NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, takes any responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product or process disclosed or represents that its use would not infringe privately-owned rights.
1 Brief Description of Main Accomplishments

Here we give a very condensed description of the main research results we have obtained with funding by this grant. Further descriptions of a large fraction of the research along with movies and talks illustrating them are in addition available on the PI’s web page at www.esam.northwestern.edu/riecke/research.html.

The overarching theme of the funded research is the formation of spatial and spatio-temporal structures as they arise in many physical, chemical, and biological systems. We have investigated theoretically various aspects of such patterns and have moved forward from relatively simple patterns to quite complex, spatio-temporally chaotic structures. The research funded by this grant played a major role in the recent awarding of a Research Award from the Alexander-von-Humboldt Foundation (Germany) to the PI.

1.1 Localized Structures

A quite striking phenomenon in pattern formation is the appearance of localized patterns, i.e. patterns that extend only over a small portion of a much larger, spatially homogeneous system. The main, tantalizing question has been: what keeps these structures from spreading over the whole system? We have addressed this and related questions for a number of physical systems.

Two results have been particularly important:

1. For the localized wave pulses observed experimentally in convection in water-alcohol mixtures by Kolodner et al., Ahlers et al., and Steinberg et al. we have been able to derive a new set of coupled Ginzburg-Landau equations that captures the main features of these pulses, i.e. their exceedingly slow drift and a new aspect of the localization mechanism (refs. [6,7,9,14] in the publication list below).

2. In electroconvection localized waves (‘worms’) have been observed by Ahlers et al. in regimes in which standard theories only predicted the unpatterned state. Our extension of the standard theory provided a natural explanation of the experimental observations ([19,26]).

We have also provided further insight into the localization mechanism of localized waves observed in vertically vibrated granular media (‘oscillons’) ([19]).

In other situations localized structures consist of domains of one type of pattern embedded in a different structure. Again, this raises the central question: what keeps one type of pattern from invading the other one and from taking over the whole system? Our research gave answers to this question in the context of patterned flames ([1]) and in a fascinating model in which one of the two types of patterns is actually chaotic in space and time while the other is stationary ([10]). We have also predicted the appearance of ordered domains of different patterns in ferro-fluids exposed to magnetic fields ([13]), which were subsequently observed experimentally.

We have published a brief review of the various localization mechanisms ([20]).
1.2 Periodically Driven Systems

Just like a periodic driving of individual oscillators can lead to interesting new phenomena, temporally periodic driving of systems supporting traveling waves induces new patterns and dynamics. In one line of research we have shown that such a forcing induces an interaction between wave fronts that can lead to the formation of stable front pairs, which then form localized waves. Since the interaction is solely due to the periodic forcing it can be tuned ([29]), in contrast to the mechanisms relevant in the systems discussed in Sec. 1.1 above.

Resonant periodic driving can excite standing waves. We have extended our previous analysis to two-dimensional systems with anisotropy ([2]). Our results match quite well experiments by Rehberg et al.

Motivated in particular by the oscillons found in vertically vibrated granular material we developed a simplified model for granular material under vertical vibrations ([18]). It provided insight into the experimentally observed dispersion relation of the surface waves excited by the shaking and into the mechanism that localizes the oscillon.

1.3 Quasi-Patterns

In one line of research we investigated quasi-patterns within the weakly nonlinear framework of Ginzburg-Landau equations. We identified the relevant instabilities of these patterns and gave insight into the defects that arise from these instabilities ([27]).

1.4 Spatio-Temporal Chaos

In recent years we have focussed our efforts on patterns that are complex in space and time, i.e. they are spatio-temporally chaotic.

One important question for such complex patterns is their characterization in terms of simpler quantities, which will give intuitive insight into the workings of the patterns. We have introduced a new approach for the characterization of spatio-temporally chaotic patterns that are dominated by point defects. Since often the motion of the defects is responsible for the disorder in the system we investigated the statistics of their trajectories. In a model system we were quite successful and identified a non-equilibrium transition between two chaotic states: one exhibits a slowly decaying correlation function revealing an almost ordered stripe structure on average, while the other is strongly disordered with a rapidly decaying correlation function. We found that the statistics of the defect trajectories exhibits qualitatively different decays in these two regimes (exponential vs. power-law) ([28]). Subsequently, we have applied this approach to the classic complex Ginzburg-Landau equation and confirmed the power-law behavior ([36]).

