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**FLOW AND DISPLACEMENT OF
BINGHAM PLASTICS IN POROUS MEDIA**

Topical Report

By
C. Shah, H. Kharabaf and Y. Yortsos

July 1995

Performed Under Contract No. DE-FG22-93BC14899

University of Southern California
Los Angeles, California

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FLOW AND DISPLACEMENT OF BINGHAM PLASTICS IN POROUS MEDIA

by

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ABSTRACT

Bingham plastics, which exhibit a finite yield stress at zero shear rate, have been used to model the flow behavior of certain heavy oils at reservoir conditions (Barenblatt et al., 1990). In such fluids, the onset of flow and displacement occurs only after the applied pressure gradient exceeds a minimum value. Understanding the flow behavior of such fluids has been limited to phenomenological approaches (Barenblatt et al., 1990, Wu et al, 1992). In this paper, we present numerical simulations and experimental visualization of flow and immiscible displacement of Bingham plastics in porous media using micromodels.

First, we describe a novel pore network simulation approach to determine the onset of flow. The dependence of the critical yield stress on the pore-size distribution is discussed. Visualization experiments of the constant-rate immiscible displacement of Bingham plastics in glass micromodels and Hele-Shaw cells are next presented. The process is subsequently simulated in a pore network. Experiments are successfully simulated with the pore network model. We discuss the effect of the yield stress and injection rate on the displacement patterns. We also propose a classification of the displacement patterns, similar to that for Newtonian displacement (Lenormand, 1989).

INTRODUCTION

Heavy oils at reservoir conditions often display non-Newtonian flow characteristics. Depending on temperatures and shear rates, bitumen and heavy oils have been found to be pseudoplastic, dilatant or Newtonian (Dealy, 1979, Poon and Kisman, 1992). Based on studies on bitumen and heavy oil, Dealy (1979) indicated that there was a mild degree of intramolecular and intermolecular aggregation between longer asphaltene molecules, leading to a non-Newtonian behavior. More recently, Poon and Kisman (1992) reported that mixtures of heavy oil and water with a small amount of sand behave as non-Newtonian and offered possible explanations for observed shear thinning behavior. Many authors in the Russian literature have approximated the rheology of heavy oils as that of a Bingham fluid (e.g. see Barenblatt et al., 1990, Bedrikovetsky, 1995). Bingham fluids possess a nonzero yield stress, so that flow takes place only after the applied pressure gradient exceeds a certain minimum value. Other examples where such flow may occur in porous media, include foam (Rossen and Mamun, 1993), ground water flow in certain clayey soils, and drilling and hydraulic-fracturing fluids (Wu et al., 1992).

There is a limited understanding of single- and multi-phase flow of non-Newtonian fluids in porous media. In the context of reservoir engineering, most existing studies are phenomenological and consist of solving effective continuum equations. For example, Wu et al. (1992) investigated the transient flow of Bingham fluids in porous media, including single- and multi-phase flow, using an *ad-hoc* extrapolation of the single-capillary expressions for Bingham flow. Similar continuum approaches have been taken in the Russian literature (see Bedrikovetsky, 1995). More fundamental work on the effect of the pore structure on the fluid flow behavior has recently been conducted by Mamoun and Rossen (1993) (see also references therein), in the context of foams, Sahimi (1993) and Shah and Yortsos (1995a) in the context of power-law fluids. However, still incomplete is the understanding of many aspects of single-phase flow in Bingham plastics, as well as of displacement processes involving non-Newtonian fluids. Such (processes) are of direct practical interest, for example in relation to mobility control involving non-Newtonian fluids (often of the power-law type), or in the recovery of oils of Bingham plastic rheology. Studies to explore the micromechanics of such processes at a more fundamental level are needed.

