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**Experimental and Theoretical Analysis of Flashing Instability
for Next Generation Natural Circulation Reactors**

Rizwan-uddin
Department of Nuclear, Plasma and Radiological Engineering
University of Illinois at Urbana-Champaign
103 s. Goodwin Ave., Urbana, Il61801
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The project titled above has been very successfully completed according to schedule. This final report is based on several papers that have appeared or are scheduled to appear in archival journals and in conference proceedings. A list of publications that have resulted from this project is included at the end of the executive summary.

EXECUTIVE SUMMARY

The project had four parts: 1) Modeling and simulation of nonlinear dynamics in forced BWRs using reduced order models; 2) Modeling and simulation of nonlinear dynamics in natural circulation BWRs using reduced order models; 3) Comparison of results with those obtained using large scale system codes; and 4) Experiments to investigate natural circulation flashing phenomenon.

PART I

In the first part, stability and bifurcation analyses of forced circulation BWRs have been carried out using a reduced order two-channel model developed earlier by Karve et al. To parameterize azimuthal asymmetry in core loading, an amplification factor F is introduced into the model to vary azimuthal mode feedback coefficients. Bifurcation analysis code BIFDD and numerical integration are used to analyze the reduced order model comprised of modified twenty-two ODEs.

Results are presented for effects of azimuthal asymmetry (as parameterized by the amplification parameter F) on characteristics of oscillations. Analysis of eigenvectors corresponding to the two pairs of complex conjugate eigenvalues with the largest and second largest real parts, suggest that one of these pairs is responsible for in-phase oscillations and the other for the out-of-phase oscillations. Specifically, the pair of complex conjugate eigenvalues for which the magnitude of the element for the fundamental mode (first azimuthal mode) in the corresponding eigenvectors is much larger than the element for the first azimuthal mode (fundamental mode) determines the stability of the fundamental model (first azimuthal mode).

For a uniform core without azimuthal asymmetry ($F = 1$), as a bifurcation parameter (total pressure drop) is varied, the pair of eigenvalues corresponding to the fundamental mode first crosses the imaginary axis, thus making the system unstable and leading to in-phase oscillations. However, for azimuthally asymmetric cores (corresponding to large values of F) and small inlet subcooling, the pair of eigenvalues corresponding to the first azimuthal mode, whose real part is the second largest for $F = 1$ case, approaches the

vertical axis faster (as a bifurcation parameter is varied) than those corresponding to the fundamental mode, thus becoming the dominant pair of eigenvalues. This consequently leads to out-of-phase oscillations. Results of bifurcation analyses show that both sub- and super-critical bifurcation can occur for large as well as small azimuthal asymmetry, depending upon values of other operating parameters. Changes in characteristics of oscillations (in-phase or out-of-phase; super- or sub-critical bifurcation) therefore result along the stability boundary. Numerical integrations confirm the results of stability and bifurcation analyses.

PART II

In the second part, a new reduced order model to simulate the dynamics of natural circulation features in current and next generation of nuclear power plants has been developed. Dynamical analysis this new natural circulation BWR model that allows local pressure dependence of water saturation enthalpy and includes fundamental and first azimuthal modes for neutronics is carried out. Stability boundaries (SBs) and 7.5% oscillation curves (representing natures of bifurcation along SBs) are plotted in inlet subcooling—external reactivity parameter space. Stability and bifurcation analyses show that, both in-phase and out-of-phase oscillations as well as supercritical and subcritical bifurcations can occur along the SBs.

PART III

In the third part of this project, the system code RAMONA as well as the reduced order model were employed for the stability analysis of a specific operational point of the Leibstadt NPP. This was carried out to assess the ROM's applicability and limitations in a quantitative manner. These comparisons show that although the ROM could adequately predict certain characteristics, it is not able to correctly predict others because of the highly simplified reactor core geometry, the uncertainties in evaluating the design and operating parameters, and the simple reactivity feedback model employed.

In the context of a detailed local bifurcation analysis carried out using RAMONA in the neighbourhood of the chosen Leibstadt operational point, a bridge is built between the ROM and the system code. This has been achieved through interpreting RAMONA solutions on the basis of the physical mechanisms identified in the course of applying the ROM (Part I). This leads, for the first time, to the identification of a subcritical Poincaré-Andronov-Hopf (PAH) bifurcation using a system code. As a consequence, the possibility of a so-called correspondence hypothesis is suggested to underline the relationship between a stable (unstable) limit cycle solution and the occurrence of a supercritical (subcritical) PAH bifurcation in the modeling of boiling water reactor stability behaviour.

PART IV

In the fourth part of this project we investigated the nonlinear dynamics of natural-circulation, boiling two-phase flows using a two-phase flow loop at the Delft University of Technology in The Netherlands. Experiments were carried out in the unstable operating region of the facility for various system pressures and for different frictions at the exit of the riser section. It appears that the boiling two-phase flow undergoes the well-known Feigenbaum scenario, the period-doubling route toward chaotic behavior. Evaluation of the recorded signals using nonlinear time series analysis methods indicates the occurrence of chaotic density-wave oscillations.

CONCLUSIONS

As can be seen from the summary of the four parts of this work, a very substantial amount of valuable research has been carried out on forced as well as natural circulation flashing two-phase flow system. Substantial leverage was obtained with this award in working with international groups working on this topic.

List of Publications that resulted from work based on the NEER award

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A. Dokhane, D. Hennig, Rizwan-uddin, R. Chawla, **A Parametric Study of Heated Channels with Two-Phase Flow Using Bifurcation Analysis**, *Trans. Amer. Nucl. Soc.*, **86**, 131-133 (2002).

Quan Zhou and Rizwan-uddin, **Bifurcation Analysis of Nuclear-Coupled Thermal Hydraulics of BWRs Using BIFDD**, *Trans. Amer. Nucl. Soc.*, **86** 191-193 (2002).

Quan Zhou and Rizwan-uddin **Impact of Modeling Assumptions on Stability and Bifurcation Analyses of BWRs**, *Trans. Amer. Nucl. Soc.*, **86** 249-250 (2002).

Quan Zhou and Rizwan-uddin, **Bifurcation Analysis of Nuclear-Coupled Dynamics of BWRs Using BIFDD**, in the Proc. Of the Int. Conf on Nuclear Engineering (ICONE-10), Arlington, VA, April 14-18 (2002).

A. Dokhane, D.Hennig, R. Chawla and Rizwan-uddin, **Stability And Bifurcation Analyses Of Two-Phase Flow Using A Drift Flux Model And Bifurcation Code BIFDD**, in the Proc. Of the Int. Conf on Nuclear Engineering (ICONE-10), Arlington, VA, April 14-18 (2002).

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A. Dokhane, D. Hennig, Rizwan-uddin, R. Chawla , **Bifurcation Analysis of Density Wave Oscillations using a Drift Flux Model**, in the Proc. of PHYSOR-2002, International Conference on the new Frontiers of Nuclear Technology Reactor Physics, Safety and High-performance Computing,, Seoul, Korea, October 7-10, American Nuclear Society, 2002.

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Dokhane, A., D.Hennig, R.Chawla, Rizwan-uddin, **Nonlinear Stability Analysis of Boiling Water Reactors on the Basis of System Codes and Reduced Order Models**, in the Proc. of JKT-03, Germany, April 2003.

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R. Zboray, W.J.M. de Kruif, T.H.J.J. van der Hagen and Rizwan-uddin, **Investigating the Nonlinear Dynamics of Natural-Circulation, Boiling Two-Phase Flows**, *Nucl. Tech.*, **146**, 244-256, (2004).

A. Dokhane, D.Hennig, Rizwan-uddin, R. Chawla, **Semi-Analytical Bifurcation Analysis of Two-Phase Flow in a Heated Channel**, *International Journal of Bifurcation and Chaos*. **15**, 8, 2005.

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Quan Zhou and Rizwan Uddin, **An Integrated Model of Natural Circulation Loop under Low Pressure**, in *Trans. Amer. Nucl. Soc.*, **91** (2005).

Quan Zhou and Rizwan Uddin, **Stability Analyses of an Integrated Neutronics-Thermal Hydraulics Model of Natural Circulation BWR under Low Pressure**, submitted for publication in Proc. M & C 2005, Int. Topical Meeting on Mathematics and Computations, Supercomputing, Reactor Physics and Nuclear and Biological Applications, Avignon, September 2005.

Abdelhamid Dokhane, Dieter Hennig, Rizwan-uddin, Rakesh Chawla, **BWR Stability And Bifurcation Analysis Using Reduced Order Models And System Codes: Part I – Reduced Order Model Investigations of In-Phase and Out-of-Phase Oscillations Modes**, submitted for publication in Nuclear Science and Engineering.

Abdelhamid Dokhane, Dieter Hennig, Rizwan-uddin, Rakesh Chawla, **BWR Stability And Bifurcation Analysis Using Reduced Order Models And System Codes: Part II - Direct Comparisons Resulting In Identification Of A Subcritical PAH Bifurcation Using A System Code** , submitted for publication in Nuclear Science and Engineering.

PART I

NONLINEAR DYNAMICS IN FORCED CIRCULATION BWRs

Preprint of a Paper to Appear in

Nuclear Science and Engineering

In-phase and Out-of-phase Oscillations in BWRs: Impact of Azimuthal Asymmetry and Second Pair of Eigenvalues

Authors and affiliation

Quan Zhou

Department of Nuclear, Plasma and Radiological Engineering, University of Illinois
103 S. Goodwin Avenue, Urbana, IL61801, U.S.A.

Rizwan-uddin[†]

Department of Nuclear, Plasma and Radiological Engineering, University of Illinois
103 S. Goodwin Avenue, Urbana, IL61801, U.S.A.

Correspondence address

Quan Zhou

Department of Nuclear, Plasma and Radiological Engineering, University of Illinois
103 S. Goodwin Avenue, Urbana, IL61801, U.S.A.
Tel. 001-217-244-7600, Fax. 001-217-333-2906
E-mail: quanzhou@uiuc.edu

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**IN-PHASE AND OUT-OF-PHASE OSCILLATIONS IN BWRs: IMPACT OF
AZIMUTHAL ASYMMETRY AND SECOND PAIR OF EIGENVALUES**

Quan Zhou and Rizwan-uddin

Department of Nuclear, Plasma, and Radiological Engineering

University of Illinois

103 South Goodwin Avenue

Urbana, Illinois 61801 U.S.A.

quanzhou@uiuc.edu, rizwan@uiuc.edu

ABSTRACT

Stability and bifurcation analyses of BWRs have been carried out using a reduced order two-channel model developed earlier by Karve et al. To parameterize azimuthal asymmetry in core loading, an amplification factor F is introduced into the model to vary azimuthal mode feedback coefficients. Bifurcation analysis code BIFDD and numerical integration are used to analyze the reduced order model comprised of modified twenty-two ODEs.

Results are presented for effects of azimuthal asymmetry (as parameterized by the amplification parameter F) on characteristics of oscillations. Analysis of eigenvectors

corresponding to the two pairs of complex conjugate eigenvalues with the largest and second largest real parts, suggest that one of these pairs is responsible for in-phase oscillations and the other for the out-of-phase oscillations. Specifically, the pair of complex conjugate eigenvalues for which the magnitude of the element for the fundamental mode (first azimuthal mode) in the corresponding eigenvectors is much larger than the element for the first azimuthal mode (fundamental mode) determines the stability of the fundamental model (first azimuthal mode).

For a uniform core without azimuthal asymmetry ($F = 1$), as a bifurcation parameter (total pressure drop) is varied, the pair of eigenvalues corresponding to the fundamental mode first crosses the imaginary axis, thus making the system unstable and leading to in-phase oscillations. However, for azimuthally asymmetric cores (corresponding to large values of F) and small inlet subcooling, the pair of eigenvalues corresponding to the first azimuthal mode, whose real part is the second largest for $F = 1$ case, approaches the vertical axis faster (as a bifurcation parameter is varied) than those corresponding to the fundamental mode, thus becoming the dominant pair of eigenvalues. This consequently leads to out-of-phase oscillations. Results of bifurcation analyses show that both sub- and super-critical bifurcation can occur for large as well as small azimuthal asymmetry, depending upon values of other operating parameters. Changes in characteristics of oscillations (in-phase or out-of-phase; super- or sub-critical bifurcation) therefore result along the stability boundary. Numerical integrations confirm the results of stability and bifurcation analyses.

1. INTRODUCTION

Boiling water reactors (BWRs) have been found susceptible to thermal-hydraulic and neutronics instabilities under certain adverse operating conditions. Such instability events, characterized by large amplitude power and flow oscillations, have been reported in several commercial BWR power plants since the mid 70's. These include TVO [1], Caorso [2], Lasalle [3], Laguna Verde [4], etc. Several stability tests have also been conducted in operating BWRs [5-7]. Clearly, such instabilities must be avoided, and if they at all occur need to be timely detected and quickly suppressed. To achieve these safety objectives, a deeper understanding of the characteristic features of various instability phenomena and feedback mechanisms leading to the instabilities is needed

Event classification process has led to identification of two different kinds of oscillations [8]. The first one is called in-phase or core-wide oscillation. The power and flow in the entire reactor core in this case oscillate with almost the same frequency and phase. The second one is the out-of-phase or regional oscillation, where different parts of the core may oscillate out of phase, but the power and flow of the reactor as a whole stay stationary or vary with very small amplitude. Another classification among various self-sustained oscillations is based on oscillation amplitude. Some instability events lead to oscillations that continue to increase in amplitude until scram, while others are found to saturate at certain amplitudes. The latter is usually called stable limit cycle oscillations.

The root cause of these instabilities has been a topic of study for many decades. It has long been understood that small perturbation of flow rate in two-phase heated channel flow can cause changes in local vapor generation rate and therefore in two-phase mixture density. Transmission of the perturbation along the channel produces a local pressure drop, which may in-turn intensify the flow rate perturbation under certain operating conditions. Such a self-amplified thermal-hydraulic feedback process can produce so-called density wave oscillations (DWOs) [8]. Apart from pure thermal-hydraulic features, in BWRs, neutronics is coupled with thermal-hydraulics through void and fuel temperature reactivity feedbacks. Variations in temperature and void fraction lead to transients in reactor power. This additional feedback between thermal-hydraulics and neutronics adds to the dynamical behavior of the system, and necessitates coupled analyses of the instability phenomena.

2. PREVIOUS WORK

Large scale and high fidelity commercial codes are often used to study coupled transient simulations. Takigawa et al. [9] simulated the out-of-phase oscillations observed in Caorso-1 with a three-dimensional code, TOSDYN. Araya et al. [10] studied the instability event that occurred in Lasalle-2 using RETRAN. Hennig [11] carried out analyses of instability events and tests in Ringhals, KKL, and Forsmark employing three-dimensional BWR transient simulation code, RAMONA. Although such detailed simulations and analyses are very helpful

in understanding specific events, they are cumbersome, time-consuming, and of limited value in determining stability boundaries in large parameter spaces, and in predicting parametric trends. For these reasons, reduced order models are developed and studied.

Numerical simulations and stability analyses of reduced order models have been performed by several researchers. March-Leuba et al. [12] analyzed a very simple model first developed by Otaduy [13], consisting of point reactor kinetics equations and lumped equations for heat transfer. This analysis was, later, extended to a two-channel model with fundamental as well as first azimuthal modes for neutronics analysis [14]. Karve et al. [15] developed a BWR model which consists of modal neutronics, fuel rod heat conduction and heated channel thermal hydraulics. Stability analyses in multiple parameter spaces were performed and numerical simulations were used to deduce the nature of oscillations [16]. A different reduced order BWR model was developed by Muñoz-Cobo et. al [17, 18] who used λ -mode, rather than ω -mode as used by Karve et al, for the neutronics part and lumped models for heat conduction and channel thermal-hydraulics. Numerical simulations were performed [18], and out-of-phase oscillations were excited and analyzed by increasing void feedback coefficient of the azimuthal mode.

Compared with the rather large number of stability analyses, bifurcation techniques have not been exploited to their fullest extent in the analyses of instabilities in BWRs—at least partly due to the fact that the algebraic complexity of “analytical” bifurcation analysis, such as that in Lindstedt-Poincaré technique, increases dramatically with the number of equations in the

reduced order models. For example, rather simple reduced order models have been used by Tsuji et al.[19] and Muñoz-Cobo et al. [20]. Bifurcation characteristics were determined in these studies as a function of only a single parameter, consequently limiting the scope of these analyses to a small region of the rather large parameter space.

In this paper a new bifurcation analysis of the more detailed and hence complicated reduced order BWR model developed by Karve et al. [15] is presented. Stability and bifurcation characteristics of the model in large parameter spaces are determined by using a bifurcation analysis code BIFDD and using numerical integration techniques. In section 3, the two-channel two-mode model developed by Karve et. al [15] is briefly reviewed and some minor modifications are introduced. In section 4, Poincaré-Andronov-Hopf (PAH) bifurcation theory [21, 22] and the bifurcation analysis code BIFDD [23] are reviewed. Results for uniform and non-uniform cores are separately presented and discussed in sections 5 and 6, respectively. Paper is summarized in the last section.

3. MODEL DESCRIPTION

The reduced order two-channel BWR model developed by Karve et. al [15], in which each channel represents one half of the reactor core, is comprised of three sub-models: neutronics, fuel rod heat conduction, and heated channel thermal-hydraulics. A modal neutron kinetics, with fundamental and first azimuthal modes, is employed. The governing neutron kinetics equations are coupled with those of heat conduction and heated channel thermal hydraulics via

fuel rod temperature feedback coefficients and void feedback coefficients.

The paper focuses on the neutron kinetics sub-model in this section due to its relevance to the modification introduced in this study. The other two sub-models, fuel rod heat conduction and channel thermal-hydraulics, are only briefly reviewed. Complete details of the sub-models can be found in reference [15] and [16].

3.1 NEUTRON KINETICS

The modal expansion method is used to develop the governing ordinary differential equations from the matrix form of the one-group neutron diffusion equation and one equation for delayed neutron precursors. These are written as

$$\frac{d\bar{Y}}{dt} = \bar{M} \bar{Y} \quad (1)$$

where $\bar{M} = \begin{bmatrix} \nu \nabla \cdot D(\bar{r}, t) \nabla - \nu \Sigma_a(\bar{r}, t) + (1 - \beta) \nu \Sigma_f(\bar{r}, t) & \mathbf{I} \\ \nu \beta \Sigma_f & -\mathbf{I} \end{bmatrix}$, $\bar{Y}(\bar{r}, t) = [N(\bar{r}, t), C(\bar{r}, t)]^T$, and $N(\bar{r}, t)$ and

$C(\bar{r}, t)$ are neutron density and precursor concentration.

