UCLA—ENG 7715
JUNE 1977

OPTIMAL EXTRACTION OF GEOTHERMAL ENERGY

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GEOTHERMAL ENERGY**

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June, 1977

**This report is based on work performed for the
U.S. Energy Research and Development Administration
under Contract E(04-3)-34, Project Agreement 248.**

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ACKNOWLEDGEMENTS

We wish to acknowledge the advice and assistance of the many individuals who have contributed to this study. We are grateful to Professor Paul Witherspoon of the University of California, Berkeley, for directing us to a specific hydrothermal reservoir model and for his useful suggestions when the project was in its formative stages. Dr. Chin Fu Tsang of Lawrence Berkeley Laboratory has been an active and enthusiastic participant in the gradual development of the project, rendering invaluable help in adapting the hydrothermal model to fit the economic model. Dr. Tsang has contributed scores of hours to this study, for which we are most appreciative.

Much of the early computational work and data gathering was done by Mr. Sashi Mozumder. We are indebted to him for these efforts as well as many valuable suggestions as the work progressed. We also appreciate the good work of Mr. James Kranyak, who was responsible for the later programming.

We are grateful to Professors Paul Kruger and Henry Ramey of Stanford University for providing an opportunity to expose our conceptual ideas to constructive criticism of practicing geothermal engineers at the First and Second Stanford Geothermal Workshops. Professors William Brigham of Stanford University and William Yeh of UCLA contributed several valuable technical suggestions as our work progressed.

We greatly benefited from the advice of Professor Donald Edwards of UCLA who contributed to our understanding of heat transfer processes. Ms. Helen Lupear of Keenen Pipe and Supply Company,

Messrs. Craig Brown and George Crabtree of Peerless Pump Division (FMC Corporation), Fred Behzadassiri of Joy Company, Charles Hayden of Trane Company, James Breese of Walter Perkins Company and Gil Lombard of San Diego Gas and Electric Company, supplied us with equipment cost data and technical information. Their assistance is gratefully acknowledged.

Since September, 1976, this project has been supported by the U.S. Energy Research and Development Administration. We appreciate this assistance, and we thank Dr. John Salisbury and Mr. Clayton Nichols of ERDA for their cooperation. We are also grateful to the Lawrence Berkeley Laboratory for financial support during the summer of 1976.

This work was begun as a seed project supported by the University of California Water Resources Center at the University of California, Davis. We are indeed grateful to J. Herbert Snyder, Professor and Director of the Center, for his willingness to support the preliminary investigations that led to this report.

Finally, we thank Cynthia Mothersole and Debra De Luca who patiently typed several versions of this manuscript.

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1. INTRODUCTION

This study is concerned with the optimal extraction of energy from a hot water geothermal field. In view of the relative "commercial" availability of the many energy sources alternative to geothermal, it is possible that a socially "best" extraction policy may not include producing geothermal energy as fast as current technology will permit. Rather, a truly "optimal" policy will depend on, among other things, the costs and value of geothermal energy in the future and the analogous values of other energy sources. Hence, a general approach to this problem would make the policy contingent on pertinent information on alternative sources. A good example of this approach is given in Manne's (1976) Energy Technology Assessment Model, where he points out that "Each energy source has its own cost parameters and introduction date, but is interdependent with other components of the energy sector." [Manne (1976), p. 379]. But by their large dimensions, such relatively macro-analyses tend to preclude a close look at the specific technology of a particular energy source. Accordingly, since an explicit treatment of the technology of a process is important in developing meaningful resource management models, we substitute for a macro model the increasing value over time of the energy extracted. In this context we seek an extraction rate (and an economic life) that maximizes the net discounted value of the energy extracted.

The determination of the rate of geothermal energy extraction is important for both conceptual and practical policy reasons. A great deal of conceptual work has been done on the theory of socially

optimal natural resource extraction. This has been summarized in a working paper by Peterson and Fisher (1976) and in the proceedings of a symposium on exhaustible resources in the Review of Economic Studies (1974).¹ However, as Peterson and Fisher point out:

"In their current state, these models are excellent vehicles for teaching concepts and techniques of dynamic optimization, especially in the presence of externalities. Unfortunately, they cannot be used to manage actual natural resources, because their functional forms are too simple and their empirical content too low." (Peterson and Fisher, 1976, p. 17).

Accordingly one goal of the present study is the introduction of some physical and empirical "reality" into the theoretical optimal extraction literature, providing a bridge between theory and practical application. To this end we have chosen a particular extractable resource and technology, non-electrical hot water geothermal energy production, and we have introduced two specific relationships into our extraction model. The first is an explicit hydrothermal expression which quantitatively describes the relationship between rate of heat energy extraction and the "quality" of the remaining unextracted energy. In particular, for the case of a production-reinjection geothermal well doublet with homogeneous aquifer (the Gringarten-Sauty model (1975)), the expression describes the relationship between geothermal brine pumping rate and the temperature of the production well brine over time (when spent brine is reinjected to avoid environmental damages).² Given these explicit

¹See also Heal (1976), Solow and Wan (1976), Pearce and Rose (1975), Schulze (1974), Gordon (1967), Hotelling (1931), Herfindahl (1967), and Smith (1968).

²We note that our optimization model can be modified to accommodate other hydrothermal models. However, the assumption of reinjection is essential.

functional relationships and the important theoretical framework of the resource economists, our second goal is to synthesize these two into an analytical tool for engineering-economic design of a geothermal heat extraction process.

The practical policy implications of this work pertain to the economics of geothermal extraction as well as decisions on how rapidly and how long private and public-sector geothermal fields should be developed. These are questions of relevance to both private energy companies as well as federal government agencies such as the U.S. Geological Survey and the U.S. Bureau of Land Management -- the agency that inventories and leases land for mineral exploration.

Specifically, we will address ourselves to the following questions:

- (a) What is the economic worth of a reservoir and in what manner and to what extent is this value dependent on physical geohydrological parameters (such as initial equilibrium temperature, permeability and porosity) as well as economic parameters (such as growth rate in the value of energy, market interest rate, royalty and land rent, equipment and operational costs, and cost of wells and their expected lives)?
- (b) At what rate and for what duration of time should a geothermal reservoir be exploited and to what degree should the brine be cooled before reinjection in the aquifer in order to maximize the present worth of profits?
- (c) To what extent are these decisions dependent on the pertinent geohydrological and economic parameters? In

particular, how are these decisions affected by variations (uncertainty) in these parameters (especially interest rates and rates of growth in energy value)?

- (d) Which of the geohydrological parameters are more important in the engineering design of the geothermal facility? How cost-beneficial would it be to obtain additional information regarding these parameters before the extraction facilities are designed?
- (e) To what extent can regulatory agencies influence the economics and rate of exploitation of geothermal energy by manipulating factors like royalty, lease terms and land rents?

With these general and more specific objectives in mind, we proceed, in the remainder of this paper, to present some background information on hot water geothermal systems, and describe the physical relationship between extraction rate and temperature over time. Next, we review some fundamental principles of resource allocation and then present our economic model for selecting optimal steady-state extraction rate, reinjection temperature and economic life of the project when the extracted energy is used for non-electrical steam production. We then analyse the relationship between the cost of each component of the production and surface equipment and our decision variables. Using these costs and data for a typical aquifer, we present the results of our optimization and attempt to answer the questions discussed above. We conclude with a discussion of these results.

2. BACKGROUND

By hot water geothermal system we refer to a saturated aquifer bounded top and bottom by impermeable aquicludes (See Figure 1). The water in the aquifer (before any pumping) is in thermal equilibrium with both aquicludes and with the aquifer matrix. The heat source is assumed to be from magmatic intrusions into the earth's outer crust. Although a thermal gradient exists everywhere from the center to the surface of the earth, a geothermal field, also referred to as an anomaly, may be detected (actually defined) by an unusually steep local thermal gradient near the earth's surface. By local, we mean a kilometer or two in all (horizontal) directions from the point where the gradient is largest.

With respect to the next few decades, the heat source (the intrusion) is assumed to be infinite. However, the rate of heat transfer across the aquicludes to the aquifer matrix and fluid is governed by the rate of thermal conductivity, which is relatively low, and therefore, the energy of the geothermal field is effectively nonrenewable in the next few decades.

From a strictly hydraulic point of view, the aquifer may be renewable, in the sense that any ground water system is renewable. Even here, though, if the aquifer is over-depleted, the matrix might be permanently compacted, reducing future hydraulic yield and inducing subsidence of the over burden. Since this subsidence may be highly undesirable for unrelated reasons, it will be necessary to reinject the spent brine; hence the introduction above of a production-reinjection doublet. Good examples of potential

damage from subsidence are found in the California Imperial Valley, where substantial subsidence could seriously disturb the vertical alignment of the irrigation canals, and in Wairaki fields in New Zealand, [Atherton et al. (1976); Stilwell and Hall (1975)].

The hydraulic operation of a hot water geothermal system may be described as follows. Water is pumped from the production well (at (1) in Fig. 1) up to the surface where it enters the heat exchanger (2). After it leaves the heat exchanger (3), it is piped to the reinjection well (4) and pumped back into the aquifer (5). It then moves through the aquifer toward point (1). The flow is turbulent from point (1) to point (5), and laminar from (5) to (1). Initially, when pumping begins, the entire aquifer is in equilibrium at temperature T_0 . Water is pumped out at that temperature and cooled to T_1 as the energy is extracted. After the water is pumped back into the aquifer, it is heated by the aquifer matrix as it moves from (5) to (1). For some period the water will be heated back to T_0 by the time it arrives at point (1).

As heat is transferred from the aquifer matrix to the fluid, the temperature of the matrix decreases. It follows that a point in time, τ , will come, when the matrix can no longer heat the fluid to T_0 by the time the fluid reaches point (1). When this happens, the production well temperature at (1) (and hence at (2)) will begin to drop. If we denote the time-variable production well temperature as T_0^t , this process of temperature degradation over time can be plotted as shown in Figure 2. The point in time, τ , when temperature begins to decline below T_0 is called "breakthrough," referring to the time when

Fig. 2. The Temperature vs Time Plot for a Given Flow Rate

the reduced fluid temperature breaks through to the production well. Breakthrough is inversely proportional to Q , the water flow rate, as we shall see shortly. The post "breakthrough" rate of decrease in temperature also depends on Q and reinjection temperature, T_1 .

We have now described the fundamental relationship between temperature and time for a given Q . As $T_0^t \rightarrow T_1$, the cost (per BTU) of heat extraction increases and there is a time when it is no longer "economical" to extract more heat, even though there may be a great deal of heat energy left in the aquifer. The question, then, is how much heat to extract "now," and how much "later," given that the cost per BTU extracted will increase as $T_0^t \rightarrow T_1$ (even though pumping rate is constant with time). Of course this depends on the relative value of the extracted energy "now" and "later" and the costs of extraction. We will discuss this general economic question later.

3. THE HYDROTHERMAL RELATIONSHIP

We begin by mathematically defining the relationship between production well temperature and time for a given flow rate. Hot brine is withdrawn at the rate Q and reinjected at the same rate. The temperature of the reinjected fluid at time t is denoted by T_1^t .