To provide better insight to these problems, we have studied the flow and displacement of certain non-Newtonian fluids. In Shah and Yortsos (1995a) and (1995b) we have investigated flow processes involving power-law fluids. In this paper, the focus is on fluids exhibiting a Bingham plastic rheology. Bingham plastics are a special type of non-Newtonian fluids, which exhibit a finite yield stress at zero shear rate and they correspond to the extreme case of pseudoplasticity. They are commonly described by the simple model (Bird et al., 1960)

$$\tau = \tau_o + \mu_p \dot{\gamma}, \text{ when } \tau > \tau_o, \quad (1)$$

and

$$\dot{\gamma} = 0, \text{ when } \tau < \tau_o, \quad (2)$$

where τ and $\dot{\gamma}$ denote shear stress and shear rate, respectively, and τ_o and μ_p are yield stress and plastic viscosity, respectively (Fig. 1). For a value of shear stress larger than τ_o , the resistance to shear caused by the tendency for the particles to build a structure fails completely, facilitating the onset of flow. In some practical applications, the plastic viscosity,

μ_p , also depends on the shear rate. Such fluids (Herschel-Bulkley fluids; see Al-Fariss and Pinder, 1987) are known as generalized plastics, and upon onset of flow they obey

$$\tau = \tau_o - m\dot{\gamma}^n \quad (3)$$

which generalizes (1) to arbitrary n . Drilling muds and some oils are good examples of generalized plastic fluids. As our emphasis in this paper is near the onset of flow, however, we shall consider only cases with $n = 1$.

To understand flow and displacement of Bingham plastics in porous media, we investigate their behavior in model porous media, such as pore networks and micromodels. Using a pore network of pores and throats we investigate by computer simulation the onset of flow of a single-phase Bingham fluid. The critical pressure gradient for the onset of flow is studied and its dependence on the system parameters is analysed. Flow patterns are compared to Newtonian fluids and power-law fluids with a small power-law index. Subsequently, we investigate the displacement of Bingham plastics, both experimentally and by numerical simulation. Flow visualization experiments for the displacement of a Bingham plastic by a Newtonian fluid in Hele-Shaw cells and glass bead packs are reported. Then, we develop a pore network model to simulate such displacements in a drainage process. Flow patterns are analyzed as a function of the yield stress and injection rates. The experimental results in glass bead packs are compared with the numerical results. Finally, the characterization of the displacement patterns in terms of a phase diagram similar to Lenormand (1989), is discussed

Single-Phase Flow in Pore Networks

First, the computer simulation of single-phase flow of Bingham plastics in network-like porous media was undertaken. In the simulations, the porous media were represented by a 2-D network of throats (bonds) and pores. Without loss in generality, we assume that the pore throat radius, r , is distributed with a pore size distribution, $\alpha(r)$, but that the pore length, l , is constant. Other assumptions in the simulations are that there is no pressure drop in the network nodes and that pressure drops are due to flow in the throats only. Thus, excess pressure drop associated with entrance and exit flow in each capillary was neglected. We expect these assumptions to be valid at very low flow rates, but to become progressively restrictive at higher rates, where the converging-diverging geometry of the pore structure dominates the macroscopic characteristics of the flow.

For flow in each capillary we take the expression (Bird et al, 1960)

$$q_i = \frac{\pi r_i^4}{8\mu_p l} \left(1 - \frac{4}{3} \left(\frac{\tau_o}{\tau_i} \right) + \frac{1}{3} \left(\frac{\tau_o}{\tau_i} \right)^4 \right) \Delta P_i \quad \text{when } \tau_i > \tau_o, \quad (4)$$

and

$$q_i = 0 \quad \text{when } \tau_i < \tau_o \quad (5)$$

where q_i is the volume flow rate through the capillary, across which the pressure drop ΔP_i is applied, and τ_i is the wall shear stress in the capillary

$$\tau_i = \frac{\Delta P_i r_i}{2l} \quad (6)$$

Equation (6) shows that the minimum applied pressure gradient (threshold) to mobilize flow in a capillary is

$$\Sigma_i \equiv \Delta P_i = 2\tau_o \frac{l}{r_i} \quad (7)$$

and depends on its particular geometrical characteristics. Thus, in the context of the network, the problem is essentially equivalent to one of distributing these thresholds. In particular, we may normalize by the overall pressure drop, ΔP , and define the normalized thresholds

$$\sigma_i = \frac{2\tau_o L}{r_i \Delta P} \quad (8)$$

where $L = Nl$ is the length of the network. The dimensionless threshold corresponding to the mean size $\langle r \rangle$

$$\sigma_o = \frac{2\tau_o L}{\langle r \rangle \Delta P} \quad (9)$$

will be used as one of the key dimensionless parameters. Larger values of σ_o imply a larger yield stress, τ_o , or a small applied pressure gradient. The other relevant parameter of interest is a measure of the pore size disorder, specifically the ratio $r_{max}/\langle r \rangle$.