Following the ω -mode approach, the eigenvalue w_{ik} and eigenfunction $\bar{f}_{ik}$ of $\bar{M}_0$ satisfy

$$\begin{aligned} \bar{M}_0 \bar{f}_{ik} &= w_{ik} \bar{f}_{ik} \\ \bar{f}_{ik} &= \begin{bmatrix} 1 \\ \nu \beta \Sigma_f / (w_{ik} + \mathbf{I}) \end{bmatrix} \mathbf{y}_k, \quad i=0,1, \quad k=0,1 \end{aligned} \quad (2)$$

where, $\mathbf{y}_0$ is the fundamental mode, $\mathbf{y}_1$ is the first azimuthal mode, and $\bar{M}_0$ is the matrix $\bar{M}$ evaluated at the steady state. The expansion of the time and space-dependent density vector

$$\bar{Y}(\bar{r}, t) = u_{00}(t)\bar{F}_{00}(\bar{r}) + u_{01}(t)\bar{F}_{01}(\bar{r}) + u_{10}(t)\bar{F}_{10}(\bar{r}) + u_{11}(t)\bar{F}_{11}(\bar{r}) \quad (3)$$

is substituted into equation (1) and the inner-product of both sides is taken with the adjoint vector $\bar{F}_{ik}^+$. After some simplifications, this step leads to the final form of the modal kinetics equation based on β -modes

$$\begin{aligned} \frac{dn_k(t)}{dt} &= \mathbf{w}_{ik}n_k(t) + (\mathbf{w}_{1k} - \mathbf{w}_{0k})u_k(t) + \sum_{m=0}^1 \frac{\mathbf{r}_{km}}{\Lambda_k} n_m(t) \\ \frac{du_k(t)}{dt} &= \mathbf{w}_{1k}u_k(t) + \sum_{m=0}^1 \frac{\mathbf{r}_{km}}{\Lambda_{1k}} n_m(t) \end{aligned} \quad k = 0, 1 \quad (4)$$

where $n_k(t) = u_{0k}(t) + u_{1k}(t)$, $u_k(t) = u_{1k}(t)$ and $\Lambda_{ik} = \frac{\langle \bar{F}_{ik}^+(\bar{r}), \bar{F}_{ik}(\bar{r}) \rangle_V}{\langle \mathbf{y}_k(\bar{r}), \nu \mathbf{u} \Sigma_f(\bar{r}) \mathbf{y}_k(\bar{r}) \rangle_V}$. n_0 and n_1 are

non-dimensional neutron densities in the fundamental and first azimuthal modes, respectively.

Λ_{ik} here is the effective neutron generation time, where $\langle \rangle_V$ denotes the inner product over the volume of the core. The reactivity is given by

$$\mathbf{r}_{km}(t) = \sum_{l=0}^1 \mathbf{r}_{ext, kml} - c_{a, kml}(\mathbf{a}_l(t) - \mathbf{a}_{0,l}) - c_{D, kml}(T_{avg, l}(t) - T_{avg, 0, l}) \quad (5)$$

where $\mathbf{r}_{ext, kml}$ is the control-rod induced reactivity, and $\mathbf{a}_l(t)$ and $T_{avg, l}(t)$ are average void fraction and average fuel temperature of channel l , respectively. The form of the void and fuel temperature feedback coefficients are given by

$$c_{a, kml} = c_{a, l} \frac{\langle \mathbf{y}_k(\bar{r}), \nu \nabla \cdot \nabla \mathbf{y}_m(\bar{r}) \rangle_{V_l}}{\langle \mathbf{y}_k(\bar{r}), \nu \mathbf{u} \Sigma_f(\bar{r}) \mathbf{y}_k(\bar{r}) \rangle_V} \quad (6)$$

$$c_{D, kml} = c_{D, l} \frac{\langle \mathbf{y}_k(\bar{r}), \nu \mathbf{y}_m(\bar{r}) \rangle_{V_l}}{\langle \mathbf{y}_k(\bar{r}), \nu \mathbf{u} \Sigma_f(\bar{r}) \mathbf{y}_k(\bar{r}) \rangle_V} \quad (7)$$

where $\langle \rangle_{V_l}$ is the inner product over the volume of the part of core represented by channel l ;

and $c_{a, l}$ and $c_{D, l}$ are the typical values of void and fuel temperature feedback coefficients.

It is clear that the feedback coefficients $c_{a,kml}$ and $c_{D,kml}$, the eigenvalues w_{km} and the effective neutron generation time Λ_{ik} depend upon the inner products of the fundamental and first azimuthal modes. Karve [16] evaluated these coefficients using two modes. In cylindrical coordinate system, these modes are given by

$$y_0(r, z) = J_0(2.4048 r / R) \sin(pz / L), \quad (8)$$

$$y_1(r, q, z) = J_1(3.1834 r / R) \sin(pz / L) \sin(q) \quad (9)$$

One disadvantage of evaluating the feedback coefficients with the above two equations is that they are valid only for cores that are not far from uniform configurations. However, there are several factors that can lead to non-uniform cores, which in-turn will lead to modes that may be quite different from the fundamental and first azimuthal modes given by equations (8) and (9). These factors include: non-uniform burn-up, asymmetric control rod insertion, z-dependent void fraction, and variations in core inlet temperature between different (lumped) channels. In fact, it is suspected that such non-uniformity may be the cause of out-of-phase or local oscillations. For example, Miró et al. [5] found that radial power distributions with a peak at the center generally tend to generate in-phase oscillations; while bowl-shaped radial power distributions, with a local minimum at the center, tend to produce out-of-phase oscillations. Quantitative representation of different power distributions in a non-uniform core requires a modal decomposition approach like that developed by Verdú et al. [24]. In this approach, modes specific to non-uniform core configurations are obtained and used to generate parameters in the neutronics model. Disadvantage of this approach is that it is fairly

complicated, time-consuming, and case specific. Here, for purpose of qualitative studies, we adopt the parametric approach used earlier by Muñoz-Cobo et al. [18], introducing a factor F to represent effects of different core configurations. The modifications needed for Karve et al.'s model are modest, almost trivial, in this case. To account for the effect of non-uniform cores, the azimuthal mode feedback coefficients are multiplied by an amplification factor F , that is,

$$c_{a,1ml,nu} = F \cdot c_{a,1ml,u} \quad (10)$$

$$c_{D,1ml,nu} = F \cdot c_{D,1ml,u} \quad (11)$$

where, $c_{a,1ml,nu}$ and $c_{D,1ml,nu}$ are feedback coefficients of a non-uniform core; and $c_{a,1ml,u}$ and $c_{D,1ml,u}$ are feedback coefficients of a uniform core. Although there are other parameters that are affected by the non-uniformity, their impacts on the strength of modes are assumed minimal, and their values are held constant for different core configurations.

3.2 HEAT CONDUCTION AND CHANNEL THERMAL HYDRAULICS

Heat conduction and channel thermal hydraulics in Karve's model are developed using variational method. Due to large temperature gradient, a two-region piecewise quadratic temperature distribution is assumed in the fuel rod. Starting from the time-dependent heat conduction equations in cylindrical coordinates, variational principle is used to derive the set of ODEs for the expansion parameters of the quadratic profile [16].

In developing the heated channel thermal hydraulics model, quadratic spatial variations are assumed for water enthalpy in the single-phase region, and for steam quality in the two-phase part. Homogeneous two-phase flow model is used to model the two-phase flow. Ordinary differential equations (ODEs) for the expansion coefficients of the quadratic spatial variation are derived from the energy equations in the single phase and two-phase regions using the weighted-residual approach. The momentum equations are integrated over the entire length of the flow channel to obtain the ODE for the inlet velocity.

The heat conduction and channel thermal-hydraulic sub-models are coupled with the neutronics sub-model by the heat generation rate

$$q_l^*(t) = c_q (n_0(t) + \mathbf{x}_l n_1(t)) \quad (12)$$

where c_q is a constant that converts the dimensionless neutron density to power density.

Volume factor $\mathbf{x}_l = \langle \mathbf{1}, \mathbf{y}_l(r) \rangle_{V_l} / \langle \mathbf{1}, \mathbf{y}_0(r) \rangle_{V_l}$ denotes the contribution of channel l to the total power.

The BWR model of Karve et al. [15] is comprised of 22 non-linear ODEs. The vector of phase variables consists of $[a_{1,0}(t), a_{2,0}(t), s_{1,0}(t), s_{2,0}(t), v_{\text{inlet},0}(t), T_{11,0}(t), T_{12,0}(t), T_{21,0}(t), T_{22,0}(t), a_{1,1}(t), a_{2,1}(t), s_{1,1}(t), s_{2,1}(t), v_{\text{inlet},1}(t), T_{11,1}(t), T_{12,1}(t), T_{21,1}(t), T_{22,1}(t), n_0(t), u_0(t), n_1(t), u_1(t)]$, where, a, s , and T are expansion coefficients of water enthalpy in the single phase region, steam quality in the two-phase region and fuel rod temperature distributions, respectively.

4. POINCARÉ-ANDRONOV-HOPF BIFURCATION (PAH-B) ANALYSIS

A brief review of Poincaré-Andronov-Hopf bifurcation (PAH-B) theorem and Floquet theory, which form the mathematical basis of bifurcation analysis applied here, are given in this section. Conditions stated in the theorem, if satisfied, guarantee the existence of periodic solutions to set of non-linear differential equations. Stability of the periodic solutions is determined by the Floquet theory.

4.1 PAH-B THEOREM

General form of a set of autonomous nonlinear ODEs is given by

$$\frac{d\bar{X}(t)}{dt} = \bar{G}(\bar{X}(t), \mathbf{I}) \quad (13)$$

where, $\bar{X}(t)$ is vector of phase variables, $\bar{G}$ defines a set of nonlinear equations, and $\mathbf{I}$ is an operating parameter. The "steady state", or so-called fixed point of the system, $\bar{X}_{ss}$ satisfies

$$\bar{G}(\bar{X}_{ss}, \mathbf{I}) = 0. \quad (14)$$

It is well known that stability of the fixed point is determined by eigenvalues of the Jacobian of the governing ODEs. The fixed point of the system is stable only if real parts of all the eigenvalues of the Jacobian are negative. Because the Jacobian evaluated at the fixed point depends on the operating parameters continuously, varying $\mathbf{I}$ may change the system from stable to unstable states, or vice versa. A critical value of the operating parameter $\mathbf{I}_{critical}$ separating the stable from the unstable systems can be determined by setting the real part of the largest eigenvalue of the Jacobian exactly equal to 0.

When operating in stable (unstable) region of the parameter space, linear analysis predicts an *infinitesimal* perturbation to decay (grow). However, results of linear analysis are not necessarily applicable to perturbations that are larger than infinitesimal. Moreover, whether amplitude of growing oscillations will saturate—leading to stable periodic solution—is also not answered by linear analysis. Bifurcation theory provides answers to some of these questions—at least in the vicinity of the stability boundary. Poincaré-Andronov-Hopf bifurcation theorem predicts the existence of periodic solutions, branching from the fixed points, when I is close to its critical value and the following two conditions [21] are satisfied:

a) a pair of pure imaginary eigenvalues $(\pm i\omega_0)$ of the Jacobian of the system exists at

$I = I_{critical}$ and the real parts of all the other eigenvalues are negative.

b) the real parts of the complex conjugate eigenvalues with zero real part cross the imaginary

axis with non-zero speed as I is varied, i.e. $\left. \frac{d(\text{real part})}{dI} \right|_{I=I_{critical}} \neq 0$.

The periodic solutions, which are indexed by a constant ϵ , are given by

$$\bar{X}(t;T) = \bar{X}_{ss} + \epsilon e^{\frac{2\pi i}{T}t} \bar{P} + O(\epsilon^2) \quad (15)$$

where $\bar{X}_{ss}$ is the fixed point of the system, $\bar{P}$ is the eigenvector corresponding to the pure imaginary eigenvalue $i\omega_0$ normalized such that its first non-zero element is equal to one, and ϵ is the amplitude of the periodic solution relative to the eigenvector $\bar{P}$. According to PAH theorem, the period T , and value of I corresponding to this periodic solution, can be expanded in ϵ as [21]

$$T = \frac{2p}{w_0} (1 + t_2 e^2 + O(e^4)) \quad (16)$$

$$I = I_{critical} + m_2 e^2 + O(e^4) \quad (17)$$

Hence, the oscillation amplitude e increases as the square root of the distance from critical parameter value, $e \propto \sqrt{I - I_{critical}}$.

4.2 FLOQUET THEORY

The stability of the periodic solutions, whose existence is guaranteed by the PAH bifurcation theorem, is determined by the Floquet theory [22]. Stability is determined in a way similar to that of fixed points, i.e. by introducing a perturbation to the periodic solution, and then analyzing whether the perturbation grows or decays. Analysis finally leads to the evaluation of *Floquet exponents*. The perturbations decay if all the Floquet exponents have negative real parts. Otherwise they grow in magnitude. The Floquet exponent β , can also be expanded in terms of e as

$$\beta = b_2 e^2 + O(e^4) \quad (18)$$

If $b_2 > 0$, the periodic solution is unstable leading to what is called subcritical PAH bifurcation. Otherwise, if $b_2 < 0$, the periodic solution is stable and the corresponding bifurcation is called supercritical PAH bifurcation. For a given set of parameter values, the bifurcation is either supercritical or subcritical but not both.

In the case of supercritical PAH-B, stable periodic solutions exist in the immediate neighborhood of the stability boundary in the unstable region. Hence, in this region where the

fixed points are unstable, the dynamical system will evolve to stable periodic solutions. In the case of subcritical PAH-B, there exist unstable periodic solutions in the neighborhood of the SB on the stable side. Hence, although a small perturbation may finally decay to the fixed point, a large one (greater than the amplitude of the periodic solution) will lead to growing amplitude oscillations. The existence of subcritical PAH-B shows that for finite amplitude perturbations, stability analysis may lead to non-conservative conclusion. Consequently, special attention must be paid even when the system is operating on the stable side near the stability boundary.

4.3 BIFDD

Bifurcation analysis requires evaluation of m_2 , t_2 and b_2 . Analytical evaluation of m_2 , t_2 and b_2 , even for reduced order simple models, is a very tedious task and quickly becomes almost impossible with increasing number of differential equations in the model. Another drawback of the analytical bifurcation studies is that each is specific to a bifurcation parameter, and must be repeated if the impact of a different parameter is to be studied. Numerical integration of the governing set of ODEs is also time consuming and leads to results valid for specific set of parameter values. Due to the limitations on the two approaches (numerical integration and analytical bifurcation studies), there has been limited investigation of the large parameter space even for simple models of BWRs.

Analytical bifurcation analysis carried out *numerically* is an attractive alternative to the two

approaches described above [25]. In this approach the governing set of nonlinear equations are neither integrated numerically in time nor treated entirely analytically. Rather, the *analytical reduction* to the Poincaré normal form via the center manifold theorem is carried out *numerically* [23]. This approach, which will henceforth be called *analytic-numeric approach*, allows accurate and efficient evaluation of the entire parameter space of interest. General-purpose bifurcation codes such as BIFDD [23] have been developed to perform analytic-numeric bifurcation analysis of set of ODEs and ODEs with delays. For a given set of nonlinear ODEs (or ODEs with delays) and the corresponding Jacobian matrix, the code determines the critical value of the bifurcation parameter, nature of the bifurcation, and the oscillation amplitude when operating close to the critical value. BIFDD allows any parameter of choice in the model to be selected as the bifurcation parameter, and hence the entire parameter space can easily be spanned.

Bifurcation analysis when carried out analytically or via an analytic -numeric approach, yields—for all other parameters fixed—the critical value of the bifurcation parameter $I = I_{critical}$, frequency of oscillation ω_0 corresponding to the critical value, and parameters m_2 , t_2 and b_2 . A negative (positive) value of b_2 indicates a supercritical (subcritical) PAH bifurcation, i.e., stable (unstable) limit cycle oscillations as the bifurcation parameter is varied across the critical value. t_2 is a correction factor to the oscillation frequency, and m_2 relates the oscillation amplitude to the value of the bifurcation parameter through equation (17).

The set of 22 nonlinear ODEs and the Jacobian matrix (evaluated with the help of *Mathematica*) of the BWR model are supplied to the bifurcation code BIFDD via a user subroutine. A parameter is selected as the bifurcation parameter. By incrementally varying a second parameter and repeating the calculations for the critical value of the bifurcation parameter, one can easily generate stability boundaries and determine the nature of the bifurcation along those boundaries in two-dimensional parameter spaces. This can further be extended easily to generate stability-surfaces in three or higher dimensional parameter spaces.

5. RESULTS: UNIFORM CORE ($F = 1$)

For uniform core ($F = 1$), using BIFDD we first determined the stability boundary (SB) in the $N_{sub} - \Delta P_{ext}$ parameter space for $r_{ext} = 0$, where, N_{sub} is the non-dimensional subcooling number, ΔP_{ext} is the non-dimensional pressure drop over the flow channels, and r_{ext} is the control rod induced fundamental mode reactivity. The SBs we obtain are identical to those obtained earlier by Karve et al. [15]. A SB for $r_{ext} = 0$ is shown in Figure 1 (a). Results of the bifurcation analysis are presented in Figure 1(b) in the form of constant amplitude oscillation curve in the $N_{sub} - (\Delta P_{ext} - \Delta P_{ext,critical})$ parameter space. For given N_{sub} , $\Delta P_{ext,critical}$ is the (critical) value of ΔP_{ext} on the SB (shown in Figure 1 (a)). In this space, $\Delta P_{ext} - \Delta P_{ext,critical} > 0$ correspond to the stable region. The curve plotted in Figure 1 (b)

corresponds to the points in the parameter space whose index constant of the periodic solution e is 0.075, that is, the amplitudes of the periodic solutions on this curve, given by the magnitude of $e\bar{P}$, are 7.5% of the magnitude of the eigenvector. For this reason, this curve is also called the *7.5% oscillation curve*. The PAH bifurcation (PAH-B) theorem states that the bifurcation is *supercritical* when this curve is in the unstable region, and *subcritical* when it is in the stable region. Hence, in the upper part of the stability boundary for $N_{sub} > 1.85$, where the oscillation curve is in the stable region, the bifurcation is subcritical. Thus, the small amplitude perturbations will decay, but “large” amplitude perturbations, greater than the amplitude of the periodic solution, will grow. For $0.5 < N_{sub} < 1.85$, the oscillation curve is in the unstable region and the bifurcation is supercritical. The periodic solution is hence stable, and any finite but small perturbation will evolve to this stable periodic solution. For even lower values of N_{sub} (< 0.5), the 7.5% oscillations curve returns to the stable region, and the bifurcation is subcritical. The back and forth transition between supercritical and subcritical PAH-B in the low N_{sub} region might explain the fact that stable amplitude oscillations are observed in some BWR instability cases, and growing amplitude oscillations (until scram) in others.

Figures 2(a) and 2(b) show the SB and 7.5% oscillation curve in $N_{sub} - r_{ext}$ parameter space, where r_{ext} is chosen as the bifurcation parameter. ΔP_{ext} is held constant at 8.2. Note that $r_{ext} - r_{ext,critical} < 0$ is the stable region. Again subcritical PAH-B occurs in region of very high and low N_{sub} values, while supercritical PAH-B occurs for intermediate values of N_{sub} . SB and

constant amplitude oscillations curves in $r_{ext} - \Delta P_{ext}$ parameter space are shown in Figures 3(a) and 3(b), where ΔP_{ext} is chosen as the bifurcation parameter. N_{sub} is held constant at 0.6. Figure 3(b) shows that very large or very small values of r_{ext} will lead to subcritical PAH-B, but intermediate values ($-0.0155 < r_{ext} < +0.0130$) lead to supercritical PAH bifurcation.

To investigate the time evolution, and validate the results obtained using BIFDD, three points are selected in the parameter space for numerical simulations. These are shown in Figure 1. $(N_{sub}, \Delta P_{ext})$ at points 1, 2, and 3 are (2.2303, 3.9840), (1.0000, 8.1000), and (1.0000, 8.5000), respectively. Figures 4, 5, and 6 show the results of numerical simulations for these points. Point 1 is in the subcritical bifurcation region on the stable side of the SB. According to PAH theorem, large perturbations, when operating at point 1 in the parameter space, may lead to increasing amplitude oscillations. Point 2, however, is in the supercritical bifurcation region on the unstable side of the SB. Hence perturbations are expected to evolve to stable amplitude oscillations. Point 3 is in the supercritical bifurcation region but on the stable side of the SB. Hence, perturbations are expected to decay. Time evolution plots of n_0 , n_1 and $v_{inlet,0}$, where $v_{inlet,0}$ is the non-dimensional inlet velocity of channel 0, shown in Figures 4, 5, and 6 confirm these predictions. Moreover, the amplification factor F determines the relative strength of reactivity feedback of the fundamental and first azimuthal modes. Hence, for F close to one, the feedback coefficients of the azimuthal mode are small, and BWR is likely to develop in-phase or global oscillations. Numerical simulations for $F = 1$ show that the time evolution of phase variables plotted in Figures 4, 5, and 6 are, as expected, almost pure in-phase oscillations,

where amplitude of n_1 is much smaller than that of n_0 . However, if F is large, the feedback coefficients of the azimuthal mode are also large, and BWR will more likely experience out-of-phase or regional oscillations. Results for $F > 1$ are presented in the next section.