Most previous studies of spatio-temporal chaos had been based on stripe-like patterns. We were therefore interested in investigating other possibilities. Specifically, we considered systems in which a hexagonal pattern undergoes a transition to a disordered state. We have shown that under quite general conditions breaking the chiral symmetry of a system that
exhibits hexagonal patterns will induce a transition to oscillating (‘whirling’) hexagons, which quite generally do not exhibit ordered oscillations but temporally and spatially chaotic ones ([23,24,25]).

We have applied this knowledge to non-Boussinesq convection, which is the classic system exhibiting hexagonal patterns. We first considered such hexagonal convection at low Prandtl numbers, which necessitated the extension of their standard description to include large-scale flows generated by deformations of the pattern ([31]). Then we broke the chiral symmetry by considering convection systems that are rotated about a vertical axis. In a model system we found an interesting new chaotic state. It is dominated by penta-hepta defects that induce the nucleation of other defects, which then leads to very different statistics compared to the standard distributions obtained previously in the complex Ginzburg-Landau equation and a number of other systems ([32,33]).

Then we turned to direct simulations of the fluid equations underlying convection with rotation. Focussing on water as the working fluid, we first identified in the absence of rotation the restabilization of hexagons at larger heating, which was not expected from the classic, weakly nonlinear theory ([38]). With rotation we found three interesting regimes. We first confirmed quantitatively that the complex behavior expected for the whirling hexagons in the weakly non-Boussinesq regime indeed also arises in the full fluid simulations. This provides us now finally with a second experimentally accessible system for one of the complex states of the two-dimensional complex Ginzburg-Landau equation ([39]). For somewhat larger non-Boussinesq effects, we obtained a new chaotic state that is not characterized by defects but rather by bursts in the oscillation amplitude ([39]). Both of these states exhibit disorder only in the oscillation amplitude; the hexagonal lattice is defect-free. For strong non-Boussinesq effects the strong whirling action leads to large deformations of the hexagonal lattice, which therefore in most cases breaks up. The resulting new chaotic state is dominated by the interplay between the whirling and the formation of defects in the hexagonal lattice ([34]). We have also investigated the effect of non-Boussinesq effects on fluids with low Prandtl number, which introduces further dynamics ([41]).

Quite recently we have started a new project in which we aim at characterizing spatio-temporally chaotic states through the statistics of various geometric aspects of the pattern. We expect that this approach will give us and other theoretical and experimental researchers a new, effective to identify transitions between different spatio-temporally chaotic states ([40]).

2 Publications

Refereed publications

1. A. Bayliss, B.J. Matkowsky and H. Riecke,
   *Structure and Dynamics of Modulated Waves in Cellular Flames*,
2. H. Riecke, M. Silber and L. Kramer,  
*Temporal Forcing of Hopf Bifurcations in Anisotropic Systems*,  

3. H. Herrero and H. Riecke,  
*Front Structures in a Real Ginzburg-Landau Equation Coupled to a Mean Field*,  

4. D. Raitt and H. Riecke,  
*Domain Structures and Zig-Zag Patterns Modeled by a Fourth-Order Ginzburg-Landau Equation*,  

5. D. Raitt and H. Riecke,  
*Domain Structures in Fourth-Order Phase and Ginzburg-Landau Equations*,  

6. H. Herrero and H. Riecke,  
*Bound Pairs of Fronts in a Real Ginzburg-Landau Equation Coupled to a Mean Field*,  

7. H. Riecke,  
*Attractive Interaction Between Pulses in a Model for Binary-Mixture Convection*,  

8. H. Riecke,  
*Solitary Waves under the Influence of a Large-Scale Field*,  

9. H. Riecke and W.-J. Rappel,  
*Coexisting Pulses in a Model for Binary-Mixture Convection*,  

10. G.D. Granzow and H. Riecke,  
*Phase Diffusion in Localized Spatio-Temporal Amplitude Chaos*,  

11. S.G.K. Tennakoon, C.D. Andereck, J.J. Hegseth, and H. Riecke,  
*Temporal Modulation of Traveling Waves in the Flow Between Rotating Cylinders With Broken Azimuthal Symmetry*,  

