. All the layers except the third layer, are homogeneous and isotropic elastic media. The third layer is 200-m thick and contains parallel, vertical fractures which are as tall as the layer, one grid cell thick (5m) and run the entire width of the model. Table 1 lists the properties of each layer. We generated a series of models with the following fracture spacings: no fractures, 10m, 25m, 35m, 50m, and 100m. More complex models, one with two sets of fractures with different spacing and orientation and the other with a Gaussian distribution of spacing, were also computed. All models use a 40Hz Ricker wavelet as the seismic source.

On the left side of Figure 1 are the shot records for the 50m fracture spacing case acquired in directions normal and parallel to the fractures. To the right of each shot record is its velocity analysis. It is clear on the shot record normal to the fracture direction that there is no coherent stackable energy

in or below the reservoir level (300 ms). However, for the parallel case, there are many coherent events that can be seen primarily below the base reservoir reflection on the shot record and observed on the velocity analysis. In the direction parallel to the fractures, the seismic energy seems to be guided by the aligned fractures and the resulting scattered energy is more coherent and similar to the direct P wave reflection. This same pattern of azimuthal variation in the modeled scattered wavefields is observed for all the model results.

For shot records acquired parallel to the fractures, the coherent, ringing scattered events are seen across the near to mid offset ranges. These traces stack together constructively when a reasonable NMO velocity correction is applied. For shot records acquired normal to the fracture direction, the observed scattered wavefield is greatly disruptive with significant back scattered, diffraction-like events. These traces do not stack together well with any NMO velocity. From these observations, the strike of the fracturing may be determined by identifying the azimuthal acquisition direction with shot records containing coherent, ringing energy which are enhanced the most when stacked.

Figure 1 Left two plots show muted seismic shot records for the model with 50m fracture spacing. The top left plot shows the shot record normal to fractures, the left bottom plot shows the shot record parallel to the fractures. The right two plots show the velocity spectra for the normal shot record (top) and the parallel shot record (bottom).

Integrated Amplitude Analysis of Scattered Waves

Having identified the orientation of the fractures using the method described immediately above, we next examine how to extract the fracture spacing or density from the scattered field. Close inspection of the shot records acquired normal to the fracture direction reveals a beating or resonance phenomenon. The pattern of beating is different for each of the fracture spacing cases, and is most prominent on the near to middle offset range of traces. To study this, we computed the integrated absolute value of the amplitudes for each trace in a given shot record in an offset and velocity controlled time window through the scattered energy. The far offset traces were omitted from the analysis. The integrated amplitudes will be affected by the AVO response of the top and bottom reflection of the reservoir. However, this effect will be a predominantly long wavelength term on the integrated amplitudes versus offset. The approximate effect of the AVO term is removed by subtracting the best fit linear trend with offset from the integrated amplitudes prior to spectral analysis.

The top panel of Figure 2 shows the integrated trace amplitude versus offset for the 50m fracture case. The analysis window followed the NMO velocity of the reservoir. The bottom panel of Figure 2 shows the wavenumber amplitude spectrum computed from the fast Fourier transform of the detrended amplitudes. Clearly visible in the wavenumber spectrum is a peak at 0.02/m, which

corresponds to the fracture spacing of 50m. The same analysis was done on the other fracture spacing cases. Spectral peaks correctly identified the 100m, 50m and 35m fracture spacings, however due to the combination of source frequency content, receiver spacing and reservoir velocity, the 25m and 10m spacing cases were not resolvable. For the model of a Gaussian distribution of fractures with a mean spacing of 50m, a broader spectral peak was still present 0.02/m.

Figure 2 Top) The integrated absolute value of the amplitudes of the 50m fracture case acquired in the normal direction. Bottom) Wavenumber spectra of the integrated amplitude plot shown on top. Note the peak for the 50m spacing (0.02 fractures/m).

Interval Scattering Using a Transfer Function

In model data with a simple, temporally separated reflectivity sequence, one might expect to be able to observe ringing, scattered wavetrains or coda waves. However, in field data where it is difficult to visually observe where the source wavelet ends and where the next reflected event starts, one might expect to have a more difficult time observing these scattered wavetrains. In addition, if there are fractured zones or other scatterers in the overburden, those scattered waves will contaminate, or overprint, the scattered energy from our zone of interest in the reservoir. So there is a need to develop a method of computing an interval estimate of scattering that can handle such complicated reflectivity series.

We developed an interval scattering estimation method that calculates the transfer function for a zone of interest using deconvolution. The method is simple for the case of two isolated reflections, one from the top of the reservoir and the other from the bottom of the reservoir. The transfer function is computed by deconvolving the top reflection (as the input) from the bottom reflection (as the output). The transfer function then characterizes the effect of scattering only in the interval of interest. A simple pulse shape indicates no scattering, while a long ringy transfer function captures the scattering within the reservoir interval. Notice that any contamination from scattering above the interval will be present in both the input and output reflections and thus will be excluded from the transfer function. For more realistic reflectivity series, we modified the method to use autocorrelations of longer time windows containing the wavetrains above (as input) and below (as output) the interval of interest for the deconvolution process. This approach can be applied to a single zone or in a continuous fashion down each trace. The presence of fractures should be detectable from observed ringy transfer functions computed for each time interval.

This method can be used on both pre-stack and stacked data. If it is applied to data stacked in different azimuthal directions, the interval transfer function should exhibit more ringiness in the direction parallel to fracturing. The method was applied to supergathers of field data acquired over a fractured reservoir using an ocean bottom cable. Results from an area in the field near a well location show strong temporal and azimuthal variations of the transfer function (Figure 3). The results indicate high degrees of scattering at a travel time between 2.2 and 2.5 seconds (the reservoir interval), with a dominant bearing of 80°-100°. Core data from the well indicates a high degree of fracturing with orientations in agreement with this scattering analysis.

Figure 3. Scattering as derived from the interval transfer function computed for a supergather of seismic data over a fractured reservoir. The reservoir interval is in the time range of 2.2 to 2.5 seconds on the vertical axis. Red values high scattering. Blue values show lower scattering, or a more compact operator. Azimuthal variations in the transfer function at a given analysis time reveal the fracture orientation, while the amplitude of the scattering operator may indicate fracture density.

Conclusions

Large-scale zones of fracturing control fluid flow in certain reservoirs, and such zones can scatter seismic energy depending on the fracture density and compliance, as well as the spacing relative to the seismic source wavelength. This scattered wave energy contains information about the fracture properties. Using numerical modeling data we developed two methods of analyzing scattered wave energy from fractured reservoirs. The spectral content of the integrated trace amplitude of pre-stack data can provide an estimate of the mean spacing of major fracture zones. A deconvolutional process that measures the ‘ringiness’ of the transfer function can be used to estimate fracture orientation and density from azimuthal stacked data. Initial application of the method to field data shows promising results.

Table 1. Parameters for model

Layer	Thickness (m)	Vp (m/s)	Vs (m/s)	Density (g/cc)
1	200	3000	1765	2.2
2	200	3500	2060	2.25
3	200	4000	2353	2.3
4	200	3500	2060	2.25
5	200	4000	2353	2.3

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Appendix B

Spatial Orientation and Distribution of Reservoir Fractures from Scattered Seismic Energy

by

M. Willis, R. Rao, D. Burns, J. Byun, and L. Vetri
(Expanded Abstract, 2004, 74th SEG Annual Meeting)

Spatial orientation and distribution of reservoir fractures from scattered seismic energy

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Summary

Details of a new method (Willis et al, 2004) for determining the reflection and scattering characteristics of seismic energy from subsurface fractured formations are presented. The method is based upon observations made from 3D finite difference modeling of the reflected and scattered seismic energy over discrete systems of vertical fractures. Regularly spaced, discrete vertical fractures impart a ringing coda type signature to any seismic energy which is transmitted through or reflected off of them. This signature varies in amplitude and coherence as a function of several parameters including: 1) the difference in angle between the orientation of the fractures and the acquisition direction, 2) the fracture spacing, 3) the wavelength of the illuminating seismic energy, and 4) the compliance, or stiffness, of the fractures. This coda energy is greatest when the acquisition direction is parallel to the fractures, the seismic wavelengths are tuned to the fracture spacing, and when the fractures have low stiffness. The method uses surface seismic reflection traces to derive a transfer function which quantifies the change in an apparent source wavelet propagating through a fractured interval. The transfer function for an interval with low scattering will be more spike-like and temporally compact. The transfer function for an interval with high scattering will ring and be less temporally compact. When a 3D survey is acquired with a full range of azimuths, the variation in the derived transfer functions allows the identification of subsurface areas with high fracturing and the orientation (or strike) of those fractures. The method was calibrated with model data and then applied to a field with a fractured reservoir giving results which agree with known field measurements.

Introduction

The way in which fractures in rock formations affect the seismic waves propagating through them depends on the fractures' mechanical parameters, such as compliance and saturating fluid, and on their geometric properties, such as dimensions and spacing. If the fractures and fracture spacing are small relative to the seismic wavelength, then the fractures cause the reservoir rock to behave like an equivalent anisotropic medium with a symmetry axis normal to the strike of the 'open' fractures. Resulting seismic reflections from the top and bottom of a fractured reservoir will display amplitude variations with offset and azimuth (AVOA) (e.g., Lynn et al., 1996). If the fractures and fracture spacing are close in size to the seismic

wavelength, then the fractures will scatter the seismic energy causing a complex seismic signature or coda. This seismic signature will vary as a function of the orientation of the seismic acquisition relative to the fracture orientation. The scattered seismic energy not only provides information about the fracture orientation, but can also be analyzed to provide information about the fracture spacing and fracture density. Observations by previous researchers have demonstrated that scattering effects are sensitive to fracture density in field data as well as numerical and laboratory experiments (e.g., Ata and Michelena, 1995). In this paper we present the details of a method to estimate fracture orientation we introduced earlier this year (Willis et al, 2004).

Modeling

We used the 3-D elastic finite difference code developed by Lawrence Berkeley National Laboratory (Nihei et al., 2002) to model a simple reservoir geometry consisting of five horizontal layers. All the layers except the third layer, are homogeneous and isotropic elastic media. The third layer is 200-m thick and contains parallel, vertical fractures which are as tall as the layer, one grid cell thick (5m) and run the entire width of the model. (Willis et al 2004, list the model parameters.) We generated a series of models with the following fracture spacings: no fractures, 10m, 25m, 35m, 50m, and 100m. All models use a 40Hz Ricker wavelet as the seismic source.

On the left side of Figure 1 are the shot records for the 50m fracture spacing case acquired in directions normal and parallel to the fractures. To the right of each shot record is its velocity analysis. It is clear on the shot record normal to the fracture direction that there is no coherent stackable energy in or below the reservoir level (300 ms). However, for the parallel case, there are many coherent events that can be seen primarily below the base reservoir reflection on the shot record and observed on the velocity analysis. In the direction parallel to the fractures, the seismic energy seems to be guided by the aligned fractures and the resulting scattered energy is more coherent and similar to the direct P wave reflection. This same pattern of azimuthal variation in the modeled scattered wavefields is observed for all the model results.

For shot records acquired parallel to the fractures, the coherent, ringing scattered events are seen across the near to mid offset ranges. Figure 2 shows the results of applying

Orientation and distribution of reservoir fractures

Figure 1: Left two plots show the seismic shot records for the model with 50m fracture spacing. The top left plot show the shot record normal to fractures, left bottom plot shows the shot record parallel to the fractures. The right two plots show the velocity spectra for the corresponding shot records on the left.

NMO and stacking the model traces in different azimuthal directions. The trace labeled “normal” corresponds to the stack of the traces in the top left plot of Fig. 1. The trace labeled “parallel” corresponds to the stack of traces in the bottom left plot of Fig 1. In between these two traces are those stacked traces corresponding to 10 degree increments between normal and parallel directions to the fractures. (For comparison, the bottom trace labeled “control” is the trace from the model with no fractures.) For shot records acquired normal to the fracture direction, the observed scattered wavefield is greatly disruptive with significant back scattered, diffraction-like events. These traces do not stack together well with any NMO velocity. From these observations, the strike of the fracturing may be determined by identifying the azimuthal acquisition direction with shot records containing coherent, ringing energy which are enhanced the most when stacked.

Methodology

In the model data we generated, it might be possible to directly measure the scattered wavefield from the isolated reflectors in the reservoir zone. However, in field data we expect to have a more difficult time observing these scattered wavetrains due to the nearly continuous nature of the subsurface reflectivity. In addition, if there are fractured zones or other scatterers in the overburden, those scattered waves will contaminate, or overprint, the scattered energy from our zone of interest in the reservoir. So we start with the concepts from existing methods of

Figure 2: Plot showing the azimuthal stacks of traces from the 50m fracture spacing model. The traces represent azimuth stacks starting in the direction parallel to fracturing (top), and then increasing in 10 degree increments until normal to the fractures. The bottom trace shows the stack for the model without a fractured layer.

seismic wavelet estimation to obtain two apparent (or temporally local) source wavelets from the reflection time series – one from above the fractured zone (the “input” wavelet) and one below it (the “output” wavelet). These wavelets are represented by their autocorrelations obtained from windowed portions of the reflection time series above and below the fractured zone of interest.

We then compute the time domain transfer function between the autocorrelations of the two extracted wavelets. The transfer function is computed by deconvolving the autocorrelation of the input wavelet from the autocorrelation of the output wavelet. This transfer function characterizes the effect of scattering in the interval of interest, between the two windowed portions of the trace. A simple pulse shaped transfer function indicates no scattering, while a long ringing transfer function captures the scattering within the reservoir interval. Notice that any contamination from scattering above the interval will be present in both the input and output extracted wavelets and thus will be excluded from the transfer function.

This method can be used on both pre-stack and post-stacked data. If it is applied to data stacked in different azimuthal directions, the interval transfer function should exhibit greater ringing in the direction parallel to fracturing. Figure 3 shows the transfer functions derived for the stacked traces in Figure 2. Clearly evident on the transfer function for the parallel direction is its ringiness away from

Orientation and distribution of reservoir fractures

the zero lag while in the normal direction the transfer function is comparatively compact.

In order to quantify the amount of transfer function ringiness or non-compactness, we created a scattering index, SI, defined by:

$$SI = \sum_{i=0}^m |t_i| i^n$$

where i is the time lag, t_i is the transfer function (time domain) amplitude at lag i , n is an exponent, typically equal to unity, and m is a lag at which there is no more significant energy in the transfer function. (It is also possible to normalize the scattering index based upon its energy and interval time sample or other such criteria.) The more the transfer function rings, the larger the value of the scattering index. If the transfer function is a simple spike, i.e. representing no scattering, then the scattering index attains a value of zero.

Figure 4 shows the scattering index values for models run with 25, 35, 50 and 100m fracture spacings. These results show that there is a clear maximum of the scattering index in the parallel direction. It is also clear that in the non-parallel directions the scattering index is not zero but fluctuates about a smaller, but fairly consistent value.