6 RESULTS: NON-UNIFORM CORE ($F > 1$)

Results of numerical simulations for $N_{sub} = 1.0000$ and $\Delta P_{ext} = 8.5000$ (corresponding to point 3 in Figure 1 (a)) for $F = 2.0, 3.5, 4.6$ and 5.0 , are shown in Figures 7, 8, 9 and 10. These results suggest that the first azimuthal mode is becoming increasingly less stable as F is increased. Although there is no significant change in the decay ratio of n_0 , that of n_1 increases as F is increased. Decay ratio of n_1 is greater than 1.0 for $F = 5.0$, implying an unstable first azimuthal mode. Moreover, *beating* of inlet velocity of the left channel $v_{inlet,0}$, observed in Figures 9 (c) and 10 (c), suggests that period of the out-of-phase component is slightly different from that of in-phase component.

Figure 11 shows SBs and $\epsilon = 0.025$ curves for $F = 1.0, 2.0, 3.5, 4.6$ and 5.0 . The SBs for $F = 1.0, 2.0, 3.5$ and 4.6 cannot be distinguished from each other. The 2.5% oscillation curves for these four cases, shown in Figure 11 (b), are also almost indistinguishable. However, SB and 2.5% oscillation curve for $F = 5.0$ when $N_{sub} < 1.177$ differ from corresponding curves for lower values of F . Moreover, at $N_{sub} = 1.177$ the SB for $F = 5$ case has a discontinuity in its

derivative and there is a corresponding jump in the 2.5% oscillation curve. Results of the numerical simulations in Figures 6~10, and bifurcation analyses shown in Figure 11 (a) and (b) cannot be explained only by a single pair of complex conjugate eigenvalues. As F is increased these results suggest that other pair(s) of complex conjugate eigenvalues are increasingly more strongly influencing the dynamical system.

To identify the cause of the discontinuities and to determine the role of additional pairs of eigenvalues in system dynamics when operating close to the SB, the bifurcation code BIFDD was modified to evaluate all the eigenvalues of the Jacobian matrix. Figure 12 shows pairs of complex conjugate eigenvalues with the largest and the second largest real parts. These eigenvalues correspond to a point on the SB in Figure 1(a) with $N_{sub} = 1.0000$ and $\Delta P_{ext} = 8.2085$, which is on the SB on a horizontal line that passes through the point 3. Eigenvalue with the largest real part for $F = 1$ is denoted by e_1 and the second pair is denoted by e_2 . For clarity real eigenvalues that may exist between them are not shown in Figure 12. By definition, when the operating point is on the SB, the real part of the largest pair of eigenvalues is always zero. For $F = 1$, the pair e_2 with the second largest real part is far from the imaginary axis, but approaches it as F is increased. Real part of e_2 for $F = 4.6$ is close to zero, and hence both e_1 and e_2 almost equally influence the time evolution. The imaginary parts of eigenvalues, which determine frequencies of oscillations associated with them, are slightly different (4.1537 and 4.2220 respectively), hence explaining the *beating* phenomena observed in the numerical simulations of $v_{inlet,0}$ in Figure 9(c). The ratio of the two frequencies correctly predicts the

period of the *beat*. As F is increased to 5.0, the pair e_2 crosses the pair e_1 . For $F = 5$ (and $N_{sub} < 1.177$), e_2 is the pair with the largest real part. Thus the SB in Figure 11 (a) for $N_{sub} < 1.177$ is determined by e_2 . Moreover, for these parameter values the bifurcation characteristics of the system are also determined by this eigenvalue. Thus, the discontinuity in the slope of SB at $N_{sub} = 1.177$ (Figure 11 (a), $F = 5.0$ case), and the jump in the 2.5% oscillation curve at these parameter values (Figure 11 (b)) are due to the fact that while the pair e_1 determines the SB and the bifurcation curve for $N_{sub} > 1.177$, it is e_2 that determines the SB and the bifurcation curve for $N_{sub} < 1.177$.

To identify the relationship between the two pairs of eigenvalues and in-phase and out-of-phase oscillations, the elements corresponding to n_0 and n_1 in the eigenvectors corresponding to eigenvalues e_1 and e_2 for different values of F are shown in Tables 1 and 2. In eigenvector corresponding to e_1 , the magnitudes of the elements of n_0 , shown in the Table 1, are much larger than those of elements of n_1 . However, the trend is reversed for eigenvalue e_2 (Table 2). As F is changed, the ratio of the magnitude of element of n_0 to n_1 for eigenvalue e_1 , and the ratio of magnitude of element of n_1 to n_0 for eigenvalue e_2 remain large. Because the magnitudes of the elements of the eigenvector determine the amplitudes of oscillations of different variables (equation (15)), these results show that the oscillation amplitude of n_0 in the oscillation produced by the eigenvalue e_1 , is much larger than that of n_1 , and hence the oscillations are dominated by the in-phase component. Also, the oscillation amplitude of n_1 will be much larger than that of n_0 in oscillations produced by the eigenvalue e_2 , and hence the

oscillations will be primarily out-of-phase.

It is clear that the two pairs of complex conjugate eigenvalues — with the largest and second largest real part — are responsible for the fundamental and the first azimuthal mode (in-phase and out-of-phase) oscillations. Moreover, either one of these—depending upon the parameter values—may be the dominant one. To avoid confusion, the pair of eigenvalues e_1 for which magnitudes of element of n_0 in the corresponding eigenvector is much larger than that of element of n_1 , will be called the fundamental mode eigenvalues ($a_f + i w_f$), and the other pair e_2 will be called the first azimuthal mode eigenvalues ($a_a + i w_a$). Figure 13 (a) shows the SB in $N_{sub}—\Delta P_{ext}$ space for $F = 5.0$. The SB (solid line) in Figure 13 (a), on which the real part of the rightmost eigenvalues is zero, is comprised of two curves that meet at the point T ($N_{sub} = 1.177$, $\Delta P_{ext} = 7.618$). Above this point, the eigenvalues with the largest real part along the SB are the fundamental mode eigenvalues, that is, $a_f = 0$ and $a_a < 0$. However, for $N_{sub} < 1.177$ (below the point T), the eigenvalues with the largest real part along the SB are the azimuthal mode eigenvalues ($a_a = 0$ and $a_f < 0$). The dotted line in Figure 13(a) denotes the boundary along which the real part of the second rightmost pair of complex conjugate eigenvalue is zero. (This means that the rightmost pair of complex conjugate eigenvalue along the dotted line has positive real part). The dotted curve is also comprised of two branches. Along the branch above the point T, the real parts of the azimuthal mode eigenvalues are zero ($a_a = 0$ and $a_f > 0$). However, along the lower branch, the real parts of the fundamental mode eigenvalues are zero ($a_f = 0$ and $a_a > 0$). Hence, the parameter space is divided into four parts, denoted by letters A,

B, C and D. Region C and A are respectively the stable and unstable regions (for both in-phase and out-of-phase mode oscillations). In region B, the fundamental mode (in-phase) is unstable while the first azimuthal mode (out-of-phase) is stable. While in region D, the first azimuthal mode (out-of-phase) is unstable, but the fundamental mode (in-phase) is stable. These results are conceptually similar to those obtained using a simple model by March-Leuba et al. [14]. Out-of-phase oscillations in reference [14] were obtained using a different set of boundary conditions than those used for in-phase oscillations. All results reported here are obtained using the same set of boundary conditions.

Results of the bifurcation analyses for $F = 5$ are presented in Figure 13 (b) in the form of 5% oscillation amplitude curve in $N_{sub} - (\Delta P_{ext} - \Delta P_{ext,critical})$ space. The solid line corresponds to the SB (solid line) in Figure 13 (a), while the dotted curve corresponds to the dotted curve in Figure 13 (a) (i.e., to the second rightmost eigenvalue crossing the imaginary axis). [The bifurcation is subcritical for $N_{sub} > 1.874$.] In Figure 13 (b), a jump in the 5% oscillation curve (solid line) occurs at the value of N_{sub} corresponding to point T. The discontinuity exists because of the degeneracy at this value of N_{sub} , where both the fundamental and first azimuthal mode eigenvalues have zero real parts. The 5% oscillation curve for $N_{sub} > 1.177$ is determined using the fundamental mode eigenvalues, while for $N_{sub} < 1.177$ they must be determined using the first azimuthal mode eigenvalues. Note that the 5% oscillation curve for each pair of eigenvalue is continuous over the entire range of N_{sub} . However, only a segment (the solid part) of each is relevant. (Condition (a) of PAH-B theorem (section 4.1) is violated along the dotted

line.) Though the PAH-B theorem is strictly applicable only along the solid lines in Figure 13 (a), results of the numerical simulations show that the conclusions regarding the stable and unstable periodic solutions may be applicable even to the second pair of complex conjugate eigenvalues that cross the imaginary axis *right behind* the first pair.

To even more explicitly identify the role of F in determining the nature (in-phase or out-of-phase) oscillations, Figures 14 (a-f) show SBs in the F — ΔP_{ext} parameter space for $N_{sub} = 4.50, 3.25, 1.90, 1.40, 1.30$ and 1.18 , respectively. The solid and dashed lines are respectively the boundaries along which the real part of the fundamental and first azimuthal mode eigenvalues are zero. System stability is determined by the boundary on the right. For all values of N_{sub} the fundamental mode boundary is independent of F . The first azimuthal mode boundary does vary with F ; much more strongly for smaller values of N_{sub} than for larger values. For $N_{sub} = 4.50$ and $F = 1.00$, the system becomes unstable due to fundamental mode. However, as F is increased for $1.50 < F < 3.53$, it is the first azimuthal mode that becomes unstable first as the stability boundary is crossed. Boundaries for the two modes in this region are close and hence both types of oscillations are likely to be manifested as the SB is crossed. For $F > 3.53$, once again it is the fundamental mode that determines the stability of the system. Similar behavior is observed for $N_{sub} = 3.25$ case (Figure 14 (b)). Hence for $2.32 < F < 3.20$ the two boundaries almost overlap leading to the degenerate case. For $F < 2.32$ and $F > 3.20$, stability is determined by the fundamental mode. As N_{sub} is decreased to 1.90 (Figure 14 (c)) and 1.40 (Figure 14 (a)), the azimuthal mode boundary stays to the left of the fundamental

mode boundary for all values of F , and hence it is only the fundamental mode that becomes unstable in the vicinity of the SB. However, as N_{sub} is decreased further to 1.30 (Figure 14 (d)) and then to 1.18 (Figure 14 (e)), the boundary for the first azimuthal mode shifts to the right. For these two values of the subcooling number, for some intermediate range of F ($5.48 < F < 7.37$ for $N_{sub} = 1.30$, and $5.03 < F < 10.29$ for $N_{sub} = 1.18$) it is the first azimuthal mode that determines the stability of the system.

Numerical simulations were carried out for parameter values at points 1 ($N_{sub} = 2.2303$, $\Delta P_{ext} = 3.9000$), 2 ($N_{sub} = 2.2303$, $\Delta P_{ext} = 4.1000$), 3 ($N_{sub} = 0.8000$, $\Delta P_{ext} = 7.8500$), and 4 ($N_{sub} = 0.8$, $\Delta P_{ext} = 8.2000$) shown in Figure 13(a). Time evolutions of n_0 and n_1 are shown in Figures 15~18. Points 1 and 2 are above the separation point T. At point 1 (Figure 15), in-phase oscillations result leading to increasing amplitude of n_0 and decreasing amplitude of n_1 . Point 2, however, is in the region C, where both fundamental and azimuthal modes are stable. Therefore, small perturbations produce decreasing amplitude oscillations of both n_0 and n_1 (Figure 16). However, n_0 decreases much slower than n_1 , causing oscillations that are primarily in-phase. Points 3 and 4 are below the separation point T. For point 3, both the fundamental and azimuthal modes are unstable, but it is n_1 whose amplitude increases much faster than that of n_0 (Figure 17). The oscillations are hence pre-dominantly out-of-phase. At point 4, out-of-phase oscillations occur, because the fundamental mode is stable while the first azimuthal mode is unstable (Figure 18). Clearly the dominant oscillations in the high and low N_{sub} regions are characteristically different. In the high N_{sub} region, the in-phase oscillations

are dominant because the critical eigenvalues on the SB correspond to the fundamental mode. In the low N_{sub} region, however, the out-of-phase oscillations are dominant because the critical eigenvalues correspond to the azimuthal mode. In the region close to the separation point T, effects of both, the fundamental and azimuthal mode eigenvalues, are evident, and oscillations in this region are a blend of in-phase and out-phase. Thus, numerical simulations confirm the findings of the numeric-analytic bifurcation analysis.

7 SUMMARY

Stability and bifurcation analyses of a two-channel BWR model is carried out. To capture the effects of asymmetry in core loading and in neutron flux, an amplification factor F is introduced in a reduced order model earlier developed by Karve et al. [15] to vary the azimuthal mode feedback coefficient. Stability and bifurcation characteristics are studied using the code BIFDD, modified to determine the entire spectrum of eigenvalues—rather than only the largest one. For azimuthally symmetric flux shape ($F = 1$), only in-phase oscillations are found in all regions of parameter space studied. Both sub- and supercritical bifurcations can occur. It is found that bifurcation characteristics can switch from sub- to super- and back to subcritical over a narrow range of a single operating parameter (such as inlet subcooling). This might explain the fact that stable amplitude oscillations are observed in some BWRs while growing amplitude oscillations in others.

As the amplification factor F is increased, spectrum of eigenvalues calculated by BIFDD

shows that real part of the second pair of complex conjugate eigenvalues increases and approaches the imaginary axis. The eigenvectors of the two pairs of complex conjugate eigenvalues suggest characteristically different oscillations. The oscillations produced by the pair of complex conjugate eigenvalues with the largest real part (for relatively smaller values of F) are mainly in-phase, and those produced by the complex conjugate eigenvalues with the second largest real part are mostly out-of-phase. For this reason, the former is called fundamental mode (in-phase) eigenvalue, and the latter is called azimuthal mode (out-of-phase) eigenvalue.

Stability boundaries in F — ΔP_{ext} parameter space show that for large or small values of N_{sub} , azimuthal mode eigenvalues can cross the fundamental mode eigenvalues. In this case, the system becomes unstable due to fundamental mode when F is large or small. The system becomes unstable, however, due to the first azimuthal mode for intermediate values of F . Therefore, oscillations will change from in-phase to out-of-phase and back to in-phase as F is increased. For intermediate values of N_{sub} , azimuthal mode eigenvalues do not cross the fundamental mode eigenvalues as F is varied. Hence, dominant oscillations are only in-phase.

For $F = 5$, boundaries and bifurcation curves of the eigenvalues with the largest and second largest real parts in N_{sub} — ΔP_{ext} parameter space show that above certain value of inlet subcooling, the fundamental mode eigenvalue reaches the imaginary axis (as the imposed pressure drop is increased) before the first azimuthal mode eigenvalue. For these high values of

inlet subcooling in-phase oscillations dominate. However, for smaller values of inlet subcooling out-of-phase oscillations dominate due to the azimuthal mode eigenvalue that reaches the imaginary axis first as the bifurcation parameters is varied. Hence, the parameter space is divided into four regions separated by boundaries of the two pairs of eigenvalues. Bifurcation results show that both sub- and super-critical bifurcation can occur along the stability boundaries for in-phase as well as out-of-phase oscillations. Numerical integrations in these regions confirm the results of the stability and bifurcation analyses.

The coupled model analyzed here clearly shows that it is capable of predicting in-phase and out-of-phase oscillations, as well as stable amplitude and growing amplitude oscillations. Thus, various types of instabilities and oscillations observed in operating reactors or those observed under controlled conditions have been successfully simulated by a single reduced order model.

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IN-PHASE AND OUT-OF-PHASE OSCILLATIONS IN BWRs: IMPACT OF AZIMUTHAL ASYMMETRY AND SECOND PAIR OF EIGENVALUES

Quan Zhou and Rizwan-uddin

Figure Captions

Figure 1. Results of stability and bifurcation analyses for uniform core, (a) stability boundary;
(b) 7.5% oscillation curve in $N_{sub} - (\Delta P_{ext} - \Delta P_{ext,critical})$ parameter space

Figure 2. Results of stability and bifurcation analyses for uniform core, (a) stability boundary;
(b) 7.5% oscillation curve in $N_{sub} - (\mathbf{r}_{ext} - \mathbf{r}_{ext,critical})$ parameter space

Figure 3. Results of stability and bifurcation analyses for uniform core, (a) stability boundary;
(b) 7.5% oscillation curve in $\mathbf{r}_{ext} - (\Delta P_{ext} - \Delta P_{ext,critical})$ parameter space

Figure 4. Time evolutions at point 1 in Figure 1 (a) ($N_{sub} = 2.2303$, $\Delta P_{ext} = 3.9840$), for $F = 1.0$.
(a) n_0 ; (b) n_1 ; (c) $v_{inlet,0}$

Figure 5. Time evolutions at point 2 in Figure 1 (a) ($N_{sub} = 1.0000$, $\Delta P_{ext} = 8.1000$), for $F = 1.0$.
(a) n_0 ; (b) n_1 ; (c) $v_{inlet,0}$

Figure 6. Time evolutions at point 3 in Figure 1 (a) ($N_{sub} = 1.0000$, $\Delta P_{ext} = 8.5000$), for $F = 1.0$.

(a) n_0 ; (b) n_1 ; (c) v_{inlet0}

Figure 7. Time evolutions for $N_{sub} = 1.0000$, $\Delta P_{ext} = 8.5000$ and $F = 2.0$. (a) n_0 ; (b) n_1 ; (c) v_{inlet0}

Figure 8. Time evolutions for $N_{sub} = 1.0000$, $\Delta P_{ext} = 8.5000$ and $F = 3.5$. (a) n_0 ; (b) n_1 ; (c) v_{inlet0}

Figure 9. Time evolutions for $N_{sub} = 1.0000$, $\Delta P_{ext} = 8.5000$ and $F = 4.6$. (a) n_0 ; (b) n_1 ; (c) v_{inlet0}

Figure 10. Time evolutions for $N_{sub} = 1.0000$, $\Delta P_{ext} = 8.5000$ and $F = 5.0$. (a) n_0 ; (b) n_1 ; (c)

v_{inlet0}

Figure 11. Stability boundaries and $\epsilon = 0.025$ curves for $F = 1.0, 2.0, 3.5, 4.6$ and 5.0 in

$N_{sub} - \Delta P_{ext}$ space.

