High Resolution Prediction of Gas Injection Process Performance for Heterogeneous Reservoirs

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Abstract

This report outlines progress in the third quarter of the second year of the DOE project “High Resolution Prediction of Gas Injection Process Performance for Heterogeneous Reservoirs”. High order finite difference schemes for one-dimensional, two-phase, multicomponent displacements are investigated. Numerical tests are run using a three component fluid description for a case when the interaction between phase behavior and flow is strong. Some currently used total variation diminishing (TVD) methods produce unstable results. A third order essentially non-oscillatory (ENO) method captures the effects of phase behavior for this test case. Possible modifications to ensure stability are discussed along with plans to incorporate higher order schemes into the 3DSL streamline simulator.

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1. Executive Summary

Performance evaluation of multicontact miscible and near-miscible gas injection processes can be assessed through conventional finite difference (FD) compositional simulation. However, low-resolution compositional simulation is adversely affected by numerical dispersion and may fail to represent geological heterogeneities adequately. High-resolution simulation is too expensive in computation time. The number of components can be reduced but at the price of less accurate representation of phase behavior. Hence, the use of FD simulators in such studies is subject to limitations that can be quite significant in some field settings.

In the previous quarterly report, a method was proposed for combining the use of one-dimensional (1D) method-of-characteristics (MOC) solutions for multicomponent gas injection with a representation of flow along streamlines. The 1D analytical solver allows any number of components to be present in the injected gas as well as in the reservoir fluid and includes the effects of volume change on mixing.

High order finite difference (HOFD) schemes are investigated in this report to further extend the capabilities of compositional streamline simulation. Several versions of HOFD schemes are tested on a ternary displacement that is strongly effected by phase behavior. Numerical solutions are compared against reference MOC solutions. Some currently used total variation diminishing (TVD) methods produce unstable results. A third order essentially non-oscillatory (ENO) method captures the effects of phase behavior more accurately for this test case. Modifications necessary to ensure stability continue to be investigated. Future work will address the efficiency of the methods and their incorporation into the 3DSL streamline simulator, developed at Stanford University [1] (1997).

2. Introduction

Many field scale gas injection processes are dominated by reservoir heterogeneity and phase behavior. Hence, numerical methods for modeling these systems should be designed to capture both effects. Many investigators have shown that streamline methods can represent the effects of heterogeneity (see [1] and references therein). In the previous quarterly report, gas injection was simulated by a streamline method using one-dimensional (1D) method-of characteristics (MOC) solutions along streamlines. MOC solutions provide fast, accurate representation of the interaction between phase behavior and two-phase flow for multicomponent displacements [2] (2001). The resulting method was shown to have good potential for predicting the performance of gas injection processes.

The MOC streamline method, like other simulation techniques, has limitations. One important assumption required to apply the MOC theory is that compositions along streamlines are initially constant. While this assumption is often reasonable at time $t = 0$, it is not valid when streamlines are updated to account for changes in mobility or well status. To satisfy the assumption that initial conditions are constant, compositions must be mapped to the updated streamlines at time $t = 0$ rather than the current time t_n .

Compositions are then moved from time $t = 0$ to time $t_n + \Delta t$ along the new streamlines. FD schemes are not limited by constant initial conditions. Compositions can be mapped to the new streamlines at time t_n and advanced to time $t_n + \Delta t$. The restarting of compositions at time $t = 0$ when streamlines are updated is a potential source of error in the MOC streamline algorithm. The magnitude of the error is related to the difference in position between new and old streamlines. For flow systems that are dominated by heterogeneity, streamlines may move little and the MOC streamline is reasonable. For systems dominated by other mechanisms such as gravity and changing well conditions, however, the approach is unlikely to be appropriate.

FD simulation of gas injection processes in 1D is computationally challenging because displacement efficiency is governed by phase behavior. Run times grow dramatically as the number of components included in phase behavior calculations is increased. Real reservoir fluids contain hundreds of components and hence the number of components included in a simulation is reduced to obtain manageable run times. While representation in terms of four to six components leads to relatively fast computations, most investigators have needed 12 to 15 components to represent phase behavior with reasonable accuracy. The reduction of components is a source of error in the simulation that cannot be eliminated through grid refinement. In addition, discretization errors present in any finite difference scheme feed back into the phase behavior calculations and lead to errors in component displacement. When first order FD methods are used, these errors can be large [3] (2002). Despite this performance the use of first order methods combined with coarse grids remains widespread. The high resolution methods outlined in the following sections offer the potential of improved solution quality and reduced simulation run times.