Results on field data

In early 2000, a 3D/4C seismic survey was collected over the Emilio Field, located in the central part of the Adriatic Sea, near the eastern coast of Italy. The reservoir unit is a fractured carbonate which from borehole studies suggests the presence of two orthogonal fracture sets oriented ENE and NNW (Angerer et al, 2002). Recent studies have investigated this 3D seismic data using PP and PS wave anisotropy to identify fracture characteristics of the reservoir level (Vetri et al, 2003; Gaiser et al, 2002).

The near to mid range offsets of the preprocessed PP data (Vetri et al, 2003) were stacked in eighteen different azimuthal orientations from East to West using 20 degree wide overlapping ranges. This process created eighteen 3D stacked volumes. The transfer functions and scattering indices for the formation zone were computed for each of these stacked volumes. The scattering indices were sorted and directions for those with the highest ranges of values are plotted in Figure 5, giving a map view of the location and direction of possible fractures determined by this method. Figure 6 shows histogrammed rose plots of the fracture distributions corresponding to the sixteen boxes outlined in green in Figure 5. In several boxes there appears to be no preferred orientation while in many of the boxes the ENE and NNW directions are strongly evident.

Figure 3: Plot showing the transfer functions corresponding to azimuthal stacks of traces (in Fig. 2) from the 50m fracture spacing model. The transfer functions correspond to azimuth starting in the direction parallel to fracturing (top), and then increasing in 10 degree increments until normal to the fractures. The bottom trace shows the transfer function for the model without a fractured layer.

Azimuthal Scattering Indices for Four Fracture Spacings

Figure 4: Polar plot of the azimuthal variation of scattering indices derived from the transfer functions of the 25, 35, 50 and 100m fracture spacing models. The scattering index is largest in the direction parallel to the fracture orientation. The largest scattering index is for the 35m fracture spacing, while the smallest shown is for the 25m spacing.

Orientation and distribution of reservoir fractures

Conclusions

Large-scale zones of fracturing control fluid flow in certain reservoirs, and such zones can scatter seismic energy depending on the fracture density and compliance, as well as the spacing relative to the seismic source wavelength. This scattered wave energy contains information about the fracture properties. Using numerical modeling data we developed a method of analyzing scattered wave energy from fractured reservoirs. A deconvolutional process that measures the 'ringiness' of the transfer function can be used to estimate fracture orientation from azimuthally stacked data. The application of this method on field data provide fracture orientation estimates which generally agree with previous borehole and PS anisotropy studies in the Emilio field.

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Figure 5. The spatial orientation and distribution of fractures over the Emilio field as indicated from scattering indices of multiple azimuthal stacks. The black and red bars show the orientations for the top two bins of highest scattering indices. The green grid shows the location of the histogram plots in figure 6.

Figure 6. Angular histograms plots of the fracture distributions shown in figure 5. Each plot represents the corresponding subportion of the field cut up by the green dashed lines in Figure 5. The top of each plot represents fractures aligned North.

Appendix C

**Investigation of a Fractured Reservoir Using P-wave AVOA Analysis:
A Case Study of the Emilio Field with
Support from Synthetic Examples**

by

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and M. N. Toksoz**

(Expanded Abstract, 2004, 74th SEG Annual Meeting)

Investigation of a fractured reservoir using P-wave AVOA analysis: a case study of the Emilio Field with support from synthetic examples

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Summary

Much evidence has been given for the utility of P-wave azimuthal AVO (AVOA) analysis for vertically fractured reservoirs modeled as HTI media. Although these investigations typically conclude that P-wave AVO information alone cannot quantify absolute fracture parameters (Lynn and Cox, 2003), there is some value in this relatively inexpensive method. In this investigation, we aim to characterize a fractured reservoir in the Emilio Field using P-wave data from a 3D OBC survey. A non-linear least squares inversion method is developed to recover azimuthal AVO parameters using a surface-fitting approach similar to that employed by Hall and Kendall (2003). Synthetic data generated by a 3D finite difference code is also analyzed in order to better understand the behavior of this routine under various fracture and overburden conditions.

Introduction

The Emilio Field is situated about 25 km offshore the east coast of Italy in the central Adriatic Sea. This area is characterized by a NNW trending anticlinal feature formed by E-W compression. The reservoir target is located at a depth of 2850m in fractured limestone. Fractures at the reservoir level are thought to be mostly near-vertical and aligned with the direction of maximum compressive stress, which is slightly north of east.

Analysis of several seismic surveys over this area has attempted to characterize the anisotropic behavior in the Emilio Field due to fractures (Gaiser *et al.*, 2002; Vetri *et al.*, 2003). These investigations focus on shear wave birefringence and velocity analysis as anisotropy indicators, though AVO is also discussed. Results show anisotropic behavior near the reservoir level and are corroborated with borehole and production well information.

In this study, P-wave azimuthal AVO analysis is applied to the Emilio 3D OBC field data. A 3D finite difference model is also analyzed using the same routine to help quantify the ambiguity in AVO inversion results.

AVOA inversion method

The basis for the azimuthal AVO inversion used here comes from Rüger (1998). P-wave reflection amplitudes at

near-to-intermediate angles of incidence are given by Equation (1).

$$R_{pp} \approx A + [B^{iso} + B^{ani} \cos^2(\varphi_k - \varphi_{sym})] \sin^2 \Theta \quad (1)$$

‘A’ is the standard AVO intercept that corresponds to zero offset reflection. The AVO gradient, ‘B’, has been split into two parts, B^{iso} and B^{ani} , to reflect the azimuthal behavior of this term. The azimuthal variation in the AVO gradient comes through the $\cos^2(\varphi_k - \varphi_{sym})$ term where φ_k represents the recorded source-to-receiver azimuths and φ_{sym} is the symmetry axis (fracture normal) of the HTI media.

An iterative, non-linear least squares inversion routine is used to fit a surface to reflection data using the four-term AVOA model in Equation (1). The inversion involves the calculation of the Jacobian ($\mathbf{A} = \partial M_i / \partial \tilde{x}_i$) at each iteration. M_i is the reflectivity model given in Equation (1) and $\tilde{x}_i$ represents the four model variables A, B^{iso} , B^{ani} , and φ_{sym} . Starting with an initial guess of the four unknown parameters, updates are made according to Equation (2).

$$d\tilde{x}_i = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T (y - M_{i-1}) \quad (2)$$

The vector y represents the amplitudes extracted from the data for each reflector. At each iteration, the solution $\tilde{x}_i$ is updated and the RMS error is calculated between the measurements, y , and the current model, M_i . When the error becomes sufficiently small, the solution has converged to $\tilde{x}_i$.

Due to the non-linear nature of Equation (1) there are two sets of solutions for the four AVOA parameters that have values for φ_{sym} separated by 90 degrees. In one case, B^{iso} and B^{ani} will have the same sign, indicating maximum AVO response in the fracture normal direction. In the other case, B^{iso} and B^{ani} have opposite signs and the maximum response is along the fracture strike.

This non-uniqueness can be accounted for if the general fracture orientation or AVO behavior is known. Synthetic data analysis can help to describe how these features behave under various conditions. Shen *et al.* (2002) show synthetic examples of gas saturated fractures exhibiting a maximum response normal to the fractures, while water saturation has a maximum response in the strike direction.

Examples

Finite difference modeling

Investigation of a fractured reservoir using P-wave AVOA analysis

A nine-layer 3D model (Figure 1a) with alternating fast and slow formations given in Table 1 is constructed for the finite difference forward modeling. The middle three layers have vertical fractures oriented normal to the x-axis embedded in the background material. The fracture density and aspect ratio are 0.1 and 0.01, respectively.

Table 1. Material properties for the synthetic model

	V_P (m/s)	V_S (m/s)	ρ (g/cm ³)
fast formation	4000	2352.9	2.5
slow formation	3000	1764.7	2.4

The wide azimuth-offset distribution for this dataset is displayed in Figure 1b. CMP data created from the finite difference results with this model are shown in Figure 1c after NMO, spherical divergence correction, and mute.

For each reflection in Figure 1a the data are inverted for the four AVOA parameters using the non-linear inversion routine discussed above. Results of the inversion are shown in Figure 1d-e. Figure 1d shows model predictions for the azimuthal reflection amplitudes (left) and azimuthal variation in the AVO gradient (right) for a reflection exhibiting anisotropic behavior due to the fractures. Figure 1e displays all four AVOA parameters at each reflector. General trends expected from the alternating lithology and fractured layers can be seen, though distortion due to the overburden is evident in the deeper layers.

Emilio field data

A similar procedure is followed for the 3D/4C OBC data from the Emilio Field. Figure 2a shows the azimuth-offset distribution for a collection of CMP gathers used in the AVOA analysis. Several gathers combined into a single CMP (Figure 2b) help to improve the signal-to-noise ratio. Results of the application of the AVOA inversion routine on a sliding window through this gather are shown in Figure 2c. Although the results are much noisier, there is an interesting trend of nearly 0° for ϕ_{sym} starting near 1.8s. This corresponds to the expected fracture orientation and may be indicative of the stronger anisotropic signal in this region.

Conclusions

Azimuthal AVO analysis of wide offset-azimuth P-wave data can provide useful information about anisotropy due to fractures. These datasets have become readily available and do not require complicated processing procedures. To overcome the uncertainties involved with these results, it is helpful if other information is included to constrain the problem. In this case, general knowledge about the reservoir and inferences from synthetic results are useful.

Further analysis of larger portions of the Emilio dataset is needed to validate the methods presented here. Calibration with other synthetic models will help to understand the behavior of this routine under various conditions.

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Investigation of a fractured reservoir using P-wave AVOA analysis

Figure 1: Synthetic model and AVOA results. (a) Nine-layer synthetic model with properties given in Table 1. (b) Azimuth-offset distribution for the synthetic model. (c) Finite difference results for synthetic model after NMO, spherical divergence correction, and mute. (d) Example of inverted AVO behavior for an interface that exhibits anisotropy. Left side shows amplitude behavior as a function of offset and azimuth, right side shows the azimuthal variation of the AVO gradient, correctly oriented near zero degrees. (e) AVOA inversion results for all eight reflections showing the effect of the alternating layers and fractures, though overburden effects are present.

Investigation of a fractured reservoir using P-wave AVOA analysis

Figure 2: Emilio field data and AVOA results. (a) azimuth-offset distribution for the CMP 'supergather' used for the AVOA analysis. (b) Selected window of CMP data used in the analysis. (c) Inversion results for the four AVOA parameters along the CMP gather. A sliding analysis window is used to capture events on the gather.

Appendix D

Seismic Scattering Attributes to Estimate Fracture Density: A Numerical Modeling Study

by

F. D. Pearce

(S.M. Thesis, 2003, Massachusetts Institute of Technology)

Seismic Scattering Attributes to Estimate Reservoir Fracture Density: A Numerical Modeling Study

by

Frederick D. Pearce

B.S. Geological Engineering
University of Idaho, 2001

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Seismic Scattering Attributes to Estimate Reservoir Fracture Density: A Numerical Modeling Study

by

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Abstract

We use a 3-D finite difference numerical model to generate synthetic seismograms from a simple fractured reservoir containing evenly-spaced, discrete, vertical fracture zones. The fracture zones are represented using a single column of anisotropic grid points. In our experiments, we vary the spacing of the fracture zones from 10-meters to 100-meters, corresponding to fracture density values from 0.1- to 0.01-fractures/meter, respectively. The vertical component of velocity is analyzed using integrated amplitude and spectral attributes that focus on time windows around the base reservoir reflection and the scattered wave coda after the base reservoir reflection. Results from a common shot gather show that when the fracture zones are spaced greater than about a quarter wavelength of a P-wave in the reservoir we see 1) significant loss of amplitude and coherence in the base reservoir reflection and 2) a large increase in bulk scattered energy. Wavenumber spectra for integrated amplitude versus offset from the time window containing the base reservoir reflection show spectral peaks corresponding to the fracture density. Frequency versus wavenumber plots for receivers normal to the fractures separate backscattered events that correspond to spectral peaks with positive wavenumbers and relatively narrow frequency ranges. In general, backscattered events show an increase in peak frequency as fracture density is increased.

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1. Introduction

Fractures are common within the subsurface and play a critical role in the mechanical and fluid flow properties of earth materials. In regions where the maximum compressive stress is vertical, open fractures will tend to be oriented vertically. The ability to interpret properties such as fracture spacing, orientation, and fluid content associated with such fracture systems is vital to the effective extraction and management of reservoir resources and the monitoring of contaminant migration. In particular, the ability to delineate zones of high fracture density is a key component in developing reservoir-scale fluid flow models.

The way in which fractures affect seismic waves depends on mechanical parameters, such as compliance and saturating fluid, and on their geometric properties, such as dimensions and spacing. When fractures are small relative to the wavelength of the seismic waves, the waves will be only weakly affected by the fractures. If fractures have characteristic lengths on the order of the wavelength then there will be scattering of energy due to the presence of the fractures. Similarly, fractures spaced much closer than the wavelength of the seismic waves are not individually sampled and result in a homogeneous, anisotropic medium. When fractures are spaced further away (about a quarter of a wavelength or more) the seismic waves will interact with the fractures and scatter appreciable energy.

Observations by previous researchers have demonstrated that scattering effects are sensitive to fracture density in real data as well as numerical and laboratory experiments (Ata and Michelena, 1995 and Rathore et al., 1994). In addition, numerical modeling of fractured media has proven to be an efficient way to explore the feasibility of using seismic waves to estimate fracture parameters. In particular, the applicability of finite difference methods to simulate wave propagation in a fractured elastic medium has been well documented in the

literature (Shen and Toksoz, 2000, Daley et al. 2002, Nihei et al., 2002, Wu et al. 2002, Groenenboom and Falk, 1999, and Nakagawa et al. 2002).

Most previous work in estimating fracture parameters from seismic reflection data has focused on using an equivalent medium approach where the fractures themselves or the spacing between the fractures is assumed to be small relative to the wavelength of the seismic wave. In this paper, we explore the characteristics of reflected and scattered elastic waves that may be used to develop seismic attributes sensitive to fracture spacing. A 3D finite difference method is used to simulate wave propagation through a simple fractured reservoir which is modeled as a single horizontal layer containing parallel, evenly-spaced fractures. Previous work has shown that the finite difference method is accurate over a wide range of scattering regimes and can account for all converted waves and their interactions (Shen et al., 2002). Synthetic seismograms are generated along transects perpendicular and parallel to the fracture orientation and we analyze spectral characteristics of the wavefield scattered from the fracture zones. Our objective is to investigate how the intensity of seismic scattering changes with the spacing of discrete fractures, and develop potentially diagnostic seismic attributes sensitive to the fracture spacing. Our analysis focuses on the P-wave reflection from the bottom of the reservoir and the coda waves that follow the primary reflections.

2. Background

The importance of fractures in the production and stimulation of reservoirs has resulted in considerable efforts to develop methods for fracture detection and characterization using seismic waves. In particular, offset dependent attributes are useful in characterizing vertical fractures because of the variation in incidence angle of waves impinging on the fractures.