Figure 12. Pairs of eigenvalues with the largest and second largest real parts for $N_{sub} = 1.0000$,

$\Delta P_{ext} = 8.2085$. $F =$ (a) 1.0; (b) 2.0; (c) 3.5; (d) 4.6; (e) 5.0

Figure 13. (a) Stability boundary and the boundary along which the real part of the second

largest pair of eigenvalue is zero ($F = 5$); (b) 5% oscillation amplitude curves corresponding to

the eigenvalues with the largest and second largest real parts ($F = 5.0$)

Figure 14. Stability boundaries in $F—\Delta P_{ext}$ space for $N_{sub} =$ (a) 4.50; (b) 3.25; (c) 1.90; (d) 1.40; (e) 1.30; (f) 1.18

Figure 15. Time evolution of n_0 and n_1 for parameter values corresponding to point 1 in Figure 13 ($N_{sub} = 2.2303$, $\Delta P_{ext} = 3.9000$, $F = 5.0$)

Figure 16. Time evolution of n_0 and n_1 for parameter values corresponding to point 2 in Figure 13 ($N_{sub} = 2.2303$, $\Delta P_{ext} = 4.1000$, $F = 5.0$)

Figure 17. Time evolution of n_0 and n_1 for parameter values corresponding to point 3 in Figure 13 ($N_{sub} = 0.8000$, $\Delta P_{ext} = 7.8500$, $F = 5.0$)

Figure 18. Time evolution of n_0 and n_1 for parameter values corresponding to point 4 in Figure 13 ($N_{sub} = 0.8000$, $\Delta P_{ext} = 8.2000$, $F = 5.0$)

(a)

(b)

Figure 1. Results of stability and bifurcation analyses for uniform core, (a) stability boundary, (b) 7.5% oscillation curve in $N_{sub} - (\Delta P_{ext} - \Delta P_{ext,critical})$ parameter space

(a)

(b)

Figure 2. Results of stability and bifurcation analyses for uniform core, (a) stability boundary,

(b) 7.5% oscillation curve in $N_{sub} - (\mathbf{r}_{ext} - \mathbf{r}_{ext,critical})$ parameter space

(a)

(b)

Figure 3. Results of stability and bifurcation analyses for uniform core, (a) stability boundary,

(b) 7.5% oscillation curve in $\mathbf{r}_{ext} - (\Delta P_{ext} - \Delta P_{ext,critical})$ parameter space

(a)

(b)

(c)

Figure 4. Time evolutions at point 1 in Figure 1 (a) ($N_{sub} = 2.2303$, $\Delta P_{ext} = 3.9840$), for $F = 1.0$

(a) n_0 ; (b) n_1 ; (c) $v_{inlet,0}$

(a)

(b)

(c)

Figure 5. Time evolutions at point 2 in Figure 1 (a) ($N_{sub} = 1.0000$, $\Delta P_{ext} = 8.1000$), for $F = 1.0$

(a) n_0 ; (b) n_1 ; (c) v_{inlet0}

(a)

(b)

(c)

Figure 6. Time evolutions at point 3 in Figure 1 (a) ($N_{sub} = 1.0000$, $\Delta P_{ext} = 8.5000$), for $F = 1.0$

(a) n_0 ; (b) n_1 ; (c) v_{inlet0}

(a)

(b)

(c)

Figure 7. Time evolutions for $N_{sub}=1.0000$, $\Delta P_{ext}=8.5000$ and $F=2.0$. (a) n_0 ; (b) n_1 ; (c) $v_{inlet,0}$

(a)

(b)

(c)

Figure 8. Time evolutions for $N_{sub}=1.0000$, $\Delta P_{ext}=8.5000$ and $F=3.5$. (a) n_0 ; (b) n_1 ; (c) $v_{inlet,0}$

(a)

(b)

(c)

Figure 9. Time evolutions for $N_{sub}=1.0000$, $\Delta P_{ext}=8.5000$ and $F=4.6$. (a) n_0 ; (b) n_1 ; (c) $v_{inlet,0}$

(a)

(b)

(c)

Figure 10. Time evolutions for $N_{sub}=1.0000$, $\Delta P_{ext}=8.5000$ and $F=5.0$. (a) n_0 ; (b) n_1 ; (c)

$v_{inlet,0}$

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(b)

Figure 11. Stability boundaries and $\epsilon = 0.025$ curves for $F = 1.0, 2.0, 3.5, 4.6$ and 5.0 in

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(a)

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(a)

(b)

Figure 15. Time evolution of n_0 and n_1 for parameter values corresponding to point 1 in Figure

13 ($N_{sub} = 2.2303$, $\Delta P_{ext} = 3.9000$, $F = 5.0$)

(a)

(b)

Figure 16. Time evolution of n_0 and n_1 for parameter values corresponding to point 2 in Figure

13 ($N_{sub} = 2.2303$, $\Delta P_{ext} = 4.1000$, $F = 5.0$)

(a)

(a)

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13 ($N_{sub}=0.8000$, $\Delta P_{ext}=7.8500$, $F=5.0$)

(a)

(b)

Figure 18. Time evolution of n_0 and n_1 for parameter values corresponding to point 4 in Figure

13 ($N_{sub} = 0.8000$, $\Delta P_{ext} = 8.2000$, $F = 5.0$)

IN-PHASE AND OUT-OF-PHASE OSCILLATIONS IN BWRs: IMPACT OF AZIMUTHAL ASYMMETRY AND SECOND PAIR OF EIGENVALUES

Quan Zhou and Rizwan-uddin

Table Captions

Table 1. Elements corresponding to n_0 and n_1 in the eigenvector corresponding to e_1

Table. 2 Elements corresponding to n_0 and n_1 in the eigenvector corresponding to e_2

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	n_0			n_1			$ n_0 / n_1 $
F	Real	Imaginary	Magnitude	Real	Imaginary	Magnitude	
1.0	1.3656	4.8667	5.0546	0.004590	0.01369	0.01444	350.04
2.0	1.3615	4.8533	5.0407	0.01231	0.03922	0.04111	122.61
3.5	1.3482	4.7732	4.9600	0.03740	0.1920	0.1956	25.36
4.6	1.6890	5.1131	5.3848	-0.6087	-0.4666	0.7670	7.02

Table. 2 Elements corresponding to n_0 and n_1 in the eigenvector corresponding to e_2

	n_0			n_1			$ n_1 / n_0 $
F	Real	Imaginary	Magnitude	Real	Imaginary	Magnitude	
1.0	-0.009630	-0.01334	0.01645	2.7592	3.9449	4.8140	292.64
2.0	-0.01475	-0.03072	0.03407	2.8850	6.3208	6.9481	203.94
3.5	-0.02896	-0.1192	0.1227	2.6818	8.5689	8.9788	73.18
4.6	0.2584	0.2299	0.3459	1.8988	8.9610	9.1600	26.48

PART II

Natural Circulation Flashing in Next Generation Boiling Water Reactors

ABSTRACT

In the second part, a new reduced order model to simulate the dynamics of natural circulation features in current and next generation of nuclear power plants has been developed. Dynamical analysis this new natural circulation BWR model that allows local pressure dependence of water saturation enthalpy and includes fundamental and first azimuthal modes for neutronics is carried out. Stability boundaries (SBs) and 7.5% oscillation curves (representing natures of bifurcation along SBs) are plotted in inlet subcooling—external reactivity parameter space. Stability and bifurcation analyses show that, both in-phase and out-of-phase oscillations as well as supercritical and subcritical bifurcations can occur along the SBs.

1. Introduction

Natural circulation is a promising design feature for next generation of BWRs. Safety and economics of the nuclear reactors can be improved through simplicity and passive features of the natural circulation design [1].

One of the primary concerns for safe operation of BWRs is stability. Both natural and forced circulation BWR systems, however, are susceptible to nuclear coupled density wave oscillations (DWOs) under certain adverse operating conditions [2]. DWOs can be either in-phase or out-of-phase type [3]. The former corresponds to global power and flow oscillations, and is excited by fundamental neutronic mode. The latter corresponds to regional oscillations (usually power of half of the core oscillates out-of-phase against the other half), and is an outcome of the first azimuthal mode. For natural circulation systems, experiments and analytical studies show other instability mechanisms besides DWOs. An important one of these is flashing-induced oscillation during start-up. Flashing that is boiling in the unheated riser above the core, is caused due to local pressure drop. Flashing occurs most frequently under low system pressure during start-up. Jiang et al. [4] indicated that “the maximum void fraction caused by flashing is about 5% under a system pressure of 1.5 MPa, and about 80% under 0.1 MPa.”

Several experimental studies of the two-phase natural circulation system have been reported [5-7]. Although a few analytical studies [4, 8-9] have been carried out for natural circulation systems, they are limited in scope due to one or more of the following reasons:

- Many existing system codes are not designed to analyze natural circulation system. Those suitable to analyze the flashing phenomenon are too cumbersome for parametric studies.
- Existing reduced order models are too simplistic. Hence, some key features of system dynamics may be lost.
- There are limited reduced-order analyses of coupled models of thermal-hydraulics, fuel dynamics and neutronics.

Taking advantage of the two-mode neutronics and sophisticated fuel dynamics model, first developed by Karve et al [10], we present stability and bifurcation analyses of a two channel,

nuclear-coupled, natural circulation BWR model. Two-phase thermal-hydraulics model is based on one we reported earlier. This model has been used to study the stability of two-phase natural circulation system (without neutronics) [11].

Reduced order model is presented in Section 2. A brief review of our approach for the stability and bifurcation analysis of the reduced order model is also given. Results of stability, bifurcation and numerical simulations are presented in Section 3, and the last section concludes the paper.

2. Reduced Order Model

2.1 Thermal-Hydraulic Model

The system considered in the model is shown in Figure 1. It is comprised of two heated channels (each of which represents a half of the core), two separate risers above the channels and a common down comer. A system pressure is imposed by setting the pressure at outlet of the risers, P_{exit} to a fixed value. Some additional system parameters are also shown in the Figure 1.

Fig. 1 Schematic plot of natural circulation system

The thermal-hydraulic model is based on the following two assumptions:

- Saturation enthalpy varies linearly with local pressure. Other thermal properties of water and steam are constant.
- Two phase flow can be modelled by Homogeneous Equilibrium Model (HEM)

Although better assumptions such as quadratic pressure dependence and non-equilibrium two-phase flow models can be applied, they will greatly increase complexity and decrease computational efficiency of the model.

Three cases, based on the location of the boiling boundary, are considered separately. These are shown in Figures 2a, 2b and 2c. For case 1, water enthalpy at the core exit is less than the saturation enthalpy, and flashing occurs in the riser. For case 2, because of relatively large heat input, water starts boiling in the core. Case 3 is a degenerate case, in which water enthalpy is less than the saturation value at the channel exit, but greater at the entrance of the riser due to local pressure loss.

Non-dimensional forms of continuity, energy and momentum partial differential equations are used to derive the dynamical system. Note that pressure dependence of water saturation enthalpy leads to modified forms of energy and momentum equations [12]. A weighted residual approach is employed to reduce the PDEs to a set of ODEs. Single phase and two-phase regions along the core and riser are discretized by arbitrary numbers of equal-sized

nodes. In each node, spatially linear trial functions with time-dependent parameters for enthalpy (saturated or unsaturated), steam quality, and mixture velocity are introduced. ODEs for these time-dependent expansion parameters are derived by integration along the node. Boiling boundary and its derivative are evaluated by applying the fixed pressure condition at the outlet of riser. Details of the derivations can be found in reference [12].

Fig.2 Profiles of water enthalpy and saturation enthalpy for the three cases: 2a) case 1; 2b) case 2; 2c) case 3.

2.2 Fuel Dynamics and Neutronics Model

The fuel dynamics and neutronics models of Karve et al. [10] are used to couple with the thermal-hydraulic model. These models retain many key elements of the dynamics of BWR systems, such as gap heat conduction between fuel pellet and cladding and different heat convection coefficients for single-phase and two-phase regions, yet are sufficiently simple for large scale parametric analyses. A piece-wise quadratic expansion with time-dependent parameter is used to represent temperature profiles in the fuel pellet. ODEs of time-dependent parameters are obtained by application of the variational principle. The neutron and precursor densities are represented by fundamental and first azimuthal modes with time-dependent coefficients. ODEs for the four neutronics variables (n_0 , u_0 , n_1 , and u_1) are derived through the β -mode approach. Retaining the first azimuthal mode in the model allows the simulation of out-of-phase oscillations. ODEs of thermal-hydraulic model and fuel dynamic and neutronics model are coupled via heat flux (represented by $N_{pch}(t)$, which is a non-dimensional number proportional to the total heat flux inputted into the channel), c_a , the void reactivity feedback coefficient, and c_D , the fuel Doppler reactivity feedback coefficient.

2.3 Poincaré-Andronov-Hopf Bifurcation (PAH-B) Theory and BIFDD

Stability of fixed points of a set of autonomous ODEs, such as those of reduced order model of BWRs, is determined by its eigenvalue of the Jacobian at the steady-state. If the real parts of all the eigenvalues are negative, the fixed point is stable, otherwise, unstable. This result is further extended by PAH-bifurcation theory [13], which states that if the right-most eigenvalues are a complex conjugate pair and they cross the imaginary axis with non-zero speed as an operating parameter is changed, there will be periodic solutions (oscillations) in the vicinity of the SB. Frequencies and amplitudes of the periodic solutions are determined by the imaginary parts of the eigenvalues and the distance of the operating point from the SB. In particular, the periodic solutions can be either stable, called supercritical PAH-B, or unstable,

called subcritical PAH-B. From the safety point of view, subcritical PAH-B is more problematic than supercritical PAH-B, since in the former case the unstable periodic solution exists on the stable side of the SB, and given large enough perturbation the system even when operating on the stable side of the SB may evolve into growing amplitude oscillations.

In this study (as in several previous studies), a stability and bifurcation analysis code BIFDD [14] is utilized to analyze the natural circulation BWR model. Given analytic forms of ODEs and Jacobian, BIFDD evaluates SBs as well as the nature of bifurcation along them.

3. Results

3.1 Effect of Nodalization Schemes

Coupling among the thermal-hydraulics, fuel dynamics and neutronics introduces extra feedbacks, which are crucial to the dynamics of the system. However, accurate evaluations of the feedback signals, the void fraction in the channel and fuel temperature rely on nodalization scheme described in section 2.1. The higher the number of nodes in each single-phase or two-phase region, the more accurate is the estimate of the void fraction and fuel temperature. Large number of nodes on the other hand dramatically increase the number of phase variables, leading to computational inefficiency. Therefore, an analysis of the impact of the number of nodes on the results of the stability analysis is carried out first.

Stability boundaries (SBs) are calculated by BIFDD for different nodalization schemes in the so-called subcooling—reactivity parameter space (Figures 3 and 4) for parameter values that roughly correspond to the Dodewaard natural circulation BWR. System pressure of these two calculations are $p_{exit} = 7.0$ MPa and $p_{exit} = 0.4$ MPa, respectively. Here, the x axis, r_{ext} , is the external reactivity introduced by control rod movement. The y axis, ΔT_{inlet} , is the inlet subcooling (saturation temperature minus inlet temperature). Different nodalization schemes are characterized by N , which is the number of nodes in each (single phase and two-phase) region.

Figure 3 shows that at high system pressure, as the number of nodes increases, the SBs converge, but insufficient nodalization may cause large deviation in both the area of stable regions and in the shapes of SBs from those obtained using sufficiently large number of nodes. Compared to the SB of $N = 16$ case, the one with $N = 1$ significantly over-estimates the area of the stable region. As N is increased, the SBs tend to converge. Good convergence of SB is achieved in regions of low and high r_{ext} for the $N = 8$ case, but the SB may be over-estimated by as much as 10% in the middle range of r_{ext} .

For the low system pressure case (0.4 MPa), Figure 4 shows a much better convergence. Even the SB for the $N = 4$ case is very close to that of the converged SB. SBs for the $N = 8$ and $N = 16$ cases are too close for most values of r_{ext} to be distinguished from each other. Results presented in the rest of the paper were obtained using $N = 8$.

Fig. 3 SBs for different size nodes in $\Delta T_{inlet} — r_{ext}$ parameter space for $P_{exit} = 7.0$ MPa

Fig. 4 SBs for different size nodes in $\Delta T_{inlet} — r_{ext}$ parameter space for $P_{exit} = 0.4$ MPa

3.2 In-Phase and Out-Of-Phase Oscillations

A parameter space is divided into stable and unstable regions by a stability boundary. In the unstable region at least one (real or complex conjugate pair) eigenvalues has positive real part. Figure 5 and 6 show SBs (solid lines) in $\Delta T_{inlet} — r_{ext}$ space for system pressure of 7.0 MPa and 0.4 MPa, respectively. Nature of instability that results, as these SBs are crossed, may however vary significantly along different parts of the SB. As shown earlier for forced circulation BWR system, the two rightmost pair of eigenvalues can be associated with in-phase and out-of-phase oscillations [15]. The relative magnitude of the elements in the corresponding eigenvectors indicate as to which mode—fundamental or first azimuthal—will become unstable as that eigenvalue crosses the imaginary axis. It was shown that the

eigenvalue pair that corresponds to the first azimuthal mode may cross the imaginary axis before the eigenvalue that correspond to the fundamental mode [15]. Hence, even for the natural circulation system it is important that along with the SB a second “boundary”—along which the second pair of complex conjugate eigenvalue has zero real part—be also plotted. Relative magnitude of the elements in the eigenvector can then be used to identify whether it is the in-phase or the out-of-phase mode that will become unstable as the SB is crossed. Figures 5 and 6 also show the “second” boundaries (dotted lines), on which the real part of the second largest pair of eigenvalue is zero. For low system pressure (Figure 6), the two curves do not cross, and the boundary for the azimuthal mode is always in the unstable region, where the fundamental mode is unstable. However, Figure 5 shows that the boundary corresponding to the largest eigenvalue for the fundamental mode and that corresponding to the first azimuthal mode cross each other at point X ($r_{ext} = -0.0335$, $\Delta T_{inlet} = 8.65$ K). Hence, the SB is comprised of a segment along which the stability is lost due to the fundamental mode (to the right of point X) and a small segment (to the right of point X) along which the first azimuthal mode is crossed.

Fig. 5 SB and boundary of the second largest real part when $P_{exit} = 7.0$ MPa

Fig. 6 SB and boundary of the second largest real part when $P_{exit} = 0.4$ MPa

Characteristic difference of oscillations associated with the eigenvalues of the largest and the second largest real parts is further illustrated by analyzing the eigenvalues and the

corresponding eigenvectors at four points on the SBs and the boundaries of the second largest real parts. Point 1 ($r_{ext} = -0.0278$, $\Delta T_{inlet} = 20.0$ K) and point 2 ($r_{ext} = -0.0306$, $\Delta T_{inlet} = 20.0$ K) are on the stability map of system with $p_{exit} = 7.0$ MPa; point 3 ($r_{ext} = 0.0124$, $\Delta T_{inlet} = 2.0$ K) and point 4 ($r_{ext} = 0.0179$, $\Delta T_{inlet} = 2.0$ K) are on the stability map of system with $p_{exit} = 0.4$ MPa. Table 1 lists the pure imaginary eigenvalues of the Jacobian E_{imag} and magnitudes of elements corresponding to $n_0(t)$ and $n_1(t)$, EV_{n_0} and EV_{n_1} in the eigenvectors at these points. [Here, $n_0(t)$ is the time-dependent coefficient of the fundamental neutron mode, and $n_1(t)$ is the time-dependent coefficient of the first azimuthal mode.]

Table 1 E_{imag} , EV_{n_0} and EV_{n_1} at four operating points shown in Figure 5 and 6

	Point 1	Point 2	Point 3	Point 4
E_{imag}	$\pm 2.306 i$	$\pm 1.709 i$	$\pm 3.030 i$	$\pm 4.079 i$
EV_{n_0}	0.156	0.170×10^{-11}	0.00972	0.110×10^{-13}
EV_{n_1}	0.943×10^{-12}	0.0595	0.324×10^{-13}	0.00530

For the points on the SB (points 1 and 3), the magnitude of $n_0(t)$ is much larger than those of $n_1(t)$; for the points on the boundaries corresponding to eigenvalue with the second largest real parts (point 2 and 4), the magnitude of $n_1(t)$ is much larger than that of $n_0(t)$. Results listed in the Table 1 suggest very large amplitudes of oscillations of $n_0(t)$, but much smaller amplitudes of oscillations of $n_1(t)$ corresponding to the pure imaginary eigenvalues at operating points 1 and 3. The pure imaginary eigenvalues at these two points, thus, are called in-phase or fundamental mode eigenvalues. Similarly, amplitudes of oscillations of $n_1(t)$ are very large, while those of $n_0(t)$ are very small at operating points 2 and 4. These eigenvalues are hence called out-of-phase or azimuthal mode eigenvalues. Therefore, a characteristics change in the nature of oscillation must be expected due to eigenvalue crossing at point X (Figure 5). To the left of point X, segment A-X is the SB due to out-of-phase oscillations. To the right of point X, segment X-B-C is the SB due to in-phase oscillations.

