Scaling Rules For Linac PreInjector Design

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Abstract

Proposed designs of the prebunching system of the NLC and TESLA are based on the assumption that scaling the SLC design to NLC/TESLA requirements should provide the desired performance. A simple equation is developed to suggest a scaling rule in terms of bunch charge and duration. Detailed simulations of prebunching systems scaled from a single design have been run to investigate these issues.

1 Introduction

The NLC and TESLA prebuncher designs are based on that of the SLC since many of their performance requirements are similar to the SLC, and the SLC source has run successfully for eleven years. Understanding how prebuncher performance changes as input parameters change is helpful for both design and operation. As an example, it would be useful for a designer to understand where a good starting point is close to desired performance before simulation. Another example, an operator may roughly want to know how injector performance will change as charge is raised, or how poor performance due to an increase in charge may be compensated by some other input parameter. Space charge forces in the preinjector are studied with and without prebunchers in order to understand these considerations.

2 Space Charge Considerations

The important differences, for the prebunching system, between a bunch coming from the NLC/TELSA cathodes and that of the SLC are the bunch length, the total charge, and the energy. The NLC and TESLA require different bunch lengths then the SLC design. The first thing to note is that the frequency of the cavity fields in the prebunchers must be scaled appropriately. This will keep the phase space of the bunch the same between the two designs. The simple rule is that the frequency must scale inversely with the time duration of the bunch. That is:

\[
\frac{f_{\text{req}1} \Delta t_1}{f_{\text{req}2} \Delta t_2} = 1
\]  

(1)

If the time duration of the bunch is changed then the charge density of the bunch also changes. This will modify space charge effects inside the bunch.
Similarly, increasing the amount of charge in the bunch will increase space charge effects. Increasing the bunch energy will increase the distance between electrons in the laboratory frame, decreasing space charge effects. It would be useful to be able to predict how performance might scale as these bunch parameters are changed. An equation that can be used as an indicator of how performance will scale is developed from relativistic kinematics and Coulomb’s Law. Beginning with the differential force on a test charge at the end of a cylinder of charge:

\[
dF_z = \frac{q}{4\pi\epsilon_0} \frac{Q}{\Delta r^2} r\cos(\phi) drd\theta \frac{dz}{z^2 + r^2}
\]  

(2)

If this equation is integrated over the cylinder of charge, shown in figure one, it will yield the total force on the test charge from the entire bunch.

\[
F_z = \frac{qQ}{4\pi\epsilon_0 r_o (\delta + \Delta)}
\]

(3)

We can use the following:

\[
F_z = \gamma m \ddot{z}
\]

(4)

To develop a proportional relationship between the longitudinal acceleration and injector parameters charge, energy, and bunch length. That is,

\[
\ddot{z} \propto \frac{Q}{\Delta \gamma}
\]

(5)

The length of the bunch can be substituted for the time length by:

\[
\Delta = \beta c \Delta t
\]

(6)

Rewriting this in terms of energy using,

\[
\beta = \sqrt{1 - \gamma^{-2}}
\]

(7)

and inserting equation six into five:
\[ \dot{z} \propto \frac{Q}{\Delta t} (\gamma^2 - 1)^{\frac{1}{2}} \]  

This implies that to keep the space charge force the same, from one design to the next, the above equation must be held roughly constant. Putting this statement into an equation:

\[ \left( \frac{Q_2 \Delta t_1}{Q_1 \Delta t_2} \right)^2 \frac{\gamma_1^2 - 1}{\gamma_2^2 - 1} = 1 \]  

Simulations have been run to test space charge considerations in the prebunching system and to test if the above equation is a reasonable indicator of scaled injector performance.

3 Multivariate Simulation With PARMELA/ROOT

Many simulations must be run over a large range in order to test the validity and accuracy of any prebuncher effects and design rules involved with scaling the SLC to the NLC. It is useful to have a technique to scan over many variables over a desired range in one large simulation and be able to analyze a large amount of data in a way that is very conformal to the needs of the study. This method of simulation has been developed to test how the performance of the prebunching system changes under different input parameters. It is a useful technique for calculating tolerances, sensitivities, and running large parameter scans.

A looping control program is used to simulate UCLA PARMELA over the range of each input parameter that is being scanned. The output of each simulation is dumped into a data array. A macro inside the data analysis framework ROOT is used to convert the data array into a form that ROOT can understand. ROOT is used for analysis of the information. Figure Two shows a flow diagram of the technique.