12. A.A. Golovin, A.A. Nepomnyashchy, L.M. Pismen and H. Riecke,  
*Steady and Oscillatory Side-Band Instabilities in Marangoni Convection with Deformable Interface*,  
13. D. Raitt and H. Riecke,  

14. H. Herrero and H. Riecke,  

15. G.D. Granzow and H. Riecke,  

16. G.D. Granzow and H. Riecke,  

17. H. Riecke and G.D. Granzow,  

18. J. Eggers and H. Riecke,  

19. C. Crawford and H. Riecke,  

20. H. Riecke,  

21. H. Riecke and L. Kramer,  

22. F. Sain and H. Riecke,  

23. B. Echebarria and H. Riecke,  


35. A. Rojin, H. Riecke, S.A. Solla,  
*Self-sustained activity in a small-world network of excitable neurons*  

36. C. Huepe, H. Riecke, K.E. Daniels, and E. Bodenschatz,  
*Statistics of Defect Trajectories in Spatio-Temporal Chaos*  

37. M. Higuera, H. Riecke, and M. Silber,  
*Near-Resonant, Steady-State Mode Interaction: Periodic, Quasi-Periodic and Localized Patterns*  

38. S. Madruga, H. Riecke, and W. Pesch,  
*Reentrant Hexagons in non-Boussinesq Convection*  
J. Fluid Mech. (accepted).

39. S. Madruga, H. Riecke, and W. Pesch,  
*CGL Defect Chaos and Bursts in Hexagonal Rotating non-Boussinesq Convection*  

40. H. Riecke and S. Madruga,  
*Geometric Diagnostics of Complex Patterns: Spiral-Defect Chaos*  
to be submitted to Chaos.

41. S. Madruga and H. Riecke,  
*Non-Boussinesq convection in gases in the strongly non-linear regime*  
to be submitted to Phys. Rev. E.

Unrefereed Publications:

1. A. Bayliss, B.J. Matkowsky and H. Riecke,  

2. A. Bayliss, B.J. Matkowsky and H. Riecke,  
*Symmetries in Modulated Traveling Waves in Combustion: Jumping Ponies on a Merry-Go-Round*,  

3. H. Riecke and H. Herrero,  
*Interacting Fronts in a Model for Binary-Mixture Convection*,


### 2.1 Presentations of DOE-Funded Research

The results obtained with funding through this grant have been presented at various conferences and seminars by the PI and his collaborators:

#### 2.1.1 Invited Talks at Conferences


2. *Binary-Fluid Convection*  
   Mini-Course consisting of 8 lectures at the Summer Institute for Geophysical Fluid Dynamics of the Woodshole Oceanographic Institute, (jointly with Dr. P. Kolodner, Bell Laboratories of Lucent Technologies) 7/8/96-7/13/96.

3. *Large-scale Description of Localized Spatio-temporal Chaos in a Model for Parametrically Driven Waves*,  


5. *Models for Oscillons and for Hexagons with Rotation*,  
   Workshop *Pattern Formation in Dissipative Continuous Systems*, Hüinfeld, Germany, September 1998.
   3 Lectures at Summer School *Waves, Vortices, Dunes: Structures in Laboratory 
   and Nature* of the Wilhelm und Else Heraeus-Foundation, University of Magdeburg, 
   Germany, September 1998.

   In this course the PI gave an introduction to the theory of spatially extended dynamical 
   systems and pattern formation. The course consisted of 3 two-hour lectures and covered 
   also results of the research that was funded by this grant in previous years.

7. **Transition from Ordered to Disordered Defect Chaos**, 
   International conference *Computer-Aided Analysis of Dynamical Structures and 
   Defects*, Max-Planck Institute for Physics of Complex Systems, Dresden, Germany, 
   July 1999.

8. **Worms in Electroconvection and the Localization of Waves by Modulation**, 
   International Conference *Pattern Formation in Liquid Crystals PFLC '99*, Bayreuth, 
   Germany, September 1999.

9. **Localized Waves in Dissipative Pattern-Forming Systems**, 
   Conference on Nonlinear Problems in Applied Mathematics, U. Notre Dame, South 
   Bend, IN, April 2000.