For the first τ years ($0 \leq t \leq \tau$), $T_0^t = T_0^0$ where $T_0^0 = T_0$ is the initial equilibrium temperature of the unexploited reservoir and τ denotes the breakthrough time. The breakthrough time is inversely proportional to Q and is described by the following relationship [see Tsang et al. (1976)]:

$$\tau(Q) = t_u/6, \quad (1)$$

where t_u is a unit for time,

$$t_u = \frac{2 \pi h D^2 \rho_a c_a}{8760 Q \rho_f c_f}, \quad (2)$$

and h is the aquifer thickness (m), D the well separation (m), Q the pumping rate (m^3/hr) and ρ_a , ρ_f , c_a , c_f the densities and specific heats of the aquifer matrix and the fluid respectively. The relationship between the heat capacities of the aquifer matrix, rock structure and the fluid is given by,

$$\rho_a c_a = \phi c_f \rho_f + (1-\phi) \rho_R c_R \quad (3)$$

where ϕ is the porosity of the aquifer.

For the purpose of this analysis, we neglect the temperature drop in surface pipes so that the temperature of the fluid entering the heat exchanger is T_o^t and the injection temperature, T_1^t , equals the temperature of the fluid leaving the heat exchanger. The temperature after breakthrough is determined by a function $\bar{g}(T_1^t, t/t_u)$ which gives the ratio of the temperature drop through the heat exchanger experienced by the brine at time t , to that at time zero:

$$\frac{T_o^t - T_1^t}{T_o - T_1} = \bar{g}(T_1^t, t/t_u). \quad (4)$$

We will later show that the variation in T_1^t is small and hence $\bar{g}$ can be approximated by a function g which is valid for invariant T_1 .

We now consider the thermodynamics of heat transfer on the surface, assuming the energy will be used for non-electrical steam production (e.g. space heating). Hot water from a building complex enters the heat exchanger at the saturation temperature of steam (at

the desired pressure) T_g (at 6 in Figure 1), is heated and leaves the heat exchanger as steam at T_g (7), is circulated throughout the building complex losing heat to the building in the process of phase change, and returns as condensate to the heat exchanger (6).

Aside from piping heat losses, there are only two ways in which heat can leave the doublet system (includes reservoir as well as surface equipment): by transfer to the steam cycle, and by heat loss from the heat exchanger. A realistic model should incorporate this heat loss, and we shall return to this detail in Section 4.1. For now, though, we will neglect heat losses in the heat exchanger and assume all the heat removed from the brine at any given time is used to generate steam.

The effectiveness, ϵ , of a heat exchanger is defined in terms of the hot and cold brine temperatures, T_o^t and T_i , and the temperature of the cold side of the exchanger, T_s . The maximum transferable heat (given an infinite exchange area) is $Q c_f \rho_f (T_o^t - T_s)$. However, since heat transfer becomes very expensive as T_i approaches T_s , the heat actually transferred is usually less than the maximum defined above. Accordingly ϵ is defined as the heat actually transferred divided by the maximum amount of heat that could be transferred given an infinite transfer surface area. [Edwards et al. (1973) p.253]:

$$\epsilon = \frac{Q c_f \rho_f (T_o^t - T_i)}{Q c_f \rho_f (T_o^t - T_s)} = \frac{T_o^t - T_i}{T_o^t - T_s} \quad (5)$$

For a given Q , equation (5) implies that ϵ increases to unity as $T_i \rightarrow T_s$. In addition, the effectiveness of a heat exchanger is a

function of the number of transfer units NTU(t) at time t,
[Edwards et al. (1973) page 243],

$$\epsilon = 1 - e^{-NTU(t)}, \quad (6)$$

where

$$NTU(t) = \frac{k(t)A}{Q}, \quad (7)$$

and

$$k(t) = 0.00488 U(t)/c_f \rho_f$$

U(t) is the overall heat transfer coefficient at time t, (BTU/hr-ft²-°F), $c_f \rho_f$ is the heat capacity (cal/cc°C) and A is the heat exchanger area (m²). The units of k are m/hr making NTU in (7) dimensionless.¹

We are now ready to discuss the variation in T_1^t with time.

Combining (5) and (6) yields

$$\frac{T_o^t - T_1^t}{T_1^t - T_s} = \frac{1 - e^{-NTU(t)}}{e^{-NTU(t)}} = e^{NTU(t)} - 1. \quad (8)$$

From (4), we have $T_o^t - T_1^t = (T_o - T_1) \bar{g}$. Substituting this in (8) gives

$$\frac{T_o - T_1}{T_1 - T_s} = \frac{e^{NTU(t)} - 1}{\bar{g}}$$

which yields

$$\frac{T_o - T_1}{T_o - T_s} = \frac{e^{NTU(t)} - 1}{\bar{g} + e^{NTU(t)} - 1}. \quad (9)$$

¹ We have generally preferred to use metric units. However, for cases where data are commonly available in British units, we have used this system of units and included the appropriate conversion factors in the equations as coefficients.

Since $\bar{g}$ is a monotone non-increasing function of t with the range of $[0,1]$, equation (9) yields the range of T_1^t . For $t \leq \tau$, $\bar{g} = 1$ and

$$T_1^t = T_g + (T_o - T_g)e^{-NTU(0)}. \quad (10)$$

As $t \rightarrow \infty$, $\bar{g} \rightarrow 0$ and $T_1^t \rightarrow T_g$. The variation of T_1^t is therefore small and $\bar{g}$ can be approximated by a function g which assumes T_1 does not vary with time. Using the results of the Gringarten-Sauty model, an expression for g has been developed [Tsang et al. (1976)] and is given by,

$$g(t/t_u) = \begin{cases} 1 & \text{if } t \leq \tau \\ \gamma_1 e^{-\phi_1 t/t_u} + \gamma_2 e^{-\phi_2 t/t_u} + \gamma_3 e^{-\phi_3 t/t_u} & \text{if } t > \tau \end{cases} \quad (11)$$

where $\phi_1 = 0.0138$, $\phi_2 = 0.656$, $\phi_3 = 8.006$, $\gamma_1 = 0.338$, $\gamma_2 = 0.337$ and $\gamma_3 = 1.368$.

In the remainder of this paper we will assume a reinjection temperature T_1 that is constant with time. Equation (4) can therefore be written as

$$\frac{T_o^t - T_1}{T_o - T_1} = g(t/t_u). \quad (12)$$

However, although T_1 may be assumed constant with time, its value obviously affects heat removed per unit of time (for a given Q) and hence discounted net revenues. That is, lower values of T_1 yield greater heat removals per time but cause the field to cool more rapidly. Furthermore, to achieve lower values of T_1 (for a given T_g), larger and hence more expensive heat exchangers are required. To see

this, note from equations (5), (6) and (7) that for a given Q,

$$\frac{T_o - T_i}{T_o - T_s} = 1 - e^{-k(0)A/Q} \quad (13)$$

yielding

$$A = \frac{Q}{k(0)} [\ln(T_o - T_s) - \ln(T_i - T_s)] \quad , \quad (14)$$

implying that $A \rightarrow \infty$ as $T_i \rightarrow T_s$.

4. THE ECONOMIC MODEL

We begin this section with the definition of the costs and benefits associated with extraction of geothermal heat energy. By "cost", we shall mean the "opportunity cost" to society of resources (steel, concrete, pumps, well drilling services, etc.) used in extracting geothermal energy, resources that could have been put to some other alternative use. The amount that some other party would be willing to pay in order to procure the services of these "social resources" will be called their "opportunity cost". We will further assume there are no primary "externalities" associated with this energy extraction. This is a good assumption, because spent brines will be reinjected, preventing subsidence and escape to the atmosphere of noxious gases. Secondary impacts, such as population influx to build and operate the geothermal system are assumed small and are neglected.

Turning to the value or benefits of the extracted energy, there are at least two ways to proceed. The first is to assume demand for the energy is price sensitive, using the area under the

demand curve as an index of "willingness to pay" and hence social benefit or value [Hotelling (1938)]. This is the appropriate approach if demand as perceived by the energy producer is at least somewhat price-elastic. Alternatively, if there are other sources of energy (including imports), then we define benefits as the cost to the customer of the least expensive alternative energy source other than geothermal. For example, a customer using geothermal energy to generate steam for space heating has the option of generating steam with an oil or coal-fired boiler, each of which also has some social opportunity cost. The lowest of these costs is therefore taken as the "price" or value of the geothermal energy. Of course in a purely competitive energy market, this least expensive price is the only market price, and the geothermal energy company is simply a price-taker attempting to maximize profits. We shall consider the optimal extraction of geothermal energy using the latter benefit measure.

The essential factor in the theory of optimal extraction is time. Indeed, the major question is "how much now and how much later?". In order to structure a framework in which to examine this question, we need to explicitly state how time affects the costs and benefits of the extraction process, and how time affects our perception of these costs and benefits. We now briefly discuss both of these topics, beginning with the latter.

Assuming we can establish some time series of costs and benefits associated with the extraction process, we can proceed toward answering the "how much now, how much later" question by first

determining the relative values (as seen from the present) of a dollar now and a dollar later. It is generally accepted that a dollar today is worth more today than a dollar a year from now (we shall temporarily disregard the impact of inflation for now, and subsequently show that it need not be considered at all for the purposes of this paper). In this sense, we tend to "discount" future value. Specifying the exact weighting between "now" and "later" is a matter of subjective judgment or preference of the individual. It is rooted in the individual's attitude toward the present and the future, and revealed in such actions as saving vs. spending (consuming). Identifying the rate at which the individual discounts future dollars is relatively straight forward.

Similarly, it is relatively easy for the firm to establish its discount rate as the highest possible rate of return on alternative investment. If the present worth of the revenues minus the costs of some activity is positive at this discount rate, then the firm should undertake this activity.

But things are not quite so simple when public resources are being expropriated and "used", as would be the case in the present study if the geothermal resources were in the public domain. Many authors have written extensively on the problem of specifying the correct weighting of dollars now vs. dollars later for public sector resource allocation. While this research has generated several useful insights into this problem, many of them seem to be contradictory, and the serious student of this problem is bound to be somewhat perplexed in his attempt to discover the "most sensible"

approach to the correct social discount rate for public sector resource allocation.

We can approach the problem from the perspective of traditional economic theorists. Drawing on basic welfare theorems (and ultimately a whole philosophy on political economy), economists recognize that the "private market" does not always produce the "right" amount of all goods and services. Certain goods and services are best produced by the public sector (usually governments). The question then is how much of each resource should be diverted from the private sector to produce these "public goods." The concern here is that the social value of goods that could have been produced privately with these diverted resources would exceed the social value of the goods produced in the public sector. To prevent this, economists pose that the social discount rate be set equal to the private market rate of interest. Only if a project has a positive net present worth at this rate should it be undertaken in the public sector.

However, there are several problems with this simple rule. Beginning with the more mechanical, we note that there is no single market rate of interest. Indeed, the best we can find here is an arbitrary composite of rates of return on various forms of investment and debt, with differing risk and differing maturity (in time). Secondly, it can be shown that different perceptions of risk (Smith vs. Jones attitude toward risk, public vs. private attitude toward risk) influence attitudes toward the future. Thirdly, and perhaps of most importance to this study is the problem of inter-generational

equity; if resources are limited, then it is always the present generation that determines how much they shall use and hence how much shall remain for future generations. And this is a subjective matter involving a trade-off, at a collective level, of altruistic vs. hedonistic attitudes. Even if a social discount rate could be estimated based on this consideration, there is no reason to believe it would conform to the rates estimated using the other approaches.

From the above discussion, it is clear that the problem of the "correct" discount rate is a difficult one indeed. Since we have not undertaken to solve this conundrum in this paper, our only recourse is to incorporate the time value of money and resources into our study on a parametric basis. We shall consider a range of discount rates, presenting results for several values within this range.