2.1 Equivalent Medium

Much effort has focused on measuring amplitude variations with offset (AVO) for seismic reflections from a fractured reservoir (Gray et al., 2002, Perez et al., 1999, and Shen et al., 2002). Amplitude variation with offset and azimuth (AVOA) has been widely used to identify the orientation of vertical fractures with variable levels of success (Ruger, 1998; Mallick et al., 1998; Perez et al., 1999; Shen and Toksoz, 2000; Minsley et al., 2003). In these applications, the fractures are assumed to be small relative to the wavelength of the seismic waves. This allows the fractured medium to be modeled using an equivalent anisotropic medium (Schoenberg and Douma, 1988). Others have looked at shear wave data for fracture analysis (Lynn et al., 1995; Gaiser et al., 2002).

Shen et al. (2002) used numerical experiments and field data from a fractured reservoir in Venezuela to demonstrate the advantages of using attributes based on frequency variations with offset and azimuth (FVOA) in addition to AVOA based attributes to further constrain the interpretation of parameters related to the presence of vertically aligned fractures.

2.2 Discrete Fracture Zones

Recently, researchers have begun to focus on relaxing the assumption of relatively small fracture spacing by incorporating the effects of discrete fractures. Shen and Toksoz (2000) used stochastic models to generate heterogeneous fracture density distributions. They examined the effect of such fracture density variations using both time-domain and frequency-domain attributes and found that the presence of the fractures was more easily identified in the attributes associated with the base reflector. They concluded that first-order effects of fractures on FVO and AVO were controlled by the mean fracture density used in their stochastic fracture representation; however, they did not investigate systematic variations in fracture spacing.

Daley et al. (2002) conducted finite difference modeling of discrete fracture zones aimed at identifying scattering mechanisms and resolvability issues. They concluded that fracture tip diffractions and P-to-S conversions are the dominant scattered events from discrete fracture zones observable in surface seismic data, and identified fracture stiffness, spacing, and spatial scale as important parameters in controlling the resolvability of discrete fracture zones. Although their fractured reservoir model is similar to the one used in this paper, they do not specifically address the development of scattering-based attributes sensitive to variations in fracture density.

Nakagawa et al. (2002) developed a hybrid numerical method that combines the plane wave method and the finite element method in order to examine the characteristics of the scattered wavefield from a layer containing discrete, vertical, evenly spaced fractures. However, their paper is more concerned with the presentation of the model and doesn't address variations in background velocities between fractured and unfractured layers or the fracture spacing.

3. Modeling

As discussed in the previous section, a great deal of research has focused on the applicability of finite difference methods to modeling seismic scattering phenomena. In particular, Nihei et al. (2002) test the performance of the Coates and Schoenberg (1995) equivalent medium anisotropic cell approach for FD modeling of discrete fractures. The finite difference method is advantageous when modeling the scattering effect of discrete fractures because of its stability over a wide range of material property contrasts and its ability to model all wave types with minimal numerical dispersion and anisotropy. In addition, synthetic

seismograms can be generated at any point within the model domain facilitating the simulation of surface seismic acquisition geometries.

3.1 Model Configuration

We use the time domain, staggered grid finite difference code developed by Lawrence Berkeley National Laboratory (Nihei et al., 2002), which solves the 3-D, anisotropic stress-velocity formulation of the elastic wave equation (Levander, 1988). This formulation accounts for geometric spreading but does not include intrinsic attenuation. The finite difference operator is 4th order accurate in space and 2nd order accurate in time. The model uses constant grid spacing in all directions of 5-m and a time step of .5-msec. For all model runs, the source is a Ricker wavelet with a center frequency of 40-Hz.

3.2 Model Geometry

Our numerical experiments use a simple reservoir geometry consisting of three horizontal layers. The first and third layers bound the reservoir and are homogeneous and isotropic with the same material properties. The second layer, which is 100-m thick, contains the fractures. Figure 1 shows the model geometry for a fracture density of .02-frac/m (i.e. 50-m fracture spacing) and the background velocities used for all model runs. Although absorbing boundary conditions are incorporated into the model, the boundaries are also placed far enough away so that any boundary reflections are not recorded in the seismograms during the time interval of interest.

The model uses a standard surface seismic reflection acquisition geometry, however, the source and receivers are embedded within the first layer to eliminate free surface effects. Receivers are placed in the crack normal and crack parallel directions with the first receiver fixed at the source. The receiver spacing is 20-m with the maximum source-receiver offset at 800-m which corresponds to a maximum angle of incidence of 45° relative to the top of the reservoir.

The fractures begin at a distance in the x-direction of 5-m from the edge of the grid and are then placed in even increments across the reservoir as dictated by the fracture density. The source is also fixed at a point above the reservoir corresponding to a distance in the x-direction of 550-m from the edge of the model domain. As a result, the actual locations of fractures relative to receivers are different for models with different fracture densities.

3.3 Fracture Representation

We incorporate discrete vertical fracture zones into the reservoir layer using a single column of anisotropic grid points. The elastic properties of the discrete fractures are calculated using the Coates and Schoenberg (1995) thin anisotropic grid cell approach. The fractures are assumed to have negligible mass and thickness relative to the seismic wavelength in accordance with the linear slip model of Schoenberg (1980) for imperfect interfaces. The fractures are given equal normal and tangential stiffness of 8×10^8 -Pa/m (Nihei et al., 2002). We assume fracture zones are bed terminated and have constant fracture density across the reservoir for each of our simulations.

4. Results

We generated synthetic data using the finite difference code and model geometry presented in the previous section. Given the source peak frequency of 40-Hz and the P- and S-wave velocities, the wavelengths in the reservoir are about 100-m and 60-m for P- and S-waves, respectively. Synthetic seismograms were produced for the no fracture case as well as for five different fracture density values: 0.01, 0.02, 0.0286, 0.04, and 0.1-frac/m (corresponding to fracture spacing values of 100, 50, 35, 25, and 10-m).

For the purposes of describing the model output, we will focus on two time windows of the time series. The first travel time window is bounded by the two coherent P-wave reflections and the second window contains all data after the estimated P-wave reflection from the base of the reservoir. We will focus on the vertical component of velocity for receiver arrays oriented normal and parallel to the fractures

4.1 Receiver Array Normal to Fractures

Figure 2 shows the vertical component of velocity recorded at each receiver position normal to the fractures for each fracture density case. The no fracture case is useful for identifying the main P- and S-wave reflections from the top and bottom of the reservoir. The first time window contains only the coherent P-wave reflections. However, careful observation shows a third event separates from the 2nd P-wave reflection at large offsets. The event is associated with the downgoing P-wave reflected as an S-wave at the base of the reservoir, and then converted back to a P-wave upon leaving the reservoir. Because of this conversion, the event has a slightly slower apparent velocity than the primary P-wave reflection from the base of the reservoir.

The dominant events in the second time window are the S-wave reflections, which show rapid increases in amplitude as offset increases because we are looking at only vertical particle velocities from our receivers. The S-wave reflection from the top of the reservoir has a relatively large amplitude while the S-wave from the base of the reservoir has a much smaller amplitude at least partially due to the additional conversions to P-waves as seen in the first time domain.

As we begin to incorporate fractures into the reservoir, we see a drastic change in the character of the time series. For the .01-fractures/meter case, the first time window shows the P-wave reflection from the top of the reservoir remains relatively unaffected while the P-wave

reflection from the base of the reservoir shows significant interference from scattered events starting at about 400-m offset. The apparent velocity of the second P-wave reflection decreases owing to the presence of the more compliant fractures.

In the second time window, we see that both S-wave reflections have become difficult to follow. The S-wave reflection from the top of the reservoir is affected by interference from scattered waves and shows better coherence at larger offsets where its amplitude is large. However, the S-wave reflection from the base of the reservoir is completely drowned out by scattering from the fractures. Overall, the second time window is dominated by many different events derived from interactions with the fractures. Close observation shows these events seem to focus at offsets between 200-m and 600-m and at travel times after the S-wave reflection from the top of the reservoir.

As we increase the fracture density to .02-fractures/meter, the same trends are observed. In the first time window, the P-wave reflection from the top of the reservoir shows an appreciable decrease in amplitude with offset. The P-wave reflection from the base of the reservoir becomes more incoherent at smaller offsets and is more difficult to follow from receiver to receiver. The decrease in amplitude of both P-wave reflections can be attributed to a combination of effects: a decrease in the impedance contrast between the two layers, reflected energy losses from interactions with the fractures, and to interference from scattered waves with relatively short travel paths.

In the second time window, the S-wave reflection from the top of the reservoir is only observable at very large offsets and the S-wave reflection from the base of the reservoir is absent. The scattered events have a very similar appearance to those for the .01-fractures/meter case except there doesn't seem to be a focus of scattered energy in the mid-offset receivers.

For the .0286-fractures/meter case, the first time window shows the P-wave reflection from the top of the reservoir is being interfered with at much smaller offsets (about 400-m). In addition, we observe a substantial increase in the amplitude of events not associated with the primary reflections at large offsets within the first time window. These observations suggest shorter travel paths and larger amplitudes for these scattered events.

The second time window contains a relatively large amplitude S-wave reflection from the top of the reservoir whose amplitude reaches a maximum between 400-m and 500-m offsets. In fact, scattered events tend to have larger amplitudes and arrive at earlier times than the previous cases. Again, these observations support shorter travel paths and larger amplitudes for scattered events.

The .04-fractures/meter case shows a sudden change in the character of the time series. In the first time window, the P-wave reflection from the top of the reservoir is still interfered with at larger offsets but the magnitude of the interference is smaller. The P-wave reflection from the base of the reservoir becomes much more coherent; however, it is now interfered with by an event that arrives at almost the same time even at zero offset and having a very similar move-out behavior. In general, we can now clearly observe that the apparent velocity of the P-wave reflection from the base of the reservoir has decreased substantially from the case with no fractures.

For the second time window, we see that the S-wave reflection from the top of the reservoir is clear and its amplitude increases with offset. The amplitude of the scattered events has decreased substantially and there are patches within the time series where no noticeable scattered energy is present. However, the weak scattered events that remain show similar move-out trends to those observed in the previous fracture density cases.

In the .1-fractures/meter case, we see almost the same behavior as the no fracture case. In the first time window, we only observe the P-wave reflections from the top and bottom of the reservoir. The amplitude of the P-wave from the top of the reservoir decreases more rapidly with offset than the no fracture case. The move-out velocity of the P-wave reflection from the bottom of the reservoir is much smaller. In the second time window, the S-wave reflection from the top of the reservoir is larger in amplitude. Almost no scattered energy is present within either time window. These observations are consistent with the equivalent medium representation that is applicable when fracture spacing is much smaller than the wavelength of the seismic waves.

4.2 Receiver Array Parallel to Fractures

The vertical component of velocity for receivers parallel to the fractures can be observed for each fracture density case in Figure 3. As we expect, the no fracture case has identical time series data as obtained from receivers in the normal direction. For the .01-fractures/meter case, we observe very little change in the first time window. Both P-wave reflections are coherent and the apparent velocity of the P-wave reflection from the base of the reservoir is slightly slower than the no fracture case.

For the second time window, the S-wave reflection from the top of the reservoir is observable at only large offsets. The scattered events are much more coherent from receiver to receiver and seem to have move-out trends similar to the P-wave reflections. In addition, the scattered events have smaller amplitudes than those obtained from the receivers normal to the fractures.

The .02-fracture/meter case shows similar trends to the .01-fractures-meter case. The P-wave reflection from the top of the reservoir is coherent across all offsets. However, the P-wave reflection from the base of the reservoir is difficult to follow at large offsets and has significant

interference from scattered events at small offsets. The S-wave reflection from the top of the reservoir can only be observed at large offsets. Again, we see that the scattered events have move-out trends similar to the P-wave reflections.

For the .0286-fractures/meter case, we again see that the P-wave reflection from the top of the reservoir is unaffected. However, the P-wave reflection from the base of the reservoir loses coherence at an offset of about 400-m. The S-wave reflections can not be distinguished; however, there is a focusing of large amplitude scattered energy around the time interval of the S-wave reflections. In general, this scattered energy has a similar move-out trend but is more concentrated particularly at small offsets.

In the .04-fractures/meter case, both P-wave reflections are very coherent and there is little scattered energy present. In general the .04-fractures/meter case is similar to the .1-fractures/meter case with a couple of exceptions. The .04-fractures/meter case has some small interference of the P-wave reflection from the base of the reservoir as compared to the .1-fractures/meter case. Also, the .04-fractures/meter case has enough residual scattered energy to blur the S-wave reflection from the base of the reservoir while this event is clear in the .1-fractures/meter case.

4.3 Snapshots

Snapshots of the wavefield in the presence of discrete fracture zones highlight the different scattering events and their interactions. Figure 4 shows the vertical component of velocity after .45-seconds for a fracture density of .02-fractures/meter. We see that the P-wave reflection from the top of the reservoir remains relatively coherent, while the P-wave reflection from the base of the reservoir is much more effected by the presence of the discrete fractures. This figure also illustrates the difference in coherence of the scattered wavefield normal and

parallel to the fractures. Normal to the fractures, interference of scattered waves from the fractures results in a complex wavefield (Figure 4a). Fractures act as secondary sources with observable forward and backscattering as well as multiple scattering events. Parallel to the fractures the wavefield is much more coherent and the fractures appear to act as waveguides (Figure 4b).

5. Analysis

The purpose of this section is to present seismic attributes developed to quantify the effect of variations in fracture density in our models.

5.1 Velocity Anisotropy

In general, aligned fractures will result in velocity anisotropy, which is often used as a measure of fracture intensity. Therefore, we examined the effect of variable fracture density on the normal move-out (NMO) velocity estimated by following the P-wave reflections as a function of offset (note: although our models are generated for a single source position with multiple receivers, and therefore are common shot gathers, we treat them as if they were a CMP gather). In our simple model, the NMO velocity of the upper layer will simply be the P-wave velocity specified for the upper layer, 3000-m/s. However, energy reflecting from the base of the reservoir will pass through both layers and potentially fractures as well. Therefore, the NMO velocity of the P-wave reflection from the base of the reservoir should depend on the fracture density and this effect should be strongest when measured normal to the fractures.

To estimate the NMO velocity, we pick the closest peak in amplitude associated with the P-wave reflection from the base of the reservoir at every offset and use the hyperbolic move-out equation to fit the time values of these peaks using a linear least squares regression. Figure 5a

shows the best-fit velocity curve for the .02-fractures/meter case and shows appreciable variation in the peak amplitudes about the best-fit model.

Using this approach, we calculated NMO velocities parallel and perpendicular to the fractures for each fracture density case (Figure 5b). In the parallel direction, the NMO velocities remain relatively constant although slight variations exist at fracture density values between .01-fractures/meter and .0286-fractures/meter. Conversely, the NMO velocities in the normal direction decrease as the fracture density increases and again we see irregular fluctuations in NMO velocity for the fracture density values between .01-fractures/meter and .0286-fractures/meter.