10. **Spatio-temporal Chaos and Defects**, 

11. **Complex Structures in Rotating Convection**, 
    Rocky Mountains Workshop on *Dynamics and Bifurcations of Patterns in Dissipative 

12. **Persistence and Failure in a Small-World Network of Integrate-and-Fire Neurons** 
    plenary talk at *Understanding Complex Systems 2004*, U. Illinois, Urbana-Champaign, 

13. **Persistence and Failure in Networks of Excitable Neurons with Short- and Long- 
    range Connectivity**, 
    ICAM Workshop *Frontiers in Biological Physics III: Neurobiology Workshop*, 

14. **Complex Structures in Rotating Non-Boussinesq Convection**, 
    in *Theoretical Aspects of Pattern Formation*, Workshop of Newton Institute for 

### 2.1.2 Invited Talks in Mini-Symposia

1. **Parity-Breaking Bifurcation in Cellular Flames**, 
   Int. Conference on Nonlinear Dynamics and Pattern Formation in the Natural En-


Lecture at workshop concluding the multi-university research initiative on pattern formation (‘Schwerpunkt’, funded by the German Research Foundation DFG), Hünfeld, Germany, September 1998.

4. *Transition from Ordered to Disordered Defect Chaos*,

5. *A Simple Continuum Model for Vertically Vibrated Granular Media*,
Summer Meeting of the Canadian Association of Physicists, Fredericton, Canada, June 1999.


7. *Instabilities and Spatio-Temporal Chaos in Hexagon Patterns with Rotation*,
March Meeting of the American Physical Society, Minneapolis, MN, March 2000.

8. *Localized Waves in Dissipative Pattern-Forming Systems*,

9. *Instabilities and Spatio-Temporal Chaos in Hexagon Patterns with Rotation*,
Sectional Meeting of the American Mathematical Society, Lafayette, LA, April 2000.

10. *Side-band instabilities and defects in quasipatterns*,
SIAM Conference on Dynamical Systems, Snowbird, Utah, May 2001

11. *Penta-Hepta Defect Chaos in Hexagons with Rotation*,
SIAM Conference on Dynamical Systems, Snowbird, Utah, May 2003

2.1.3 Contributed Talks at Conferences

1. *Localized Spatio-Temporal Chaos in Parametrically Excited Waves*,
talk of graduate student G. Granzow at the March Meeting of the American Physical Society, St. Louis, Missouri, March 96.

2. *On the stability of standing waves and traveling rectangles*,
March Meeting of the American Physical Society, St. Louis, Missouri, March 96.
3. Instability and Dynamics of Standing Waves and Traveling Rectangles,
   49th Annual Meeting of the Division of Fluid Dynamics of the American Physical
   Society, Syracuse, New York, November 1996.

4. Phase Diffusion in Localized Spatio-Temporal Amplitude Chaos of Parametrically
   Excited Waves,

5. Complex Dynamics of Spatial Structures,
   15th Symposium on Energy Engineering Sciences, DOE, Argonne National Labora-
   tory, Argonne, IL, May 1997.

6. Instabilities of Hexagon Patterns in a Model for Rotating Convection,
   poster by student F. Sain at SIAM Conference on Dynamical Systems, Snowbird,


8. Complex Patterns,

9. Oscillon-type Structures and Their Interaction in a Swift-Hohenberg Model,
   talk by student C. Crawford, March Meeting of APS, Los Angeles, March 1998.

10. Localization of Waves without Bistability: Worms in Nematic Electroconvection,
    March Meeting of APS, Los Angeles, March 1998.

11. A Continuum Model for Vibrated Sand,
    March Meeting of APS, Los Angeles, March 1998.

12. Localization of Non-Hysteretic Waves: Worms in Electroconvection,
    talk in mini-symposium Convection in Nematic Liquid Crystals, 51st Annual Meeting

13. Instabilities of Hexagons in a Model for Rotating Convection,
    talk by student Fil Sain, Meeting of the Division of Fluid Dynamics of the APS,

14. Longwavelength instabilities in rotating convection,
    talk by postdoctoral associate Blas Echebarria, at Meeting of the Division of Fluid


16. Dispersion relation and oscillons in a continuum model for vibrated granular media,
17. *Dispersion relation and oscillons in a continuum model for vibrated granular media*,

18. *Localized traveling waves under the influence of temporal modulation: growing worms and pulses*,
talk by student Catherine Crawford, at the 5th SIAM Conference on *Applications of Dynamical Systems*, Snowbird, May 1999.