We now consider the effect of time on the costs and benefits of the geothermal energy extraction problem, a subject far less frustrating and complicated than the question of perceptions of these costs and benefits. As Hanke, Carver and Bugg (1975) point out, it is appropriate to disregard inflation in a dynamic analysis, if real (as opposed to inflated) costs and benefits are used, and the discount rate is not compensated for inflation. Conversely, if one of these is inflation compensated, then they must all be. Accordingly, we will use "real" benefits, costs and discount rates. However, since we must now go further and deeper for each BTU of energy consumed, we shall formulate our model for the general case

where the real value of energy is allowed to increase with time.¹

We will use two kinds of relationships for the increase in real value of energy:

$$(a) P_t = P_0 e^{rt},$$

$$(b) P_t = P_0 (1+rt) \quad (15)$$

where P_t = price (value) of energy at some time, t ,

P_0 = price (value) of energy at time $t = 0$,

r = rate of increase of real energy price per year.

We can now state the general economic problem of geothermal energy extraction. The question of when and how much energy to extract from a geothermal aquifer depends on the relative benefits and costs of the energy now and into the future. On one hand, the real value of the energy increases with time as outlined above. This suggests that extraction should be postponed to a time when the net social value (benefits minus costs) is greater. On the other hand, pumping energy costs increase at the same rate, and a positive discount rate discounts these greater future values, so the rate of increase in value of energy and the discount rate work against each other in determining when and how much energy to extract. Furthermore, the temperature-time profile for a particular pumping rate, Q , implies a significant trade-off between energy

¹We are effectively assuming away the "technological fix" here. This is probably rather conservative. As the rents on remaining in place resources increase, potential returns to speculative capital investment in the research and development market grow large, and substantial attempts at technical innovation are made. In this way, technical innovation is shown to be consistent with, and a natural outcome of, the model of a purely competitive economy [see Barnett and Morse (1963)]. Whether such a market exists and functions in the public interest is, of course, another matter.

obtained now and later. If energy is extracted rapidly at first, the temperature will decrease rapidly, seriously diminishing the quality of the heat in the future. Moreover, for a given pumping rate, more heat can be extracted by lowering the reinjection temperature.¹ However, for achieving lower reinjection temperatures we require larger and hence costlier heat exchangers.

4.1 The Model with Exponentially Growing Energy Value (The Exponential Growth Model).

In this section, we structure the model assuming the value of energy increases exponentially with time, and that the price (value) of energy at time zero can be computed, based on the cost of alternative sources of energy. We shall refer to this version of the model as the "Exponential Growth Model." Note that the producer is a price taker, so price is a given and we can sell all extracted energy at that price. From (15) we have

$$P_t = P_0 e^{rt},$$

where P_t is the price (value) of energy at time t , $t > 0$, and r is the continuous annual rate of increase of real energy price with time. At the end of this section, we present a method for the computation of P_0 .

The amount of heat recovered per unit time is the product of the flow rate, heat capacity of the fluid and the temperature drop experienced by the hot brine in the heat exchanger. For the first τ years, this temperature drop is $T_0 - T_1$. From that time until the termination of the project at time L , the temperature drop is governed by equation (12). Since a certain amount of heat is lost in

¹Note that by (12), $T_0^t = T_0 g + T_1(1-g)$. This implies that T_0^t decreases as T_1 decreases. However, $T_0^t - T_1$, which determines the amount of heat recovered, increases as T_1 decreases.

the heat exchange, we limit the approach of the brine temperature at the heat exchanger inlet to δ °C of the brine outlet temperature ($\delta > 0$). This restricts the "optimal life" of the project to L_δ , where L_δ is such that

$$T_o - T_1 = \delta \quad (16)$$

Note that by (12), equation (16) yields

$$g(L_\delta/t_u) = \frac{\delta}{T_o - T_1}, \quad (17)$$

which implies that for given Q and T_1 , L_δ is a decreasing function of δ . Our optimization problem is therefore

$$\begin{aligned} \text{Maximize } \Pi = & (1-\eta) \int_0^{\tau(Q)} 34.76 P_o e^{rt} Q c_f \rho_f (T_o - T_1) e^{-it} dt \\ & + (1-\eta) \int_{\tau(Q)}^L 34.76 P_o e^{rt} Q c_f \rho_f (T_o - T_1) g(t/t_u) e^{-it} dt \\ & - C(Q, T_1, L) \end{aligned} \quad (18)$$

subject to

$$g(L/t_u) \geq \frac{\delta}{T_o - T_1}$$

$$Q \geq 0,$$

where

η = royalty for geothermal lease paid as a percentage of the value of produced energy.

Q = extraction rate (m^3/hr),

c_f = specific heat of the fluid (cal/gr °C),

ρ_f = fluid density (gr/cm³) ,

P_o = assumed energy price (\$/MBTU) ,

i = discount rate

τ = breakthrough time (years) ,

L = project life (years) ,

$C(Q, T_1, L)$ = cost function describing the present worth of
total capital and operating costs,

and 34.76 is a conversion factor to yield revenues in dollars per year. By taking time in days, we could have obtained a closer approximation to the discounted profits. However, for simplicity, we compute time in years.

Let

$$\beta = \pi h D^2 \rho_a c_a / 26280 c_f \rho_f$$

$$\tau = \beta / Q$$

$$t_u = 68 / Q \quad (19)$$

$$\psi_j = \phi_j / 68$$

$$\alpha = i - r$$

$$a = 34.76 P_o c_f \rho_f (1 - \eta)$$

where ϕ_j are the exponential parameters of g in equation (11). For $t > \tau$, equation (11) can therefore be written as

$$g(t/t_u) = \gamma_1 e^{-\psi_1 Q t} + \gamma_2 e^{-\psi_2 Q t} + \gamma_3 e^{-\psi_3 Q t}. \quad (20)$$

Thus, the integral in (18) reduces to

$$\Pi = a Q (T_o - T_1) \int_0^{\beta/Q} e^{-\alpha t} dt$$

$$+ aQ(T_0 - T_1) \int_{\beta/Q}^L (e^{-\alpha t} \sum_{j=1}^3 \gamma_j e^{-\psi_j Q t}) dt$$

$$- C(Q, T_1, L),$$

yielding

$$\Pi = aQ(T_0 - T_1) \left[(1 - e^{-\alpha \beta/Q})/\alpha + \sum_{j=1}^3 \gamma_j \frac{(e^{-(\psi_j Q + \alpha)\beta/Q} - e^{-(\psi_j Q + \alpha)L})}{\psi_j Q + \alpha} \right] - C(Q, T_1, L). \quad (21)$$

4.2 The Model with Linearly Growing Energy Value (The Linear Growth Model)

Since the assumption of exponentially increasing value for energy may tend to overestimate the value for the long-run, we present an alternative model for the rate of increase of energy. Specifically, we let

$$P_t = P_0(1+rt), \quad (22)$$

where P_0 and r are the parameters defined in section 4.1. We shall refer to this variation of the model as the "Linear Growth Model." As in the previous section, we continue to assume that price (value) is a given to the firm. In this case our optimization problem becomes:

$$\begin{aligned} \text{Maximize } \Pi = (1-\eta) \int_0^{\tau(Q)} 34.76 P_0 (1+rt) Q c_{ff} (T_0 - T_1) e^{-it} dt \\ + (1-\eta) \int_{\tau(Q)}^L 34.76 P_0 (1+rt) Q c_{ff} (T_0 - T_1) g(t/t_u) e^{-it} dt \\ - C(Q, T_1, L), \end{aligned} \quad (23)$$

subject to

$$g(L/t_u) \geq \frac{\delta}{T_o - T_1}$$

$$Q \geq 0 .$$

Invoking (19), we can write

$$\begin{aligned} \Pi = aQ(T_o - T_1) & \left[\int_0^{\beta/Q} (1+rt)e^{-it} dt + \int_{\beta/Q}^L (1+rt)g(t/t_u)e^{-it} dt \right] \\ & - C(Q, T_1, L) . \end{aligned} \quad (24)$$

Now, using (20), we can evaluate (24) to obtain

$$\begin{aligned} \Pi = aQ(T_o - T_1) & \left\{ \frac{1-e^{-\beta/Q}}{1} - r \left[\frac{e^{-\beta/Q}}{1^2} \left(1 + \frac{\beta}{Q} \right) - \frac{1}{1^2} \right] \right. \\ & + \sum_{j=1}^3 \gamma_j \frac{e^{-(\psi_j Q+1)\beta/Q} - e^{-(\psi_j Q+1)L}}{\psi_j^{Q+1}} \\ & - r \sum_{j=1}^3 \gamma_j \frac{e^{-(\psi_j Q+1)L} [(\psi_j Q+1)L+1]}{(\psi_j Q+1)^2} \\ & \left. + r \sum_{j=1}^3 \gamma_j \frac{e^{-(\psi_j Q+1)\beta/Q} [(\psi_j Q+1)\beta/Q + 1]}{(\psi_j Q+1)^2} \right\} \\ & - C(Q, T_1, L), \end{aligned} \quad (25)$$

which gives Π in terms of our decision variables.

Note that in equations (21) and (25), the lifetime L , has been assumed to be greater than the breakthrough time, τ . When $L \leq \tau$, equations (21) and (25) have to be accordingly modified. This modification will be discussed in section 6.

4.3 Determining P_o

We have defined P_o as the cost of the least expensive alternative method of producing one million BTU's of steam. At the present time, this alternative is producing steam in a boiler heated by fuel oil. The components comprising the cost are capital, fuel and operating costs. Based on empirical cost data [Hayden (1976)].

Original Capital Cost = \$55/Boiler HP,
which equals \$1643/MBTU/hr, as each Boiler HP is equivalent to 0.033475 MBTU/hr. Taking the lifetime of the equipment as 25 years, the annual capital cost is obtained by multiplying the original capital cost by CRF (i,25), where CRF (i,n) is the capital recovery factor when the interest rate is i and the lifetime of the equipment is n. We can compute CRF from:

$$CRF(i,n) = \frac{i(1+i)^n}{(1+i)^n - 1} + m \quad (26)$$

where m represents cost of insurance, and local taxes expressed as a percentage of original capital costs. Therefore:

$$\text{Annual Capital Cost} = 1643 \cdot CRF(i,25) \text{ \$/yr/MBTU/hr.}$$

The fuel cost (No. 2 fuel oil at 15¢/gallon) is \$.66/10³ lbs. of 5 psi steam. Thus:

$$\text{Fuel Cost} = \$.66/\text{hr/MBTU/hr} = 5782 \text{ \$/yr/MBTU/hr.}$$

In addition, the operation of pump and boiler fan costs \$125/Boiler HP per year. Hence:

$$\text{Operating Cost} = 125 \text{ \$/yr/Boiler HP}$$

$$= 3734 \text{ \$/yr/MBTU/hr.}$$

The value of the energy P_o , is therefore

$$P_o = 1643 \text{ CRF}(1,25) + 5782 + 3734 \text{ (\$/yr/MBTU/hr) ,}$$

yielding

$$P_o = .1876 \text{ CRF}(1,25) + 1.086 \text{ \$/MBTU.} \quad (27)$$

5. THE COST FUNCTION

In this section we develop the cost function, $C(Q, T_1, L)$. The components of the cost function are 1) costs for wells and casing and their maintenance, 2) well assemblies, 3) pumps and their operation, 4) pipes and pipe cleaning 5) heat exchangers and 6) rent and salaries. In this section we develop the relationship between the costs of each piece of equipment and our decision variables.