As we pointed out in the previous section, the presence of discrete fractures in our model affects the coherence of the P-wave reflections because of interference from scattered events. Therefore, as our observations indicate, we expect to see the greatest variability in NMO velocity for fracture density values associated with the greatest amount of scattering. This is important because using velocity anisotropy alone to estimate the fracture density associated with large discrete fractures may lead to erroneous results.

5.2 Bulk Scattering

In addition to velocity anisotropy, it is also important to quantify the amount of scattering present in the different fracture density cases. To calculate the amount of scattered energy in each time series output, we select a time window that incorporates as much of the data as possible while giving an equal time duration for each receiver (Figure 5c). We then take the absolute value of the windowed amplitudes and sum them to get a single integrated amplitude value. This procedure is done for the normal and parallel directions and for each fracture density value.

Figure 5d shows the resulting integrated amplitude values as a function of fracture density and orientation. The shapes of the integrated amplitude curves are nearly identical in the normal and parallel case. However, the normal case has much larger integrated amplitude values in the fracture density range between .01-fractures/meter and .0286-fractures/meter. The integrated amplitude values rise sharply as you move away from the no fracture case, reach a peak at the .0286-fractures/meter case, and decline more gradually towards the .1-fractures/meter case. The fracture density of .0286-fractures/meter obtains a value almost three times that of the case with no fractures in the normal direction. The .1-fractures/meter case falls to the no fracture case in the normal direction and actually falls below the no fracture case in the parallel direction, due to a change in AVO response.

5.3 1D Spectra of Integrated Amplitude versus Offset

We have demonstrated that the strongest scattering of seismic energy occurs for the fracture cases between .01-fractures/meter and .0286-fractures/meter. Now we will extend the integrated amplitude approach to develop an offset dependent attribute sensitive to fracture spacing within this strong scattering regime. We will focus on the P-wave reflection from the base of the reservoir and construct a robust window to capture this event and scattered events with similar arrival times. As we have shown, the P-wave reflection from the base of the reservoir is difficult to follow because of interference from scattered events. For this reason, we use a relatively large window based upon the NMO velocity of the P-wave reflection from the top of the reservoir. However, the P-wave reflection from the top of the reservoir does not sample the fractures directly and so will be excluded from the window. The same window is used for all fracture density cases making it robust in its application. Amplitude values within

the window are rectified and then summed to give a single integrated amplitude value for each offset.

Figure 6 shows the plots of integrated amplitude versus offset for each of the fracture density values with receivers normal to the fractures. In the absence of strong scattering, as in the no fracture case, we see that the curves show a smooth line dominated by the AVO response of the P-wave reflection from the base of the reservoir along with secondary effects from P to S conversions as described in the Data section. For the fracture density range associated with strong scattering, we also observe an AVO trend in the integrated amplitudes with offset; however, there are also periodic fluctuations about the AVO trend. These fluctuations are the greatest for the .01-fractures/meter and .02-fractures/meter case and are substantially less in the .0286-fractures/meter case.

Figure 7 displays the same integrated amplitude versus offset plots parallel to the fractures. We see the same trends in these plots but they lack the oscillations about the AVO trend. The difference between the normal and parallel cases is consistent with our earlier observation that scattered events in the parallel case tend to have move-out trends similar to the primary reflections. Therefore, we conclude that the interference patterns observed in the integrated amplitude versus offset from the normal direction may have a direct correlation with fracture density.

To test this hypothesis, we transform the integrated amplitude versus offset data in Figure 6 to the wavenumber domain using the Fast Fourier Transform (FFT). Since the resulting spectra are dominated by the AVO trend in the data, we remove this trend by normalizing the spectra. If the method is to be robust, the fracture density estimate must be insensitive to the spectrum used to normalize. To address this issue, we normalize the spectrum obtained from

each of the strong scattering fracture density values (i.e. .01- .02-, and .0286-fractures/meter) with each of the spectra obtained from the weak scattering regime (Figure 8). Several important observations can be made from the resulting normalized spectra:

- 1) Peaks in the normalized spectra correspond to the input fracture density values for the .01-fractures/meter and .02-fractures/meter cases.
- 2) The ability to resolve the .0286-fractures/meter spacing is beyond the resolution of the analysis, which is determined by the receiver spacing.
- 3) The method is robust with respect to the spectrum used to remove the dominant trend and gives accurate estimates even when the .04-fracture/meter spectrum is used.

5.4 2D Spectra of Time Series versus Offset

The previous analysis method focuses on the time window around the P-wave reflection from the base of the reservoir. However, we have demonstrated that a great deal of energy arrives much later than the P-wave reflections. Attempts at using the integrated amplitude method presented in the previous section for this second time window, resulted in erroneous estimates of the fracture density based on spectral peaks because of the potential superposition of many different scattered events.

To better understand the complexity present in this second time window, we use the 2D FFT to analyze the time series versus offset data. For each fracture case, the same window is applied that captures all the data following the first time window presented in Figures 6 & 7. The traces are zero padded to maintain the same absolute time for all traces. In addition, a cosine taper is applied to smooth the edges of the windowed data.

Figure 9 shows the results of the 2D FFT for each fracture density value when the receiver array is normal to the fractures. In the no fracture case, all of the spectral peaks in

amplitude are associated with negative wavenumbers. The peaks also have several distinguishable ridge-like structures whose slopes are in the range of the NMO velocities of the S-wave reflections from the top and bottom of the reservoir.

When we examine the high scattering cases, the spectral peaks now have both positive and negative wavenumbers and we lose the coherent ridge-like structures seen in the no fracture case. Of particular interest are the patterns in spectral peaks associated with the positive wavenumbers. These peaks are associated with events that are arriving across the receivers in the opposite direction of the reflections. The spectral peaks associated with positive wavenumbers also have narrow bandwidth. Therefore, we conclude that spectral peaks with positive wavenumbers are associated with backscattered events from the fractures.

We observe systematic changes in the nature of the backscattering as the fracture density changes. The .01-fractures/meter case seems to have multiple peaks with the largest peak at a wavenumber of $.01\text{-m}^{-1}$ and a frequency of about 40-Hz. Overall, the .01-fractures/meter case has four identifiable peaks that seem to have increasing frequency with increasing wavenumber.

The .02-fractures/meter case has the same spectral peak at a wavenumber of $.01\text{-m}^{-1}$ but lacks the other spectral peaks seen in the .01-fractures/meter case except for the peak at the largest frequency. The .0286-fractures/meter case shows a strong peak at a wavenumber of about $.014\text{-m}^{-1}$ with a frequency close to 60-Hz. The .04-fractures/meter case has a peak at a wavenumber of about $.018\text{-m}^{-1}$ with a frequency of about 70-Hz. Also, the negative wavenumber peaks for the .04-fractures/meter case seem to be showing more of the ridge-like structure observed in the no fracture case.

The .1-fractures/meter case has a very similar pattern to the no fracture case. There are no spectral peaks in the positive wavenumber domain. The negative wavenumber domain shows

ridge-like structure with velocities in the range of the S-wave reflections. However, the trend of these ridges is slightly different owing to the change in mechanical properties of the reservoir due to the presence of the fractures.

Figure 10 shows the results of the 2D FFT for each fracture density when the receiver array is parallel to the fractures. In all of the fracture cases, we observe no spectral peaks in the positive wavenumber domain. For the fracture density values from .01-fractures/meter to .04-fractures/meter, we see that the smooth, ridge-like structure has been significantly disrupted. In addition, close comparison between figures 9 & 10 shows that the spectral peaks in the positive wavenumber domain for the direction normal to the fractures can also be seen in the negative wavenumber domain for the direction parallel to the fractures. This observation is consistent with the fractures acting as secondary sources and scattering energy in all directions and this scattered energy having move-out trends in the parallel direction similar to the reflections.

6. Discussion and Conclusions

We used a finite difference code to model scattering from discrete, vertical fractures with constant fracture spacing across the domain. Observations from vertical velocity time series show that scattering along receiver transects normal to fractures contain more complex wave patterns while receiver transects parallel to fractures tend to only have hyperbolic move-outs similar to the primary P-wave reflections. This implies that, if fractures behave as scatterers, stacking data obtained normal to fractures may remove valuable information regarding fracture parameters while stacking data parallel to fractures may retain such information (Willis et al., 2003).

Analysis of NMO velocities associated with the P-wave reflection from the base of the reservoir show that using velocity anisotropy to estimate fracture intensity might lead to erroneous results when fractures strongly scatter seismic waves. For our numerical experiment, this strong scattering regime occurs between fracture densities of .01-fractures/meter and .0286-fractures/meter. Given the background velocities in our reservoir, this strong scattering regime dominates when fracture spacing is greater than about a quarter wavelength of a P-wave in the reservoir. This observation agrees with well-established theory on scattering of waves by heterogeneities (Aki & Richards, 2002).

We use a robust window about the P-wave reflection from the base of the reservoir to calculate integrated amplitude values as a function of offset. When receivers are normal to the fractures, we observe periodic fluctuations about the general AVO trend within the strong scattering regime. Transforming this information to the wavenumber domain yields peaks in the spectra that correspond to the fracture density of the model, which can be easily observed when the general trend is removed. The peaks in the normalized spectra are not sensitive to the spectrum used to remove the general trend.

However, two important issues in using this method were identified when attempting to resolve the fracture density of .0286-fractures/meter. The spatial resolution of this method is controlled by the spacing of the receivers. For a specified maximum fracture density to be resolved, the receiver spacing must be at least half of that maximum fracture density as governed by the Nyquist criterion.

Another consideration is that the amplitudes of the fluctuations in the .0286-fractures/meter case are much smaller than in the other two cases that were successful. This may be due to the window of the data not capturing enough of the scattered energy responsible for the

interference patterns that lead to the fracture density estimation. In fact, this observation can be somewhat validated by examining the time series for the .0286-fractures/meter case, which show a lack of almost any events within the time window used for this analysis. Instead, this case shows strong scattered energy just below the window used in this analysis. Therefore, the optimal window used for this method may depend on the fracture density and may require additional constraints.

We have focused our more detailed spectral analysis on the time window around the P-wave reflection from the base of the reservoir. However, we have shown that a great deal of energy is scattered and results in events that arrive significantly later than the primary P-wave reflections. To capture information about these scattered events, we use the 2D FFT to generate frequency versus wavenumber plots for each fracture density case. When this method is applied to data in the direction normal to the fractures spectral peaks associated with backscattering from the fractures can be separated. These backscattered spectral peaks have narrow bandwidth and in general have increasing peak frequency as the fracture density increases. In addition, the same spectral peaks are present in the direction parallel to the fractures; however, they arrive with similar move-out trends as the reflections. Therefore, identifying these scattering events in the frequency – wavenumber domain may provide information regarding fracture density as well as the orientation of the fracture zones.

In this paper, we have only investigated the effect of variations in the spacing between discrete fractures. For this model, the effect of fracture stiffness, fracture zone thickness (in this study the fracture zones were set to the grid size of 5 meters), heterogeneous fracture spacing, and reservoir thickness on the scattered wavefield should also be investigated. In addition, we

have only focused on one component of the wavefield. It would be beneficial to look at horizontal components as well in order to investigate the relative strength of P and S wave scattering.

Examining a simple model such as the one presented in this paper allows for many fundamental observations related to scattering from discrete fracture zones. Lessons learned from this effort should be used to develop more complex models that incorporate other fundamental effects in real seismic data. Such complexities include additional layers above the reservoir, intrinsic attenuation, and layer anisotropy.

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Figure 1 – The model geometry for the .02-fractures/meter case showing the source and receiver locations, fracture locations, and velocity and density parameters

Figure 2 – Vertical component of velocity normal to fractures for each fracture density case

Figure 3 – Vertical component of velocity parallel to fractures for each fracture density case

Figure 4 – Snapshots of the vertical component of velocity in the normal (a) and parallel directions (b) at a time of .45-seconds

Figure 5 – a) shows the traces for the .02-fractures/meter case where the bold line represents the best fit NMO velocity, b) plot with the best fit NMO velocity for each fracture density in the normal and parallel directions, c) shows the windowing used to calculate integrated amplitudes summed across all offsets in d) for the normal and parallel case

Figure 6 – Integrated amplitude as a function of offset for receivers normal to fractures. Lines show window used in calculations.

Figure 7 – Integrated amplitude as a function of offset for receivers parallel to fractures. Lines on traces show window used in calculations.

Figure 8 – Spectra of integrated amplitude vs. offset curves in Figure 6. a) Spectra used for normalizing (from left to right) - no fracture spectrum, .1-fractures/meter spectrum, and .04-fractures/meter spectrum. b) Normalized spectra for the .01-fractures/meter case, c) Normalized spectra for the .02-fractures/meter case, and d) Normalized spectra for the .0286-fractures/meter case

Figure 9 – 2D FFT for each fracture density value with receiver array normal to fractures

Figure 10 – 2D FFT for each fracture density value with receiver array parallel to fractures

Appendix E

Fluid Flow Modeling in Fractures

by

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Fluid Flow Modeling in Fractures

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Abstract

In this paper we study fluid flow in fractures using numerical simulation and address the challenging issue of hydraulic property characterization in fractures. The methodology is based on Computational Fluid Dynamics, using a finite-volume based discretization scheme. Steady-state, viscous, laminar flow simulations for a Newtonian fluid are carried out in both 2D and 3D fracture models. In 2D, flow is analyzed in single fractures, series and parallel combination of fractures, inclined fractures, intersecting fractures, mixed networks, and in real (rough-surface) fractures. In 3D, flow is simulated in both uniform and variable aperture fracture models. To characterize each fracture model with an *equivalent hydraulic aperture*, equations are derived for all possible scenarios followed by comparison and validation with results derived from flow simulation.

Based on the fracture models analyzed, the following are some of the important findings: 1) For fractures connected in series, the *equivalent hydraulic aperture* is a weighted harmonic mean of cubed apertures of all fractures. 2) For fractures connected in parallel, the *equivalent flow* is simply the sum of all flows through individual fractures. 3) If a fracture is inclined with respect to the axis of pressure gradient, then the amount of flow will be reduced by a factor of *cosine* of the inclination angle. 4) Any network of randomly intersecting fractures can be replaced by a single fracture to give flow equivalence; the aperture of that equivalent fracture will roughly be close to the aperture of the fracture in the network that cuts across the boundaries (inlet and outlet) in the most continuous fashion and have the smallest inclination (with respect to the pressure gradient axis). 5) For hydraulic characterization purposes, fluid flow in fractures can be sufficiently modeled using both *Stokes* and *Navier-Stokes* equations for flow *Reynolds number* upto approximately 100.

