Study of Synchro-betatron Coupling in IPNS Upgrade RCS

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Introduction

The 2-GeV rapidly-cycling synchrotron (RCS) of the proposed 1-MW spallation neutron source upgrade has 16 dispersion-free straight sections and eight straight sections with a dispersion of approximately 1 m. Six of the ten rf cavities are located in dispersion-free straight sections, and the remainder are located in the non-dispersion-free straight sections. The possibility of exciting synchro-betatron resonances is investigated in this paper. It is found that the effect of the coupling is not important in the RCS.

Synchro-betatron resonances are driven in single beams by position-dependent energy gains from the accelerating fields. A primary cause is the dispersion at the location of the rf cavities. Longitudinal and transverse motions are coupled due to a transverse deflection associated with a particle crossing the rf cavity gap. This changes the equilibrium orbit and excites betatron oscillations. The resonance condition for coupling is given by

$$kv_x \pm lv_y \pm mv_s = n$$

where k , l , m , and n are positive integers, and v_x , v_y and v_s are the betatron and synchrotron tunes, respectively. In proton machines in general, and in the RCS in particular, $v_s \ll v_{x,y}$. Therefore, the synchro-betatron resonances ($m \geq 1$) manifest as satellites of the integer ($k + l = 1$) resonance, and the resonance condition for coupling becomes

$$v_{x,y} \pm mv_s = n \quad (1)$$

Here, the $\pm$ signs refer to the sum and difference resonances, respectively.

Growth Rate

On resonance, according to Suzuki,¹ the maximum growth rate of the energy spread per revolution, $\delta(\Delta E/E)$, and the maximum betatron oscillation amplitude change per revolution, δx , are given by

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$$\delta\left(\frac{\Delta E}{E}\right)_{\max} = \frac{\beta^3 |\alpha_m|}{cpR\beta_x} \frac{\phi_{\max}^m x}{2^m (m-1)! |\eta| (\Delta E/E)} \frac{v_s}{A_\eta}, \quad [\text{rev}^{-1}] \quad (2)$$

$$\text{and} \quad \delta x_{\max} = \frac{|\alpha_m|}{cp\beta} \frac{\phi_{\max}^m}{2^m m!} A_\eta, \quad [\text{m rev}^{-1}] \quad (3)$$

where R is the average synchrotron radius, β and γ are the standard time-varying relativistic factors, $\eta = \gamma_t^{-2} - \gamma^{-2}$ is the slip factor, γ_t is the transition energy, p is the beam momentum, E is the beam energy, $x = \sqrt{\epsilon_x \beta_x / \pi}$ is the betatron oscillation amplitude, β_x is the lattice parameter at the cavity, and $\phi_{\max}$ is the maximum phase of the synchrotron oscillations. The Taylor expansion coefficient, α_m , is given by

$$\alpha_m = \begin{cases} -(1)^{m/2} \sin \phi_s & m \text{ even} \\ -(-1)^{(m+1)/2} \cos \phi_s & m \text{ odd} \end{cases}$$

The parameter A_η depends on the rf voltage and the dispersion function, and is given by

$$\begin{aligned} A_\eta &= \sqrt{(D_{cn} - F_{sn})^2 + (D_{sn} + F_{cn})^2} \\ D_{cn} &= \sum_j eV_j \eta_{xj} \cos n\phi_{xj} \quad D_{sn} = \sum_j eV_j \eta_{xj} \sin n\phi_{xj} \\ F_{cn} &= \sum_j eV_j F_j \cos n\phi_{xj} \quad F_{sn} = \sum_j eV_j F_j \sin n\phi_{xj} \\ F &= \eta'_x \beta_x + \eta_x \alpha_x \end{aligned} \quad (4)$$

where, for each cavity j , V is the accelerating voltage, α_x , β_x , and η_x are the Courant-Snyder lattice parameters, and ϕ_x is the betatron phase advance. The sum is over one turn.