The simulations were conducted by solving for the pressure field in the network using a mass balance at each node i ,

$$\sum_j q_j = 0 \quad (10)$$

where j runs over all nodes connected to node i , and q_j is given from (4)-(5). To solve (10) we used successive relaxation methods, with fairly stringent convergence criteria. Results have been obtained for various network sizes, not exceeding 100×100 , and for various size distributions. In all flow patterns shown in the subsequent Figures, the normalized flow in each bond is denoted with a level of greyness corresponding to the amount of flow rate through the bond. Thus, black color represent the maximum flow rate, white color corresponds to little or no flow. Only flow through the bonds is shown, flow through the nodes connecting two bonds being depicted in white.

Figure 2 shows the flow pattern for the case of a Newtonian fluid ($\sigma_o = 0.0$) for a uniform distribution of dimensionless bond sizes in the dimensionless interval $[0.01-1.99]$. Such relatively wide distributions are expected to characterize many natural porous media. We observe that the flow is spread over the entire network, and we may identify a connected cluster, resembling a percolation cluster, through which most of the flow occurs. Figure 3 shows the corresponding flow pattern for a Bingham plastic fluid with $\sigma_o = 0.21$. Now, it is evident that flow occurs over a smaller part of the network, as a larger number of throats is non-flowing due to the higher value of σ_o . The pattern becomes more ramified, as the yield stress further increases (see Figures 4 and 5, where the values of the dimensionless yield stress are 1.05 and 1.36, respectively). In Figure 4, flow occurs only through a limited network of connecting bonds. Even more pronounced are these effects for $\sigma_o = 1.36$, where the entire flow occurs through a minimum number of connected bonds (Fig. 5a). The remaining part of the network has very limited participation to flow and behaves as non-flowing. One should point out that these results are similar to those for shear thinning fluids, in the limit when

the value of the power-law index, n , is small. For example, flow patterns for the single-phase flow of a shear thinning fluid with $n=0.07$, for the same network size and distribution, are shown in Figure 5b. We observe a striking similarity between the two patterns. Additional results with different bond size distributions are discussed in Shah (1994).

At a critical value, σ_{oc} , flow ceases from taking place, indicating the existence of an overall threshold in the applied pressure drop, $\Delta P_c \sim 1/\sigma_{oc}$, which must be exceeded for flow to occur. Figure 6 shows a plot of the flow rate, normalized with its value corresponding to $\sigma_o=0$ (Newtonian case), versus the dimensionless yield stress. For the particular simulations, the dimensionless critical yield stress is found to be 1.36. Beyond a certain size, the critical yield stress was found not to depend on the network size. This leads to the important result that the minimum value of the macroscopic pressure gradient for the onset of flow, $\frac{\Delta P_c}{L} = \frac{2\sigma_{oc}}{\langle r \rangle}$, is size-independent. On the other hand, σ_{oc} depends strongly on the particular pore size distribution used and on the network connectivity. For example, the threshold for the onset of flow in a network of capillaries of the same size can be readily shown to be $\sigma_{eff} \equiv \sigma_{oc} = 1$, that for a bundle of capillaries is $\sigma_{oc} = \frac{r_{max}}{\langle r \rangle}$ (hence, it would be equal to 2 in the particular case here), while for a chain it is $\sigma_{oc} = \frac{r_{min}}{\langle r \rangle}$ (hence it would be equal to 0.01 in the particular case here).

Using a different method for obtaining the critical path, Kharabaf (1995) has studied its dependence on various geometric properties. Figure 7 shows how the critical path changes as the distribution of thresholds (equivalently, of the ratio l/r) changes by a simple translation of the distribution. Fig. 7a corresponds to a uniform distribution in $[0,1]$ and it has features similar to the previous (Fig. 5). However, as the mean of the distribution is displaced to the right (Fig. 7b), the path becomes less tortuous. In this case the critical path is essentially a straight line extending across the sample, in which case the minimum pressure gradient would be controlled by the largest threshold (smallest radius) of the distribution (Kharabaf, 1995).