1. Introduction

In the crust, fractures occur at various scales; they are important in hydrogeology, engineering geology, and in petroleum engineering. Fractures can act as hydraulic conductors, providing easy pathways for fluid flow, or barriers that prevent flow across them. From a geologist's perspective, a fractured reservoir is a reservoir with structural discontinuities resulting from a given paleostress history, while for a reservoir engineer it is a structural discontinuity affecting flow. Most hydrothermal-geothermal systems are found in fractured rock masses. Fractures play important role in water aquifers too. A recent estimate suggests that fractures are important in about 60% of the world's hydrocarbon reservoirs, although 'fracture denial' is not an uncommon phenomenon through the oil industry. Geologists and engineers alike are increasingly faced with evaluating the role of fractures to underpin development decisions. Fractures present both problems and opportunities for exploration and production from petroleum reservoirs. In most cases the fractures are usually important because of *permeability* rather than *porosity*. Matrix porosity stores the hydrocarbons, and fractures provide permeable pathways for the transport of hydrocarbons to producing wells.

For this research, we carried out steady-state flow simulation of a single-phase fluid using Computational Fluid Dynamics in various fractures and fracture-like idealized geometries to show how some of the fracture parameters influence fluid flow in them, and we also present quantitative analysis of fracture hydraulic conductivity and its sensitivity to some key fracture parameters.

One area of application of fluid flow modeling in fractures is the reservoir simulation of fractured petroleum reservoirs. The unexpected production behavior of many fields arising from an insufficient consideration of fracture effects on flow emphasized the need for better characterizing the fractures at various scales and transferring the meaningful part of this information to field simulation models. Nowadays, a study of fractured reservoir, from detection to full field simulation, often consists of a multi-disciplinary, integrated workflow

(Bourbiaux *et al.* 2001). The steps of such a workflow typically are: (I) Constrained modeling of the geological fracture network (Cacas *et al.* 2001), (II) Characterizing the hydrodynamic properties of the network (Sarda *et al.* 2002), (III) Choosing an appropriate flow simulation model (Jourde *et al.* 2002; He *et al.* 2001; Henn *et al.* 2000; Dershowitz *et al.* 2000; Lough *et al.* 1997; Koudine *et al.* 1998; Sarda *et al.* 1997), and (IV) Simulating reservoir flow behavior (Thomas *et al.* 1983; van Golf-Racht, 1982).

For reservoir simulation, equivalent flow properties are assigned to each reservoir cell. The determination of such effective or equivalent parameters is the most important task for simulating reservoir flow and predicting future performance yet it still is one of the most challenging tasks for the fractured reservoir simulation. For example, in a *dual porosity* or a *dual porosity-dual permeability* simulation model (Barenblatt *et al.* 1960; Warren and Root, 1963; Kazemi *et al.* 1976; van Golf-Racht, 1982; Adler and Thovert, 1999; Wu and Pruess, 2000; Consentino *et al.* 2001), an equivalent fracture permeability (or *transmissivity*) tensor is assigned to each reservoir grid cell. These estimates are used in the beginning of a simulation. In the continual process of reservoir simulation these initial estimates are perturbed and updated to match the production data as new data become available (a procedure called *history matching*). The purpose is to have a “good” reservoir model at a given time that matches the historical reservoir performance to a level that is acceptable for making reliable future forecasts. However, like any data-fitting process the final *history-matched* reservoir model is not necessarily unique! In other words, a family of permeability models may provide equally acceptable matches to past reservoir performance but may yield significantly different future predictions. To help reduce the uncertainty associated with fracture flow models, there is a need for improving the fundamental understanding of the physics of fluid flow in the complex reservoirs. In this study, we aim to enhance our understanding of the actual physics of fluid flow in fractures using Computational Fluid Dynamics methods (Chung 2002). In the following sections we will discuss the flow simulation methodology and give details of simulation results obtained for different fracture geometries.

2. Methodology

We use numerical solution of fluid flow equations to model flow in fractures. The basic physics of fluid flow can be described by the equations of mass and momentum conservation. The equations of motion for a single-phase, Newtonian, and incompressible (constant density) fluid can be written (in scalar form) as following:-

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (2a)$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (2b)$$

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (2c)$$

where, u , v , w are x , y , z components of velocity respectively. Equation 1, known as the *Continuity Equation*, and Equation 2(a-c), known as the *Navier-Stokes Equation* completely describe the motion of an incompressible fluid in a continuum media in 3D (Wilkes, 1999). The assumption of incompressibility is acceptable for liquids (e.g. water, gas-free oil) under typical subsurface conditions. (Ertekin *et al.*, 2001). The compressibility effect is important for transient problems, since it contributes to the storativity of the rock/fluid system (de Marsily, 1986). However, since the relationship between fracture geometry and our key parameter of interest, *hydraulic conductivity*, is most readily studied using steady-state flow, we will ignore transient effects, and assume that the fluid density is constant. The relevant boundary conditions for the *Navier-Stokes* equations include the ‘no-slip’ conditions, which specify that at any boundary between the fluid and a solid, the velocity vector of the fluid must be equal to that of the solid (Paterson, 1983). This implies that at the fracture walls, not only the normal component of the velocity equals to zero, but the tangential component vanishes as well. At fluid inlet(s) and outlet(s), pressure and/or velocity boundary conditions may be specified (Paterson, 1983).

In this study, we are primarily interested in characterizing the *hydraulic conductivity* of a fractured medium, which leads to characterizing fracture *permeability*.¹ Fracture permeability is generally defined under the assumption of steady-state flow under a uniform macroscopic pressure gradient (van Golf-Racht, 1982). We will model fluid flow using the steady-state form of the *Navier-Stokes* equation, which in vector form, can be expressed as:

$$\rho(\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P + \mu \nabla^2 \mathbf{u} \quad (3)$$

The equation is a nonlinear partial differential equation with no general solution. Generally, in order to solve it analytically, a number of assumptions will have to be made (Brodkey, 1967). The assumption of an incompressible fluid or steady-state condition is not enough to allow a general solution, because the equation is still nonlinear. One major problem in solving *Equation (3)* is the presence of the advective term, $(\mathbf{u} \cdot \nabla) \mathbf{u}$. In certain cases while modeling subsurface flow, this term is either very small, in which case it can be neglected, or vanishes altogether (Sherman, 1990). When the advective terms of the *Navier-Stokes* equation drop out, a much simpler form is obtained, which can be solved easily (Curie, 2003). Very slow (laminar), viscous flow is an example to that, and to model that fluid motion, mathematically linear *Stokes* equations can be used (Fox and McDonald, 1998; Sherman, 1990):

$$\nabla P = \mu \nabla^2 \mathbf{u} \quad (4)$$

Equation (4) can also be used to simulate flow in fractures. It is valid when the fracture walls are parallel, and is a good approximation for very low speed flows even when the fracture walls are not completely parallel. At low speeds, the effects of non-linearity – in other words the importance of advective terms – are mostly felt when there are sharp corners or bends in the fracture geometry, or when the flow is analyzed at a very fine scale near the fracture walls that have high-frequency roughness. However, from our fundamental flow simulation work in fractures we have seen that these effects do not perturb the mostly laminar and developed flow pattern that exists elsewhere in the flow path. Although eddies that are characteristics of turbulent flow can be seen near those surface irregularities, their effect on the entire flow can be evaluated by comparing results from *Stokes* solution with those of *Navier-Stokes*.

For numerical solution of steady-state *Navier-Stokes* equation, we use a commercial implementation of Computational Fluid Dynamics (CFD) techniques, named FLUENT². The code uses a *finite volume* based technique to convert the governing mathematical equations to algebraic equations that can be solved numerically (Versteeg and Malalasekera, 1995). Further details about the discretization of the flow variables and different solution schemes used by the code can be found in the FLUENT documentation and also in the literature (e.g. Patankar, 1980; Issa, 1986; Berth and Jespersen, 1989; Holmes and Connell, 1989). The code is supplemented by a proprietary CAD based geometry construction and meshing engine, which allows users to build and mesh complex flow models to be used by the solver. Various features of this package makes it a more sophisticated tool than our in-house developed code.

3. Numerical Flow Simulation

3.1 Parallel Plate Model

The simplest model of flow through a rock fracture is the parallel plate model (Huitt, 1955; Snow, 1965). This is the only fracture model for which an exact calculation of the hydraulic conductivity is possible; this calculation yields the well-known ‘cubic law’ (Witherspoon *et al.*, 1980). The derivation of the cubic law begins by

¹ The term ‘*hydraulic conductivity*’ is used to quantify the flow transmission capacity of a single continuum (e.g. fracture), while the term ‘*permeability*’ is used to describe the average flow capacity of larger systems, such as a porous-permeable rock, or a network of fractures.

² <http://www.fluent.com>

assuming that the fracture walls can be represented by two smooth, parallel plates, separated by an aperture h (Figure 1). Flow takes place in the space between these parallel plates – from inlet to outlet as marked by the arrows – with a commonly used boundary condition: constant static pressures at inlet and outlet. The flow space remains bounded by impermeable and rigid fracture walls (*no-slip* boundary conditions) elsewhere. The fracture width is expressed as W , and the distance between the inlet and outlet (fracture length) is l . This system creates a uniform pressure gradient which lies entirely in the plane of the fracture, resulting in a unidirectional flow through the system. The flow in this case, is in the x-direction, therefore only the x-component velocity, u , exists (i.e. $v = 0$; $w = 0$).

The analytical solutions for pressure and velocity are (Wilkes, 1999):-

$$P(x) = P_i - \frac{x}{l}(P_i - P_o) \quad (5)$$

$$u(z) = \frac{1}{2\mu} \left(\frac{P_i - P_o}{l} \right) z(h - z) \quad (6)$$

It is worth mentioning that Stokes equation (Equation 4) also yields the same solutions. The velocity profile, as given by Equation (6), is parabolic (Figure 2).

The total volumetric flux through the fracture, for a width W , is found by integrating the velocity across the fracture from $z = 0$ to $z = h$, resulting in:

$$Q_x = -\frac{Wh^3}{12\mu} \left(\frac{P_o - P_i}{l} \right) \quad (7)$$

The average velocity is found by dividing the flux by the cross-sectional area, Wh :

$$\bar{u} = \frac{Q_x}{Wh} = -\frac{h^2}{12\mu} \left(\frac{P_o - P_i}{l} \right) \quad (8)$$

Darcy's law for flow through porous media, in one dimension, can be written as:

$$Q = -\frac{kA}{\mu} \left(\frac{P_o - P_i}{l} \right) \quad (9)$$

The cross-sectional area A is equal to Wh . From Equations (7) and (9), the permeability of the fracture can be identified as:

$$k = \frac{h^2}{12} \quad (10)$$

The product of the permeability and area, also known as *transmissivity*, is equal to:

$$T \equiv kA = \frac{Wh^3}{12} \quad (11)$$

The dependence of T on h^3 is the essence of the well-known cubic law.

Although exact solutions for steady-state pressure and velocity distributions are easily found for parallel plate type flow models, yet we begin by numerical solutions of flow equations for the same fracture model. This facilitates us with necessary validation of the Computational Fluid Dynamics code by checking its results against analytical (true) solutions.

The fracture geometry is defined by its aperture h and length l . We begin our study with 2D modeling, therefore $W = 1$ is assumed. Some 3D cases will be discussed later. In the parallel plate model, the rigid walls (fracture surfaces) are smooth and plain and have no rugosity. In steady-state conditions, the inlet and outlets are held at constant pressures, and a fully-developed flow occurs through the system. *Figure (3)* shows the results (velocity and pressure distribution) from numerical simulation in a parallel plate model. The velocity profile as seen is parabolic, with maximum value at the center, which conforms to the theory. Maximum velocity in a parallel plate model can be theoretically found by setting $z = h/2$ in *Equation (6)*:-

$$u_{\max} = -\frac{h^2}{8\mu} \left(\frac{P_o - P_i}{l} \right) \equiv \frac{3}{2} \bar{u} \quad (12)$$

For this and all subsequent simulations, we use an incompressible fluid with $\rho = 0.8$ g/cc, and $\mu = 5$ cp. For the model shown in *Figure (3)*, ($h = 2$ mm, $l = 80$ cm, $P_i = 200039.8$ Pa, $P_o = 200000$ Pa) there is excellent agreement between the simulation results and analytical results of the flow variables:-

	Numerical solution	Analytical solution
Mean u (m/s)	0.0033	0.0033
Mean v (m/s)	0	0
Max u (m/s)	0.005	0.005
Flow rate Q (m ³ /s)	6.68x10 ⁻⁶	6.64x10 ⁻⁶

The Reynold's number³ for this problem is found as, $Re \approx 2$.

In further analysis with parallel-plate type fracture models, we investigated the effect of aperture on the volumetric flow rate through the system. For the same length l and the same inlet/outlet pressure conditions, we generated different parallel-plate fracture models by varying the aperture h , and simulated flow through each of them. It was seen that the mean velocity $\bar{u}$ varied as h^2 , and the volumetric flow rate varied as h^3 , as predicted by the theory.

3.2 Flow Simulation in Variable Aperture Fractures

The cubic law was derived under the assumption that the fracture consisted of a region bounded by two smooth, parallel plates. Real rock fractures, however, have rough walls and variable apertures. Furthermore, there are usually regions where the two opposing faces of the fracture wall are in contact with each other. Since transmissivity is proportional to h^3 , fluid flow in a variable-aperture fracture under saturated conditions will tend to follow paths of least resistance, which is to say paths of largest aperture, and thereby depart from the rectilinear streamlines of the parallel plate model. In order to use the cubic law to predict transmissivity of a real rock fracture, one could assume that *Equation (11)* still holds if the aperture h is replaced by an equivalent aperture h_{eq} . Therefore, a more "generalized cubic law" which can be applied to any fracture geometry can be expressed as:

$$T \equiv k_f A = \frac{w h_{eq}^3}{12} \quad (13)$$

where, k_f denotes fracture permeability. In a *dual porosity-dual permeability* reservoir simulation scheme, this k_f is used as the equivalent fracture permeability of a single simulation grid block. Therefore, if the correct h_{eq} from a distribution of fracture apertures could be determined, then a reasonable approach would be to use the following expression to characterize the equivalent fracture permeability for a reservoir simulation task:

³ $Re = \frac{\rho \bar{u} L_c}{\mu}$. For internal flow in a rectangular duct, $L_c = D_h$, hydraulic diameter: $4 \times \text{Cross-sectional area} / \text{Wetted}$

perimeter. For a very long and very wide channel with rectangular cross-section, $Re = \frac{\rho \bar{u} \times 2h}{\mu}$ - this is the formula that is

used to compute Reynold's number for the 2D simulations in this study. Experimental evidence suggests that laminar flow may persist up to $Re \sim 2300$ for internal flows.

$$k_f = \frac{h_{eq}^2}{12} \quad (14)$$

Computation of the correct equivalent hydraulic aperture, h_{eq} , requires solution of Navier-Stokes equations in fracture geometries that include varying aperture and obstructed regions (Zimmerman and Bodvarsson, 1996). For any fracture system, we first compute the volumetric flow rate Q [m^3/s] from our Computational Fluid Dynamics based flow simulation, and then use that to compute the equivalent hydraulic aperture of that fracture system using the following simple relation:

$$h_{eq_{simulation}} = \sqrt[3]{\frac{12Q_{simulation}\mu l}{(P_i - P_o)}} \quad (15)$$

3.2.1 Fracture Connected in Parallel

A *parallel combination* of n equi-length fractures implies that they all are subject to the same pressure gradient (analogous to resistors connected in parallel in electrical circuits experiencing equal amount of potential difference across them). For such a case, using the same principle of computing equivalent resistance (or conductance) of parallel resistors, a rule for computing the equivalent hydraulic aperture for fractures connected in parallel can be established as:

$$h_{eq_{parallel}} = \sqrt[3]{\sum_{i=1}^n h_i^3} \quad (16)$$

The equivalent (or total) flow rate through the system will simply be the sum of flow rates through each individual fractures:

$$Q \equiv \frac{h_{eq}^3}{12\mu} \left(\frac{P_i - P_o}{l} \right) = \sum_{i=1}^n Q_i \quad (17)$$

To test the validity of this rule, we do a simulation with three distinct fractures connected to parallel. While they have different apertures (2 mm, 0.5 mm and 1 mm), their lengths are same (80 cm) and are connected to the same inlet and outlet (pressure differential between inlet and outlet is 190 Pa). The pressure and velocity distribution from the simulation are plotted on *Figure 4*. The numerical solution yields a flow rate $Q = 3.615 \times 10^{-5} m^3/s$, which gives $h_{eq} = 2.09$ mm according to *Equation (15)*. The network rule, according to *Equation (16)*, predicts $h_{eq} = 2.09$ mm, which is exactly equal to the result obtained from actual flow simulation. Therefore, it is verified that *Equation (16)* provides a good estimate of the equivalent fracture hydraulic aperture of a fracture network where individual members are arranged in parallel with the axis of pressure gradient.