We will denote the maximum flow rate from each production well by $\bar{Q}$. This upper limit is determined by two factors. The first is the assumption in the Gringarten-Sauty hydro-thermal model that the flow into the production well be laminar. Laminarity is indexed by the Reynolds number

$$N_R = \frac{Vd}{\nu}, \quad (28)$$

where V , the specific discharge is given by

$$V = \frac{Q}{2\pi r_w h}. \quad (29)$$

In the above relationships, r_w is the well radius, d the average grain diameter and ν the kinematic viscosity. Hence, once the properties of the field are determined, the first limit on flow

rate, $\bar{Q}_1$ will be known.

The second limit is a function of the technology of geothermal brine pumps. These vertical pumps are limited both by their technical capacities (maximum flow rate) and the drawdown generated in the production well, which is in turn dependent on the flow rate. The steady state drawdown for the production well is given by [De Wiest (1967) p 249]:

$$\Delta P = \frac{Q}{2\pi Kh} \ln \frac{D}{r_w} . \quad (30)$$

The hydraulic conductivity, K, is obtained from

$$K = 1.1653 \times 10^{-11} \frac{k\gamma}{\mu} . \quad (31)$$

where γ is the specific weight of water (lb_f/ft^3), μ the absolute viscosity ($\text{lb}_f\text{sec}/\text{ft}^2$), k the intrinsic permeability of the aquifer (millidarcies) and 1.1653×10^{-11} a conversion factor, so that the units of K are m/hour. Again, once the height of the aquifer, the intrinsic permeability and the temperature of the hot brine are known, this second maximum flow rate, $\bar{Q}_2$, that would be consistent with current pump technology can be determined. $\bar{Q}$ is the minimum of $\bar{Q}_1$ and $\bar{Q}_2$.

We will denote by S the present value of total salaries and rents for the geothermal reservoir paid during the life of the project, i.e.

$$S = \int_0^L (\text{Annual Rents} + \text{Annual Salaries}) e^{-it} dt. \quad (32)$$

Note that S is independent of the extraction rate. Since there are certain fixed costs that must be paid for each doublet, the total

cost function, $C(Q, T_1, L)$, is a step function of Q (see figure (3)), with jumps equal to the present value of well and overhead assembly costs plus fixed capital costs of pumps and heat exchangers. Let $q(Q, T_1, L)$, $0 \leq Q \leq \bar{Q}$, be the cost function describing the present value of total costs (excluding rents and salaries) associated with one doublet. Then, suppressing the dependence of C and q on T_1 and L , we can write

$$\begin{aligned} C(Q) &= 0 & \text{if } Q &= 0 \\ &= q(Q) + S & \text{if } 0 < Q \leq \bar{Q} \\ &= q(\bar{Q}) + q(Q - \bar{Q}) + S & \text{if } \bar{Q} < Q \leq 2\bar{Q} \end{aligned}$$

and in general

$$C(Q) = nq(\bar{Q}) + q(Q - n\bar{Q}) + S \quad \text{if } n\bar{Q} < Q \leq (n+1)\bar{Q} \quad (33)$$

for $n = 1, 2, \dots$

In our analysis, we take the useful life of pumps and heat exchangers as ten years and that of pipes and well assemblies as 25 years. We will later demonstrate that the well life is a crucial determining factor of the economic life of a reservoir. Since the life of a geothermal well may vary from field to field, we will let well life be an input parameter. We will assume that payments for the cost of each type of equipment and accrued interest are distributed uniformly over the lifetime of the equipment. Furthermore, each piece of equipment (with the exception of the wells) has a salvage value equal to a percentage of its remaining unpaid costs, if it is sold before its lifetime is up.

For each doublet, let

WC = Cost of wells and casing,

WL = Useful life of wells,

Figure 3. Total Discounted Costs vs Pumping Rate for Given Project Lives

PM	= Total cost of the vertical and horizontal pumps,
WA	= Cost of well assemblies,
WM	= Annual well maintenance costs,
PO(t)	= Operating costs of pumps as a function of time,
HE	= Cost of a heat exchanger,
PP	= Cost of pipes,
PC	= Annual pipe cleaning costs,
L	= Life of the project,
L_1	= The smallest multiple of 10 containing L,
L_2	= The smallest multiple of 25 containing L,
L_3	= The smallest multiple of well life containing L,
s_1	= salvage value of pumps as a percentage of their remaining payments,
s_2	= salvage value of heat exchangers as a percentage of their remaining payments
s_3	= salvage value of pipes as a percent of their remaining payments,
s_4	= salvage value of well assemblies as a percentage of their remaining payments.

The total cost function of one doublet is therefore

$$\begin{aligned}
 q(Q, T_1, L) &= 0 && \text{if } Q = 0 \\
 &= \int_0^L [(PM+HE)CRF(1,10) + (WA+PP)CRF(1,25) \\
 &\quad + (WC)CRF(1,WL) + WM + PC + PO(t)]e^{-it} dt \\
 &\quad + [(1-s_1)PM + (1-s_2)HE] CRF(1,10) \int_L^{L_1} e^{-it} dt
 \end{aligned} \tag{34}$$

$$\begin{aligned}
& + [(1-s_3)PP + (1-s_4)WA] \text{CRF}(1,25) \int_L^{L_2} e^{-it} dt \\
& + (WC) \text{CRF}(1,WL) \int_L^{L_3} e^{-it} dt \quad \text{if } 0 < Q < \bar{Q},
\end{aligned}$$

where $\text{CRF}(1,.)$ is the capital recovery factor defined in (26).

The last three terms give the present value of the extra costs associated with project termination prior to completion of lifetime cycles of various equipment components.

To better visualize these extra termination costs, suppose L , the life of the project is 23 years and the useful life of wells, WL , has been assumed to be 10 years. Then L_1 and L_3 are 30 year and $L_2 = 25$ years. The wells which have been drilled in the beginning of the 21st year still have a useful life of seven years. Since wells do not have any salvage value, we assume the remaining payments on the well cost become immediately payable. The present value of this cost is

$$WC \cdot \text{CRF}(1,10) \int_{23}^{30} e^{-it} dt.$$

The other equipment however, have a salvage value. As an example consider pipes which have a life of 25 years. The present value of the unpaid cost is

$$PP \cdot \text{CRF}(1,25) \int_{23}^{25} e^{-it} dt.$$

Since a percentage of this cost, namely s_3 , can be recovered by resale, the termination cost for pipes is

$$(1-s_3)PP \cdot CRF(1,25) \int_{23}^{25} e^{-it} dt.$$

In the remaining parts of this section, we will obtain relationships describing the costs for pipes, pipe cleaning, pumps, operation of pumps and heat exchangers as functions of our decision variables Q and T_1 .

5.1 Pipe Costs

The data in Table 1 have been supplied by a leading pipe manufacturer [Lupear, (1976)] for the cost of steel pipe. We will assume a flow velocity v of 6 ft/sec. Both the flow velocity and the pipe specification are made in accordance with standard industrial practice which is based on suboptimization analyses [Lombard, (1976)]. Multiplying the flow rate Q (in m^3/hr) by 16.938 gives the flow rate in in^3/sec . From

$$Q = \pi d^2 v / 4,$$

and $v = 72$ in/sec, we obtain

$$\begin{aligned} d &= (4 \times 16.938 Q / \pi \times 72)^{0.5} \\ &= .5473 Q^{0.5} \text{ inches} \end{aligned} \quad (35)$$

A polynomial regression of degree two gives the following relationship between C^P , the cost/ft and d :

$$C^P = 0.1337 d^2 + 0.737 d - 1.33 \quad (36)$$

Substituting the value for d from (35) in (36), and multiplying by 3.28 (ft/m) and D (the separation distance in meters) yields the pipe cost:

$$\text{Pipe Cost (\$)} = k_p D [0.1313 Q + 1.323 \sqrt{Q} - 4.36], \quad (37)$$

Table 1
Pipe Cost Data

d (diameter inches)	C^P(Cost, \$/ft)
4	4.08
6	6.89
8	13.06
10	19.99
12	28.92
14	33.98
16	42.93
18	55.89
20	67.30
Cost of ASTM - Grade B Steel Pipe - Schedule 60	
Source: H. Lupear, 1976	

where k_p is a cost multiplier to reflect pipe support and installation costs.

We will assume an additional cost for pipe cleaning. The pipe cleaning cost, PC is proportional to the length of the pipeline.

Hence

$$PC = p_c \cdot D \quad (38)$$

where p_c is the estimated annual cost of cleaning one meter of pipe.

5.2 Pump Costs

Because of the very large drawdown generated in the production well at rather high production rates (see equations 30 and 31), it is not economical and probably infeasible, for a single pump to lift the brine from the aquifer, pump it through the pipeline and heat exchanger to the injection well and overcome the pressure build-up in the injection well. We therefore require that two pumps be used for each doublet. The first will be a vertical turbine pump installed in the production well. This pump will lift the brine to the surface and send it through the piping system and heat exchanger to a second pump, the latter being a horizontal pump capable of pumping the brine back to the aquifer through the injection well.

In this section, we obtain a function describing the relationship between extraction rate and total pumping costs. In parts (a) and (b) we deal with the costs of the production and injection pumps respectively, and in part (c) we discuss the pump operating costs.

5.2.1 The Production Pump

The vertical turbine pump in the production well will discharge the brine to a surface piping system. The pump consists of five component assemblies: 1) the Drive, an electric motor, 2) The Discharge Assembly, on which the motor is mounted, 3) the vertical Lineshaft, 4) the Column Assembly, through which the lineshaft extends and 5) the Bowl Unit.

In this section, we present our estimation of the cost of each individual component. The pump cost, PM, is the sum of these costs and the cost of the horizontal pump discussed in part (b).¹

(i) The Motor Cost

The cost of the electric motor is a function of its brake horsepower, as given in Table 2. A linear regression gives the following relationships between motor cost and horsepower (with a correlation coefficient of 0.998):

$$\text{Motor Cost (\$)} = 14.97 \text{ HP} + 1907.1. \quad (39)$$

We next consider the horsepower requirement of the drive.

When the rotating energy of the drive is transmitted through the lineshaft, some of this energy is lost in the lineshaft bearings by mechanical friction. For each RPM, the shaft horsepower loss per 100 feet of lineshaft can be obtained from pump manufacturers' bulletins as a function of shaft diameter. On the other hand, once the total hydraulic downthrust and the horsepower requirement

¹The authors gratefully acknowledge the assistance of Craig Brown and George Crabtree of FMC Corporation, Peerless Pump Division, who supplied cost data and design features of Peerless pumps.

Table 2
Pump Motor Costs

Horsepower, HP	Cost, \$
250	5578
300	6387
350	7212
400	8039
450	8890
500	9622
600	10440
700	12024
800	13741
900	15459
1000	17176
Cost of 1760 RPM, 60 cycle, General Electric motors	
Source: G. Crabtree, 1976	

of the pumping unit are known, the shaft diameter can be determined. The horsepower in equation (39) is the sum of the horsepower requirement (brake horsepower to pump, BHP) and the horsepower loss.