Parameter space in Figure 5 is divided by these boundaries of fundamental and first azimuthal mode eigenvalues into four regions. In region I, both the fundamental and first azimuthal modes are stable. Small perturbation will cause decreasing amplitude oscillations. In region II, the fundamental mode is unstable and the first azimuthal mode is stable. Small perturbation will lead to in-phase oscillations. In region III, the first azimuthal mode is unstable and the fundamental mode is stable. The oscillations are therefore out-of-phase. In the last region (IV), both modes are unstable. Oscillations are therefore a combination of in-phase and out-of-phase modes.

For low system pressure, instead of four regions, three regions of the parameter space are identified. Both modes are stable in region I; fundamental mode is unstable and first azimuthal mode is stable in region II; and both modes are unstable in region III (Figure 6).

3.3 Bifurcation Analyses

In addition to in-phase and out-of-phase, the oscillation in a BWR can also be characterized by whether they evolve to stable periodic oscillation or to unstable growing amplitude oscillation [13]. As discussed in section 2.3, PAH bifurcation theorem provides us with conditions that lead to on or the other. According to PAH-B theorem, if the periodic solution (oscillation) is in unstable region, its amplitude is stable and the bifurcation is supercritical; if it is in stable region, the amplitude is unstable and the bifurcation is subcritical. Nature of PAH-B (supercritical or subcritical) along the SB can be represented by fixed amplitude

oscillation curves. Figure 7 and 8 show 7.5% oscillation curves for different segments of the SBs of Figure 5 and 6 in $\Delta T_{inlet} - (r_{ext} - r_{ext,critical})$ parameter space. The amplitude of oscillations of the limit cycle along these curves is about 7.5% of the magnitude of eigenvector elements. In Figure 7a, for the out-of-phase segment A-X, the oscillation curve is in the unstable region, indicating a supercritical PAH-B along the SB. From the out-of-phase segment (A-X) of the SB to in-phase segment X-B, oscillation curve has a jump from the unstable side to the stable side. For $\Delta T_{inlet} < 28.91$ K, the oscillation curve of segment X-B is in the stable region, and type of PAH-B is subcritical. For $28.91 < \Delta T_{inlet} < 31.42$ K (point B), the oscillation curve returns to the unstable region, and consequently type of PAH-B along this segment of the SB is supercritical. For segment B-C, Figure 7c show that oscillation curve along the B-C branch is always in the unstable region. Type of PAH-B along this segment is hence supercritical. Similar back-and-forth transitions from sub- to supercritical bifurcations are observed for the low system pressure case. In Figure 8a, the type of bifurcation is supercritical along the D-E branch of the SB. Along the E-F branch of the SB (Figure 8b), when $\Delta T_{inlet} > 2.762$ K, the bifurcation is supercritical; while for $\Delta T_{inlet} < 2.762$ K, it is subcritical.

Fig. 7 7.5% oscillation curve along three segments of the SB in Figure 5

Fig. 8 7.5% oscillation curve along two segments of SB in the Figure 6

For the out-of-phase segment A-X (Figure 7a) and low ΔT_{inlet} region of the in-phase segment B-C (Figure 7c), deviation of r_{ext} required for the 7.5% amplitude, are much smaller than those for the segment X-B and high ΔT_{inlet} region of the segment B-C, indicating that much larger amplitude oscillations (in-phase or out-of-phase) may result for the same deviation along the SB of low subcooling than along that of high subcooling. Large amplitude oscillation at lower inlet subcooling is due to longer two-phase regions in the channel and riser. Similar trend can be found in the low system pressure case (Figure 8).

The change of characteristics of bifurcation (supercritical or subcritical) and change of characteristics of oscillations (in-phase or out-of-phase) along the SB, thus, reveal complexity

of the dynamic behavior of the coupled natural circulation BWR system.

3.4 Sensitivity Analysis for Low System Pressure Case

Effect of operating parameters other than ΔT_{inlet} and r_{ext} on the SBs is investigated. Since behavior of two-phase natural circulation systems under high system pressure conditions (close to 7.0 MPa) is relatively better understood, we focus on sensitivity of operating parameters when the system pressure is relatively low. To study effect of system pressure, we plot SBs in $\Delta T_{inlet} - r_{ext}$ parameter space for five different values of P_{exit} (Figure 9a). Impact of a change in P_{exit} on systems with relatively large pressure ($P_{exit} = 1.5, 1.0$ MPa) is different from that on systems at low pressure ($P_{exit} = 0.4, 0.3, 0.2$ MPa). Stable region decreases as P_{exit} decreases if the system pressure is relatively high, especially for small values of r_{ext} . Thus, a drop in P_{exit} has a destabilizing effect under these conditions. If the system pressure is relatively low, however, a drop in P_{exit} causes the stable region to increase. Therefore, instead of destabilizing, it has a stabilizing effect on the dynamics of the system. Comparing these results with those of pure thermal-hydraulics model [12], where a drop in system pressure is a destabilizing factor, this new stabilizing effect can be attributed to the coupling between system thermal-hydraulics and neutron kinetics. At low system pressure, without neutronics coupling, small perturbation can quickly grow due to two-phase flow instability. In the coupled system, however, void fraction feedback acts as a stabilizing force. Perturbations in void fraction lead to negative feedback of reactivity, which suppresses thermal-hydraulic perturbations.

Fig. 9 Sensitivity analysis results for different operating parameters. Out-of-phase oscillation SBs are indicated by OP. A change in slope of the SB indicates transition to in-phase SB.

Figure 9b shows the effect of riser length L_r , for $P_{exit} = 0.4$ MPa case. In natural circulation system, increasing L_r increases the driving force leading to higher flow rate. Therefore, it has a strong stabilizing effect. However, Figure 9b also shows that out-of-phase oscillations are possible at low reactivity for long risers. Effect of pressure loss coefficient at riser outlet $k_{r,out}$ is not as strong as those for P_{exit} and L_r .

(Figure 9c), but SBs indicate the possibility of out-of-phase oscillations for low reactivity if $k_{r,out}$ is high. Figure 9d shows the effect of an increase in pressure loss at the outlet of the core. Increasing $k_{c,out}$ generally tends to cause a loss of stability. The last operating parameter studied is the pressure loss coefficient at the channel inlet, $k_{c,in}$. As shown in Figure 9e, increasing of $k_{c,in}$ has a strong stabilizing effect.

3.5 Numerical Simulations

Numerical simulations are carried out to gain further insight into the dynamics of the coupled system as well as to examine the results obtained using BIFDD. Eight points on the stability maps of high and low pressure systems are chosen for numerical simulations. These points are shown in Figures 10a and 10b. Table 2 lists operating parameters as well as characteristics of bifurcation and nature of oscillations (in-phase or out-of-phase) predicted by BIFDD.

Fig.10 Operating points for numerical simulations in the SBs, 10a) $P_{exit} = 7.0$ MPa; 10b) $P_{exit} = 0.4$ MPa

Table 2 Operating points for numerical simulations

	P_{exit} (MPa)	r_{ext}	ΔT_{inlet} (K)	Characteristics of oscillations	Characteristics of PAH-B
Point a	7.0	-0.03413	6.0	Out-of-phase	Supercritical
Point b	7.0	-0.03400	6.0	Out-of-phase	Supercritical
Point c	7.0	-0.02781	20.0	In-phase	Subcritical
Point d	7.0	-0.00840	20.0	In-phase	Supercritical
Point e	7.0	-0.00823	20.0	In-phase	Supercritical
Point f	0.4	0.01237	2.0	In-phase	Subcritical
Point g	0.4	-0.02570	3.8	In-phase	Supercritical
Point h	0.4	-0.02540	3.8	In-phase	Supercritical

Points a and b are close to out-of-phase segment of the SB in Figure 10a. Point a is in the unstable region, while point b is in stable region. Bifurcation along this segment of the SB is supercritical. Figure 11 shows results of numerical simulation for parameter values corresponding to point a. As expected, stable amplitude limit cycle of $n_1(t)$ and very small amplitude of $n_0(t)$ result. Figure 12 (for point b) shows decreasing amplitude oscillations for both $n_0(t)$ and $n_1(t)$. Points c, d and e are close to in-phase segment of the SB. Bifurcation at point c is subcritical; while those for points d and e are supercritical. Figure 13, corresponding to point c, shows increasing amplitude of $n_0(t)$ (and decaying $n_1(t)$) for large

perturbation even though point c is in the stable region consistent with the subcritical nature of the bifurcation. For point d (in stable region) and e (in unstable region), perturbations lead to decreasing amplitude oscillations (point d) and stable limit cycle oscillations (point e) for $n_0(t)$ while decreasing amplitude oscillations result for $n_1(t)$ in both cases (Figure 14 and 15).

For the case of low system pressure, Figures 16, 17, and 18 show that oscillations at points f, g, and h are in-phase as suggested by the stability analysis. Large perturbation at point f, which is in the stable region, causes increasing amplitude oscillation consistent with the subcritical bifurcation there. Perturbations at point g (unstable region), and h (stable region) cause limit cycle and decreasing amplitude oscillations, respectively, for $n_0(t)$. Numerical simulations of system with high and low system pressures agree with predictions of BIFDD satisfactorily.

11a

11b

Fig. 11 Time evolutions of $n_0(t)$ and $n_1(t)$ at point a in Figure 10a

12a

12b

Fig. 12 Time evolutions of $n_0(t)$ and $n_1(t)$ at point b in Figure 10a

13a

13b

Fig. 13 Time evolutions of $n_0(t)$ and $n_1(t)$ at point c in Figure 10a after large amplitude perturbation. System is stable for small amplitude perturbations

14a

14b

Fig. 14 Time evolutions of $n_0(t)$ and $n_1(t)$ at point d in Figure 10a

15a

15b

Fig. 15 Time evolutions of $n_0(t)$ and $n_1(t)$ at point e in Figure 10a

16a

16b

Fig. 16 Time evolutions of $n_0(t)$ and $n_1(t)$ at point f in Figure 10b after large amplitude perturbation. System is stable under small amplitude perturbations

17a

17b

Fig. 17 Time evolutions of $n_0(t)$ and $n_1(t)$ at point g in Figure 10b

18a

18b

Fig. 18 Time evolutions of $n_0(t)$ and $n_1(t)$ at point h in Figure 10b

4. Summary

A new natural-circulation BWR model that allows local pressure dependence of water saturation enthalpy and includes fundamental and first azimuthal modes for neutronics has been developed. ODEs of this model are derived through weighted residual and variational principle approaches. A nodal-size sensitivity analysis shows that proper nodalization is important to accurately capture the SB. Both in-phase and out-of-phase oscillations as well as supercritical and subcritical bifurcations can occur along

the SBs for both high and low system pressures. Sensitivity analysis of various operating parameters is also carried out. It is shown that increasing system pressure may lead to a more stable or a less stable system depending upon the value of the absolute system pressure. Increasing pressure loss coefficient at the outlet of core or riser has a destabilizing effect, while increase of pressure loss coefficient at the core inlet has a stabilizing effect. Numerical simulation results confirm findings of stability and bifurcation analyses.

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PART III
STABILITY USING SYSTEM CODES
(Based on a paper submitted for publication in Nuclear Science
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BWR STABILITY AND BIFURCATION ANALYSIS USING REDUCED ORDER MODELS AND SYSTEM CODES: DIRECT COMPARISONS RESULTING IN IDENTIFICATION OF A SUBCRITICAL PAH BIFURCATION USING A SYSTEM CODE

Abdelhamid DOKHANE^{1,2}, Dieter HENNIG^{1,3}, Rizwan-uddin⁴, Rakesh CHAWLA^{1,2}

¹*Laboratory for Reactor Physics and Systems Behavior, Paul Scherrer Institute
CH-5232, Villigen, Switzerland.*

²*Ecole Polytechnique Fédérale de Lausanne (EPFL)
PHB-Ecublens, CH-1015 Lausanne, Switzerland.*

³*Currently: Heckelberger Ring 4, D-13055 Berlin, Germany.*

⁴*Department of Nuclear, Plasma, and Radiological Engineering,
University of Illinois, 103 S. Goodwin Ave. Urbana, IL 61801, USA*

Correspondence Addresses:

Abdelhamid Dokhane

PSI, OVGA 311,

CH-5232, Villigen, Switzerland.

Tel: ++41563102352, Fax: ++41563102327

E-mail: abdelhamid.dokhane@psi.ch

and

Dieter Hennig

Heckelberger Ring 4, D-13055 Berlin

Tel: ++493097897266.

E-mail: dieter@dhennig-berlin.de

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ABSTRACT

The system code RAMONA, as well as the BWR reduced order model (ROM) described and applied in Part I, are employed for the stability analysis of a specific operational point of the Leibstadt nuclear power plant. This has been done in order to assess the ROM's applicability and limitations in a quantitative manner. Thus, comparisons of the results obtained using the ROM with those of RAMONA show that, although the ROM could adequately predict certain characteristics, it is not able to correctly predict some others because of the highly simplified reactor core geometry, the uncertainties in evaluating the design and operating parameters, as because of the limitations of the feedback reactivity model employed.

In the context of a detailed local bifurcation analysis carried out using RAMONA in the neighbourhood of the chosen Leibstadt operational point, a bridge is built between the ROM and the system code. This has been achieved through interpreting RAMONA solutions on the basis of the physical mechanisms identified in the course of applying the ROM (Part I). This leads, for the first time, to the identification of a subcritical Poincaré-Andronov-Hopf (PAH) bifurcation using a system code. As a consequence, the possibility of a so-called correspondence hypothesis is suggested to underline the relationship between a stable (unstable) limit cycle solution and the occurrence of a supercritical (subcritical) PAH bifurcation in the modeling of boiling water reactor stability behaviour.

I. INTRODUCTION

A new BWR reduced order model (ROM) was developed and applied, in Ref. 1 (Part I), to simulate global and regional oscillations for typical values of BWR operating and design parameters. Thereby, stability and bifurcation analyses were performed using the bifurcation analysis code BIFDD². Stability boundaries and the nature of PAH bifurcation were determined and presented in a suitable two-dimensional parameter-state space. From an in-depth investigation of the elements of the eigenvectors of the system, it was shown that analysing the properties of the elements of the eigenvectors corresponding to the pairs of complex eigenvalues with the largest and second largest real parts gives detailed information on the type of oscillation mode, *i.e.* in-phase or out-of-phase, without solving the corresponding set of ODEs.

The principal objective of the current exercise is to compare the ROM results with those obtained using a large system code, *viz.* RAMONA³, for the same OP. The direct comparisons between the ROM and the system code, with its much higher degree of NPP modeling detail, have allowed important conclusions to be drawn regarding the ROM's applicability and limitations.

Various previous studies using reduced order models⁴⁻⁹, including the analysis reported in Ref. 1 (Part I) with the present ROM, show that both sub- and supercritical PAH bifurcations can be expected during the loss of system stability in a BWR. These in fact being the only bifurcation types encountered in such analyses while crossing the stability boundary^a.

In BWR stability analysis using system codes, stable limit cycle oscillations have been observed and reported¹⁰⁻¹², as has also been the case for certain BWR stability tests performed at NPPs such as Leibstadt and Ringhals^{13,14}. To the authors' knowledge, unstable

^a Deep inside the unstable region, a cascade of period-doubling bifurcations may exist as reported for instance in Ref. 7.

limit cycles, however, have not been observed or reported in numerical studies using system codes. Clearly, unstable limit cycles, in a numerical analysis, cannot be achieved by marching forward in time. Their existence can only be inferred by studying the system response to different amplitude perturbations.

Within the framework of reduced order model semi-analytical bifurcation analysis, results clearly distinguish stable fixed points with an unstable limit cycle around them from unstable fixed points with stable limit cycle around them. With this information in hand, confirmation of the results using numerical integration is straightforward. Thus, for an unstable fixed point, whatever the initially induced perturbation amplitude, oscillations always grow in amplitude. On the other hand, for unstable limit cycles, the initial perturbation amplitude plays an important role in determining the behaviour of the system, *viz.* for small perturbation amplitudes the oscillations decay to the fixed point that exists inside the unstable limit cycle, whereas for a large enough perturbation amplitude the oscillations grow in amplitude. Such an exercise using a system code is very time consuming. However, if semi-analytical bifurcation analysis results of an “approximate” ROM are available, they can be judiciously used to help “zoom-in” the area of interest (subcritical bifurcation) for a more detailed analysis using the system code.

In this paper, in Section II, the ROM is implemented for a particular operational point (OP) of the Leibstadt nuclear power plant (NPP) in Switzerland, and stability and bifurcation characteristics such as stability boundary, nature of bifurcation and type of oscillation mode are determined. In Section III of the present paper, the same Leibstadt OP, as that analysed using the ROM, is analysed using the system code RAMONA³. The primary focus in Section III is to address the RAMONA solutions in the light of the physical mechanisms identified in the course of applying the ROM. Thus, a detailed numerical bifurcation analysis is carried out with the system code by carefully examining the solutions obtained at several different OPs in the neighbourhood of the reference

Leibstadt OP. A link is thereby established between the ROM and RAMONA, resulting in the identification of a subcritical PAH bifurcation. To the authors' knowledge, this is the first demonstration of a subcritical bifurcation using a large scale BWR system code.

A summarized assessment of the present ROM against the results obtained with the system code is given in Section IV, while final conclusions are drawn in Section V.

II. STABILITY AND BIFURCATION ANALYSIS OF THE KKLC7_REC4 OP USING THE REDUCED ORDER MODEL

In this section, as indicated earlier, the stability behaviour of a specific OP of Leibstadt NPP is analysed using the current reduced order model. This OP corresponds to the so-called cycle 7 record 4 (*kklc7_rec4* OP), with 60.5% power and 36.7% mass flow rate. The OP is located in the plan's exclusion area^b, in the power-flow plane, and is an OP for which a stability measurement was carried out during cycle 7 reactor start-up in September 1990 showing growing out-of-phase oscillation amplitudes.

For the ROM modeling, the operating and design parameters for this OP have been evaluated suitably. For instance, the single and two-phase friction factors, the two-phase multiplier, the fuel heat capacity, the thermal fuel conductivity and the gap conductance have all been calculated using the correlations used in RAMONA. The inlet and exit loss coefficients have been adjusted to include the spacer pressure losses in the channel at the inlet and exit. The void and Doppler feedback reactivities have been calculated based on the methodology proposed in Ref. 1. The complete set of design and operating parameters for *kklc7_rec4* OP is given in Ref. 15.

Figure 1 gives the stability boundary as predicted by the reduced order model for *kklc7_rec4* OP in the $N_{sub} - DP_{ext}$ plane. At the *kklc7_rec4* OP:

^b a conservatively defined region in the power-flow map where the reactor is not allowed to operate during normal operating conditions.

- The subcooling enthalpy is $125 \cdot 10^3 J \cdot Kg^{-1}$, which corresponds to a subcooling temperature of $23.4 K$, and a subcooling number N_{sub} of 1.55 (all the dimensionless quantities are defined in Ref. 15).
- The total pressure drop across the core is $0.4497 \cdot 10^5 N/m^2$. This corresponds to a dimensionless total pressure drop across the core DP_{ext} of 8.57.