Also revealed from this simulation is that most of the flow is through the thickest fracture. The 2 mm fracture occupies 57% of the total available flow area in the system, but it transports about 87% of the total flow. Therefore, a general conclusion for parallel networks is that, when there are fractures of different apertures in a parallel network, a single fracture that is flow equivalent to that network will have an aperture magnitude (h_{eq}) close (and slightly larger) to the aperture of the thickest fracture (*thickest = largest aperture magnitude*) of the original network.

3.2.2 Fractures Connected in Series

In a series combination of fractures, the same Q prevails through each member of the fracture network. Again, using the series connection of resistors analogy, the following formula can be used to compute the equivalent fracture aperture:

$$h_{eq_{series}} = \frac{1}{\sqrt[3]{\sum_{i=1}^n \frac{l_i}{l} \left(\frac{1}{h_i^3} \right)}} \quad (18)$$

where, l_i and h_i are the length and aperture of the i^{th} fracture section; l is the total length of the series network, i.e. distance between inlet and outlet. It is noted that the derivation of this expression is based on the assumption that each fracture section of the network has the ideal parallel plate geometry; therefore n is the total number of “parallel plate” fracture members connected in series.

To predict the equivalent hydraulic aperture by *Equation (18)*, information about the length (l_i) of individual fracture section that has uniform aperture (h_i) is required. For real complicated reservoir fractures, such a precise description of fracture geometry involving knowledge of l_i would be very restrictive, if not impossible. The longitudinal variation of apertures in real fractures will be much more high frequency, severe and consequently very difficult to determine. Therefore, it should be noted that *Equation (18)* will have limited use in characterizing equivalent fracture hydraulic aperture. It could only be applied for relatively simple geometries, or where the aperture field varies smoothly along the fracture length.

To test the validity of *Equation (18)*, a flow simulation is carried out in a series network of fractures with three distinct fracture elements having different aperture magnitudes (0.5 mm, 1 mm, and 2 mm). The equivalent aperture of the network is computed from the simulation derived flow rate (Q) using *Equation (15)*, which is then compared to the theoretically predicted equivalent aperture magnitude given by *Equation (18)*. We do this for a number of series networks, which we generate by simply varying the length fractions and topology of connection of the three fracture elements. The following table summarizes the model properties of couple of series cases and also shows the comparison between equivalent aperture values computed from numerical simulation and network formula:

	l_i cm	h_i mm	h_{eq} (mm) simulation:Eqn.15	h_{eq} (mm) theory:Eqn.18
Case a	45	2	0.822	0.819
	15	0.5		
	20	1		
Case b	15	2	0.597	0.594
	45	0.5		
	20	1		

[Case a: $\Delta P = 114.8$ Pa, $Q = 1.33 \times 10^{-6} \text{ m}^3/\text{s}$]

[Case b: $\Delta P = 300.1$ Pa, $Q = 1.33 \times 10^{-6} \text{ m}^3/\text{s}$]

While constructing each of these models, total length was kept fixed ($l = 80$ cm). Velocity distribution obtained by flow simulation for *Case a* - mainly showing regions of geometry variations - is shown by *Figure 5*. Velocity distribution for *Case b* (not plotted) exhibits identical characteristics. As seen from these cases, the series network formula (*Equation 18*) has good accuracy in predicting equivalent hydraulic apertures. The theoretical expression for equivalent aperture (*Equation 18*) assumes fully-developed laminar flow in each fracture section and does not take into account the non-linearity produced due to geometry change. In reality, non-linearity in flow variables do occur at the intersection of two different fracture sections; however the flows in these models are so slow that these non-linear behaviors are very localized and do not affect the flow in other parts of the model. Therefore, the effects of such geometry variation are negligible in terms of their influence on velocity distribution in the entire model, and hence we see a very good agreement between the equivalent aperture magnitudes predicted by theory and computed from simulation.

3.2.2.1 Length of fracture members in a series network unknown

From *Equation (18)*, it is seen that the equivalent hydraulic aperture of a series network is a weighted harmonic mean of the cubic apertures of participating fracture members, the weights being the length fractions (l_i/l) of respective members. In reality, however, these length fractions are difficult to get. Therefore, given only a distribution of apertures $\{h_i\}$, it is important to know the likely range of equivalent aperture magnitude the fracture network may have. For example, we take an 80 cm long fracture composed of three discrete fracture elements

having apertures 0.5 mm, 1 mm and 2 mm. We assume that the lengths of these individual elements are unknown. We employ a routine to compute all possible values of equivalent hydraulic apertures for different combinations of length fractions of individual members (i.e. using *Equation 18* for each realization). With 1 cm accuracy in length, there are 3081 possible combinations of length fractions and the histogram of equivalent apertures has a shape as shown by *Figure 6*. The equivalent aperture of this series network is minimum ($min = 0.5039 \text{ mm}$) when the thickest fracture section is the shortest (1 cm) and the thinnest section is the longest (78 cm); and it is maximum ($max = 1.6219 \text{ mm}$) when the thickest section is longest (78 cm) and the thinnest section is shortest (1 cm) (illustrated by *Figure 7*). This, of course, is intuitive and obvious, but what is not so obvious is where the mean of equivalent apertures will be located. In this example, the *mean* is calculated as 0.7548 mm . It is found that more than one combination results in such an equivalent aperture. These are:

l_1, h_1	l_2, h_2	l_3, h_3	h_{eq}
7, 2	16, 0.5	57, 1	0.755
15, 2	17, 0.5	48, 1	0.755
23, 2	18, 0.5	49, 1	0.755
31, 2	19, 0.5	30, 1	0.755
39, 2	20, 0.5	21, 1	0.755
47, 2	21, 0.5	12, 1	0.755
55, 2	22, 0.5	3, 1	0.755

We further investigate the issue of how the range of equivalent aperture values change as we include additional fracture elements. We repeat our analysis by changing our original 3-tier aperture configuration as follows: 1) Adding a fracture section of a relatively small aperture, 0.1 mm (this could represent a case when the void space in fractures are sealed due to chemical or other reactive process during transport), 2) Adding a fracture section of a relatively large aperture, 1.9 mm (this could represent a case when fractures are propped open naturally due to fluid pressure or artificial stimulation such as *hydrofrac*), 3) Adding both small (0.1 mm) and large aperture (1.9 mm) fractures. The total length remains the same (80 cm) in each of the cases, and we combine the set of fractures in series in all possible ways (by varying the length fraction of individual members) and compute the equivalent fracture apertures for each different scenario. We summarize some statistics of the results in the following table:

$\{h_i\}$	Min h_{eq}	Max h_{eq}	Mean h_{eq}	Std. h_{eq}
$\{0.5, 1, 2\}$	0.504	1.622	0.755	0.179
$\{0.5, 1, 2, \underline{0.1}\}$	0.101	0.428	0.189	0.069
$\{0.5, 1, 2, \underline{1.9}\}$	0.506	1.621	0.834	0.201
$\{\underline{0.1}, 0.5, 1, \underline{1.9}, 2\}$	0.102	0.428	0.204	0.073

The histogram from each of these experiments has a shape similar to that of *Figure 6*. When the lengths of individual fractures are not known, this type of analysis can provide useful bounds for equivalent hydraulic parameter estimates.

3.2.2.2 Effect of Constrictions

For practical purposes, very often fractures are modeled as media bounded by *parallel plates* with rough fracture walls represented by straight line segments (*Figure 8*). As the effect and importance of fracture *apertures* (distance between fracture walls) are widely discussed, little is known about the role of *constrictions* in fracture modeling. In this paper, we use the term ‘constriction’ to define the wall-to-wall distance at the plane of intersection between two fractures (*Figure 8*). In this section, we do some flow simulation to investigate the effects of constrictions on flow through fractures. We present simulation results from two cases where we have the same three fractures (2 mm, 0.5 mm, 1 mm) connected in series, but different types of connectivity.

Figure 9 plots the geometry and connectivity of one of the models and also shows some simulation results. The sequence of wall-to-wall distance (*mm*) can be written as: $S1 \equiv \{2 \underline{0.5} 0.5 \underline{0.5} 1\}$, where the underlined numbers indicate constrictions, c_i and the rest just being the apertures, h_i . The steady-state flow solution for this

model yields a total flow rate, $Q = 1.33 \times 10^{-6} \text{ m}^3/\text{s}$ for a pressure differential $\Delta P = 114.9 \text{ Pa}$ between the inlet and outlet, which – by Equation (15) – gives an equivalent aperture of 0.822 mm for this model. This is in good agreement with the simulation results of previously modeled *Case a* (Figure 5), which essentially had the same fracture members having same length and aperture but connected in a slightly different manner. We present another model (Figure 10 [a]) where the sequence of wall-to-wall distance (in mm) is: $S2 \equiv \{0.5 \underline{0.25} 2 \underline{0.75} 1\}$, the underlined numbers being the constriction magnitudes, c_i . Some of the simulation results for this model are presented on Figure 10 [b,c,d]. The steady-state flow solution for this model yields a total flow rate, $Q = 1.33 \times 10^{-6} \text{ m}^3/\text{s}$ for a pressure differential $\Delta P = 306.6 \text{ Pa}$ between the inlet and outlet, which – by Equation (15) – gives an equivalent aperture of 0.5927 mm for this model. The model has same $\{l_i, h_i\}$ as *Case b* discussed under Section 3.2.2 (Fractures Connected in Series), but the magnitudes of constrictions are different. The slight difference in results between this one and *Case b* is due to this geometry difference at intersections. To relate the issue of constriction with fluid motion, we note that in the first model of this section (sequence S1) both constriction values are equal to at least one of their adjacent aperture values, however in the second model (sequence S2) the constrictions are not equal to any of their adjacent apertures. (We label the constrictions of sequence S1 and sequence S2 as ‘smooth’ and ‘non-smooth’ respectively). To better understand how velocity magnitudes vary at different sections of the fracture (including at intersections/constrictions), a plot of x-velocity at different constant-x surfaces for the model described by sequence S2 is given on Figure 11.

Due to the possibility of stark velocity contrast at ‘non-smooth’ constrictions at fracture intersections, we propose that the calculation of equivalent hydraulic aperture must also take into account the effect of constrictions. Therefore, we suggest the following modification to Equation (18):

$$h_{eq_series} = \frac{1}{\sqrt[3]{\sum_{i=1}^n w_{hi} \left(\frac{1}{h_i^3} \right) + \sum_{i=1}^{n-1} w_{ci} \left(\frac{1}{c_i^3} \right)}} \quad (19)$$

where, w_{hi} is the weight associated with aperture h_i (it is the length fraction, l_i/L), and w_{ci} is the weight associated with constriction c_i . For most cases, $w_{ci} \ll w_{hi}$. In the limiting cases where constrictions are ‘smooth’, i.e. they are equal to one of the adjacent aperture magnitudes (for example sequence S1), w_{ci} can be treated as zero, and Equation (19) reduces to Equation (18). However, when constrictions are ‘non-smooth’, i.e. their magnitudes are different than any of the adjacent aperture magnitudes (for example sequence S2), use of w_{ci} may be necessary. Work is currently under way to understand more about the effects of constrictions and connectivity of series fractures on fluid flow.

3.2.3 Combination of Series and Parallel Fractures

In this section, we present simulation results for a fracture network having a combination of series and parallel connections (Figure 12). Fractures $f2, f3, f4$ and $f5$ are connected in parallel, and their combination is connected in series with fracture $f1$. The system has only one inlet, but four different outlets. The flow solution gives a total flow rate $Q = 1.334 \times 10^{-6} \text{ m}^3/\text{s}$ for $\Delta P = 116.4 \text{ Pa}$, which yields an equivalent aperture of 0.8194 mm . Using theoretical expressions for series and parallel connections (Equations 16 & 18) the equivalent hydraulic aperture is predicted as 0.7539 mm . The difference is explained by the fact that the simulation takes into account all the non-linearity that are present in the real model, however the network formulae consider each fracture member as ideal parallel-plate model and the non-linearity at the intersection of fractures are not accounted for by the theoretical expressions. The flow simulation provides more accurate estimate of equivalent aperture magnitudes as it models the real physics of fluid flow rather than any idealized approximation.

Difficulty arises when the network topology is not known. In such a situation, a reasonable estimate of the equivalent hydraulic aperture could be the following:

$$h_{eq_series} \leq h_{eq} \leq h_{eq_parallel} \quad (20)$$

Inequality (20) gives an expression for the upper and lower bounds of the equivalent hydraulic aperture, h_{eq} , for any network of connected fractures. The upper bound, h_{eq_series} , is the equivalent aperture if all the participating fractures in the network were connected in series, while the lower bound $h_{eq_parallel}$ is again the equivalent aperture if all the participating fractures in the network were connected in parallel. In reservoir simulation, where the connectivity of fracture networks is not well known, these bounds could be used; such bounds may be useful in a stochastic simulation. For the model shown in this paper (*Figure 12*), h_{eq_series} and $h_{eq_parallel}$ are computed as 2.023 mm and 0.2906 mm respectively, which readily verifies *Inequality (20)* as h_{eq} was computed as 0.7539 mm by using the network formulae.