The brake horsepower (BHP) is given by [Peerless, Bulletin B-141 p.15]:

$$\text{BHP} = \frac{\text{Capacity (gpm)} \cdot \text{Total Head(ft)} \cdot \text{specific gravity}}{3960 \cdot \text{Vertical Pump Efficiency}} \quad (40)$$

The total head in equation (40) is the sum of the distance the brine has to be lifted and friction losses in the system. Friction losses consist of the loss caused by the skin friction as the water rises in the column pipe as well as friction losses in the heat exchanger and the pipeline system. We will denote the friction losses by b . Of course, for accurately estimating b , the flow rate Q (which determines the pipe and column diameter and indirectly the shaft size and hence friction losses in pipes and column) and heat exchanger area A (which with Q determines the head losses in the heat exchanger) must be known. Since we are seeking these quantities (and the magnitude of b is small compared to the total head), we assume a value for b in the cost function. Once the optimal Q and A are known, b can be more accurately estimated and if its value is significantly different from the original b , Q and A can again be computed based on the new value for b . Our experience indicates that the value for b can be safely estimated at 20 m. (The pressure drop in a 12 inch 1000 ft. standard steel pipe with $Q=500 \text{ m}^3/\text{hr}$ is 3.05 m [Peerless, Brochure EM77, p.11] and the skin friction in a 12" column of length 765 ft. containing a shaft of 2-3/16" diameter is 4.9 m. [Peerless, Bulletin B-185,

p.81]. The remaining 12.05 m is ample enough for the pressure drop in the heat exchanger [Perry (1950), p.391]).

Let z be the static level of the brine, that is, the vertical distance in meters between the discharge and the free pool when no water is being pumped. The total head is therefore the sum of the drawdown ΔP , z and b . From equation (40) therefore,

$$\begin{aligned} \text{BHP} &= \frac{Q \times 264/60 [\Delta P(Q) + z + b] 3.28 \times \text{sp. gr.}}{3960 \times \text{Eff}_V} \\ &= \frac{0.003644 Q \text{ sp. gr.}}{\text{Eff}_V} [\Delta P + z + b], \end{aligned} \quad (41)$$

where ΔP is a function of Q and is given by equation (30).

Our next step is the computation of shaft horsepower losses. From the horsepower ratings table for AISI-1045 threaded lineshaft - 1760 RPM [Peerless, Bulletin B-185, p.85], we note that the HP rating for a given shaft diameter does not significantly vary over a wide range of the thrust values (1000-20000 lbs). We take the values corresponding to 10000 lbs. thrust, and thus relate the shaft diameter to BHP. Since both HP loss and shaft and tubing costs are related to shaft diameter [Peerless, Bulletin B-185, p.84, 87; Crabtree (1976)], we can also relate the latter two quantities to the brake horsepower. The data is given in Table 3.

Taking the mean value of the shaft HP loss column (2.34 HP/100 ft), the estimated HP loss is

$$\text{HP Loss} = 0.0234 (\Delta P + z), \quad (42)$$

which when added to the BHP obtained in equation (41), gives the total horsepower required of the drive. Hence, from (39), (41) and

(42) we obtain

$$\begin{aligned} \text{Motor Cost} = 14.97 \left[\frac{0.003644 Q \text{ sp. gr.}}{\text{Eff}_V} (\Delta P + z + b) \right. \\ \left. + 0.0234 (\Delta P + z) \right] + 1907.1 \end{aligned} \quad (43)$$

(ii) The Discharge Assembly Cost

This component constitutes a minor portion of the pump costs. In the cost estimation, we take the cost of a 10 x 10 F Standard Fabricated Steel Head, which is recommended by the pump manufacturer for our range of flow rates.

(iii) Cost of Shaft and Enclosing Tube

The vertical lineshaft is enclosed in a tube and extends downward through the column assembly to the bowl unit. As mentioned earlier, the shaft diameter (which determines the tube diameter) is a function of the horsepower requirement of the drive. The last column of Table 3 gives the combined cost of shaft and its enclosing tube per foot of lineshaft length. A linear regression yields the following relationship between the shaft and tube cost, and the horsepower requirement (with correlation coefficient of 0.967):

$$\text{Shaft Cost (\$/ft)} = 0.112 \text{ HP} - 3.089.$$

Therefore,

$$\begin{aligned} \text{Shaft Cost (\$)} = \left\{ 0.112 \left[\frac{0.003644 Q \cdot \text{sp gr}}{\text{Eff}_V} (\Delta P + z + b) \right. \right. \\ \left. \left. + 0.0234 (\Delta P + z) \right] - 3.089 \right\} [3.28 (\Delta P + z)] \end{aligned}$$

which simplifies to

$$\begin{aligned} \text{Shaft Cost (\$)} = [0.001339 Q \frac{\text{sp gr}}{\text{Eff}_V} (\Delta P + z + b) \\ + 0.0768 (\Delta P + z) - 10.132] [\Delta P + z]. \end{aligned} \quad (44)$$

Table 3
Shaft and Tube Costs

BHP	Shaft Diam. ¹ inches	Tube Diam. ¹ inches	Shaft HP Loss ¹ BHP/100 ft.	Shaft & Tube ² Dollar/ft
50.9	1	1-1/2	.53	15.0
81.6	1-3/16	2	.72	19.5
188	1-1/2	2-1/2	1.25	26.2
281	1-11/16	3	1.4	34.6
443	1-15/16	3	1.9	36.9
560	2-3/16	3-1/2	2.3	50.8
767	2-7/16	4	2.9	92.0
1051	2-11/16	5	3.4	163.2
1387	2-15/16	5	4.2	163.2
1814	3-3/16	5	4.8	168.4

1. Bulletin B-185, Peerless Pump Division, FMC Corp., p. 84-87

2. G. Crabtree, 1976

(iv) The Column Assembly Costs

As in the computation of pipe costs, the flow rate determines the size and hence the cost of the column assembly. The setting (the length of the column assembly) is the sum of the static level z , the drawdown and four meters of section pipe connected to the bowl unit. The cost per meter of the column assembly is given by the terms in brackets in equation (37). Hence Column Assembly Cost (CAC) is given by

$$CAC(\$) = [z + 4 + Q \ln (D/r_w)/2\pi Kh] [0.1313 Q + 1.323 \sqrt{Q} - 4.36] \quad (45)$$

(v) The Bowl Unit Cost

The bowl unit or the pumping element consists of one or more pumping stages. Each stage lifts a given quantity of water by a given height and consists of a bowl case and an impeller which rotates at the speed of the drive. The cost of the bowl unit is proportional to the number of required pumping stages and hence to the lift and capacity.

Although any two reference points can be used in the extrapolation of bowl unit cost, we take as our reference points the cost of two bowl units for which data is readily available from pump manufacturers.

Let c_1 be the cost of a bowl unit capable of lifting $50 \text{ m}^3/\text{hr}$ (220 gpm) of water a distance of $\Delta P(50)+z$ meters and c_2 the cost of a bowl unit capable of lifting $250 \text{ m}^3/\text{hr}$ (1100 gpm) of water a distance of $\Delta P(250)+z$ meters.

The Bowl Unit Cost (BUC) is therefore

$$\begin{aligned}
BUC &= c_1 + \frac{(c_2 - c_1) \cdot ([\Delta P(Q) + z] - [\Delta P(50) + z])(Q - 50)}{([\Delta P(250) + z] - [\Delta P(50) + z])(250 - 50)} \\
&= c_1 + \frac{(c_2 - c_1) [(Q - 50) (\ln D/r_w) / 2\pi Kh] (Q - 50)}{[(250 - 50) (\ln D/r_w) / 2\pi Kh] (250 - 50)} \\
&= c_1 + (c_2 - c_1) (Q - 50)^2 / 40000.
\end{aligned} \tag{46}$$

5.2.2 The Injection Pump

The cost of this pump is considerably less than the production pump. We require a horizontal pump with a discharge head of ΔP to overcome the pressure buildup in the injection well. The drive is supplied by an electric motor.

(1) The Pump Cost

The pertinent data [Brown (1976)] is shown in Table 4.

Table 4

Horizontal Pump Data

Capacity (m ³ /hr)	Discharge Hd. (Feet)	Pump Spec. Number	Pump Cost (Dollars)	Base Dollars	Total Dollars
50	76	AD-11	800	400	1200
250	380	TU-15	4000	2000	6000
500	760	TU-22	8500	4000	12500
Source: C. Brown, 1976					

The discharge heads in the second column correspond to the drawdown generated by flow rates in the first column (based on a typical field data). The last column indicates that the cost is proportional to the flow rate Q . Hence

$$\text{Horizontal pump cost } (\$) = 24Q. \quad (47)$$

(11) The Motor Cost

The horsepower of the horizontal pump can be determined from

(40) yielding

$$\text{BHP}_H = \frac{0.003644 Q \cdot \text{sp gr}}{\text{Eff}_H} \Delta P. \quad (48)$$

Substituting in equation (39) yields the cost of the horizontal motor, MC_H :

$$\text{MC}_H(\$) = \frac{0.0546 Q \cdot \text{sp gr}}{\text{Eff}_H} \left[\frac{Q \ln(D/r_w)}{2\pi Kh} \right] + 1907.1 \quad (49)$$

5.2.3 Pump Operating Costs

Let R_t be the price of electricity (\$/kwh) at time t , supplied to the motors of the vertical and horizontal pumps. Since the price of energy is assumed to increase with time, equation (15) yields

$$R_t = R_o e^{rt} \quad \text{for the model of Section 4.1}$$

$$R_t = R_o (1+rt) \quad \text{for the model of Section 4.2}$$

Then, pump operating cost at time t , $\text{PO}(t)$ is given by

$$\text{PO}(t) = (\text{HP}_V + \text{HP}_H) 0.7457 \times 8760 R_t,$$

which, combined with equations (41), (42) and (48), gives

$$\begin{aligned} \text{PO}(t) = R_t k_m \left[\frac{23.8 Q \cdot \text{sp gr}}{\text{Eff}_V} (\Delta P + z + b) + 152.86 (\Delta P + z) \right. \\ \left. + \frac{23.8 Q \cdot \text{sp gr}}{\text{Eff}_H} \cdot \Delta P \right], \end{aligned} \quad (50)$$

where k_m is a multiplier indicating annual maintenance costs of pumps and their motors.

5.3 Heat Exchanger Costs

The heat exchanger cost is proportional to heat exchanger area A. The cost estimates in Table 5 were supplied by a manufacturer [Breese (1976)] for heat exchangers with stainless tubes and cast iron shell:

Heat Exchanger Area (ft ²)	Cost (dollars)
5000	75000
2500	40000

Table 5

Heat Exchanger
Costs

Based on these data, the relationship between the heat exchanger cost (HE) and area A is:

$$HE = 5000 + 150.7 A$$

where A is in m². Combined with equation (14), we can write

$$HE = 5000 + 150.7 \frac{Q}{k(0)} [\ln (T_o - T_s) - \ln (T_1 - T_s)] . \quad (51)$$

5.4 Well and Well Assembly Costs

The cost for wells and their casing has to be determined based on the thickness of the impermeable strata and the aquifer for each individual field. The well assemblies (Christmas tree valves, etc.) are to some extent a function of the capacity. In our computer program, we have allowed these costs as well as an annual cost for maintenance of each pair of production injection wells to be given as inputs.

6. RESULTS

6.1 Preliminaries

In this section we present the results of our analysis of the two economic models discussed in sections 4.1 and 4.2. The objective is to find the extraction rate Q^* , project life L^* and injection temperature T_1^* which maximize the functions described by equations (21) and (25), subject to the constraint that the difference between the production and injection temperatures would remain greater than a prescribed amount, δ degrees centigrade.