3. Mathematical Model

When the volume occupied by a component is assumed independent of the phase that the component appears in, the conservation equations for multiphase, multicomponent, convective flow in one dimension are written as

$$\frac{\partial C_i}{\partial \tau} + \frac{\partial F_i}{\partial \xi} = 0, \quad i = 1, \dots, n_c - 1. \quad (3.1)$$

Here n_c is the number of components,

$$\tau = \frac{u_{inj} t}{\phi L} \quad \text{and} \quad \xi = \frac{z}{L}, \quad (3.2)$$

where u_{inj} is the injection velocity, t is the time, ϕ is the porosity, L is the length of the porous medium and z is the distance from the inlet. Injection takes place at $\xi = 0$. Dindoruk gives the full derivation [4]. Only two phases are considered in this

investigation. We ignore the presence of water to focus on phase behavior rather than the complexities of three-phase flow. Let $c_{i,L}$ and $c_{i,V}$ denote the volume fractions of component i in the liquid and vapor phases, respectively. Then the overall volume fraction of component i is given by

$$C_i = c_{i,L}(1-S) + c_{i,V}S. \quad (3.3)$$

Similarly the overall volumetric flux is

$$F_i = c_{i,L}(1-f) + c_{i,V}f. \quad (3.4)$$

Here S and f refer to the saturation and fractional flow of the vapor phase. The fractional flow function is given by

$$f(S) = \frac{k_{rV}(S)/\mu_V}{k_{rV}(S)/\mu_V + k_{rL}(S)/\mu_L}, \quad (3.5)$$

where quadratic relative permeabilities (see [2]) are used with a fixed viscosity ratio, μ_L/μ_V . The phase behavior of the system is determined by assuming that equilibrium K -values are independent of composition. This simple test situation is analyzed because a closed-form analytical solution is available and the complexity introduced by a highly nonlinear equation of state is avoided. In later sections, currently used FD schemes are shown to have limitations even for the constant K -value case.

Equation (3.1) has the matrix form

$$\frac{\partial C}{\partial \tau} + \frac{\partial F}{\partial \xi} = 0, \quad (3.6)$$

where C and F are column vectors of length n_c-1 with entries corresponding to the volume fractions and volumetric fluxes of each component. Equation (3.6) has the quasilinear form

$$\frac{\partial C}{\partial \tau} + \frac{\partial F}{\partial C} \frac{\partial C}{\partial \xi} = 0, \quad (3.7)$$

where $\partial F/\partial C$ is the Jacobian matrix of the system.

The systems (3.1), (3.6), and (3.7) are nonstrictly hyperbolic since the eigenvalues of the Jacobian matrix are real but may not be distinct. Equal eigenvalue compositions are associated with pairs of parallel right eigenvectors. These mathematical distinctions play an important role in Section 5.3 where extensions of HOFD schemes to systems with more than two components are discussed. Eigenvalues are nonnegative because components flow only in the downstream direction. This observation simplifies the calculation of the volumetric fluxes in FD schemes as shown in the following section.

The problem of solving (3.1) with constant initial and injection conditions is known as a Riemann problem. Dindoruk and Wang first obtained MOC solutions to the Riemann problem using constant K -values for three and four component systems [4] (1992), [5] (1998). Semi-analytic MOC solutions with arbitrary numbers of components were made available by Jessen *et al.* [4] (2001). The semi-analytical approach allows phase behavior to be governed by more complex equations of state and volume change on mixing to be included.