Madden (1976) investigated random networks as models of heterogeneous media and concluded that for small contrasts, networks tend toward the geometric mean. For a combination of series and parallel fractures, this implies that the *geometric mean* of a given aperture distribution, $\{h_i\}$, could be used as an estimate for the equivalent hydraulic aperture of that network. For any set of numbers, its geometric mean (*GM*) is bounded by its arithmetic mean and (*AM*) and its harmonic mean (*HM*):

$$HM \leq GM \leq AM \quad (21)$$

Inequalities (20) & (21) are analogous: h_{eq_series} and $h_{eq_parallel}$ are indeed some variants of *HM* and *AM* of $\{h_i\}$, respectively. Therefore, when deterministic facts about the fracture network are not known, using the *GM* of $\{h_i\}$ as an estimate of h_{eq} is justified. However, the accuracy of this estimate will vary from one model to another, and is subject to further analysis. For example, the *GM* of apertures of the model shown by *Figure 12* is 0.464 mm , while its theoretically predicted $h_{eq} = 0.754\text{ mm}$ – resulting in an error of about 38% if *geometric mean* is used.

We also investigate what percentage amount of total flow is carried to the outlet by each fracture leg connected in parallel for this network model. Theoretically, volumetric flow rate varies as h^3 (*Equation 7*), and therefore the thicker fractures will always be dominating in the vicinity of thinner fractures. For example, fracture *f5* constitutes only 42% opening of the total available fracture area to the outlet, yet it carries 77% of the total flow to the outlet. This has important implications in oil production, because the majority of the flow towards the well from a complex fractured reservoir will be through larger aperture fractures and only those can be targeted to boost production.

3.3 Inclined Fractures

In this section, we study flow through inclined fractures – fractures that are oriented at angle with respect to the overall (macroscopic) pressure gradient axis that controls flow, as shown by *Figure 13*.

Figure 14 shows the results of a flow simulation in a fracture model, which is embedded in a rock slab that is subjected to a horizontal pressure gradient. The surface of the fracture forms a 45° angle with the axis of pressure gradient. From visualization of the results, it becomes clear that the actual physics of flow requires the equi-pressure lines (shown by pressure contours in *Figure 14 (a & b)*) inside the fracture to have an orientation normal to the fracture walls/surfaces⁴; and as a consequence, the flow, which should be normal to the tangents drawn on equi-pressure surfaces, is aligned with the fracture orientation. The same physics should hold for all laminar flows in any orientation of a fracture. To validate this, we ran simulations in single fractures oriented at some other angles: 20° , 30° , and 70° , and found that the average direction of the velocity vectors were 20° , 30° , and 70° , respectively. Likewise, if there are multiple oriented fractures with different orientations inside a rock slab, in each of those fractures the flow will perfectly align with its channel direction and so will the pressure gradient axis, regardless of the macroscopic pressure gradient direction that exists due to the boundary conditions (inlet and outlet)

⁴ Except for very close to the inlet and the outlet. The pressure anomaly near those boundaries is a consequence of the fluid entering and leaving the system (rock) horizontally. We also studied flow by forcing the fluid to enter and leave at 45° , and found that the overall difference in flow between the two cases is negligible.

of the rock. This is an important understanding that forms the rationale to justify an equation to calculate flow rate and effective permeability for oriented fractures, which we will present shortly.

3.3.1 Numerical Experiment

To find an expression for computing steady-state flow rate in oriented fractures, we did a numerical experiment. We took a simple parallel plate fracture model having an aperture (h) of 1 mm, and oriented it at different angles inside our rock slab model (as *Figure 13*). Holding the pressures at inlet and outlet constant, we ran flow simulations in each of those oriented fracture models. Each flow simulation yields a steady-state volumetric flow rate Q . The following is a summary of this numerical experiment:

True fracture aperture h [mm]	Fracture orientation θ [degree]	$\Delta P (=P1-P2)$ [Pa]	Length of the rock slab l [cm]	Q (10^{-6} m ³ /s)
1	0	123	20	10.25
1	20	123	20	9.63
1	30	123	20	8.87
1	30	61.6	20	4.45
1	45	123	20	7.24
1	70	123	20	3.49

Analyzing these data, empirically we find the following expression that relates the flow rate to its function parameters:

$$Q = \frac{h^3 \cos \theta}{12\mu} \left(\frac{\Delta P}{l} \right) \quad (22)$$

Comparing *Equation (22)* with *Darcy's Equation*, the magnitude of permeability in the direction of macroscopic pressure gradient can be expressed as:

$$K = \frac{h^2}{12} (\cos \theta) \quad (23)$$

The essence of *Equation (22)* is that the flow rate is decreased by a factor of $\cos \theta$ due to inclination (where $-\pi/2 \leq \theta \leq \pi/2$). Similarly, permeability is reduced by the same factor as shown by *Equation 23*. A physical explanation to this is the effect of added length of fracture (the actual length of the fracture is increased by $\frac{l}{\cos \theta}$).

As explained earlier, inside the inclined fracture the pressure gradient and the velocity have same direction; therefore the only thing that makes a difference in the inclined fracture case as compared to the horizontal fracture case is just that added length, hence $\cos \theta$.⁵

⁵ We note a significant disagreement between our analysis and what currently exists in the major literature. Originally proposed by Parsons (1966), and thereafter adopted in standard reservoir engineering literature (e.g. van Golf-Racht 1982, Nelson 2001, etc.) the discriminating factor is $\cos^2 \theta$, as opposed to our proposed $\cos \theta$. The present-day literature states:

$$Q = \frac{h^3 \cos^2 \theta}{12\mu} \left(\frac{\Delta P}{l} \right), \text{ and } K = \frac{h^2}{12} (\cos^2 \theta), \text{ which may cause significant difference in estimation of these important}$$

properties, when there are large number of oriented fractures. We could not find numerical verification of these equations with *Navier-Stokes* flow simulation. While further investigation may be required to completely resolve this discrepancy, our formulation, which is backed by numerical analysis and visualization of the true physical behavior, seems much more insightful and reliable at this point.

3.3.2 Equivalent Hydraulic Aperture

The concept of equivalent hydraulic aperture (h_{eq}) can be applied in oriented fractures as well, and we use the values reported on the previous table to calculate h_{eq} for each model using Equation (15) :-

θ [degree]	True h [mm]	h_{eq} [mm]
0	1	1
20	1	0.98
30	1	0.95
45	1	0.89
70	1	0.70

The effect of fracture inclination in the hydraulic conductivity is also illustrated graphically (Figure 15).

3.3.3 Effect of Fracture Orientation in a Series Network

Applying Equation (22) in a series network, we derive the following expression for calculating the equivalent hydraulic aperture of a system which is composed of n fracture members connected in series:

$$h_{eq_{series}} = \frac{1}{\sqrt[3]{\sum_{i=1}^n \frac{l_i}{l} \left(\frac{1}{h_i^3 \times \cos \theta_i} \right)}} \quad (24)$$

Here, l_i is the projected length of the i^{th} fracture member along the direction of the macroscopic pressure gradient axis, h_i and θ_i are aperture and inclination angle of the i^{th} member respectively. The principal of deriving this expression is essentially same as that of deriving Equation (18), and for all $\theta_i = 0$, it reduces to Equation (18). Simulations were run (results not presented in this paper) in series fractures having one or several inclined members, and it was found that Equation (24) had excellent agreement with h_{eq} derived from simulation.

3.3.4 Effect of Fracture Orientation in a Parallel Network

In the case of parallel combination of fractures having arbitrary orientations, the total flow rate through the system is simply the sum of all flow rates through individual members, and the formula to compute h_{eq} can be written as follows:

$$h_{eq_{parallel}} = \sqrt[3]{\sum_{i=1}^n (h_i^3 \times \cos \theta_i)} \quad (25)$$

This equation is nothing but a more general form of Equation (16), and was verified to match simulation results quite well.

3.4 Flow Simulation in Fracture Networks

Fractured reservoir flow simulation, an application area that we are mainly interested in, requires modeling fluid flow in fracture networks. In most cases these fracture networks are formed by intersecting, oriented fractures. The reservoir models used in flow simulation are simplified representation of the actual geology of the fractured medium. The simplification procedure depends on the length scale of fractures relative to the simulation grids/blocks and also on the connectivity of those fracture sets. A schematic diagram showing some of the typical fractured reservoir models and the pertinent simulation schemes that are being used in the petroleum industry is given by Figure 16.

We present a simulation study in a model fracture network (Figure 17) in order to delineate flow in fracture networks. The complexity of the fracture network modeled in this study is comparable to the fractured

reservoir models used in real applications (e.g. medium to large scale connected fractures that use dual-porosity scheme as shown by *Figure 16*).

On *Figure 18*, we show the mesh from some of the zones of the network model. We took advantage of the flexible and adaptive meshing capacity of the Pre-Processor that we used and tried our best to create a balance between overall mesh quality and computational cost. For example, in many places we used non-uniform meshing by putting more mesh in the boundary layers of the channels and in the intersections than the other parts of the channel. As shown by *Figure 18*, both *quadrilateral* and *triangular* mesh elements were used in the computational model. This was done in order to keep the ‘*skewness*’ of the meshes at a minimum possible level so that it would not adversely affect the convergence during flow simulation.

The steady-state simulation results from same parts of the network are plotted on *Figure 19*. These plots enable us to visualize the complexity of flow in networks.

The flow simulation in this fracture network yields a volumetric flow rate, $Q = 2.51 \times 10^{-5} \text{ m}^3/\text{s}$ for a total pressure drop of 625 Pa ($P_{\text{inlets}} = 200625 \text{ Pa}$, $P_{\text{outlets}} = 200000 \text{ Pa}$) existing over a 0.5 m length in the x-direction (direction of pressure gradient axis). With these, we can characterize this network with the concept of equivalent hydraulic aperture: $h_{eq} = 1.06 \text{ mm}$ (by *Equation 15*). This essentially means that the steady-state flow response of this network will be the same as having a single 0.5 m long, 1.06 mm aperture, simple parallel-plate fracture oriented horizontally and subjected to a pressure differential of 625 Pa in a 0.5 m long reservoir grid block. It is noted that the magnitude of h_{eq} is very close to the true aperture of fracture *c* – this is because fracture *c* provides the easiest pathway for fluid all the way from inlet to outlet in the network, and so most of the flow is along this path, while others contribute very little.

In an effort to further our understanding in network fracture flows, we perturbed the given network by – (A) Increasing the aperture of fracture *f* by a factor of 2, (B) Increasing the aperture of fracture *a* by a factor of 4, and (C) Adding a straight fracture of 2 mm aperture to the network in the upper part that extends all the way from inlet to outlet. For each case, we did flow simulation and computed h_{eq} from the simulation results. In cases (A) and (B), increments in h_{eq} were insignificant (1.07 mm and 1.1 mm respectively) compared to the original network, but in case (C) a drastic change in the velocity and pressure distribution was observed, along with a significant change in h_{eq} (2.17 mm).

From these, we conclude that in a network of intersecting fractures, the fracture that cuts across the boundaries (inlet and outlet) and has the smallest inclination with respect to the pressure gradient axis will serve as the least resistive path to fluid and therefore will carry most of the flow in the network; also the equivalent hydraulic aperture of the network will have a magnitude close to (and larger than) the aperture of that fracture.

3.5 Comparison Between *Stokes* and *Navier-Stokes* Simulation

Flow simulation using *Stokes* equation is much simpler and computationally less expensive than using full *Navier-Stokes* equation. But *Stokes* equation is only valid for very slow flow. In this section, we use a series fracture model [$\{l_i, h_i\} = \{3 \text{ cm}, 2 \text{ mm}\}, \{3 \text{ cm}, 0.4 \text{ mm}\}, \{3 \text{ cm}, 1.4 \text{ mm}\}$] to determine at what ranges of *Re* the hydraulic characterization of fractures using both methods are comparable. For *Stokes* flow simulation, we use an in-house developed code (Sarkar *et al.* 2002), and for *Navier-Stokes* simulation we use the commercial package that is used for all other simulations presented on this paper. As seen from *Figure 20*, for both low and (moderately) high Re^6 cases, the *Stokes* method forces the simulator to form fully-developed velocity profiles in each fracture section. *Navier-Stokes* simulation also yields very similar uniform profiles for low *Re* case (*Figure 21(a)*), but as *Re* increases, the advective terms come into play and some vortices start to form near the sharp corners of the fracture (last section of the fracture, as seen on *Figure 21(b)*). In real fractures, where surfaces are rough and rugose, and geometry change can be abrupt, eddies like these may occur even at fluid velocities lower than the threshold magnitude required for the onset of turbulent flow. It was found, however, that these “non-linearities,” especially at

⁶ *Re* reported for these models are calculated based on the inlet mean velocity. In our case, for the same pressure gradient *Stokes* and *Navier-Stokes* simulation will yield slightly different inlet velocity profiles; hence *Re* would be slightly different too. These differences are quite normal, because the numerical algorithm and gridding schemes used by these methods are very different.

low Re values, do not make much difference in the equivalent hydraulic conductivity magnitudes (*Figure 22*). Based on this observation we conclude that, both methods (*Stokes* or *Navier-Stokes*) will give satisfactory and comparable hydraulic parameter estimates for *Reynolds number* magnitude upto 100 ($0 < Re < 100$). This probably covers most part of conceivable viscous, incompressible fluid flow in subsurface fractures at reservoir conditions – far from well-bore. The upper limit of Re for *Stokes* method to be valid will depend on how geometries change along the fracture: the cutoff number will be much less for fractures with higher amplitude higher frequency wall asperity variations than relatively smooth fractures.

3.6 Flow Simulation in Fractures with Rough Surfaces

At low flow velocities, the two main causes of deviations from the *Cubic law* are roughness of the fracture walls and asperity contact⁷ between the opposing fracture faces. In this section we present a 2D simulation in a fracture with rough surface. Flow simulation rough surfaces fractures are more suitable for laboratory scale analysis of rock fractures than reservoir scale treatment of fractures.

Rough surface fracture models for our study are generated by using the mathematical model proposed by Brown (1995). We first determined the appropriate set of modeling parameters by upscaling the values that are reported by Brown (1995), and used them to generate two 3D fracture surfaces at a length scale of our interest. (A sample 3D surface is shown by *Figure 23*). Then we made some adjustments between the separations of the two surfaces to ensure that they did not touch each other at any location. (Flow simulation in fractures with asperity contacts will be discussed later). Finally we took a 2D section and thus prepared our fracture geometry for flow simulation. Some results from our flow simulation are shown by *Figure 24*.