We begin by expressing T_1^* as a function of Q and L , thereby reducing the number of decision variables to two. It is easily seen from equations (21) and (25) that the total revenue is a decreasing linear function of T_1 , i.e. the revenue increases as $T_1 \rightarrow T_g$. This follows from the fact that a lower injection rate implies that for any given pumping rate Q , a larger amount of heat can be extracted. However, it follows from equation (14) that for a given steam temperature T_g , achieving lower values of T_1 requires larger and hence more expensive heat exchangers. Since both the cost and revenue functions are continuous in T_1 , it follows that for any given value of Q and L , the optimal injection temperature T_1^* , is achieved at the point where the marginal revenue with respect to T_1 equals the marginal cost of further reducing T_1 . In other words

$$\left. \frac{\partial R_1}{\partial T_1} \right|_{T_1^*} = \left. \frac{\partial C}{\partial T_1} \right|_{T_1^*}$$

and

$$\left. \frac{\partial R_2}{\partial T_1} \right|_{T_1^*} = \left. \frac{\partial C}{\partial T_1} \right|_{T_1^*}$$

where R_1 and R_2 denote revenues for the models of sections 4.1 and 4.2 respectively.

Note that the dependence of the cost function $C(Q, T_1, L)$ on T_1 is only through the cost for heat exchangers which is described by equation (51), and their salvage values. Denoting the sum of terms which do not contain the heat exchanger cost HE in equation (34) by $J(Q, L)$, the total cost function of one doublet can be written as

$$\begin{aligned} C(Q, T_1, L) &= J(Q, L) + \int_0^L HE \cdot CRF(1, 10) e^{-it} dt + \int_L^{L_1} (1-s_2) HE \cdot CRF(1, 10) e^{-it} dt \\ &= J(Q, L) + HE \cdot CRF(1, 10) \left[(1-e^{-iL_1}) - s_2(e^{-iL} - e^{-iL_1}) \right] / i, \end{aligned} \quad (52)$$

where J is independent of T_1 . Let

$$M \equiv \frac{150.7}{k(0)} CRF(1, 10) \left[(1-e^{-iL_1}) - s_2(e^{-iL} - e^{-iL_1}) \right] / i.$$

Substituting the value for HE from (51) in (52) and utilizing (33) we obtain

$$\begin{aligned} C(Q, T_1, L) &= nJ(\bar{Q}, L, i) + n\bar{Q}M \left[\ln(T_o - T_s) - \ln(T_1 - T_s) \right] \\ &\quad + J(Q - n\bar{Q}, L, i) + (Q - n\bar{Q})M \left[\ln(T_o - T_s) - \ln(T_1 - T_s) \right] \\ &= nJ(\bar{Q}, L, i) + J(Q - n\bar{Q}, L, i) + QM \left[\ln(T_o - T_s) - \ln(T_1 - T_s) \right]. \end{aligned}$$

Hence

$$\left. \frac{\partial C}{\partial T_1} \right|_{T_1^*} = - \frac{QM}{T_1^* - T_s}. \quad (53)$$

To obtain $\frac{\partial R_1}{\partial T_1}$, note that equation (21) can be written as

$$\begin{aligned}\Pi &= R_1(Q, T_1, L) - C(Q, T_1, L) \\ &= aQ(T_0 - T_1)\sigma_1 - C(Q, T_1, L)\end{aligned}\quad (54)$$

and equation (25) as

$$\begin{aligned}\Pi &= R_2(Q, T_1, L) - C(Q, T_1, L) \\ &= aQ(T_0 - T_1)\sigma_2 - C(Q, T_1, L),\end{aligned}\quad (55)$$

where σ_1 and σ_2 denote the terms inside the brackets in equation (21) and (25) respectively, yielding

$$\frac{\partial R_1}{\partial T_1} = -aQ \sigma_1 \quad (56)$$

and

$$\frac{\partial R_2}{\partial T_1} = -aQ \sigma_2. \quad (57)$$

Therefore equating (56) and (57) with (53) yields respectively

$$T_1^* = T_s + \frac{M}{a\sigma_1} \quad \text{for the exponential model} \quad (58)$$

and

$$T_1^* = T_s + \frac{M}{a\sigma_2} \quad \text{for the linear model.} \quad (59)$$

And so we conclude that T_1 may appropriately be expressed as a function of Q and L for purposes of optimization.

6.2 The Optimization Algorithm

The algorithm used for obtaining the optimal solution to the linear and exponential models is a grid search over values of L and Q . The lifetime, L , is varied from $L_{\min}$ to $L_{\max}$ in increments of L_{inc} . For each L , the pumping rate Q is varied from $Q_{\min}$ to $Q_{\max}$ in increments of Q_{inc} . These values of $L_{\min}$, $L_{\max}$, L_{inc} , $Q_{\min}$, $Q_{\max}$, and Q_{inc} are specified by the

decision maker according to judgment and are inputs to the computer program. For each Q and L , the δ constraint is checked to make sure the difference between the production and injection temperatures does not fall below δ degrees centigrade. In this regard, note that by equations (17) and (20), the temperature drop is a function of $Q \cdot L$. Therefore if, for a given L and some $\hat{Q}$, ($\hat{Q} \leq Q_{\max}$), $T_0^L(\hat{Q}) - T_1 \leq \delta$, it follows that $T_0^L(Q) - T_1 < \delta$ for all $Q > \hat{Q}$. Thus, the search for an optimal Q ceases at $\hat{Q}$ and resumes with $Q = Q_{\min}$ at $L + L_{\text{inc}}$. For each L and feasible Q , $T_1^*(Q, L)$ is computed from equation (58) or (59), and the values for the present worth of profits, Π are determined. The set of values (Q^*, L^*, T_1^*) that yields the maximum Π , are the optimal decision variables. For our example computations we use $L_{\min} = 0$ years, $L_{\max} = 250$ years, $L_{\text{inc}} = 1$ year, $Q_{\min} = 50 \text{ m}^3/\text{hr}$, $Q_{\max} = 5000 \text{ m}^3/\text{hr}$ and $Q_{\text{inc}} = 5 \text{ m}^3/\text{hr}$.

To determine the profit for each set of decision variables, the present worths of total revenues and costs must be computed. To begin, we estimate the cost of the necessary equipment and their salvage values as well as operating costs according to the equations developed in section 5. These costs are, of course, functions of our decision variables as well as input data. The total discounted cost for each doublet is then computed from equation (34), and total costs $C(Q, T_1, L)$ from equation (33).

As mentioned in section 4, when lifetime L is greater than the breakthrough time τ , equations (21) and (25) yield the present worth of profits. However, when $L \leq \tau$, a modified version of these equations has to be used for the exponential and linear models. The present worth of profits when $L \leq \tau$ is given by

$$\Pi = aQ(T_0 - T_1) \cdot (1 - e^{-\alpha L}) / \alpha - C(Q, T_1, L) \quad (60)$$

for the exponential model and

$$\Pi = aQ(T_0 - T_1) \left[(1 - e^{-iL}) / i - r \left\{ e^{-iL} (1 + Li) - 1 \right\} / i^2 \right] - C(Q, T_1, L) \quad (61)$$

for the linear model. These expressions are derived from equations (18) and (23) by setting the upper limits of the first integrals equal to L since $L \leq \tau$. Of course the second integrals are equal to zero.

In section 6.4 we discuss the results of our computation for both the linear and exponential models. The computation is carried out for a particular set of data which to our best judgment reflects the current value of pertinent costs. The geohydrological data have generally been chosen in the midrange of values associated with known geothermal resources. We will call these data the "basic data" and the models using these data the "basic models."

6.3 The Basic Data

The following data have been used in the analysis of the basic models (all costs are in 1976 dollars).

Thickness of Aquifer, h	100 m
Doublet Separation, D	300 m
Well Radius, r_w	0.15 m
Well Capacity, $\bar{Q}$	500 m ³ /hr
Porosity of Aquifer, ϕ	.20
Intrinsic Permeability, k	200 m.d.
Initial Equilibrium Temperature, T_o	150°C
Temperature of Generated Steam, T_s	109°C
Heat Capacity of Fluid, $\rho_f c_f$	0.92 cal/cc°C
Heat Capacity of Rock, $\rho_R c_R$	0.50 cal/cc°C
Specific Gravity of Fluid	0.9173
Overall Heat Transfer Coefficient of Fluid, $U(0)$	1000 BTU/hr ft ² °F

Friction Losses, b	20 m
Static Level of Fluid, z	0 m
Vertical Pump Efficiency, Eff_v	0.75
Horizontal Pump Efficiency, Eff_H	0.75
Pump Salvage Value as Fraction of Remaining Payments, s_1	0.40
Heat Exchanger Salvage Value as Fraction of Remaining Payments, s_2	0.40
Pipe Salvage Value as Fraction of Remaining Payments, s_3	0.40
Well Assembly Salvage Value as Fraction of Remaining Payments, s_4	0.40
Pipe Cleaning Cost, p_c	10 \$/m/year
Pipe Support Multiplier, k_p	1.25
Cost of 50 m ³ /hr Bowl Unit, c_1	125 dollars
Cost of 250 m ³ /hr Bowl Unit, c_2	3941 dollars
Pump Maintenance Cost Coefficient, k_m	1.10
Well Cost per Doublet, WC	600,000 dollars
Well Maintenance Cost, WM	6000 \$/year/doublet
Useful life of Wells, WL	25 years
Well Assembly Cost, WA	35,000 dollars
Electricity Cost in 1976, R_o	3¢/kwh
Annual Salaries	50,000 \$/year
Annual Rents	4,000 \$/year
Royalty, η	.10
Minimum allowable Temperature Difference, δ	6°C

The absolute viscosity η (in poises) of the fluid is directly computed from the Bingham formula [Bingham, (1922) p. 340]]:

$$\frac{1}{\mu} = 2.1482 \left[(T_o - 8.435) + \sqrt{8078.4 + (T_o - 8.435)^2} \right] - 120$$

Multiplying μ in poises by 0.00209 gives the viscosity in $\text{lb}_f \text{ sec/ft}^2$.

6.4 Results for the Basic Models

In this section we present our results for a set of interest rates i , and a set of rates of increases in the price of energy, r , using the basic data. As we mentioned in section 4, the determination of a single market interest rate is very difficult. In fact, it is more likely that different investors would use different interest rates, depending on their perception of risk and their best alternative investment opportunities. Accordingly, we present our results for a range of interest rates from $i=0.06$ to $i=0.15$. We also vary the value for r within a range that we feel corresponds to likely futures, namely, from zero to three per cent, enabling us to examine the sensitivity of our decision variables to changes in this important parameter as well. In addition to this general survey we discuss in some detail the results when $r=0.024$ and $i=0.10$ which we have chosen as our basic case. Energy prices are forecast to increase at 2.4 percent per year in constant dollars according to the 1977 National Energy Outlook [Federal Energy Administration (1977)], and 10 percent in the discount rate designated by the Office of Management and Budget [OMB Circular A-94].