19. *Instabilities of hexagonal patterns in the presence of rotation*

20. *Defect statistics in ordered and disordered spatio-temporal defect chaos*

21. *Defect Chaos in Hexagon Convection with Rotation*

22. *Instabilities and Spatio-temporal Chaos of Hexagonal Patterns in Rotating Convection*


poster by graduate student V. Moroz, conference *Nonlinear Dynamics and Pattern Formation* in honor of H.L. Swinney, Austin, TX, June 2000.


27. *Hexagonal Convection Patterns Coupled to a Mean Flow*,
talk to be given by associate Yuan-Nan Young, SIAM Conference on Applications of Dynamical Systems, Snowbird, May 2001.
28. Localization of Traveling Waves by An Advected Field, 

29. Side-band instabilities and defects in quasipatterns 
March Meeting of the American Physical Society, Seattle, WA, 2001

30. Instabilities and Spatio-temporal chaos in long-wave rotating Marangoni convection 

31. Near-resonant Mode Interaction: One-dimensional Periodic and Quasi-periodic Pattern 
March Meeting of the American Physical Society, Indianapolis, IN, 2002

32. Penta-Hepta Defect Chaos in a Model for Rotating Convection 
Pattern formation for the next millennium: where do we go from here, CKP Meeting (celebration of 60th birthday of P. Clavin, L. Kramer, Y. Pomeau), La Foux d’Allos, France, June 2002.

33. Whirling Chaos in Rotating Non-Boussinesq Convection 

34. Persistence and Failure in a Small-World Network of Excitable IF-Neurons 

35. Reentrant Hexagons in non-Boussinesq Convection 
talk of associate S. Madruga, 57th Annual Meeting of the Division of Fluid Dynamics of the American Physical Society, Seattle, Washington, 2004

36. CGL-Chaos and Bursts in Rotating Non-Boussinesq Convection 

37. Reentrant Hexagons in non-Boussinesq Convection 
talk of associate S. Madruga, SIAM Conference on Dynamical Systems, Snowbird, Utah, 2005.

38. Whirling and Bursting Hexagons in Non-Boussinesq Rayleigh-Bénard Convection 
talk of associate W. Pesch, workshop, St. Etienne, France, September 2005.

2.1.4 Seminars and Colloquia

1. Physics Department, Nonlinear Dynamics Seminar, University of Texas, Austin, February 1993.


5. Departamento de Fisica y Matematica Aplicada, seminar, Universidad de Navarra, Pamplona, Spain, August 1993.


8. Physics Department, colloquium, Northwestern University, November 1994.


18. Cornell Theory Center, seminar, Cornell University, March 1996.


20. IBM Yorktown Heights, seminar, November 1996.


24. Physics Department, seminar, Universität des Saarlandes, Saarbrücken, Germany, December 1999.

25. Physics Department, seminar, Universität Bayreuth, Germany, December 1999.
26. *Stability of Hexagonal Patterns in Rotating Convection*  
Seminar by postdoc B. Echebarria, Department of Condensed Matter Physics, University of the Basque Country, Leioa (Spain), September 1999.


28. Physics Department, colloquium, Northeastern University, March 2000.


31. *Patterns, Oscillons and Localization* talk by graduate student C. Crawford, seminar, mathematics department, Metropolitan State University, March 2000.

32. Interdisciplinary Seminar, University of Houston, April 2001. 4/12/01 *Non-periodic Structures in Pattern-Forming Systems*

33. Mathematics Department, Applied Mathematics Seminar, Illinois Institute of Technology, October 2001. 10/1/1 *Hexagonal patterns: secondary instabilities and disorder*


35. Physics Department, Colloquium, Northwestern University, May 2002. *Disordered Spatio-temporal Structures Far From Equilibrium*

*Spatio-temporal chaos and defects in pattern-forming systems*

*Spatio-temporal chaos and defects in pattern-forming systems*


41. Physics Department, seminar, McGill U., Montreal, Canada, May 2005. *Spatio-Temporal Chaos and Defects in Pattern-Forming Systems*