The above analysis showed how the critical pressure drop scales with the parameters of the problems. The $Q - \Delta P$ relation following the onset of flow is expected to depend in a complicated manner with these parameters. The schematic shown in Fig. 6 indicates that the macroscopic flow approximately behaves in a parabolic fashion after the onset of flow, but it approaches a Newtonian flow behavior at large Q . Near the onset, however, its behavior is quite complex. The mobilization of various paths, as the pressure gradient increases, is shown in Figure 8 following the approach of Kharabaf (1995). For a medium where thresholds are not widely distributed, we expect an effective (mean) theory to be well applicable, in which case expressions based on parallel chains of capillaries should be valid. These directions are currently being pursued.

Immiscible Displacement Involving Bingham Plastics

In parallel, we have considered the immiscible displacement of a Bingham plastic by a Newtonian fluid. Such processes should mimic waterfloods of Bingham rheology heavy oils. Flow visualization in micromodels and numerical simulation in pore networks were conducted.

1. Flow Visualization

Experimental studies were undertaken with main objective the visualization of Bingham plastic displacement in Hele-Shaw cells and glass bead packs. A Hele-Shaw cell was con-

structed consisting of two glass plates of dimensions 18 cm × 7.6 cm × 0.09 cm, separated by a silicon rubber spacer. Two ports on the top plate, one for injection and another for production were drilled. The glass bead pack was constructed similarly, except that one or more layers of closely packed glass beads were sandwiched between the two plates. The experimental apparatus is shown in Figure 9. In the displacement sequence, the models were first saturated with the Bingham plastic. The subsequent injection of the displacing fluid was done using a syringe pump operating at a constant rate. Two types of water-based mud (Shah, 1994) were used to represent the Bingham plastic rheology. The Newtonian fluids consisted of kerosene, mineral oil or air. Red dye was added to the kerosene or the mineral oil in order to visualize the interface movement clearly during the course of the experiments.

The displacements are characterized by the following dimensionless numbers: The capillary number,

$$Ca = \frac{q\mu}{\gamma} \quad (11)$$

which denotes the ratio of viscous to capillary forces, where μ is the viscosity of the displacing fluid, the mobility ratio M

$$M = \frac{\mu_p}{\mu} \quad (12)$$

which is the ratio between the two viscosities, and a third dimensionless number,

$$N_\tau = \frac{\tau_o k_1^{1/2}}{\gamma} \quad (13)$$

where k_1 is the single-phase permeability. The latter expresses the relative magnitude of the yield stress to the capillary pressure. Patterns to be obtained below will be characterized in terms of these numbers, in addition to the geometrical characteristics of the microstructure.

For comparison purposes, we briefly first recall the general features of the displacement of a Newtonian fluid by another. At conditions of unfavorable mobility ratio, the displacement is a viscous fingering pattern (Homsy, 1987), provided that the capillary number is relatively high (of the order of 10^{-5} or higher). Otherwise, capillary forces stabilize the front, which propagates in a compact fashion. Figure 10 shows the displacement of mineral oil by water in a Hele-Shaw at the following conditions $Ca = 1.5 \times 10^{-5}$, $M = 25$. Displacement is from left to right. Because of the high mobility ratio, viscous fingers of the expected behavior develop, as shown in Figure 10. Consider, next, the displacement of the first Bingham plastic, *BP1* ($\tau_o = 12$ lb / 100 ft², $\mu_p = 13$ cp) by air at conditions $Ca = 1.8 \times 10^{-7}$, $M = 764$, $N_\tau = 0.044$. Figure 11 shows that the displacing fluid penetrates and displaces *BP1* as a single, rather narrow, finger, the interface of which is quite smooth, without any evidence of tip-splitting. Figure 12 shows the displacement of the second Bingham plastic fluid, *BP2* (with a higher yield stress, $\tau_o = 25$ lb/100 ft², and $\mu_p = 19$ cp) by air at the same capillary number, $Ca = 1.8 \times 10^{-7}$, but at different viscosity and yield stress, $M = 1118$, $N_\tau = 0.088$. The fingering tendency not only remains, but the finger is narrower compared to fluid *BP1*. Tip splitting is completely suppressed, the front advancing almost as a narrow finger along the straight line joining injector and producer. Displacements of fluid *BP2* were also carried out using kerosene or mineral oil at the same flow rate, 3.08 cc/min, as previously. Figure 13 shows displacement patterns involving kerosene at the conditions $Ca = 7.5 \times 10^{-5}$, $M = 9.5$, $N_\tau = 0.3125$. Despite the significantly lower value of M , we also observe a single finger

that rapidly breaks through. This behavior is attributed to the higher value of N_r . Similar results were obtained for the displacements using mineral oil (Figure 14) for the conditions $Ca = 2.96 \times 10^{-4}$, $M = 0.9$, $N_r = 0.1184$. Even though the viscosity ratio is now close to one, indicating the absence of viscous instability, the Bingham plastic rheology leads to a finger-like (actually more to a needle-like) displacement.