6.4.1 Profits

We begin with the present worth of maximum profits, Π for the exponential and linear models respectively as shown in Tables 6 and 7. The values across the top row represent discount rates, while those in the

TABLE 6

PRESENT WORTH OF MAXIMUM PROFITS, Π^* (\$1976,\$1000)

FOR EXPONENTIAL GROWTH MODEL

$i \backslash r$	0.06	0.08	0.10	0.12	0.15
0.000	374.	317.	266.	220.	160.
0.010	554.	456.	375.	308.	226.
0.020	830.	620.	505.	412.	303.
0.024	1065.	728.	561.	457.	337.
0.030	1503.	938.	688.	537.	389.
P_o \$/METU	1.104	1.107	1.110	1.113	1.118

TABLE 7

PRESENT WORTH OF MAXIMUM PROFITS, Π^* (\$1976,\$1000)

FOR LINEAR GROWTH MODEL

$i \backslash r$	0.06	0.08	0.10	0.12	0.15
0.000	374.	317.	266.	220.	160.
0.010	543.	447.	369.	304.	223.
0.020	717.	584.	479.	393.	291.
0.024	787.	639.	523.	429.	318.
0.030	914.	721.	588.	482.	358.
P_0 \$/METU	1.104	1.107	1.110	1.113	1.118

bottom denote values for P_0 , the 1976 value of one million BTU of 5 psi pipeline steam (which depends on interest rate by equation 27). The left column contains the values for r . For each i and r the optimal profit is given in the table.

As expected, the tables show that optimal profits increase as r increases and decrease as i increases. That this would always be true can be seen by the following argument. Obviously for fixed Q and L , Π is decreasing in i and increasing in r . Let $i_1 < i_2$ and suppose Q_1, L_1 maximize $\Pi(i_1; Q, L)$ and Q_2, L_2 maximize $\Pi(i_2; Q, L)$. Then,

$$\Pi(i_1; Q_1, L_1) \geq \Pi(i_1; Q_2, L_2) > \Pi(i_2; Q_2, L_2).$$

Similarly let $r_1 < r_2$ and suppose Q^1 and L^1 maximize $\Pi(r_1; Q, L)$ and Q^2, L^2 maximize $\Pi(r_2; Q, L)$. Then

$$\Pi(r_2; Q_2, L_2) \geq \Pi(r_2; Q_1, L_1) > \Pi(r_1; Q_1, L_1).$$

For each i and r , the profits for the exponential model are higher than the linear model. This follows from the fact that $\int f(t)e^{rt}dt \geq \int f(t)(1+rt)dt$ with the equality holding when $r=0$. Note that the profits in tables 6 and 7 are identical when $r=0$.

6.4.2 Optimal Pumping Rate

We come now to the most important decision variable, Q . Tables 8 and 9 present the optimal pumping rates, Q^* , for the two models. The optimal pumping rates increase as i increases and decrease as r increases, with the values for the linear model dominating those of the exponential model (though not by much, except when $r=3.0$). Furthermore, the difference between the optimal pumping rates for the two models increases as r increases. These are consistent with intuition whereby as r is increased, we tend to extract heat more slowly, thus reserving a larger amount for

TABLE 8

OPTIMAL PUMPING RATE, Q^* (CUBIC METERS/HR)

FOR EXPONENTIAL GROWTH MODEL

$i \backslash r$	0.06	0.08	0.10	0.12	0.15
0.000	380.	390.	400.	405.	420.
0.010	365.	375.	385.	395.	410.
0.020	325.	375.	385.	395.	410.
0.024	320.	345.	385.	395.	410.
0.030	310.	340.	360.	380.	405.
P_o \$/METU	1.104	1.107	1.110	1.113	1.118

TABLE 9

OPTIMAL PUMPING RATE, Q^* (CUBIC METERS/HR)

FOR LINEAR GROWTH MODEL

i r	0.06	0.08	0.10	0.12	0.15
0.000	380.	390.	400.	405.	420.
0.010	365.	375.	385.	395.	415.
0.020	365.	375.	385.	395.	410.
0.024	365.	375.	385.	395.	410.
0.030	325.	375.	385.	395.	410.
P_o \$/METU	1.104	1.107	1.110	1.113	1.118

the future when value is higher. However as i increases, we tend to extract heat at a higher rate, leaving less for the future when energy value (although rapidly increasing) is heavily discounted. The fact that the energy values are higher for the exponential model than the linear model accounts for the decision to extract less heat when the growth rate is exponential, especially when r is large.

6.4.3 Optimal Project Life

Tables 10 and 11 present the optimal project lives for the two models. The economic lives are nonincreasing in i and nondecreasing in r , with the values for the exponential model dominating those of the linear model. Thus, as future profits are discounted more heavily, we tend to extract a greater amount of heat per unit of time over a shorter period in comparison to when the discount rate is not as high. On the other hand, when the energy is expected to rapidly increase in value with time, extraction of heat over a longer period of time is more profitable.

The cross signs on some of the lifetimes in Tables 10 and 11 indicate the beginning of a range of optimal values for L . That is, the present worth of maximum profits remains constant for lifetimes greater than the numbers indicated by a cross, until a time L_δ , after which it decreases. Although the project lives within the range of $L^\dagger$ to L_δ are equally desirable from a profit perspective, they are all associated with the same values of Q^* and A^* . Accordingly which L^* is actually "best" is of no operational importance.

For the basic model and $\delta=6$, $L_\delta \geq L_{\max}$ which we have chosen as 250 years. Therefore Π^* is effectively constant from $L^\dagger$ to 250 years for these results. We shall return to L_δ and its relationship with δ in section 6.4.5. However, we continue now with our discussion of the 'profit plateau' phenomenon.

TABLE 10

ECONOMIC RESERVOIR LIFE, L^* (YEARS)

FOR EXPONENTIAL GROWTH MODEL

i r	0.06	0.08	0.10	0.12	0.15
0.000	20.	20.	20.	20.	20.
0.010	25.	25.	25.	25.	25.
0.020	250. [†]	25.	25.	25.	25.
0.024	250. [†]	215. [†]	25.	25.	25.
0.030	250. [†]	220. [†]	165. [†]	125. [†]	25.
P_o \$/MBTU	1.104	1.107	1.110	1.113	1.118

† Denotes the start of an optimal range of values. (See section 6.4.3.)

TABLE 11

ECONOMIC RESERVOIR LIFE, L^* (YEARS)

FOR LINEAR GROWTH MODEL

$i \backslash r$	0.06	0.08	0.10	0.12	0.15
0.000	20.	20.	20.	20.	20.
0.010	25.	25.	25.	25.	20.
0.020	25.	25.	25.	25.	25.
0.024	25.	25.	25.	25.	25.
0.030	220.†	25.	25.	25.	25.
P_0	1.104	1.107	1.110	1.113	1.118
\$/MBTU					

† Denotes the start of an optimal range of values. (See section 6.4.3.)

6.4.4 Response of the Profit Function

It is now possible to discuss the response of the profit function to values of L and $Q^*(L)$, the pumping rate which maximizes $\Pi(L, Q)$ for a given L . We begin by noting that the objective functions in equations (21) and (25) tend to become independent of L as L gets large. That is, for a given Q , there exists a $\Pi(Q)$ such that

$$\lim_{L \rightarrow \infty} \Pi(L, Q) = \Pi(Q) \quad (62)$$

for both the exponential and linear models. This follows from the fact that for both models, the revenues are monotone increasing functions of L and the terms containing L approach zero as $L \rightarrow \infty$. Furthermore the cost function $C(Q, T_1, L) \rightarrow C(Q)$ as $L \rightarrow \infty$. This last statement is a consequence of the fact that by (34) we can write

$$\begin{aligned} q(Q, T_1, L) = & AC(Q) \left(\frac{1 - e^{-1L}}{1} \right) + OP(Q) \left(\frac{1 - e^{-\alpha L}}{\alpha} \right) \\ & + \sum_{j=1}^3 c_j \left(\frac{e^{-1L} - e^{-1L_j}}{1} \right), \end{aligned} \quad (63)$$

where AC indicates the annual costs (excluding pump operating costs), OP is the first year pump operating costs and the last term is the total termination costs. From (63),

$$\lim_{L \rightarrow \infty} q(Q, T_1, L) = \frac{AC(Q)}{1} + \frac{OP(Q)}{\alpha}$$

and therefore $C(Q, T_1, L)$ approaches $C(Q)$ as L gets large. On the other hand, the terms R_1 and R_2 in equations (54) and (55) depend only on Q as L approaches infinity and therefore for L large enough, Π is dependent only on Q . The validity of (62) is thus established.

Let Q' be the maximand of $\Pi(Q)$ which is defined by (62). It

follows that

$$\lim_{L \rightarrow \infty} Q^*(L) = Q',$$

which implies that for L larger than some L' , not only the pumping rate but the other quantities of interest, namely the injection temperature T_1^* , the breakthrough time τ^* and the cost function C remain effectively constant. Our computational experience indicates that for most cases, L' is around 100 years.

The asymptotic behavior of the profit function and the convergence of $\Pi(L, Q)$ to $\Pi(Q)$ and $Q^*(L)$ to Q' is illustrated in Tables 12 and 13. Here increasing values of L are given in the second column. For each L , the value of $\Pi[L, Q^*(L)]$ and $Q^*(L)$ are shown as well as the corresponding values for optimal heat exchanger area A^* , injection temperature T_1^* , breakthrough time τ^* , total costs $C(Q, L, T_1)$ and pump capacity and operating costs (all optimal with respect to the given value of L).

Note also that in the base case presented in Tables 12 and 13, ($r=0.024$, $i=0.10$), the optimal life occurs at 25 years which is the assumed well life. Because a second well cost must be incurred if project life is greater than one well life, there is always a local maximum for the profit function $\Pi[L, Q^*(L)]$ at $L = \text{well life}$. In the base case this local maximum exceeds $\Pi(Q)$ and hence $L = 25$ is optimal. However, when the value of the energy is allowed to increase at a faster rate than $r=0.024$ (or alternatively if the discount rates are small) $\Pi(Q)$ is larger than peaks attained at $L = WL$ and large lifetimes are optimal. This can be seen in Figure 4 where for $r=0.03$, $\Pi(Q) \geq \Pi(L, Q)$ for all L when $i \leq 0.12$. Note, however, that as i increases, $\Pi(Q)$, the plateau of the profit function gets closer to zero, so that there is an i above which the local maximum at the well life dominates $\Pi(Q)$. For $r=0.03$, this interest rate is 0.15

Figure 4. Present Worth of Total Profits vs Project Life for Given Interest Rates

(as seen in Figure 4, $\Pi[25, Q^*(25)]$ dominates $\Pi(Q)$). As we further increase i , the present worth of maximum profits decreases until at $i=0.32$ when $\Pi^*=0$ (which means, incidentally, that the so-called "internal rate of return" is 32% for this particular value of r). For $i > .15$, the economic life remains at 25 years.

Note that 32% is the maximum internal rate of return in our range of growth rates. The minimum internal rate of return for the project is obtained when $r=0$. This minimum rate is 25.6 percent.

6.4.5 The Effect of δ

In section 6.4.4, the effect of δ , the minimum allowable temperature difference between production and injection temperature has been neglected. In Tables 12 and 13, the optimal pumping rates are small enough that even after 250 years, the temperature difference is still greater than 6°C , and hence the pumping rates remain unaffected during the period under consideration (250 years). In fact, for every positive δ , there is an L_δ such that $Q^*(L)$ and hence the corresponding profits, $\Pi[L, Q^*(L)]$, will decrease as L increases beyond L_δ . Thus L_δ signifies the end of the plateau region of the profit function. This phenomenon is observed in Figure 8 and 9 where for higher production temperatures (which imply higher flow rates), $Q^*(L)$ and Π decrease after the corresponding L_δ .