The estimation of the void distribution parameter and drift velocity values at this OP is based on the following justifications:

- (i) The thermal-hydraulic model validation, mentioned briefly in Ref. 1, was carried out mainly against the Saha *et al.*¹⁶ experimental data. It has been shown in Ref.15 that, for the Set I data (corresponding to an inlet velocity of $0.98 m/s$), a value of the void distribution parameter of 1.03 allows the current model to predict a SB which best fits the measurements. Because, for the *kklc7_rec4* OP, the liquid inlet velocity is also about $0.98 m/s$, $C_0 = 1.03$ is clearly a reasonable value for the void distribution parameter.
- (ii) Based on the relationship between the vapour and liquid velocities used for the slip model in RAMONA, it was found that the average slip value is 1.35 for the *kklc7_rec4* OP. This corresponds to a drift velocity $V_{gj} = 0.12$.

II.A. Semi-analytical Bifurcation Analysis

The bifurcation analysis for *kklc7_rec4* OP was carried out using the bifurcation code BIFDD. As seen in Fig. 1, the *kklc7_rec4* OP is located on the unstable side and lies very close to the stability boundary. The transformed stability boundary in the power-flow plane is shown along with the exclusion area in Fig. 2. Again, as in Fig. 1, the *kklc7_rec4* OP is located in the unstable region. In addition, Fig. 3, which shows the nature of PAH

bifurcation along the SB, indicates that for the SB branch with $N_{sub} > 2.1$, the type of PAH bifurcation is subcritical whereas, for the branch with $N_{sub} < 2.1$, supercritical PAH bifurcation is expected.

With *kklc7_rec4* located in the unstable region and lying very close to the SB branch where a supercritical PAH bifurcation is expected, one can conclude that a stable limit cycle solution should be found. Note that this OP is not too far from the SB branch where subcritical PAH bifurcation can occur. Moreover, a close look at the properties of the elements of the eigenvector corresponding to the eigenvalue with largest real part (responsible for the occurrence of the PAH bifurcation) reveals that only in-phase oscillations are expected to be observed when the system loses its stability.

Confirmation of these BIFDD predictions of the system behaviour for *kklc7_rec4* is obtained in the following subsection by numerically integrating the set of 22 ODEs of this ROM for parameter values corresponding to this OP using a MATLAB code based on the Gear's algorithm.

II.B. Numerical Simulation

Figure 4 shows the time evolution of the amplitude of the fundamental and first mode oscillations. The development of a stable limit cycle for $n_0(t)$ is clearly seen here. This is in agreement with the bifurcation analysis prediction (supercritical PAH bifurcation), as well as with the eigenvector analysis that predicts the excitation of in-phase oscillations. The time evolution of the inlet velocities of the two channels is depicted in Fig. 5. Both channels are seen to behave in the same manner, *i.e.* the two inlet velocities have the same amplitude and phase. This again confirms the excitation of the in-phase oscillation mode at this OP. The oscillation frequency from the numerical results obtained using the reduced order model is 0.63 Hz.

III. STABILITY AND BIFURCATION ANALYSIS USING RAMONA

In this section, a detailed bifurcation analysis is carried out using the system code RAMONA-5/PRESTO^f. Stability characteristics of *kklc7_rec4* OP are determined first. Then, in order to understand the system behaviour in the neighbourhood of this OP, further analyses are performed for various nearby OPs.

III.A. Stability Behaviour of the *kklc7_rec4* OP using RAMONA

The stability characteristics of *kklc7_rec4* OP are investigated in this subsection using the system code RAMONA.

Figure 6(a) shows the time series of LPRM84^d and LPRM354 for a perturbation amplitude of 2 nodes^e. A *2-node-amplitude*^f control rod perturbation means that a sinusoidal (perturbation) movement of a specific control rod bank is induced with an amplitude of 2 nodes over 1 second.

Figure 6(a) clearly shows the excitation of out-of-phase oscillations (with a frequency of 0.58 Hz), *i.e.* when the power increases in LPRM84, it decreases in LPRM354, and *vice versa*. In addition, the oscillation amplitudes of both LPRMs are increasing. The increasing amplitude oscillation in Fig. 6(a) could be indicative of: 1) a stable fixed point with an unstable limit cycle around it; 2) an unstable fixed point with a stable limit cycle^g around it; or 3) an unstable fixed point. A decreasing amplitude oscillation that has not been allowed to evolve long enough to determine if it would decay to zero or would saturate at some stable amplitude limit cycle, on the other hand, could be indicative of: 1) a stable fixed point with an unstable limit cycle around it; 2) an unstable fixed point with a stable limit

^c This version is equivalent to RAMONA-3, but runs much faster.

^d LPRM84 stands for local power range monitor (LPRM) number 8 located at axial level 4. Level 4 corresponds to the highest level.

^e This is the default value of the perturbation amplitude in RAMONA input.

^f The Leibstadt core is modelled with 25 axial nodes in RAMONA, where each node equals 15.24 cm.

^g Historically, this type of solution was ascribed to the system at this OP¹¹, with the argument that, at longer times, the growing oscillation amplitude would saturate to a stable periodic solution (stable limit cycle).

cycle around it; or 3) a stable fixed point. Unlike the usage of ROM, presented in Section II, that allows analytical or semi-analytical bifurcation analysis leading to determination of sub- or supercritical bifurcation along the SB, large scale system codes do not permit such determinations. Therefore, detailed numerical investigations—simulations over much longer period of time and/or simulations with different amplitude perturbations—must be carried out to determine the type of solution (bifurcation) and to uniquely identify the state of the system.

Thus, for an unstable fixed point, whatever the initially induced perturbation amplitude, oscillations always grow in amplitude. On the other hand, for the case of stable fixed point with an unstable limit cycle around it, the initial perturbation amplitude plays an important role in determining the behaviour of the system, *viz.* for small perturbation amplitudes the oscillations decay to the stable fixed point, whereas for a large enough perturbation amplitude the system is repelled by the unstable limit cycle and the oscillations grow in amplitude. Thus, a perturbation with a significantly smaller amplitude is introduced to determine the impact of perturbation amplitude on the system behaviour.

Figure 6(b) shows the time series of the two LPRMs at the same OP, but with a perturbation amplitude of 0.05 node instead of 2 nodes. The figure clearly shows that the oscillation amplitude decays to a stable fixed point, indicating a clear dependence on perturbation amplitude and a dramatic change of the qualitative behaviour of the system compared with the large amplitude perturbation case. This conclusively shows that the OP is a stable fixed point with an unstable limit cycle around it. Once again, it should be borne in mind that the question of the bifurcation type leading to an observed stable limit cycle or growing/decaying amplitude oscillations in the course of BWR stability analysis using large system codes has not been raised before. It should be recalled in this context that the reduced order model had predicted a supercritical bifurcation at the *kklc7_rec4* OP. This will be discussed further in a later section.

III.B. Bifurcation Analysis for Different Leibstadt OPs around *kklc7_rec4* OP

In this subsection, the stability behaviour of the Leibstadt NPP is analysed in the neighbourhood of the reference operational point *kklc7_rec4*. A detailed local investigation is carried out to study how the solution manifold of the system varies as a function of the mass flow rate, which is here considered to be the bifurcation parameter. The results obtained are then interpreted by comparing them with the results found using our reduced order model. Note that based on the experience accumulated to date using ROMs of BWRs, only sub- and supercritical PAH bifurcations have been observed and reported during the loss of BWR stability.

It should be pointed out that the intention here has been to perform a qualitative comparison between the results found using RAMONA and those found using the current ROM. In other words, the objective has been to compare how the solution manifold can vary as a function of a certain bifurcation parameter, *viz.* the mass flow rate. In effect, the stability behaviour is investigated at the following five OPs:

- 60.5% thermal power and 36.7% mass flow rate (nominal OP, *kklc7_rec4*).
- 60.5% thermal power and 37.0% mass flow rate (+0.3% F OP^h).
- 60.5% thermal power and 37.7% mass flow rate (+1%F OP).
- 60.5% thermal power and 36.4% mass flow rate (-0.3% F OPⁱ).
- 60.5% thermal power and 35.7% mass flow rate (-1%F OP).

At each of these OPs, RAMONA analyses have been carried out by inducing control rod (CR) perturbations with different amplitudes, the objective being to analyse the stability behaviour for each OP and for each CR perturbation amplitude separately. Figures 7 to 11

^h means that the mass flow for this OP is higher than that for the nominal OP by 0.3%

ⁱ means that the mass flow for this OP is less than that for the nominal OP by 0.3%

show the LPRM354 time series signals, calculated using RAMONA, for the nominal OP, +0.3%F OP, +1%F OP, -0.3%F OP and -1%F OP, respectively.

The nominal OP: 60.5% power and 36.7% mass flow

As mentioned previously, this operating point is located in the exclusion area in the power-flow map of the Leibstadt NPP. For a small amplitude perturbation (0.05-node control rod perturbation), the power decays to the stable steady-state solution as shown in Fig. 7(a), while a perturbation amplitude of 0.1-node of the control rod leads to growing oscillation amplitudes (Fig. 7(b)). Increasing the initial perturbation further to 2 nodes also results in growing amplitude oscillations (Fig. 7(c)). Mathematically, this indicates that beside the stable fixed point solution, an unstable limit cycle solution exists around this operational point.

The +0.3% F OP: 60.5% power and 37.0% mass flow

Figure 8 shows clearly that, at this operational point, the system again has two different behaviours depending on the perturbation amplitude. Thus, for 0.05-node and 0.1-node control rod perturbation amplitudes, the power oscillations are seen to decay to the stable steady-state solution (stable fixed point) as shown in Fig. 8(a-b), while, for a 2-node control rod perturbation amplitude, oscillations with growing amplitudes are observed. Therefore, an unstable limit cycle solution also exists around this OP. Moreover, while at the nominal OP a 0.1-node perturbation is enough to kick the system out of the stable fixed point's basin of attraction (growing oscillations (Fig. 7(b))), the same perturbation at +0.3%F OP is not enough to destabilize the system (decaying oscillations (Fig. 8(b))). This can be explained by the difference in the amplitude of the unstable limit cycles at the two different OPs (nominal and +0.3%F). Consequently, for a perturbation amplitude of 0.1 node or less, the system in phase space remains in the basin of attraction of the stable fixed point and is hence attracted by it (Fig. 8(a, b)).

The +1%F OP: 60.5% power and 37.7% mass flow

The behaviour of the reactor at this operational point is shown in Fig. 9. From parts (a) and (b) of this figure, it is clearly seen that the system is stable (stable fixed point solution) for both 0.05- and 2-node control rod perturbation amplitudes. In order to rule out the existence of a large-amplitude unstable limit cycle solution^j, the control rod perturbation amplitude was increased to 5 nodes^k. Figure 9(c) clearly shows that the analysed OP is indeed a stable fixed point.

The -0.3%F OP: 60.5% power and 36.4% mass flow

For the -0.3%F OP, Fig. 10(a) and (b) show that, the oscillation amplitude grows independently of the perturbation amplitude (0.05- or 2-node control rod perturbation), *i.e.* the system is unstable (unstable fixed point solution). To rule out the possibility of an unstable limit cycle with very small amplitude, a case has been analysed in which there was no induced control rod perturbation at all, *i.e.* only the numerical noise, assumed to be very small, acts as a perturbation. Results shown in Fig. 10(c) clearly confirm that the system is indeed unstable, *i.e.* an unstable fixed point is indeed the solution at this OP.

The -1%F OP: 60.5% power and 35.7% mass flow

The behaviour of the system at the -1%F OP is the same as that for the -0.3%F OP (see Fig. 11(a)-(c)), *i.e.* once again the solution is seen to be an unstable fixed point.

^j If a large amplitude limit cycle exists, larger perturbation amplitudes are needed to take the system outside the limit cycle.

^k A control rod perturbation amplitude of 5 nodes is considered to be a very large perturbation.

III.C. Interpretation and Discussion

At first glance, it may seem quite peculiar that the qualitative behaviour (solution type) of the system changes dramatically within a small range of the mass flow rate (from 37.7% to 35.7%), *i.e.* from a stable fixed point solution at the +1%F OP, to stable fixed points with unstable limit cycle solutions at the +0.3%F and nominal OPs, and then to unstable fixed point solutions at the -0.3%F and -1%F OPs. This is however quite consistent with the predictions of the PAH bifurcation theorem and clearly indicates the behaviour of a dynamical system that goes through a subcritical PAH bifurcation. Moreover, while carrying out semi-analytical bifurcation analysis using our reduced order model, such a system behaviour has been observed. Figure 12 shows the different solutions that exist close to the SB when such a bifurcation is expected for the reduced order model. Note that region in $N_{sub} - DP_{ext}$ space where the ROM predicts subcritical PAH bifurcation is different from the region around the nominal *kklc7_rec4* OP. Hence, the similarity in the variation of the solution type between the BWR system analysis with RAMONA and the reduced order model analysis is only qualitative.

Thus, points A, B, and C in Fig. 12 are located in a region where a subcritical PAH bifurcation occurs¹. Because the operational point A is in the stable region and close to the stability boundary, both a stable fixed point and an unstable limit cycle solution are found. Point B is inside the stable region but far from the SB. Here, only a stable fixed point solution is observed. Finally, point C is in the unstable region and the unstable fixed point is, therefore, the only solution of the system. Again, it needs to be emphasized that such a change of solution type happens only because of the occurrence of a subcritical PAH bifurcation. Therefore, based on the reduced order model findings, two important conclusions can be drawn:

¹ from analysis using our reduced order model in conjunction with the bifurcation code BIFDD

1. The change of the solution type from +1%F OP to -1%F OP in the RAMONA calculation can be explained only by the occurrence of a subcritical PAH bifurcation during the loss of system stability.
2. The nominal, +0.3%F, and +1%F OPs are located in the stable region, while the -0.3%F and -1%F OPs are located in the unstable region. Consequently, a local stability boundary exists between the nominal OP and the -0.3%F OP. The schema shown in Fig. 13 summarizes the results found in this study.

III.D. The Correspondence Hypothesis: Stable (Unstable) Limit Cycle vs. Supercritical (Subcritical) PAH Bifurcation

For a PAH bifurcation to occur, three conditions have to be fulfilled⁷. These conditions can easily be verified when using models represented by a system of ODEs, as is the case with reduced order models, since analytical bifurcation analysis can then be carried out using a bifurcation code like BIFDD. However, for models based on PDEs, as those used by the system code RAMONA, one does not presently have the capability to check the fulfillment of the conditions for the occurrence of a PAH bifurcation. Nevertheless, certain guidelines may be suggested by considering the following facts:

1. A supercritical PAH bifurcation is characterized by the appearance of stable limit cycle solutions inside the linear unstable region close to the SB, while a subcritical PAH bifurcation is characterized by a stable fixed point and an unstable limit cycle solution inside the linear stable region close to the SB.
2. Only sub- or supercritical PAH bifurcations have been observed and reported so far during the loss of system stability in the context of BWR stability analysis using reduced order models. Therefore, one can confidently assume that these two types of bifurcation are the only ones that can be expected to occur when a BWR loses its stability.

3. During the occurrence of a subcritical PAH bifurcation, a turning point may exist in which large amplitude stable limit cycles^m are observed in the linear unstable region.

Therefore, excluding the large amplitude stable limit cycle case, *i.e.* turning point, the following “correspondence hypothesis” is suggested in the framework of the system codes BWR stability analysis :

When a BWR system loses its stability, the observation of a stable limit cycle is indication of the occurrence of a supercritical PAH bifurcation, while the existence of an unstable limit cycle indicates the occurrence of a subcritical PAH bifurcation.

IV. REDUCED ORDER MODEL ASSESSMENT AGAINST THE SYSTEM CODE RAMONA

Comparing the *kklc7_rec4* OP predictions obtained using RAMONA (Section III.A) and the current reduced order model (Section II.B), one can draw several conclusions. Thus, firstly, the reduced order model is seen to yield a good prediction of the location of the OP with respect to the stability boundary (both tools predict the OP to be very close to SB), as well as of the oscillation frequency value. However, the discrepancy is obvious between the model results and those of RAMONA concerning the excited oscillation mode (in-phase or out-of-phase) and the nature of PAH bifurcation. This is not surprising, keeping in mind that: (a) the reduced order model is highly simplified with the entire BWR core lumped into two representative channels; and (b) the design and operating parameters used in the ROM are core-average values.

It should be stressed, however, that the main objective of ROM studies is to permit analysis of the complete solution manifold of the BWR system. Thus, as demonstrated in

^m In Ref. 18, the amplitude of the stable limit cycle due to the turning point was found to exceed 300%.

Ref. 1 (Part I), the application of a suitable ROM can provide deep generic insights into the complex processes determining BWR stability. As discussed, also in Ref. 1 (Part I), the present ROM has been developed while trying to respect two conflicting requirements, *viz.* faithful representation of the physical model (closeness to the system code RAMONA) and simplicity. Clearly, the latter feature can result in certain discrepancies, and further improvements in the ROM modeling are called for if the differences with respect to the system code predictions are considered important in a given context.

The current reduced order model predicted in-phase oscillations, while out-of-phase oscillations are predicted by RAMONA for the reference Leibstadt OP. This discrepancy is mainly because of the limitations of the feedback reactivity model for the mode coupling ($\mathbf{r}_{10}(t)$ and $\mathbf{r}_{01}(t)$) as discussed in Ref. 1. Accordingly, it is suggested that an improved model for the feedback reactivities in mode coupling be derived for future work.

The discrepancy in predicting the nature of PAH bifurcation is due mainly to the uncertainties in evaluating the design and operating parameters as core-average values. In Ref. 15, it was found that changing the value of certain parameters, *e.g.* the drift flux model parameters (C_0 and V_{gj}) or the inlet pressure loss coefficient (K_{inlet}), can change the nature of PAH bifurcation. In other words, it was observed that a SB branch that was associated with subcritical PAH bifurcation can become supercritical, or *vice versa*. This means that a small discrepancy in evaluating one or more of such parameters may lead to a wrong prediction as regards the nature of the PAH bifurcation. Also the recent ROM studies of Zhou and Rizwan-uddin¹⁹ have shown that the type of PAH bifurcation encountered can be very sensitive to the modelling assumptions made. Clearly, since the present ROM has various modelling assumptions which differ from those of RAMONA, it is not surprising to observe the discrepancy in PAH bifurcation type as predicted by the two models.

Despite the quantitative limitations of the present ROM, it has been clearly demonstrated in Section III that ROM predictions can be invaluable for the proper interpretation of the complicated results which are sometimes obtained in BWR stability analysis using large system codes.

V. SUMMARY AND CONCLUSIONS

The new BWR reduced order presented in Part I has been used to analyse a specific operational point of the Leibstadt NPP (*kklc7_rec4*). The results obtained have been compared to those of the system code RAMONA for the same OP, thus permitting a direct assessment of the performance of the current ROM in terms of both its applicability and its limitations.

It has been seen that, for the case considered, the reduced order model very well predicts the frequency of the oscillations and also localizes the analysed OP in an appropriate region close to the stability boundary. However, clear discrepancies have been found between the ROM model results and those of RAMONA as regards the prediction of the oscillation mode (in-phase, out-of-phase) and the nature of PAH bifurcation at this operational point. The inability of the ROM to predict the out-of-phase oscillation mode at this OP is due to the limitations of the feedback reactivity model for the mode coupling, while the discrepancy in predicting the nature of PAH bifurcation is mainly due to the uncertainties in evaluating the design and operating parameters adequately. These “negative” results concerning the reduced order model’s quantitative performance are in fact not surprising, since the reduced order model is highly simplified with the entire BWR core being lumped into just two representative channels, and the design and operating parameters calculated for a specific OP are core-average values.