4 Results

Two series of simulations have been run using this technique. The first run is a study of space charge effects with only the gun and solenoids. The second simulation includes a prebuncher. Figure Three is a simple diagram of the section of the injector being simulated.

The prebunching system is simulated, scanning over the bunch length, charge, bunch energy in the first run, and scanning over bunch length, charge, bunch energy, field in the buncher, and phase of the buncher relative to the launch of the gun. The scanned input parameters, for both simulation runs, along with the range of the scans are shown in tables one and two.
Figure 2: Diagram Of Method

Figure 3: Prebunching System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gun Voltage</td>
<td>120</td>
<td>200</td>
<td>kV</td>
</tr>
<tr>
<td>Bunch Length</td>
<td>0.5</td>
<td>2</td>
<td>ns RMS</td>
</tr>
<tr>
<td>Charge</td>
<td>1E10</td>
<td>5E10</td>
<td>Electrons</td>
</tr>
</tbody>
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Table 1: Parameters And Range Of Scan
4.1 Space Charge Only

Table One shows the range of input parameters simulated for a bunch traveling through a DC Gun and a set of solenoids. This set of simulations will help in understanding scaling issues without yet complicating the analysis with a prebuncher. It is useful to define the following parameter.

\[
K \equiv \left| \frac{1}{2.354\text{e}^7\Delta_t} \frac{Q}{(\gamma^2 - 1)^{\frac{1}{2}}} \right| \tag{10}
\]

Note that K is proportional to the longitudinal acceleration of a charge at the end of the bunch derived in the previous section. Higher K means stronger space charge effects inside the bunch. The set of simulations from Table One yield the plots in figures four and five.

The method of analysis described in section three makes it simple to limit the data from the simulations described in table one to only that with equal values of K. Longitudinal emittance and bunch length vs. position are used to
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<td>EField</td>
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<td>60</td>
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Table 2: Parameters And Range Of Scan

judge similar performance. Two separate cases are studied. In the first case, the data is cut such that data from simulations with K near one is kept. This corresponds to plots on the left in figures four and five. In the second case, K near five is kept, corresponding to plots on the right.

The plots in Figure four, for both cases, show horizontal lines that correspond to different simulations with different initial bunch lengths, initial energies, and charge. An arrow is drawn on each of the plots to highlight a single simulation and to guide the eye in seeing others. The only consistency between these simulations is the value of K. The growth in bunch length is parallel for each simulation plotted, which implies that the space charge force for similar K is the same. Also, the plots in figure five show for similar K roughly the same longitudinal emittance. Notice the bunch length growth and emittance growth is stronger for the higher K simulations. So, according to this set of simulations, in order to design a gun with space charge dependent performance, equation nine is a fine indicator of operation. The next step is to include a prebuncher.

4.2 With PreBuncher

The simulations described in table two are run to test how the issue of scaling changes when a prebuncher is added. Figure six is a plot of bunch length vs. position for several simulations with equal K (K=2). Most simulations with equal K demonstrate bunching at the same rate, but a noticeable amount do not bunch at the same rate at all. The circle on the plot highlights a few examples.

The addition of a prebuncher complicates the issue of scaling. If a preinjector design, scaled from another, can use the same bunch energy before the prebuncher, then it is simply a matter of changing the frequency of the buncher according to equation one in order to maintain the effect the buncher has on the phase space of the bunch. If energy changes, this changes both the bunch length in the laboratory frame and the amount of time the bunching field works on the bunch. If a stronger field is applied to designs with greater bunch energy then the focal point and bunching rate should be restored. The plots in figure seven contain simulations where a higher field is applied to higher energy bunches along with lower energy bunches with lower field. By changing the field in the buncher the focal point of the buncher is maintained as well as the bunch rate.
The plot on the right of figure seven demonstrates that transverse emittance of the bunch does not change.

The trade-off in applying a larger field for higher energy bunches is that longitudinal emittance increases. Figure ten contains simulations of higher energy bunches before and after the bunching field is increased. The plot on the left demonstrates how the focal point and bunching rate are changed. The plot on the right shows the increase in longitudinal emittance as a result of the higher field. Therefore, in order to maintain performance between two designs in a prebunching system equation nine is a fine indicator along with the fact that the bunching field will have to be raised in order to maintain the focal point and bunching rate in designs with higher bunch energies.

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Figure 7: High Energy And Low Energy Bunches With Different Bunching Fields Applied

Figure 8: High Energy Bunches Before And After Stronger Field Is Applied