The next set of experiments were made to visualize the effect of disorder on the flow pattern by carrying out displacements in glass bead packs at the same flow rate as previously. In the subsequent pictures, brown represents the Bingham plastic, red is kerosene. Figure 15 shows the displacement of fluid BP1 with kerosene at conditions $Ca = 1.2 \times 10^{-4}$, $M = 6.5$, $N_r = 0.25$. A pattern similar to viscous fingering develops. For the same flow rate, more pronounced are the results obtained for fluid BP2 (Figure 16). Here, a single sharp finger emerges right from the beginning of the displacement and penetrates through fluid BP2, leaving most of the initial fluid in place. Figure 16 shows one thin finger from the injection port to the production port through fluid BP2. The value of the higher yield stress, τ_o , makes the displacement difficult and the sweep efficiency can be quite low in such cases (Barenblatt et al, 1990). To examine effects of injection velocity, we also carried out displacements of BP1 by kerosene in the glass bead pack at the higher rate ($Ca = 2.5 \times 10^{-4}$, $M = 6.5$, $N_r = 0.52$). Figure 17 shows snapshots of this displacement. Although, this displacement is also unstable, the recovery of the displaced fluids is relatively better. We attribute this to the fact that more fluid can be mobilized as the yield stress is exceeded in many places.

2. Simulation

Subsequently we carried out pore network simulations. A network model of randomly sized bonds connected by randomly sized nodes was used for modeling such immiscible displacement in a drainage process. The constant rate drainage of a Bingham plastic by a Newtonian fluid, with no-flow boundary conditions applied at the lateral boundaries, was considered. The pressure at the outlet face is kept constant. In the simulations, the following standard assumptions were used: (i) Volume calculations apply to pore bodies only, the pressure drop occurring in pore throats. (ii) Throats can be occupied by either invading or displaced fluids, while bodies may contain both fluids. (iii) The capillary pressure in a throat is inversely proportional to the tube radius, while the capillary pressure drop in a node is negligible. (iv) Inertial and gravity effects are neglected.

The pressure fields in the network were calculated by applying mass balances at each node as described in the simulation of the single-phase flow. When a bond is occupied by a Bingham plastic, the corresponding expression for flow is given from (4) above. What is different from the single-phase flow problem is that the displaced phase can be trapped by capillary forces and that for an interface to advance, capillary forces must be exceeded. The invading fluid cannot enter a bond unless the initial fluid can be mobilized and the pressure difference across that bond exceeds a threshold pressure P_c , given by the simple Laplace law

$$P_c = \frac{2\gamma}{r_{ij}} \quad (14)$$

where γ is the interfacial tension. Hence, a particular bond is invaded if there is no trapping, the capillary pressure condition is satisfied and the shear stress exceeds the yield stress. Once the bond is invaded and the bond is occupied by a Newtonian fluid, the flow rate pressure drop relation is obtained from (4) by setting the yield stress equal to zero, or, equivalently,

by

$$q_i = \frac{\pi r_{ij}^4 \Delta P_i}{8 \mu l} \quad (15)$$

To maintain a constant injection rate through the network, the pressure P_{in} at the inlet face is calculated iteratively until convergence is obtained. The saturation, $S_i(t)$, at each node is updated at each time step as follows

$$S_i(t + \Delta t) = S_i + \Delta t V_i \sum_j q_j \quad (16)$$

where V_i is the volume of node i . The time interval Δt is calculated such that only one node is filled at each time step. During the next time step, the calculations are repeated and the pressure fields determined.