A detailed numerical bifurcation analysis was carried out using the system code RAMONA in the immediate neighbourhood of the reference OP. The main conclusion to

be drawn from this investigation is that, although the reduced order model has limitations in quantitative performance, its results are of paramount importance in analysing and interpreting the results obtained with RAMONA. It is a consequence of the “bridge” built between the ROM and RAMONA that a subcritical PAH bifurcation has been identified, for the first time, in the course of BWR stability analysis using a system code.

As a general conclusion, it should be stressed that detailed quantitative studies for specific NPP operational points are still not possible using a reduced order model, and this remains a challenge. Thus, reduced order models, with respect to detailed system codes, still need to be considered as valuable *complementary* tools, and *not* as alternatives.

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Fig. 1. Stability boundary (ROM) in $N_{sub} - DP_{ext}$ plane for operating and design parameters corresponding to the *kklc7_rec4* operational point.

Fig. 2. Stability boundary in the power-flow map for operating and design parameters corresponding to the *kklc7_rec4* operational point (ROM).

Fig. 3. Bifurcation characteristics in $N_{sub} - b_2$ plane for operating and design parameters corresponding to the *kklc7_rec4* operational point (ROM).

Fig. 4. Time evolution of the deviation of the fundamental mode amplitude in the ROM from the steady state value ($n_0 - n_{0s}$) and the first mode amplitude (n_1) at the *kklc7_rec4* operational point (n_{0s} is the steady state value for $n_0(t)$).

Fig. 5. Time evolution of the (liquid) inlet velocity in channel 1 (v_{inlet1}) and channel 2 (v_{inlet2}) predicted by ROM. The two channels oscillate in-phase.

Fig. 6. Time series of the *kklc7_rec4* LPRMs (RAMONA analysis) showing the occurrence of out-of-phase oscillations with a subcritical PAH bifurcation. **(a)** 2-node control rod perturbation amplitude. **(b)** 0.05-node control rod perturbation amplitude.

Fig. 7. Nominal OP time series of Leibstadt LPRM showing the occurrence of a subcritical PAH bifurcation: (a) 0.05-node control rod perturbation amplitude. (b) 0.1-node control rod perturbation amplitude. (c) 2-node control rod perturbation amplitude.

Fig. 8. +0.3%F OP time series of Leibstadt LPRM showing the occurrence of a subcritical PAH bifurcation: (a) 0.05-node control rod perturbation amplitude. (b) 0.1-node control rod perturbation amplitude. (c) 2-node control rod perturbation amplitude.

Fig. 9. +1%F OP time series of Leibstadt LPRM showing a stable fixed point solution: **(a)** 0.05-node control rod perturbation amplitude. **(b)** 2-node control rod perturbation amplitude. **(c)** 5-node control rod perturbation amplitude.

Fig. 10. $-0.3\%F$ OP time series of Leibstadt LPRM showing an unstable fixed point solution: (a) 0.05-node control rod perturbation amplitude. (b) 2-node control rod perturbation amplitude. (c) No control rod perturbation.

Fig. 11. -1%F OP time series of Leibstadt LPRM showing an unstable fixed point solution: (a) 0.05-node control rod perturbation amplitude. (b) 2-node control rod perturbation amplitude. (c) No control rod perturbation is induced.

Fig.12. Different types of solutions during a subcritical PAH bifurcation occurrence, as predicted using the current reduced order model.

Fig.13. Scheme showing the different solution types encountered when the mass flow is varied for the nominal Leibstadt OP.

PART IV
Experimental Investigations of
Natural Circulation Flashing
Phenomenon

INVESTIGATING THE NONLINEAR DYNAMICS OF NATURAL-CIRCULATION, BOILING TWO-PHASE FLOWS

THERMAL HYDRAULICS

KEYWORDS: *boiling two-phase flow, nonlinear dynamics, chaotic oscillations*

ROBERT ZBORAY* and WILHELMUS J. M. DE KRUIJF
Delft University of Technology, Interfaculty Reactor Institute, The Netherlands

TIM H. J. J. VAN DER HAGEN *Delft University of Technology
Interfaculty Reactor Institute and Kramers Laboratorium voor Fysische Technologie
The Netherlands*

RIZWAN-UDDIN *University of Illinois
Department of Nuclear, Plasma, and Radiological Engineering
103 S. Goodwin Avenue, Urbana, Illinois 61801*

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The nonlinear dynamics of natural-circulation, boiling two-phase flows are investigated using a two-phase flow loop. Experiments have been carried out in the unstable operating region of the facility for various system pressures and for different frictions at the exit of the riser section. It appears that the boiling two-phase flow undergoes the well-known Feigenbaum scenario, the period-doubling route toward chaotic behavior. Evaluation of the recorded signals using nonlinear time series analysis methods indicates the occurrence of chaotic density-wave oscillations.

I. INTRODUCTION

Natural-circulation, boiling two-phase flows are widely applied in industrial systems like heat exchangers and boilers. The application of natural circulation of the coolant in boiling water nuclear reactors (BWRs) has gained interest in the last decades because it results in more economical and safer designs than present-day, forced-circulation BWRs.

A major concern for the safe operation of the aforementioned industrial boiling, two-phase flow systems is to avoid the occurrence of so-called density-wave oscillations. Density-wave oscillations in heated channels have

been studied for a long time, and the progress in this field has continuously been reviewed.^{1–3} Thanks to the considerable research effort, significant success has been achieved in explaining and identifying the main physical mechanisms involved in density-wave oscillations.

The mechanism of density-wave oscillations in boiling channels can be explained in a number of equivalent ways. The most commonly accepted explanation is as follows: Channel inlet flow perturbations create enthalpy fluctuations in the single-phase region. At the boiling boundary (the elevation at which the bulk of the liquid reaches the saturation temperature and starts to boil), the enthalpy perturbations are transformed into void-fraction variations that travel with the flow through the system. Obviously, the position of the boiling boundary fluctuates as well. The combined effect of flow, void-fraction, and boiling-boundary perturbations causes fluctuations in the pressure drop across the channel. Since the total pressure drop across the channel is imposed externally by the characteristics of the system feeding the channel, variations in the pressure drop produce a feedback perturbation in the inlet flow. Under specific conditions, the pressure-drop perturbations can acquire appropriate phases, and the inlet flow oscillations become self-sustained. It is clear that transportation-delay and pressure-drop characteristics are of paramount importance for the stability of the system.

Although the current understanding of density-wave oscillations is fairly good for *linear* phenomena,⁴ more work is needed to understand *nonlinear* dynamics and *unstable* behavior. For validating codes for these operational regimes, the experimental database on the dynamics of natural circulation should be broadened. There is

*E-mail: robert.zboray@psi.ch

no confidence that state-of-the-art codes are able to reproduce within acceptable margins the behavior of natural-circulation, boiling two-phase flows, particularly in regimes of highly nonlinear, unstable behavior, where modeling assumptions might play a more pronounced role than in the more conventional, linear regime.

Several attempts to model the nonlinear dynamics of boiling flows can be found in the literature. Achard et al.⁵ have performed analytical Hopf-bifurcation analysis of a boiling channel using the homogeneous-equilibrium model. It was later extended by Rizwan-uddin and Dornig⁶ using the drift-flux model and nonuniform heater profiles. As another approach, fully numerical studies have been carried out as well. Rizwan-uddin and Dornig^{7,8} have shown that two-phase flow in a vertical heated channel subjected to periodical forcing exhibits a strange attractor. Clausse and Lahey⁹ developed a model based on the homogeneous-equilibrium approach and found limit-cycle oscillations and a strange attractor for a single channel with uniform heat flux. Extending that model, Chang and Lahey¹⁰ predicted a period-doubling sequence and chaotic oscillations for a heated channel coupled with an adiabatic riser.

As for the *experimental* investigation of the nonlinear dynamics of boiling flows, the experiments are usually restricted to the determination of the threshold of instability and to parametric studies on that, e.g., Saha et al.,¹¹ Furutera,¹² Delmastro et al.,¹³ and Kyung and Lee^{14,15} have performed an extensive measurement campaign on the stability of a natural-circulation Freon loop. They encountered a region of excursive instability inside the unstable region, at very high subcooling, which could result in "periodic flow excursions." However, to the authors' knowledge, no experimental findings have ever been reported about chaotic dynamics or period-doubling sequences in boiling systems.

In the present paper, investigations on the nonlinear dynamics of natural-circulation, boiling two-phase flows are discussed concerning density-wave instabilities. Experiments have been carried out using the Delft Simulated Reactor (DESIRE) facility, a natural-circulation loop. The facility is described briefly in Sec. II. Thereafter, measurements on the nonlinear dynamics of the system are presented. Measured time traces are analyzed using nonlinear time series methods.

II. THE EXPERIMENTAL FACILITY

II.A. Dimensions and Instrumentation

The DESIRE facility is a natural-circulation, boiling two-phase flow loop using Freon-12 as a working fluid.¹⁶ The central part of the facility consists of a 6×6 array of electrically heated rods, which is a scaled copy of one fuel assembly of a BWR. A tall riser section is placed on top of the heated assembly ("core") to enhance the natural-

circulation flow rate, which is maintained by the density difference between the vapor-liquid mixture in the assembly+riser and the liquid in the downcomer section. The scheme of the loop is shown in Fig. 1. Two sets of electrical heater rods can be used in the facility: one set with a uniform axial power profile, the other with a sinus+offset profile. For the latter, the pitch of the coils of the resistance wire is a function of the position in the rod in such a way that the axial power profile is

$$P'(z) = P \frac{\left(1 - \frac{2}{\pi} f_p\right) + (f_p - 1) \sin\left(z \frac{\pi}{L}\right)}{L \left(1 - \frac{2}{\pi}\right)}, \quad (1)$$

where

P = total power

z = axial elevation

L = length of the rods

f_p = axial power peaking factor, which has a value of 1.4.

A flow-resistance element with variable friction is installed on top of the riser section. This enables the study of density-wave oscillations in DESIRE since a high outlet (two-phase) friction tends to destabilize the two-phase flow (see below).

The facility is fully equipped with pressure, temperature, and flow sensors. A gamma-transmission setup with an ²⁴¹Am source is used to measure the area-averaged void fraction at different heights along the assembly in broad gamma beam geometry. The diameter of the gamma beam is ~ 2 cm, whereas the width of the assembly is 5.4 cm.

II.B. System Behavior in the Stable Operating Region

To quantify the linear stability of the system, the so-called decay ratio (DR) is used. The DR is defined as the ratio between two consecutive maxima of the impulse response function of the system. The DR is smaller than unity for a stable system (with a damped response), and it is equal to or larger than unity for an oscillatory, unstable system. If the system is driven by wideband (white) noise, the DR can also be extracted from the consecutive maxima of the autocorrelation function (ACF) of measured dynamical variables of the system. The ACF of signal $x(t)$ is defined as¹⁷

$$\text{ACF}(\tau)_{x(t)} \equiv \lim_{T \rightarrow \infty} \frac{\frac{1}{T} \int_0^T x(t)x(t+\tau) dt}{\frac{1}{T} \int_0^T x(t)^2 dt}. \quad (2)$$

Fig. 1. Scheme of the DESIRE facility. (a) P , T , and F denote pressure, temperature, and flow sensors, respectively. (b) The dimensions of the facility are shown. The length of the heated section is 962 mm; the liquid flows upward in this section; the length of the riser section is 1400 mm.

The ACF of the inlet mass-flow rate and void-fraction signals are used here to obtain the DR.

Experimental series have been carried out to examine the linear stability of the DESIRE facility at different settings of the riser-exit friction element.¹⁸ All of these measurements have been performed at a nominal system pressure of 11.6 bars.

As an example, measurement points for a series performed for a uniform axial power profile are given in terms of the dimensionless Zuber (or phase-change) number (N_{Zu}) and the subcooling number (N_{sub}) in Fig. 2. These two dimensionless numbers are commonly used in the analysis of boiling, two-phase flow systems. The former one is proportional to the total power to inlet mass-flow ratio, and the latter one is proportional to the inlet subcooling:

$$N_{Zu} \equiv \frac{P}{\phi(h_g - h_l)} \frac{\rho_l - \rho_g}{\rho_g}$$

and

$$N_{sub} \equiv \frac{h_l - h_{in}}{h_g - h_l} \frac{\rho_l - \rho_g}{\rho_g}, \quad (3)$$

where

P = total assembly power

ϕ = inlet mass-flow rate

h_l, ρ_l = saturated liquid enthalpy and density, respectively

h_g, ρ_g = vapor enthalpy and density, respectively

h_{in} = specific liquid enthalpy at the assembly inlet.

It can be seen from the DR values in Fig. 2a that moving toward higher N_{Zu} and N_{sub} , the DR, in general, is increasing; i.e., the two-phase flow becomes gradually less stable. For indicative purposes only, several equi-DR lines are drawn in Fig. 2a obtained by interpolating on the measurement points. Figure 2b compares the stability for two riser-exit friction settings. It shows that the DR is larger at about the same operating point for a higher riser-exit friction. This destabilizing effect of an increased two-phase (exit) friction on density-wave oscillations is well known from literature on two-phase flow dynamics.^{1,11}

Fig. 2. The stability behavior of the system showing measurement points in the N_{Zu} - N_{sub} plane for uniform power profile settings. The numerals are the measured DRs indicated near the points. Solid symbols indicate unstable points (limit-cycle oscillations, $DR = 1$); open symbols indicate stable operating points. (a) Results obtained for riser-exit friction factor $K_{ex,ris} = 122$. Several equi-DR lines are drawn in this figure (obtained by cubic interpolation of the measured values) for indicative purposes. (b) Comparison of the stability for two riser-exit friction settings; the uncertainty of the DR values is ~ 1 to 2%; the uncertainty of the operating points (N_{Zu} - N_{sub}) is ~ 4 to 5%.

Several operating points of Fig. 2 are indicated to be unstable ($DR = 1$). In these points, which lie in the unstable operating region of the facility, constant-amplitude oscillations are measured. These, so-called, limit-cycle oscillations are typical nonlinear phenomena for which the initially small-amplitude, growing oscillations are bounded by nonlinear effects when the oscillation amplitude becomes large.

The behavior of the natural-circulation two-phase flow in operating points deep in the unstable operating region is described in Sec. III.

III. INVESTIGATING THE NONLINEAR DYNAMICS

III.A. Experimental Results

The experiments in the unstable operating region have been carried out for different system pressures and for various distributions of the frictional pressure drops in the single- and two-phase regions. The latter is achieved by varying the friction coefficient of the adjustable flow resistance element at the riser exit. A sinus + offset heating-power profile has been used in all the measurements. The influence of the axial-power profile on the nonlinear dynamics of natural-circulation two-phase flows has been

investigated in Zboray et al.,¹⁹ and it is not examined further here.

The following procedure has been followed in all cases: Starting from a stable operating condition, the heating power is gradually increased, in steps, while keeping all other controllable system parameters constant and waiting at each step until transients die out and the system reaches asymptotic behavior. The following scenario has been observed in all cases with different system pressures and with various frictions at the riser exit: As the power reaches the threshold of stability, the fixed point becomes unstable; it undergoes a supercritical Hopf bifurcation,²⁰ and limit-cycle flow oscillations result. As the power reaches another critical value, the system undergoes a next bifurcation: a so-called period-doubling bifurcation. At this point, oscillations with two different amplitudes and with a period twice the period of the original oscillations arise. The occurrence of two such consecutive bifurcations indicates the onset of the well-known Feigenbaum scenario.²¹

In the Feigenbaum scenario, a cascade of successive period-doubling bifurcations takes place as a system parameter (the so-called bifurcation parameter) is being varied in a nonlinear system, and the aperiodic (chaotic) behavior starts off as the bifurcation parameter is varied beyond a critical value. After each period doubling, the number of different oscillation amplitudes is doubled,

which corresponds to the appearance of successive subharmonic components in the frequency spectrum of the oscillations. According to Feigenbaum's theory,²¹ this scenario possesses certain universal features; namely, the critical values of the successive period-doubling bifurcations converge in an asymptotically geometric manner toward the aperiodic limit. The convergence ratio is a universal number, the Feigenbaum-delta ($\delta = 4.6692 \dots$). Another universal scaling feature of the scenario is that the ratio of the amplitudes of the consecutive subharmonic components in the spectrum converges to: $\mu = 6.5573 \dots$

Representative oscillation patterns and the corresponding Fourier spectra for the aforementioned behavior of the two-phase flow in the DESIRE facility are shown in Fig. 3. The maxima and minima of the oscillations are shown in Fig. 4 as a function of the power. Running at 25.5 kW, the mass-flow rate shows a constant-amplitude, limit-cycle oscillation (Fig. 3a). The spectrum indicates that the oscillation is not sinusoidal at all but contains many higher harmonics. The maximum and the minimum of this oscillation correspond with the two symbols at 25.5 kW in Fig. 4. Figure 4 shows that the flow starts to oscillate at a power of ~ 18 kW, with increasing amplitude as the power is increased. Increasing the power from 25.5 to 28.4 kW leads to the first period doubling: The flow oscillates now with a high and a low maximum and a high and a low minimum. Consequently, Fig. 4 shows the four corresponding symbols at 28.4 kW. A peak appears in the spectrum at half the original resonance frequency (a subharmonic); its harmonics show up as well. Increasing the power just a bit more to 32.3 kW leads to chaotic behavior (Fig. 3c); correspondingly, the number of maxima and minima becomes infinite, some of which are indicated by small dots in Fig. 4. The spectrum does not contain clear resonance peaks any longer. The measured transition from limit-cycle oscillations to chaotic oscillations is illustrated also in the bifurcation diagram in Fig. 5.

The finer details of the Feigenbaum scenario—the cascade of period-doubling bifurcations—could not be detected conclusively. Because of the relatively large value of δ , the consecutive bifurcation points—after the first few period doublings—cannot be distinguished from each other in terms of the bifurcation parameter due to the finite resolution of the measurement. Because of the scaling with μ , the amplitude of the successive subharmonics (i.e., the vertical spacing of symbols in Fig. 4) becomes very small for practical (measurement) purposes after a few period doublings as well. Obviously, the presence of measurement noise diminishes further the possibility to identify subsequent bifurcations. The small spectral peaks corresponding to higher-order bifurcations disappear in the noisy background spectrum (e.g., due to the boiling noise). Nevertheless, in all cases, a gradual and clear change can be observed in the measured oscillation patterns and in the corresponding spec-

tra (a gradual broadening of the peaks and a peculiar increase in the broadband background; see Fig. 3c) as the power is increased past the first period-doubling bifurcation point.

Since the discovery of the universality features of the period-doubling route toward chaotic behavior, this scenario has been observed experimentally in a broad variety of small-scale physical systems including nonlinear electronic circuits,²² Rayleigh-Bénard cells,²³ optically bistable laser cavities,²⁴ and superfluid helium.²⁵ However, in large-scale physical, mechanical systems prone to high noise levels, even the onset of the scenario (the first period doubling) has not been reported. For example, although predicted with theoretical models by Clausse and Lahey⁹ and by Chang and Lahey,¹⁰ it has not been observed yet in boiling two-phase flow systems. To the authors' knowledge, the results presented here are the first experimental evidence suggesting that a large, boiling two-phase flow system undergoes the Feigenbaum scenario.

The influence of system pressure on the bifurcation sequence is studied by repeating the experiments for two different values of system pressure and the same riser exit friction factor. Table I summarizes the experimental findings for four experimental cases. The power at which the system starts to oscillate (P_0), the power of the first bifurcation (P_1), and the measured chaotic limit (P_∞) are given. Using the Feigenbaum-delta, the chaotic limit is also estimated from the previous critical power values assuming geometric convergence with δ as

$$P_\infty^{est} = P_1 + \frac{P_1 - P_0}{\delta} \quad (4)$$

The estimated values of the chaotic limit are somewhat lower than the actually observed chaotic limit. The discrepancy is believed to be due to the fact that in reality the convergence is asymptotically geometric.