4. Godunov's Method

The finite difference schemes considered here produce numerical solutions that approximate the average compositions of discrete cells. We assume that the discretization is uniform for simplicity. Godunov proposed a method for updating cell averages by solving Riemann problems at the cell faces [6] (1959). Given a numerical approximation solution C^n to (3.6) at time τ_n , the abstract scheme proceeds as follows (Figure 1):

1. (reconstruction) assume the solution is piecewise constant on each discrete cell,
2. (evolution) solve Riemann problems at the cell boundaries to advance the solution to τ_{n+1} , and
3. (averaging) average the resulting Riemann solutions over cells to define a numerical solution C^{n+1} at the new time level.

Figure 1: The reconstruction (left), evolution (center), and averaging (right) stages of Godunov's method.

The scheme has been successful in practice because the nonlinear propagation is captured in the evolution stage and because it can be put in conservative form, hence acquiring desirable properties [7]. The averaging step of Godunov's method smears out the details of the Riemann solutions. Therefore, the conservative form of Godunov's method combines the evolution and averaging steps by explicitly calculating the volumetric flux between adjacent cells. The conservative form is written

$$C_j^{n+1} = C_j^n - \frac{\Delta\tau}{\Delta\xi} [\bar{F}(C_j^n, C_{j+1}^n) - \bar{F}(C_{j-1}^n, C_j^n)], \quad (4.1)$$

where

$$\bar{F}(C_j^n, C_{j+1}^n) = \frac{1}{\Delta\tau} \int_{\tau_n}^{\tau_{n+1}} F(C(\xi_{j+1/2}, \tau)) d\tau, \quad (4.2)$$

is the numerical flux at the downstream face of cell j . If the time step is chosen sufficiently small that the solution at $\xi_{j+1/2}$ is unaffected by Riemann problems defined at neighboring cell faces, $C(\xi_{j+1/2}, \tau)$ will be constant for $\tau \in (\tau_n, \tau_{n+1})$ since Riemann solutions are self-similar. Moreover, since our eigenvalues are nonnegative the constant $C(\xi_{j+1/2}, \tau)$ will be equal to the upwind cell composition at time τ_n (Figure 1). Therefore,

$$\bar{F}(C_j^n, C_{j+1}^n) = F(C_j^n), \quad (4.4)$$

and Godunov's method for (3.6) takes the simple form

$$C_j^{n+1} = C_j^n - \frac{\Delta\tau}{\Delta\xi} [F(C_j^n) - F(C_{j-1}^n)], \quad (4.5)$$

which is the standard, single-point upstream weighting FD form.

For each cell at every time step, the phase behavior of the system is resolved by solving for the volume fractions of each component in both phases and the vapor phase saturation. The fractional flow of the vapor phase is then evaluated, and the flux in (4.5) is computed from (3.4). The method (4.5) can be applied directly to multicomponent systems as well as scalar conservation laws. Numerical solutions obtained using (4.5) are first order accurate. Results given in Section 6 motivate the need for higher order methods.

5. Higher Order Methods

In the first two parts of this section, we develop higher order methods for scalar conservation laws ($n_c = 2$). The scalar methodology is extended to multicomponent systems in the following subsection. Higher order methods are obtained by improving upon the reconstruction stage of the algorithm. Implementing a Godunov-type method requires flux information at the cell faces only. Therefore, we focus on making an improved approximation $C_{j+1/2}^*$ to $C(\xi_{j+1/2}, \tau)$ in (4.2). The scheme (4.5) is then modified to be

$$C_j^{n+1} = C_j^n - \frac{\Delta\tau}{\Delta\xi} [F(C_{j+1/2}^*) - F(C_{j-1/2}^*)]. \quad (5.1)$$

For smooth solutions, linear and quadratic interpolations yield second and third order approximations to $C(\xi_{j+1/2}, \tau)$, respectively. When the solution is not smooth near a cell

face, higher order interpolation gives an inaccurate approximation and large oscillations are generated. Van Leer's MUSCL (monotone upstream-centered schemes for conservation laws) approach gives a framework for eliminating these oscillations [8] (1979). Flux-based approaches are similar but not discussed in this report (see [7] for an overview). The following two subsections outline successful variants of the MUSCL approach.