The overall length (horizontal distance between inlet and outlet) for this model is 80 cm. We found $Q = 6.21 \times 10^{-5} \text{ m}^3/\text{s}$ for an overall pressure differential $\Delta P = 870.8 \text{ Pa}$ existing between inlet and outlet. (Pockets of relatively high velocity are observed at constrictions/narrow aperture sections). Next, *Equation (15)* was used to compute the equivalent hydraulic aperture for this model, yielding $h_{eq} = 1.51 \text{ mm}$. For realistic fractures with high frequency variation of apertures, doing a flow simulation is the most reliable method for estimating equivalent hydraulic parameters. Alternatively, one can approximate this rough fracture with n numbers of serially connected small parallel-plate fracture segments (the concept of *Figure 8*), and then use the formula for serial combination of fractures to estimate the equivalent hydraulic aperture. To show how this can be accomplished, we also sampled the aperture values at equal (length) intervals over the entire fracture to obtain 513 aperture samples (representing 513 fractures connected in series, each having equal lengths). The minimum, maximum, mean and std. of these samples are (in mm): 0.56, 5.19, 2.12, and 0.859 respectively. Using equal length fraction for each fracture, i.e. $l_i/l = 1/513$, the equivalent hydraulic aperture is estimated by *Equation (18)* as 1.54 mm . This is indeed in good agreement with the value obtained from flow simulation ($h_{eq} = 1.51 \text{ mm}$).

3.7 Flow Simulation in 3D

Real fractures are 3D. Although insights about the fluid motion can be gained by studying 2D models, to understand the actual flow occurring in natural fractures, 3D modeling is essential. Examples from our 3D simulation are summarized in the following.

3.7.1 Brick-shaped fracture

This is simply an extension of the 2D parallel plate model in the direction perpendicular to the 2D fracture plane. In our 2D models, we assumed the width very large compared to the aperture; however in 3D representation it has a finite width. *Figure 25* shows the 3D model and plots some simulation results. The model dimensions are: length (x) = 5 cm, width (y) = 5 cm, aperture (z) = 2 mm. For a pressure differential between inlet and outlet $\Delta P = 0.76 \text{ Pa}$, we obtained a volumetric flow rate, $Q = 1 \times 10^{-7} \text{ m}^3/\text{s}$ for this 3D model. The velocity profile is parabolic; however it gradually flattens and becomes zero as it approaches the walls. This ‘edge’ or ‘wall’

⁷ To properly understand the effects of asperity contacts, full 3D modeling is required, because in 2D analysis, asperity contacts will result in flow blockage, however in reality (3D) flow may still continue around those obstructions.

effect can be clearly seen by the flattening/dampening of velocity vectors near the walls (*Figure 25(c)*). The velocity profile, $u(y,z)$, from simulation has excellent agreement with the analytical form (*Figure 26*).

3.7.2 Flow Model with variable aperture

This is also a 3D extension of one type of the series models studied in 2D. The properties of this 3D model are: length (x) = 5 cm, width (y) = 5 cm, and apertures (z_i) = 2 mm, 0.5 mm & 1 mm. The model and some simulation results are plotted on *Figure 27*. $Q = 1 \times 10^{-7} \text{ m}^3/\text{s}$ is computed for an overall pressure drop $\Delta P = 15.2 \text{ Pa}$. Overall pressure gradient in the flow model is linear and uniform, similar to the brick-shaped model, however some localized non-linearity is seen at the intersections /sharp corners where fracture geometries change (*Figure 27[c & d]*). The concept of *equivalent hydraulic aperture* that we used in 2D can be also applied in 3D, and the 3D steady-state simulation results can be used to characterize the hydraulic conductivity of a 3D fracture in the following manner:

$$h_{eq_{simulation}} = \sqrt[3]{\frac{12Q_{simulation}\mu l}{W(P_i - P_o)}} \quad (26)$$

Theoretical expressions for computing h_{eq} for series and parallel network of 3D fractures can also be devised as follows:

$$h_{eq_{series}} = \frac{1}{\sqrt[3]{\sum_{i=1}^n \left[\frac{W}{W_i} \times \frac{l_i}{l} \left(\frac{1}{h_i^3} \right) \right]}} \quad (27)$$

$$h_{eq_{parallel}} = \sqrt[3]{\sum_{i=1}^n \left(\frac{W_i}{W} \times h_i^3 \right)} \quad (28)$$

where, W_i is the width of the i^{th} 3D fracture, and W is the width of the equivalent 3D block that we wish to use in replacement of n individual fractures.

For the *Case 3d.2* presented here, $h_{eq_{simulation}} = 0.734 \text{ mm}$, while theoretically, $h_{eq_{series}} = 0.714 \text{ mm}$. A possible explanation for this slight discrepancy is that *Equation 26* is actually an approximate rather than exact relation in view of 3D flow. *Equation 26* is just another form of *Equation 7*, which basically states that the volumetric flow rate per unit width through a 2D model can be multiplied with a width W to give the volumetric flow rate through its 3D counterpart. In reality, however, this multiplication will result in an overestimation of the actual 3D flow rate, because in real 3D flow there will be that ‘wall’ or ‘edge’ effect as was shown by *Figure 25(b)*. However, when W is much larger than h , the wall effect may be considered negligible, and it is safe to use *Equation 26*.

3.7.3 Fracture model with asperity contacts

The previous two 3D models were simple in a sense that each of them could be replaced with a 2D model and simulation could be carried out in 2D without much loss or accuracy. In fact, for transversely isotropic models, which are very often used in seismic modeling of fractures, 3D flow models can be replaced with equivalent 2D models in order to save the cost of computation. However, when the roughness of fracture surfaces are considered important in flow modeling, a full 3D simulation is essential. In this section, we show a slightly complicated 3D fracture model with varying apertures (asperity) and presence of asperity contacts (*Figure 28*). *Navier-Stokes* simulation can be used to study flow distribution around obstructions (*Figure 29*). The steady-state volumetric flow rate obtained for this 3D fracture is $Q = 1 \times 10^{-7} \text{ m}^3/\text{s}$, for $\Delta P = 6.579 \text{ Pa}$ between the inlet and outlet. With this information, the equivalent hydraulic aperture for this 3D fracture model can be computed as:

$$h_{eq} = \sqrt[3]{\frac{12Q\mu l}{W(P_i - P_o)}} = \sqrt[3]{\frac{12 \times 1e-7 \times 0.005 \times 0.05}{0.05 \times 6.579}} \cong 0.97 \text{ mm}$$

The numerical model of this 3D object consists of a total 705243 computational cells (control volumes). It is not possible to determine the series/parallel connection of these cells, and therefore no theoretical formula can be used in such cases to predict/estimate h_{eq} . The minimum, average (mean), and maximum values of these cell heights are 0.008 mm , 0.1015 mm , and 0.1919 mm respectively, however nothing conclusive can be said about the h_{eq} from these values. Therefore, doing a Computational Fluid Dynamics based flow simulation is the most reliable, and perhaps the only way to provide estimates of equivalent hydraulic parameters for real 3D fractures.

4. Conclusion

The simplicity of *cubic law* in computing fracture *transmissivity* (or *permeability*) makes it a popular choice among the reservoir simulation community. We recommend using a modified cubic law, namely “*generalized cubic law*” that relates *fracture transmissivity* (or *permeability*) to the *equivalent hydraulic aperture* of a fractured medium. In this paper, we proposed methods for computing *equivalent hydraulic aperture* for different types of fracture models.

Numerical solutions of either *Stokes* or *Navier-Stokes* equation can be used to model slow flow of viscous and Newtonian fluids (e.g. oil, water) in subsurface fractures. *Navier-Stokes* is more reliable especially when modeling flow in variable aperture or rough fractures; however based on our analysis in a few fracture models, we conclude that *Stokes* equation based modeling yields comparable hydraulic property estimates for *Reynolds numbers* upto approximately 100. This upper limit, though, will vary depending on the severity and frequency of aperture variation in the fracture.

To estimate the *equivalent hydraulic aperture*, a flow simulation in the fracture is preferred. In fact, for complicated 3D fracture geometries, or for complex 2D/3D fracture networks, flow simulation may be the only way to estimate the true *equivalent hydraulic aperture* of the fractured medium. However, when the fracture geometry is not known to build the computational model or due to other constraints a complete flow simulation cannot be performed, the following rules will serve as good alternatives in estimating *equivalent hydraulic apertures*: 1) For series fractures, using the harmonic mean of all apertures cubed, 2) For parallel fractures, using the sum of all apertures cubed, 3) For rough surface fractures, modeling it as a series combination of several parallel-plate fractures using the surface roughness information, 4) For inclined fractures, using a *cosine* correction term, 5) For fracture networks, using the aperture of the fracture that offers the shortest continuous path from inlet to outlet.

The methodology described in this paper is very robust and using this method insights were gained about the relative significance of some major fracture flow controlling factors such as *aperture*, *length*, *constriction*, *width*, *asperity contacts*, *orientation*, and *intersection*. Work is currently underway to apply the method and results described in this paper to a reservoir problem.

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Figure 1: Parallel plate flow model for fractures.

Figure 2: Parallel plate flow model in 2D, and the parabolic velocity profile.

Figure 3: Flow simulation results in a parallel plate model. From top to bottom: pressure distribution (contours), velocity distribution (contours), velocity vectors showing a parabolic velocity profile.

Figure 4: Flow simulation results for three distinct fractures connected in parallel: (a) Pressure Contours, and (b) Velocity Contours.

Figure 5: Velocity magnitude contours for Case a. (Model has very high aspect ratio, therefore to show undistorted velocity contours, only parts of the model are plotted.)

Figure 6: Histogram (plotted in 50 bins) of equivalent apertures for a series network with 3 discrete fractures having variable length fractions.

Figure 7: (For a total length of 80 cm): Upper – Series combination yielding minimum h_{eq} ;
Lower – Series combination yielding maximum h_{eq} .

Figure 8: Approximating rough fractures with smooth, *parallel plate* models.
Lines in red indicate real fracture surfaces, lines in green indicate parallel
plate representation of those surfaces.
Terms ‘aperture’ and ‘constriction’ are defined as shown.

[a]

[b]

[c]

[d]

Figure 9: [a] Model geometry: the green ovals indicate regions for which simulation results are presented. [b] (Un-filled) Pressure contours in regions identified by green ovals in [a]. Separate color-maps are used for two regions to better represent the variation of local contours. [c] (Filled) contours of Velocity magnitudes. [d] (Un-filled) contours of Stream functions that extend from inlet to outlet.

Figure 10: [a] Geometry of a fracture model, where the magnitude of constrictions are different than the magnitudes of adjacent apertures. The green ovals indicate regions for which simulation results are presented.

[b] (Un-filled) Pressure contours in regions identified by green ovals in [a] showing non-linearity due to constrictions. [c] (Filled) contours of Velocity magnitudes – large contrast in velocity magnitude at the constrictions. [d] (Un-filled) contours of Stream functions that extend from inlet to outlet.

Figure 11: X-velocity magnitudes at different locations (constant x planes) for the model shown by Figure 10[a]. The constrictions are located at $x=45$ cm (green dots) and $x=60$ cm (sky blue dots). A sharp change is noticed at $x=45$ cm compared to its adjacent fracture sections (black dots and dark blue dots). Velocity profile is parabolic in each section – a characteristic of laminar flow.

Figure 12: Combination of series and parallel fractures. **[a]** Pressure contours (also showing fracture dimensions). **[b]** Velocity contours near the intersections.

Figure 13: A fracture inside a rock slab, making an angle θ with the axis of overall pressure gradient, given by $(P_1 - P_2)/l$.

Figure 14: Flow in a 45° fracture. Pressure Contours: [a] near the Inlet, and [b] near the Outlet. Velocity Magnitude Contours: [c] near the Inlet, and [d] near the Outlet. [e] - Velocity vectors in the mid-zone, exhibiting a parabolic profile normal to the fracture cross-section.

Figure 15: Effect of fracture inclination in hydraulic conductivity estimation. The plot is for inclination angle, θ vs. equivalent hydraulic aperture, $h_{eq} (= \sqrt[3]{h^3 \times \cos \theta})$

Figure 16: Methodology of choice for reservoir flow simulation models, depending on the length-scale of fractures relative to a reservoir grid-block (adopted from Bourbiaux *et al.* 2002).

Figure 17: Total seven fractures (numbered a - g), intersecting with each other as shown to form a fracture network. The intersections are also numbered so that they can be referred in some subsequent plots. The true fracture apertures (normal distance between fracture walls) in mm are: a – 0.63, b – 0.63, c – 0.9, d – 0.7, e – 1.0, f – 0.96, g – 0.61. For flow simulation purposes, constant static pressures are specified at Inlet and Outlet faces, while the other two opposite faces represent no-flow boundaries. In each individual fracture, fracture walls are impermeable and rigid.

Figure 18: From left to right, top to bottom: mesh used at and around intersection nos. 1, 3, 5 & 7.

Figure 19: Flow simulation results: from top to bottom – around intersection nos. 1, 2, and 11.

Figure 20: Stokes equation based flow simulation results; top: for $\Delta P = 39$ Pa, bottom: for $\Delta P = 20466$ Pa.

Figure 21: Velocity magnitude contour plots from Navier-Stokes simulation: [a] for $\Delta P = 39$ Pa, and [b] for $\Delta P = 20466$ Pa.

Figure 22: Comparing hydraulic characterization of the series fracture model using two different computational (fluid dynamics) methods.

Figure 23: A computer model of a realistic 3D fracture surface.

Figure 24: 2D flow simulation in a real fracture: **[a]** Pressure contours, **[b]** Velocity contours.

[a]

[b]

[c]

Figure 25: Flow bounded by parallel plates in 3D (also known as *Poiseuille Flow*): [a] Model geometry, with meshes; [b] Velocity vectors shown in 3D; [c] Pressure distribution (contours) in 3D.

[1]

$$\begin{aligned} -b &\leq y \leq b \\ -a &\leq z \leq a \end{aligned}$$

[2]

$$u(y, z) = \frac{16a^2}{\mu\pi^3} \left(-\frac{dp}{dx} \right) \sum_{i=1,3,5,\dots}^{\infty} (-1)^{(i-1)/2} \left[1 - \frac{\cosh(i\pi z/2a)}{\cosh(i\pi b/2a)} \right] \frac{\cos(i\pi y/2a)}{i^3}$$

[3]

[4]

[5]

Figure 26: [1] 3D brick geometry (*non-circular duct*) and [2] constant rectangular cross-section normal to the flow direction. [3] Analytical expression for velocity profile (Gursul, 1998). [4] Analytical solution for velocity profile using series index i up to 200, and [5] Velocity profile obtained from flow simulation.

[a]

[b]

[c]

[d]

Figure 27: [a] 3D flow model (showing meshes as well) with variable aperture. [b] Pressure distribution in 3D. [c] x-y profile of pressure distribution plotted for a surface $z=0$ (midway between the two walls), and [d] magnified view of the region pointed by white oval on [c] to highlight the non-linearity effect due to aperture change.

Figure 28: Realistic fracture is 3D having both asperities and asperity contacts. Ovals/circles in red denote asperity contacts present in the model.

Figure 29: Contour plots along a 2D surface ($y = 0$) drawn from the 3D fracture model of Figure 33.