The pore network simulator was first used to compare experimental and theoretical results for the glass bead pack model. The network used was of size 45×100 , corresponding to the true aspect ratio of the experimental model. Comparison between experimental and simulation results for the displacement of *BP1* are shown in Figure 18. The two patterns match reasonably well, both showing consistent tendency for trapping and bypassing. Corresponding results for fluid *BP2* are shown in Figure 19. A good agreement is also noted, indicating that the numerical model gives a good description of the displacement process in such systems.

Subsequently, we used the simulator to investigate various effects. Two different cases were examined: the limit where viscous forces are negligible (low Ca) and the limit where capillary forces are negligible (high Ca). In the first case, a different simulator was used as the computation of pressure drops due to viscous forces was not required (details can be found in Kharabaf, 1995). At the low Ca limit, the patterns obtained depend on the value of the yield stress number, N_τ . Low values of the latter lead to the standard *Invasion Percolation* patterns (Figure 20a). Upon increase of the yield stress, a smaller number of pores participate in the flow process, as the yield stress prevents some from flowing (Figure 20b-20c). The invading front changes appearance and at larger N_τ values it approaches a limiting pattern, almost like the minimum path discussed previously in the case of single-phase flow. However, because the process here is one of constant rate, there are certain differences: This limiting pattern depends on the disorder of the porous medium, in contrast to the percolation pattern, which is insensitive to it. Figure 21 shows invasion fronts at breakthrough for different values of the standard deviation of the thresholds, at conditions where yield stress effects predominate. We note that the pattern is strongly affected by the variance, a higher degree of dispersion leading to a more dispersed front. By contrast, when the standard deviation decreases, the pattern eventually reduces to a needle-like finger, which has much in common with some of the patterns observed experimentally.

In the limit when capillary forces are negligible, two different limits can be distinguished. Here, it is more appropriate to introduce the ratio $N_B = Ca/N_\tau$, which is a ratio of viscous to yield stress. Clearly, at large N_B yield stress effects are negligible, and the pattern is purely viscous- of the *DLA* type when $M \gg 1$ or compact if $M \ll 1$. The more interesting behavior is at smaller N_B . The variation of the patterns is shown in Figure 22. As we increase the value of the yield stress, the pattern resembles, although it is not identical to, a viscous fingering pattern. We identify a greater degree of dispersion and viscous fingering

and the bypassing of a sizeable portion of the initial fluid in place. For even higher values of the yield stress, the displacement has all the characteristics of an unstable displacement at large M , with pronounced fingering and large stagnant regions. The flow pattern, however, is not a *DLA* pattern. Here the mechanism is somewhat different. Because of the distribution of pore sizes, the Bingham plastic flows only in such pores, where the yield stress value can be exceeded and flow can take place. Bonds that cannot flow due to large yield stress requirements behave effectively as having infinitely large viscosity, while those that contain flowing fluid have an effective viscosity that decreases as the flow rate through them increases, thereby moderating somewhat the "fingering" tendencies. The limit $N_B \rightarrow 0$ is one in which yield stress effects dominate. This pattern should be identical to the previous at low Ca and high N_τ . As before, this pattern is sensitively dependent on the pore-size distribution, but not on the viscosity ratio, M .

For a fixed M , these patterns can be classified in a phase diagram similar to Lenormand's (1989). Figure 23 shows the resulting patterns in a diagram with axes Ca and N_τ . *IP* and *DLA* (or compact) patterns dominate the left side (of low N_τ). These patterns are *not* sensitive to the pore size distribution. On the right side of the diagram (high N_τ), the yield-stress dominated pattern is obtained. This pattern is sensitive to the particular disorder of the porous medium, as explained above. Delineating the boundaries is currently in progress.

CONCLUSIONS

In this paper, we described a pore-network simulation of Bingham plastic single-phase flow and the visualization and simulation of displacements in porous media. For a wide distribution of throat sizes, numerical flow patterns for a single-phase Bingham flow were obtained. The effect of the yield stress was investigated, and a critical yield stress value was identified for the onset of bulk flow. Subsequently, we considered the displacement of a Bingham plastic fluid by a Newtonian fluid in a drainage process. It was shown that in addition to the typical parameters Ca and M , a third dimensionless parameter, N_τ , expressing the relative magnitude of yield stress to capillarity is needed to describe such displacements. When yield stress dominates the displacement, a new pattern different from percolation or viscous fingering was identified. This pattern is sensitive to the disorder in the pore size distribution. The characterization of these displacements with the use of a phase diagram was reported. Experiments in Hele-Shaw cells and glass bead packs confirmed the general trends found in the simulations.