5.1 Total Variation Diminishing Schemes

To quantify oscillatory behavior, we define the total variation (TV) of a discrete solution as

$$\text{TV}(C^n) = \sum_j |C_{j+1}^n - C_j^n|. \quad (5.2)$$

Observing that the TV of an MOC solution does not increase with time, we seek total variation diminishing (TVD) schemes with the property that

$$\text{TV}(C^{n+1}) \leq \text{TV}(C^n). \quad (5.3)$$

The TVD reconstruction makes a correction to linear interpolation that enforces property (5.3). The correction procedure is known as slope limiting within the MUSCL framework and is a two step process. First a measure of the smoothness of the solution is obtained, e.g. the ratio of successive gradients

$$\Theta_j = \frac{C_j - C_{j-1}}{C_{j+1} - C_j}. \quad (5.4)$$

The reconstructed composition is then given by

$$C_{j+1/2}^* = C_j^n + \frac{\Phi(\Theta)}{2} [C_{j+1}^n - C_j^n], \quad (5.5)$$

where $0 \leq \Phi(\Theta) \leq 2$ is a nonlinear limiter that controls the TVD correction. Sweby gives a review of limiters [9] (1984). The simple minmod limiter

$$\Phi(\Theta) = \max(0, \min(1, \Theta)), \quad (5.6)$$

gives solutions that satisfy (5.3) but are typically too diffused. Van Leer's limiter

$$\Phi(\Theta) = \frac{|\Theta| + \Theta}{1 + \Theta}, \quad (5.7)$$

is less diffusive. The Fromm limiter

$$\Phi(\Theta) = \max(0, \min(2, 2\Theta, \frac{1+\Theta}{2})), \quad (5.8)$$

can yield a third order accurate reconstruction and gives the least diffusive solutions. We discuss results using all three limiters in Section 6. $C_{j+1/2}^*$ in (5.5) depends on the fixed stencil $\{C_{j-1}, C_j, C_{j+1}\}$. For $\Theta < 0$ all limiters yield $\Phi(\Theta) = 0$, so the TVD reconstruction (5.5) reduces to first order at extrema of the solution.

5.2 Essentially Non-Oscillatory Schemes

The motivation behind essentially non-oscillatory (ENO) schemes is to extend the MUSCL approach to arbitrarily high orders of accuracy and to maintain accuracy at smooth extrema. This is accomplished by relaxing the TVD property (5.3) to

$$\text{TV}(C^{n+1}) \leq \text{TV}(C^n) + O(\Delta \xi^r), \quad (5.9)$$

where r is the order of the scheme. Small oscillations on the order of the truncation error are allowed in the reconstruction but do not degrade solution quality. High order accuracy is achieved by polynomial interpolation. We enforce the ENO property (5.9) by adapting the stencil of $C_{j+1/2}^*$. At each cell face, r candidate interpolations are performed and the least oscillatory interpolating polynomial is selected (figure 2). In practice, we make the selection by examining divided differences. Harten *et al.* give more details on the ENO reconstruction [10] (1987).

Figure 2: The second order ENO reconstruction chooses between two linear interpolations of a cell face value. Here the upwind stencil is selected because the slope of the line interpolating the upwind compositions is smaller in magnitude than the centered slope.

5.3 Extension to Multicomponent Systems

The reconstructions of the previous subsections apply only to scalar conservation laws. The extension of higher order methods to multicomponent systems ($n_c > 2$) is nontrivial.

The system (3.7) should be decoupled into n_c-1 scalar equations before the TVD or ENO reconstructions can be applied [7]. This decoupling procedure takes the form of an eigenvalue problem that has not been solved analytically for general two-phase, multicomponent flows. Numerical solutions to the eigenvalue problem are computationally expensive to obtain and potentially unreliable due to the lack of strict hyperbolicity. As mentioned earlier, semi-analytic solutions to the same eigenvalue problem have been presented in the context of constructing Riemann solutions [4] (2001). Decoupling (3.7) is equivalent to decomposing the problem into characteristic variables. Methods incorporating this characteristic reconstruction are currently under development but results are not included in this report. To avoid the complications noted above, the general practice has been to apply higher order reconstructions in a component-by-component fashion. This straightforward extension to multicomponent systems does not enforce the TVD property (5.3) or the ENO property (5.9). However, similar approaches by other investigators have produced reasonable results for some test cases [11] (1997), [12] (2001). Results using TVD and ENO versions of this componentwise reconstruction are given in Section 6.