Period doubling in experiment series 3 could not be clearly detected, and hence, the effect of system pressure on the bifurcation sequence is not clear. The effect of changing the pressure drop in the two-phase region was studied by varying the riser-exit friction factor (series 1 and 4). Decreasing the frictional pressure drop at the riser exit tends to increase the critical bifurcation values and the chaotic power limit (P_∞); it also tends to stretch the interval between the Hopf bifurcation and the period-doubling bifurcation. Similar observations can be made comparing series 1 to series 3 (although for the latter the period-doubling bifurcation point is not clear).

It is not very instructive to try to extract features of a nonlinear system by just looking at the power spectrum. To qualify and quantify the system behavior in the presumed chaotic region, methods developed for nonlinear and chaotic time series analysis are of use. These methods are described and applied in Sec. III.B.

Fig. 3. Inlet flow rate oscillation patterns and the corresponding autopower spectral density (APSD) for a system pressure of 11.6 bars and a friction factor at the riser exit of 118 ± 18 . (a) Beyond the threshold of stability (at 25.5 kW power), the appearance of numerous higher harmonic components in the APSD indicates the strongly nonlinear character of the oscillations. (b) Just beyond the first period-doubling point (at 28.4 kW power), the oscillation period is twice of that at 25.5 kW. Correspondingly, a halved frequency component (f_1 , the first subharmonic) appears in the spectrum next to f_0 . (c) Further beyond the first period-doubling bifurcation (at 32.3 kW power), the oscillations are chaotic. (The figures are based on series 1 from Table I.)

Fig. 4. The plots show the minima and maxima of the flow-rate and void-fraction oscillations as a function of power measured at a system pressure 11.6 bars and with a riser-exit friction factor of 118 ± 18 . The hollow dots show two stable fixed points at low power just under the Hopf-bifurcation point. Several measured oscillation minima and maxima depicted at 32.3 kW illustrate chaotic oscillations. The arrows indicate the regimes of different oscillation modes

Fig. 5. Bifurcation diagram showing the oscillation amplitudes as a function of the power for the different measured oscillation modes. The small dots show measured chaotic oscillation amplitudes at 32.3 kW.

III.B. Nonlinear Time Series Analysis

A vast literature exists on methods to detect and quantify chaos in measured time series.^{26,27} Three chaos descriptors are determined here for the measured signals: the Kolmogorov entropy, the correlation dimension, and the maximum Lyapunov exponent. The algorithms for determining these quantities are based on the reconstruction of the chaotic (strange) attractor in the

state-space of the system using the time-delay embedding technique.²⁸ Given the measured time series $x(i)$ with $i = 1, \dots, N$ and sampling time τ , the reconstruction vectors of dimension d are formed as

$$X(i) = (x(i), x(i+T), \dots, x(i+(d-1)T)) \quad (5)$$

where T and d are the time delay and the embedding dimension, respectively. The embedding dimension that

TABLE I

Measured Critical Values of Heating Power for the Transition to Oscillatory Behavior P_0 , for the First Period-Doubling Bifurcation P_1 , and for the Chaotic Oscillations P_∞^*

Series	Pressure (bar)	$K_{ex,ris}$	P_0 (kW)	P_1 (kW)	P_∞ (kW)	P_∞^{est} (kW)
1	11.6	118 ± 18	18.0 ± 0.3	27.4 ± 1.0	32.3	30.0 ± 1.3
2	8.4	145 ± 17	15.2 ± 0.3	24.6 ± 1.0	29.3	27.1 ± 1.4
3	11.2	145 ± 17	16.2 ± 0.5	—	31.3	—
4	12.2	110 ± 7	24.0 ± 0.4	38.2 ± 1.0	45.9	42.1 ± 1.4

*Also shown are the estimated threshold values for transition to chaos P_∞^{est} . Experimental conditions (pressure, riser-exit friction factor, and $K_{ex,ris}$) are given as well.

we use in the following analyses is typically ~ 50 with $T = 1$.

III.B.1. The Kolmogorov Entropy

The Kolmogorov entropy K characterizes the *dynamics* (time evolution) of chaotic behavior. One divides the state-space into small cells and follows the time evolution of the system using a collection of initial conditions located all within one cell. After N time steps of length τ , when the trajectories generally are already spread out over a large number of cells, the entropy S_N is calculated using

$$S_N = -\sum_r p_r \ln p_r, \quad (6)$$

where p_r is the probability (the relative frequency) that a trajectory is in cell r after N steps. The Kolmogorov entropy characterizes the rate of change in S_N :

$$K_N = \frac{1}{\tau} (S_{N+1} - S_N). \quad (7)$$

The precise definition of K is the average of K_N over the whole attractor (as $N \rightarrow \infty$) in the limit of infinite small cells size and time steps ($\tau \rightarrow 0$).

One of the most fundamental features of a chaotic system is the divergence of nearby trajectories, which manifests itself as an exponential separation of initially close trajectories as times goes on. This can be interpreted as information loss since the state of a single initial point specified with certain accuracy will not be predictable after evolving for a finite time. The Kolmogorov entropy measures the rate of information loss (or gain) along the attractor. It is usually expressed in bits per second. A positive, finite K is considered as the conclusive proof that the time series and the underlying system are chaotic. K equal to zero represents a regular, cyclic, or constant motion, while an infinite K refers to a stochastic, random phenomenon.

The maximum-likelihood estimation of the Kolmogorov entropy K_{ML} as proposed by Schouten et al.²⁹ is used in this study, which has the advantage that the uncertainty of the entropy can be easily estimated.

First, the influence of the embedding time window (embedding dimension) on the estimation of K_{ML} is examined. It is proposed³⁰ that the average cycle time of the oscillations (T_C) provides, in many cases, a robust and characteristic measure for the length of the embedding time window (i.e., $d \cdot T \cdot \tau$). The average cycle time is defined as

$$T_C = \frac{[\text{length of time series (units of time)}]}{[(\text{number of crossings of average of time series})/2]}.$$

The value of K_{ML} evaluated for a range of embedding time windows is depicted in Fig. 6. Figure 6 shows that for embedding windows larger than twice the average cycle time, the estimated value of K_{ML} is independent of

Fig. 6. The influence of the choice of the embedding time window (normalized with T_C) on the estimated Kolmogorov entropy (series 4).

TABLE II
Maximum-Likelihood Estimation of the
Kolmogorov Entropy K_{ML}

Series	Power (kW)	Void Fraction		Inlet Flow Mass Flow Rate	
		K_{ML} (bit/s)	Relative Standard Deviation (%)	K_{ML} (bit/s)	Relative Standard Deviation (%)
1	32.3	0.256	0.96	0.211	0.94
2	29.3	0.127	0.93	0.139	0.93
3	31.3	0.044	0.93	0.049	0.91
4	45.9	0.189	0.94	0.116	0.89

the embedding time window. Generally, it is expected that the quantifiers of the reconstructed attractor do not change as a function of the size of the embedding window once the window size necessary to accommodate the attractor is reached.³¹ Choosing the embedding window as twice the cycle time, the K_{ML} values evaluated for the different measurements in the four experimental series are summarized in Table II.

It might be interesting (also as a cross-check for the above results) to examine which value of K_{ML} is predicted for the different system states from stable to chaotic, i.e., as a function of the increasing heating power. This is shown in Fig. 7 for experimental series 1. The value of K_{ML} is not exactly zero for the regular oscillation patterns—like limit cycles—as it should be theoretically. This is mainly because of the influence of measurement and inherent noise to which the entropy estimate is sensitive.²⁹ For example, just after the Hopf-bifurcation point (P_0) where the oscillation amplitude is still quite small, the influence of noise is relatively strong, leading to a high entropy estimate. As the oscillation amplitude increases, the entropy estimate decreases toward zero. It again increases past the first period-doubling point (P_1) because the noise gets relatively stronger again (compared to the small amplitude subharmonics). Finally, K_{ML} jumps suddenly to a distinctly higher value than for the previous states as the power P_∞ is reached, confirming that the system's behavior is indeed chaotic.

III.B.2. The Correlation Dimension

To examine the topological characteristics of the chaotic attractor in state-space, a *geometric* type of chaos quantifier, the correlation dimension, is used. The correlation dimension is proposed by Grassberger and Procaccia³² based on the behavior of the so-called correlation sum. If one lets the system trajectory evolve for a longer

Fig. 7. Evaluation of the Kolmogorov entropy for the different system states (oscillation patterns) as a function of the heating power for series 1. The arrows indicate bifurcation points and the chaotic limit. The Kolmogorov entropy is not zero for the regular oscillation patterns (limit cycle, etc.) due to the influence of noise; however, it remains small and jumps to a significantly higher value once the chaotic limit is exceeded.

time and collects N trajectory points, then the correlation sum is defined as

$$C(R) = \frac{1}{N(N-1)} \sum_{i=1}^N \sum_{j=1, i \neq j}^N \Theta(R - |x_i - x_j|), \quad (8)$$

where Θ is the Heaviside function. If R is about the size of the attractor, then $C(R) \rightarrow 1$; if R is smaller than the smallest distance between the trajectory points, then $C(R) = 0$. The correlation dimension D_C is defined to be the number that satisfies

$$C(R) = \lim_{R \rightarrow 0} kR^{D_C}$$

or equivalently

$$D_C = \lim_{R \rightarrow 0} \frac{\log C(R)}{\log R}, \quad (9)$$

where D_C is also called the scaling index because $C(R)$ scales with it as a function of R . This generalized dimension definition can give dimensionalities that are not integers for fractal objects. Fractals play an important role in the dynamics of chaotic systems since a strange attractor, per definition, has a noninteger dimension.

Generally, measurement noise induces a bias in the observed distances of trajectories and corrupts the scaling behavior expressed in Eq. (9). To calculate D_C , we use the method proposed by Schouten et al.³⁰ for the case of noisy attractors. This method can disentangle the dimension of the underlying uncorrupted, noise-free

Fig. 8. The behavior of the correlation dimension estimate as a function of the length of the normalized embedding time window (series 4), where (a) shows that similar to the Kolmogorov entropy, the value of the correlation dimension seems to settle as the embedding time window approaches two cycle times and (b) shows the same in terms of the (not-rescaled) $\log C(R)$ versus $\log R$ curves for a sequence of several embedding time windows corresponding to (a). Note that because of normalization, the upper limit of the scaling region is at $\log l = 0$, and the lower limit of the scaling region varies around -0.5 .

attractor, assuming a bounded-amplitude noise, by re-scaling the correlation integral. The method also provides an estimation of the noise level. For the rescaling technique to work fine, the embedding dimension should be chosen to be as large as possible. In practical cases, it is suggested to be at least of the order of 50 (Ref. 30). To be sure about using an appropriate embedding, the behavior of the correlation dimension estimate is also examined as a function of the size of the embedding time window. Figure 8 shows that similar to the Kolmogorov entropy, the value of the correlation dimension seems to settle as the embedding time window approaches two times the average cycle time, which just corresponds to (for our sampling frequency of 4 Hz) an embedding dimension of ~ 50 as proposed in Ref. 30. Therefore, the correlation dimensions for the four measurement series are evaluated using this embedding window, just as the Kolmogorov entropy, in Table II. The results are presented in Table III. The estimated dimension of the chaotic attractor varies ~ 1.4 to 1.8 in almost all cases, which indicates a low-dimensional chaotic behavior. The estimated noise levels (r_n) are relatively high, up to 30% of the average absolute deviation of the signals.

Figure 9 shows the behavior of D_C as a function of the increasing heating power. A qualitatively similar trend can be observed as for K_{ML} in Fig. 7, and it can be explained similar to that. For small-amplitude limit cycles, the state-space filling effect of noise leads to a higher dimension estimate. D_C falls to ~ 1 for large-amplitude limit-cycle and period-two oscillations, with the latter being more sensitive to noise, and finally, it jumps to the

TABLE III
The Values of the Correlation Dimension and the Noise Level in the Measurement Series*

Series	Power (kW)	Void Fraction		Inlet Flow Mass-Flow Rate	
		D_C	r_n	D_C	r_n
1	32.3	1.738	0.158	1.796	0.206
2	29.3	1.746	0.273	1.831	0.242
3	31.3	1.576	0.287	1.615	0.194
4	45.9	1.203	0.345	1.405	0.301

* r_n is relative to the average absolute deviation of the signal.

value describing the strange attractor for chaotic oscillations.

III.B.3. The Maximum Lyapunov Exponent

The Lyapunov exponents measure the average exponential rate of divergence or convergence of nearby orbits in state-space and are closely related to the Kolmogorov entropy. Any system containing at least one positive Lyapunov exponent is defined to be chaotic, and the magnitude of the exponent reflects the timescale on which system dynamics become unpredictable. Given a dynamical system in an n -dimensional state-space, one monitors the time evolution of an infinitesimal n -sphere.

Fig. 9. The behavior of the correlation dimension is plotted as a function of the heating power for series 1. A qualitatively similar behavior is shown as for the Kolmogorov entropy in Fig. 7.

After time t it will be transformed to an n -ellipsoid with principal axes $p_i(t)$, and the i 'th Lyapunov exponent is defined in terms of $p_i(t)$ as

$$\lambda_i = \lim_{t \rightarrow \infty} \frac{1}{t} \log_2 \frac{p_i(t)}{p(0)} . \quad (10)$$

Thus, the Lyapunov exponents are related to the expanding or contracting nature of different directions in state-space.

The estimation of the largest Lyapunov exponent from the measured time series is based on the method proposed by Wolf et al.³³ The method, which tracks the evolution of two points on nearby orbits of the reconstructed attractor, works as follows: First, the nearest neighbor (in the Euclidean sense) of the first point of the embedded time series is located (we use the same embedding as in Secs. III.B.1 and III.B.2). The distance of the initial points is denoted by $L(t_0)$. At time t_1 , this distance increases to $L(t_1)$ as the two points evolve along their trajectories. As the separation between the two points becomes large (comparable to the size of the attractor), the algorithm replaces the second point. In this way one monitors the evolution of a single principal axis vector, and a Lyapunov exponent is estimated for each evolution before each replacement. The replacement attempts to preserve orientation in state-space—so that it keeps following the evolution of the principal axis always in an expanding direction—by trying to minimize the angle between the replacement point and the replaced point. It also minimizes the distance between the evolved first point and the replacement point. The evolution+replacement procedure is repeated until the end of the time series is reached and the set of estimated Lyapunov exponents is averaged to get

$$\lambda_1 = \frac{1}{M} \sum_{k=1}^M \frac{1}{t_k - t_{k-1}} \log_2 \frac{L(t_k)}{L(t_{k-1})} , \quad (11)$$

where M is the total number of replacement steps.

In the algorithm proposed by Wolf et al.,³³ the evolution time $t_k - t_{k-1}$ is chosen constant; however, this seems to be a poor estimation of the maximal exponent in our case. Even for an optimized value of the evaluation time, the maximum Lyapunov exponent estimates [Eq. (11)] have high relative standard deviations (200% to 300%). The large scattering in the individual exponent estimates, in addition to the influence of measurement noise, is mainly due to the fixed evolution time. Depending on the position on the attractor, a fixed evolution time can be sometimes so long that one passes from an expanding region to a folding region of the attractor (since for bounded attractors L cannot grow unlimited), which results in an underestimated or even a negative exponent for that evolution and consequently an increased scattering of the mean. On the other hand, choosing the evolution time to be very short would increase the necessary number of replacement steps, which might lead more frequently to momentary loss of following the direction of expansion at replacements. To circumvent these problems, we use variable evolution times: After each replacement we follow the evolution of the distance between the two trajectories until it starts to decrease. At that point, the next replacement step is taken; thus, one always follows the trajectories in the stretching region of the attractor, and the necessary number of replacement steps is decreased as well. To reduce the effect of noise, we reject those evolutions that are shorter than a few time steps (likely due to noise). The maximum Lyapunov exponents estimated with the modified algorithm are summarized in Table IV. Their values vary between 0.23 and 0.36 for the different series, suggesting chaotic system behavior.

TABLE IV

Estimates of the Maximum Lyapunov Exponent Obtained Using a Modified Version of the Method of Wolf et al.*

Series	Power (kW)	Void Fraction		Inlet Flow Mass-Flow Rate	
		λ_1 (bit/s)	Standard Deviation (bit/s)	λ_1 (bit/s)	Standard Deviation (bit/s)
1	32.3	0.23	0.13	0.21	0.12
2	29.3	0.33	0.25	0.30	0.29
3	31.3	0.29	0.19	0.25	0.21
4	45.9	0.36	0.24	0.26	0.18

*Reference 33.

IV. CONCLUSIONS

The nonlinear dynamics of natural-circulation, boiling two-phase flows have been investigated using an experimental two-phase flow loop. Sets of experiments have been carried out in the unstable operating region of the facility for various system pressures and for different frictions at the exit of the riser section. It is found that the two-phase flow undergoes first, a supercritical Hopf bifurcation at the threshold of stability, followed by a period-doubling bifurcation, and finally becomes chaotic as the heating power is increased, which suggests that it undergoes the Feigenbaum scenario. The finer details of the Feigenbaum scenario, i.e., the cascade of period-doubling bifurcations, could not be detected, presumably due to the influence of measurement and inherent noise. To analyze measurements in the chaotic region, nonlinear time sequence analysis methods were applied. Three types of chaos quantifiers have been calculated: the Kolmogorov entropy, the correlation dimension, and the maximum Lyapunov exponent. To find the correct embedding parameters, the influence of the embedding window is examined on the estimated Kolmogorov entropy. It is found that choosing the embedding time window as twice the average cycle time or larger, the attractor is reconstructed correctly. Using these embedding parameters, the estimated values of the Kolmogorov entropy and of the maximum Lyapunov exponent indicate chaotic system behavior. The estimated correlation dimensions suggest low-dimensional dynamics.

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Robert Zboray (MS, applied physics, Technical University of Budapest, Hungary, 1997; PhD, Delft University of Technology, The Netherlands, 2002) is research engineer at the Laboratory for Thermalhydraulics of the Paul Scherrer Institute, Switzerland. His background includes experimental analysis and modeling of the dynamics of natural-circulation boiling flows and boiling water reactor (BWR) stability.

Wilhelmus J. M. de Kruijf (MS, physics, Eindhoven University of Technology, The Netherlands, 1990; PhD, physics, Delft University of Technology, The Netherlands, 1994) was project manager of BWR research at the Interfaculty Reactor Institute, focusing on BWR stability issues. His background includes reactor physics analyses of the Doppler effect, pressurized water reactor loading pattern optimization, waste transmutation, accelerator driven systems, criticality analyses, and coupled neutronic/thermal-hydraulic transient analysis. Currently he is employed at Erasmus MC Rotterdam as a medical physicist trainee with specialization radiotherapy.

Tim H. J. J. van der Hagen (MS, Eindhoven University of Technology, The Netherlands, 1984; PhD, Delft University of Technology, The Netherlands, 1989) is a full professor in the Department of Applied Sciences of the Delft University of Technology. He heads the Department of Reactor Physics of the Interfaculty Reactor Institute of that university. He has explicit experience in BWR dynamics and specific interest in the interaction between neutronics and thermal hydraulics and in the dynamics of innovative reactor designs.

Rizwan-uddin [BS, mechanical engineering, Middle East Technical University, Ankara, Turkey, 1980; MS, 1983, and PhD, 1987, nuclear engineering, University of Illinois at Urbana-Champaign (UIUC)] is an associate professor in the Department of Nuclear, Plasma, and Radiological Engineering at UIUC. He is also affiliated with the Computational Science and Engineering Program at UIUC. His research interests include reactor operations, BWR stability, two-phase flow dynamics, reactor physics, and computational fluid dynamics.