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FIGURES

1. Shear Stress-Shear Rate Behavior of Bingham Plastics.
2. Flow Pattern of a Newtonian Fluid in a Pore Network ($\sigma_o = 0$).
3. Flow Pattern of a Bingham Plastic in a Pore Network ($\sigma_o = 0.21$).
4. Flow Pattern of a Bingham Plastic in a Pore Network ($\sigma_o = 1.05$).
5. Flow Patterns in a Pore Network: a. Bingham Plastic ($\sigma_o = 1.36$), b. Power-Law Shear Thinning Fluid ($n = 0.07$).
6. Flow Rate, Normalized By Its Corresponding Value for a Newtonian Fluid, vs. Dimensionless Yield Stress.
7. Critical Paths in a 100×100 Pore Network. Thresholds, Uniformly Distributed in: a. $[0-1]$, b. $[0.3-1.3]$, c. $[0.8-1.8]$.
8. Mobilization of Flow Paths as the Pressure Gradient Increases.
9. Schematic of Experimental Apparatus.
10. Displacement of Mineral Oil by Water in a Hele-Shaw Cell at Conditions $Ca = 1.5 \times 10^{-5}$, $M = 25$.
11. Displacement of Bingham Plastic (*BP1*) by Air in a Hele-Shaw Cell at Conditions $Ca = 1.8 \times 10^{-7}$, $M = 764$, $N_r = 0.044$.
12. Displacement of Bingham Plastic (*BP2*) by Air in a Hele-Shaw Cell at Conditions $Ca = 1.8 \times 10^{-7}$, $M = 1118$, $N_r = 0.088$.
13. Displacement of Bingham Plastic (*BP2*) by Kerosene in a Hele-Shaw Cell at Conditions $Ca = 7.5 \times 10^{-5}$, $M = 9.5$, $N_r = 0.3125$.
14. Displacement of Bingham Plastic (*BP2*) by Mineral Oil in a Hele-Shaw Cell at Conditions $Ca = 2.96 \times 10^{-4}$, $M = 0.9$, $N_r = 0.1184$.
15. Displacement of Bingham Plastic (*BP1*) by Kerosene in a Glass Bead Pack at Conditions $Ca = 1.2 \times 10^{-4}$, $M = 6.5$, $N_r = 0.25$.
16. Displacement of Bingham Plastic (*BP2*) by Kerosene in a Glass Bead Pack at Conditions $Ca = 7.5 \times 10^{-5}$, $M = 9.5$, $N_r = 0.3125$.
17. Displacement of Bingham Plastic (*BP1*) by Kerosene in a Glass Bead Pack at Conditions $Ca = 2.5 \times 10^{-4}$, $M = 6.5$, $N_r = 0.52$.
18. Snapshots of the Numerical Simulation of the Displacement of Bingham Plastic (*BP1*) by Newtonian Fluid at Conditions $Ca = 1.2 \times 10^{-4}$, $M = 6.5$.
19. Snapshots of the Numerical Simulation of the Displacement of Bingham Plastic (*BP2*) by Newtonian Fluid at Conditions $Ca = 7.5 \times 10^{-5}$, $M = 9.5$.

20. Numerical Simulation Results at *Low Ca*: a. $N_\tau = 0$ (*Invasion Percolation*) b. $N_\tau = 0.1$, c. $N_\tau = 1$.
21. Numerical Simulation Results at *Low Ca* and $N_\tau = 0.1$: a. Standard Deviation = 3, b. Standard Deviation = 2, c. Standard Deviation = 1, d. Standard Deviation = 0.1.
22. Numerical Simulation Results at *High Ca*; Effect of Yield Stress: a. $N_B = \infty$, b. $N_B = 0.018$, c. $N_B = 0.01$.
23. Phase Diagram Showing Limiting Displacement Regimes.

Figure 1

Figure 2

Figure 3

Figure 4

(a)

(b)

Figure 5

Figure 6

Figure 8

Figure 7

[0-1]

[0.3-1.3]

[0.8-1.8]

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18

Figure 19

(a)

(b)

(c)

Figure 20

(a)

(b)

(c)

(d)

Figure 21

(a)

(b)

(c)

Figure 22

Figure 23