5.4 Higher Order Accuracy in Time

In previous sections we have only considered spatial accuracy. To extend our methods to higher order accuracy in time we adopt a method of lines approach. Returning to (3.6), we make an approximation $-L(C)$ to the spatial derivative $\partial F / \partial \xi$ using either the TVD or ENO reconstruction. This leaves the ordinary differential equation (ODE)

$$\frac{\partial C}{\partial \tau} = L(C). \quad (5.10)$$

Equation (5.10) can be solved by any numerical ODE method. Scheme (5.1) is obtained by selecting forward Euler time stepping. TVD Runge-Kutta methods are commonly used for modeling conservation laws [13]. We use second and third order versions of these schemes to equate the temporal accuracy with the spatial accuracy in all of our tests.

6. Results

In this section we give results for Godunov's method, the componentwise TVD method, and the componentwise ENO method. Each method has been applied to a three component vaporizing drive with constant equilibrium K-values (Table 1). The viscosity ratio was fixed at 5 (oil/gas). Profiles for saturation and the overall volume fraction of the intermediate component are shown in Figures 3-5 after one pore volume of gas has been injected. Solution paths through compositional space are drawn on ternary diagrams. MOC profiles and paths are given for reference.

Table 1: Data used in test simulation.

	C_1	C_2	C_3
Initial oil in place	0.00	0.25	0.75
Injection gas	0.90	0.10	0.00
Equilibrium K-value	2.50	1.50	0.05

Results using Godunov's method are shown for three levels of grid refinement (Figure 3). Diffusive truncation errors cause the solution paths to be smeared towards the dilution line that connects the initial and injection compositions. This numerical diffusion causes the displacement efficiency to be underestimated. Hence, the leading shock propagates too fast in all three cases.

We tested the componentwise TVD reconstruction using the three limiters described in Section 5.1 (Figure 4). The less diffusive limiters produced unstable results. The paths in compositional space depart substantially from the MOC path. Runs with the minmod limiter were more stable although some deviation from the MOC path can be noticed. We assume that diffusive truncation errors in the minmod scheme damped much of the instability. The unstable solutions overestimated the displacement efficiency of the injection process as seen by the size and location of the leading shock.

Results are displayed for second and third order versions of the componentwise ENO reconstruction (Figure 5). The second order scheme incorrectly predicted the compositional path. This error in compositional space led to an underestimated leading shock speed. The third order scheme followed the MOC path closely. Hence, shock speeds were captured accurately in this case. At earlier time steps some significant oscillations were present in both versions. This suggests that the level of stability and accuracy seen here may not extend to all test cases.

Figure 3: First order results using Godunov's method.

Figure 4: TVD results for $N = 200$ using different limiters.

Figure 5: Second and third order ENO results for $N = 200$.

7. Discussion and Conclusions

In this report we have presented a framework for extending Godunov's method to higher order accuracy. Results demonstrated that HOFD methods can capture the effects of phase behavior on two-phase, multicomponent displacements. The test also showed the potential for instabilities to corrupt higher order solutions when componentwise reconstruction strategies are used. TVD methods combined with less diffusive limiters such as the Fromm limiter or van Leer's limiter appear to be particularly sensitive to these instabilities. For our single test case, the third order ENO scheme performed better results than other methods. Additional tests and analysis are needed before comprehensive guidelines to insure stability can be formulated for componentwise TVD or ENO schemes.

Methods based on a characteristic reconstruction procedure are currently under development. The characteristic reconstruction will be more complex and computationally expensive than the componentwise approach. However, the resulting methods may not suffer from the instabilities reported here. Semi-analytical techniques developed for solving Riemann problems may prove useful in improving the efficiency of the characteristic reconstruction.

Once a robust method has been developed or guidelines to insure the stability of current methods have been established, HOFD schemes will be incorporated into the 3DSL streamline simulator. The inclusion of HOFD schemes will allow for high resolution three-dimensional simulation that is not limited by the major assumption of the MOC streamline method. In addition, using the higher order version of 3DSL should explain discrepancies between first order FD streamline methods and the MOC streamline method.

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