Results

The RCS operates entirely below transition energy (400 MeV – 2 GeV, $\gamma_t = 5.40$); therefore, the sum resonance is unstable and the difference resonance is stable. However, growth in the betatron oscillation amplitude may be important even for difference resonances. The largest v_s is 0.003, which occurs at 4 msec in the 25-msec acceleration cycle. Here the synchrotron frequency is 4 kHz and the revolution frequency is 1.2 MHz. Using Eqn. (1) and $v_x = 6.821$ and $v_y = 5.731$, the minimum orders m for the four-sided RCS lattice² are about 60 for the sum (x-plane) and about 246 for the difference (y-plane) resonance. The estimated growth rates using

Eqns. (2) and (3) are vanishingly small for such large orders of the synchro-betatron resonance due to the factorial term in the denominator.

As an illustration, δx is evaluated at 4 msec. The relevant beam parameters for the RCS are given in Table 1, while the average lattice parameters in the rf straight sections are given in Table 2. The RCS lattice for one super-period (out of four) is shown in Figure 1, where the long, rf straight sections are marked: L1, L2, ... ,L6. The quadrupoles, QF and QD, the chromaticity-correcting sextupoles, SF and SD, and the harmonic-correcting sextupoles, S1, are also marked.

Table 1. Beam parameters for the RCS at 4 msec in the cycle

β	p (MeV/c)	x (mm)	v_s	$ \eta $	ΔE (MeV)	$\phi_{\max}$ (rad)	ϕ_s (deg)
0.755	1080	53	0.003	0.395	5	1.8	26

Table 2. Average lattice parameters in rf long straight sections

long straight section #	$\langle\beta_x\rangle$ (m)	$\langle\alpha_x\rangle$	$\langle\eta_x\rangle$ (m)	$\langle\eta'_x\rangle$	ϕ_x (rad)	F
L1	6.20	1.31	0.920	-0.335	2.64	-0.872
L2	6.22	1.32	-0.011	-0.005	4.16	-0.0456
L3	6.21	-1.32	-0.041	-0.013	5.01	-0.0266
L4	6.21	1.32	-0.041	0.013	5.70	0.0266
L5	6.22	-1.32	-0.011	0.005	6.55	0.0456
L6	6.20	-1.31	0.920	0.335	8.08	0.872

One rf cavity is placed in each long straight section as follows: (I) L1, L2, L3, L4, L5, L6; (II) L5, L6; and (III) L5, L6. The roman numeral refers to the super-period number. Each rf cavity provides 18 kV voltage. A phase advance of $\Delta\phi_x = (v_x/4)(N-1)$ is added to the betatron phase, ϕ_x , listed in Table 2, where $v_x = 6.821$ is the tune and N is the super-period number.

For the sum resonance, $n = 7$, and the value of $A_n = 50.35$ keV-m. The results for the growth in the betatron amplitude are given in Table 3 for synchro-betatron resonance orders up to $m = 5$. The mode numbers, m , we are concerned about are 60 and 246, and, clearly, the amplitude growth for these numbers is negligibly small.

Table 3. Growth of betatron amplitude for first few orders of m

m	δx (mm/rev)	δx (mm/msec)
1	4.6e-3	5.5
2	8.3e-4	1.0
3	4.2e-4	0.5
4	3.8e-5	0.04
5	1.2e-5	0.01

Conclusion

In the RCS, the minimum synchro-betatron sum and difference resonance orders are very high. The synchrotron tune would have to increase by two orders of magnitude to cause an appreciable betatron oscillation amplitude due to synchro-betatron coupling. Therefore, the effect of the synchro-betatron resonances can be neglected.

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References

1. T. Suzuki, "Synchrobetatron resonances driven by dispersion in RF cavities," in *Particle Accelerators*, **18** (1985).
2. Y. Cho, *Lattice for RCS II, Version 2 - Four-Sided Lattice*, Argonne National Laboratory Report No. NSA-94-8 (June 1994).

Figure 1. Lattice functions for one super-period in the RCS.