Note that by (12) and (20), the temperature difference is a decreasing function of $L \cdot Q(L)$. As discussed in Section 6.4.4, if no restriction existed on the temperature difference, the pumping rate $Q^*(L)$ would remain constant for $L > L'$. Since $g(L_\delta/t_u) = \delta$, it follows that as L increases beyond L_δ , the value of $Q^*(L)$ decreases so that the δ constrain is not violated.

The quantity $Q^*(L_\delta) \cdot L_\delta$ can be easily computed. Noting that the

TABLE 12

PROFIT AND DECISION VARIABLES RESPONSE TO LIFE

EXPONENTIAL GROWTH MODEL ($i = 0.10$, $r = 0.24$)

$\Pi[L, Q^*(L)]$	L	$Q^*(L)$	A^*	T_1^*	τ^*	$C(Q^*, T_1^*, L)$	PUMPS	POC
(\$10^3)	(YRS)	(m ³ /hr)	(m ²)	(°C)	(YRS)	(\$10^3)	(\$10^3)	(\$10^3)
47.51	5.	470.	447.	109.27	1.45	2021.8	96.0	655.4
345.50	10.	435.	432.	109.22	1.56	2488.7	98.0	950.1
443.58	15.	405.	396.	109.24	1.68	2740.4	113.1	1057.5
529.06	20.	395.	391.	109.22	1.72	2901.6	112.9	1157.6
560.89	25.	385.	379.	109.23	1.77	2993.9	117.0	1199.3
519.18	30.	380.	376.	109.22	1.79	3122.4	116.0	1234.6
532.02	35.	375.	371.	109.22	1.81	3151.7	116.6	1246.9
544.51	40.	370.	367.	109.22	1.84	3153.6	114.6	1244.0
550.38	45.	370.	367.	109.22	1.84	3181.2	115.7	1263.7
555.85	50.	370.	367.	109.22	1.84	3198.2	115.9	1277.3
552.71	55.	370.	367.	109.22	1.84	3216.3	116.4	1286.5
555.23	60.	365.	362.	109.22	1.86	3185.9	113.8	1259.5
556.65	65.	365.	363.	109.22	1.86	3191.2	114.0	1263.7
557.83	70.	365.	363.	109.22	1.86	3194.5	114.0	1266.5
558.52	75.	365.	363.	109.22	1.86	3196.9	114.0	1268.5
558.59	80.	365.	363.	109.22	1.86	3198.9	114.1	1269.9
558.93	85.	365.	363.	109.22	1.86	3200.0	114.1	1270.8
559.19	90.	365.	363.	109.22	1.86	3200.7	114.1	1271.4
559.35	95.	365.	363.	109.22	1.86	3201.2	114.1	1271.8
559.47	100.	365.	363.	109.22	1.86	3201.5	114.1	1272.1
559.50	105.	365.	363.	109.22	1.86	3201.7	114.1	1272.3
559.56	110.	365.	363.	109.22	1.86	3201.9	114.1	1272.5
559.60	115.	365.	363.	109.22	1.86	3202.0	114.1	1272.6
559.62	120.	365.	363.	109.22	1.86	3202.1	114.1	1272.6
559.64	125.	365.	363.	109.22	1.86	3202.1	114.1	1272.7
559.65	130.	365.	363.	109.22	1.86	3202.1	114.1	1272.7
559.65	135.	365.	363.	109.22	1.86	3202.2	114.1	1272.7
559.66	140.	365.	363.	109.22	1.86	3202.2	114.1	1272.7
559.66	145.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.66	150.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.66	155.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	160.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	165.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	170.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	175.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	180.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	185.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	190.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	195.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	200.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	205.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	210.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	215.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	220.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	225.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	230.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	235.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	240.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	245.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	250.	365.	363.	109.22	1.86	3202.2	114.1	1272.8

TABLE 13

PROFIT AND DECISION VARIABLES RESPONSE TO LIFE

LINEAR GROWTH MODEL ($i = 0.10$, $r = 0.024$)

$\Pi[L, Q^*(L)]$	L	$Q^*(L)$	A^*	T_1^*	τ^*	$C(Q^*, T_1^*, L)$	PUMPS	POC
(\$10^3)	(YRS)	(m ³ /hr)	(m ²)	(°C)	(YRS)	(\$10^3)	(\$10^3)	(\$10^3)
45.37	5.	470.	447.	109.27	1.45	2020.4	96.0	654.0
336.54	10.	435.	432.	109.22	1.56	2481.9	98.0	943.4
424.89	15.	405.	395.	109.24	1.68	2726.1	113.1	1043.4
500.12	20.	395.	389.	109.23	1.72	2877.8	112.9	1134.2
522.50	25.	385.	378.	109.23	1.77	2961.3	117.0	1167.1
472.82	30.	380.	374.	109.23	1.79	3081.3	116.0	1194.0
479.18	35.	375.	369.	109.23	1.81	3103.6	116.6	1199.4
486.68	40.	375.	369.	109.23	1.81	3135.8	117.2	1222.4
488.66	45.	370.	364.	109.23	1.84	3122.4	115.7	1205.6
491.36	50.	370.	365.	109.23	1.84	3135.1	115.9	1214.9
486.18	55.	370.	365.	109.23	1.84	3149.8	116.4	1220.8
487.18	60.	370.	365.	109.23	1.84	3154.9	116.4	1224.6
487.48	65.	370.	365.	109.23	1.84	3158.3	116.6	1227.1
487.86	70.	370.	365.	109.23	1.84	3160.3	116.6	1228.6
487.99	75.	370.	365.	109.23	1.84	3161.7	116.7	1229.6
487.65	80.	370.	365.	109.23	1.84	3163.0	116.7	1230.2
487.70	85.	370.	365.	109.23	1.84	3163.5	116.7	1230.6
487.76	90.	370.	365.	109.23	1.84	3163.8	116.7	1230.9
487.78	95.	370.	365.	109.23	1.84	3164.0	116.7	1231.0
487.80	100.	370.	365.	109.23	1.84	3164.2	116.7	1231.1
487.77	105.	370.	365.	109.23	1.84	3164.3	116.7	1231.1
487.78	110.	370.	365.	109.23	1.84	3164.3	116.7	1231.2
487.78	115.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	120.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	125.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	130.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	135.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	140.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	145.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	150.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	155.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	160.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	165.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	170.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	175.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	180.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	185.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	190.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	195.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	200.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	205.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	210.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	215.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	220.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	225.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	230.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	235.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	240.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	245.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	250.	370.	365.	109.23	1.84	3164.4	116.7	1231.2

first term in equation (20) dominates the other two ($\psi_1 = \frac{1}{47.5}$, $\psi_2 = \frac{1}{580}$, ψ_3), equation (17) can be written as

$$\gamma_1 e^{-\psi_1 Q^*(L_\delta) \cdot L_\delta} \approx \frac{\delta}{T_o - T_1}$$

yielding

$$Q^*(L_\delta) \cdot L_\delta = \frac{1}{\psi_1} \ln \frac{\gamma_1 (T_o - T_1)}{\delta} \quad (64)$$

Unless the value for δ is chosen very large (and therefore not permitting $Q(L)$ to approach Q'), $Q^*(L_\delta) = Q'$. Note from equation (64) that for fixed Q , T_o increases the value for L_δ . However, as T_o increases, $Q^*(L)$ increases for all L (and hence for L_δ), reducing the value for L_δ . The total effect of increase in T_o is that $\frac{1}{\psi_1 Q^*(L_\delta)} \ln \frac{\gamma_1 (T_o - T_1)}{\delta}$ and hence

L_δ decreases as T_o increases. Figure 9 illustrates this point.

6.4.6 Optimal Injection Temperature

Tables 14 and 15 give the optimal injection temperature T_1^* . The optimal injection temperatures decrease slightly as r increases and increase slightly as i increases. This can be explained by the fact that at higher pumping rates, it becomes more expensive to attain lower injection temperatures. However, although there is some small variation of T_1^* with r and i , the most important information conveyed by this table is that 1) the value of T_1^* is remarkably stable with respect to i and r , and 2) this value of T_1^* is very close to T_g (the temperature of 5 psi steam: 109°C). The resulting high costs of heat exchanger equipment are evidently offset by the value of the extra energy extracted by having T_1^* close to T_g .

TABLE 14

OPTIMAL INJECTION TEMPERATURE, T_1^* (DEG. CENT.)

FOR EXPONENTIAL GROWTH MODEL

<div> <div>i</div> <div>r</div> </div>	0.06	0.08	0.10	0.12	0.15
0.000	109.23	109.25	109.26	109.27	109.28
0.010	109.23	109.24	109.25	109.26	109.27
0.020	109.19	109.22	109.23	109.24	109.26
0.024	109.18	109.20	109.23	109.24	109.25
0.030	109.15	109.18	109.20	109.22	109.25
<div> <div>P_o</div> <div>\$/METU</div> </div>	1.104	1.107	1.110	1.113	1.118

TABLE 15

OPTIMAL INJECTION TEMPERATURE, T_i (DEG. CENT.)

FOR LINEAR GROWTH MODEL

$i \backslash r$	0.06	0.08	0.10	0.12	0.15
0.000	109.23	109.25	109.26	109.27	109.28
0.010	109.23	109.24	109.25	109.26	109.27
0.020	109.22	109.23	109.24	109.25	109.26
0.024	109.21	109.22	109.23	109.24	109.26
0.030	109.19	109.21	109.22	109.23	109.25
P_o \$/METU	1.104	1.107	1.110	1.113	1.118

Note from the footnote on page 20 that although the production temperature T_o^t increases as T_1 increases, the difference $T_o^t - T_1$ which determines the amount of recoverable heat at time t , decreases as T_1 increases. This confirms equations (56) and (57) which show a decrease in total revenues as T_1 increases. We conclude that, although injecting the brine at a high temperature prolongs the duration of time that the production temperature remains above a specified level, it has no effect on prolonging the economic lifetime of the project.

6.4.7 Optimal Heat Exchanger Area

Tables 16 and 17 give the optimal heat exchange areas. The optimal areas increase with i and decrease with r . As i increases and r decreases the optimal flow rates increase, thus requiring larger heat exchanger areas, even though the fluid is injected at higher temperatures.

6.4.8 Optimal Breakthrough Time

Tables 18 and 19 give the optimal breakthrough times τ^* . These times are inversely proportional to Q^* . Note that the optimal breakthrough times occur very early in the project, namely during the first two years -- long before L^* .

6.4.9 Costs

Tables 20 and 21 present total project costs and tables 22 and 23 the total operating costs for pumps. Note that the pump operating costs, which are mainly the cost of the electricity used to operate the pumps, constitute a major portion of total costs. The electricity cost, as described in section 5, increases with time at the rate of e^{rt} or $1+rt$

TABLE 16

OPTIMAL HEAT EXCHANGER AREA, A^* (SQUARE METERS)

FOR EXPONENTIAL GROWTH MODEL

$i \backslash r$	0.06	0.08	0.10	0.12	0.15
0.000	372.	378.	385.	387.	396.
0.010	358.	365.	372.	379.	389.
0.020	331.	371.	377.	384.	393.
0.024	331.	348.	379.	386.	395.
0.030	329.	349.	362.	376.	393.
P_o \$/METU	1.104	1.107	1.110	1.113	1.118

TABLE 17

OPTIMAL HEAT EXCHANGER AREA, A^* (SQUARE METERS)

FOR LINEAR GROWTH MODEL

i r	0.06	0.08	0.10	0.12	0.15
0.000	372.	378.	385.	387.	396.
0.010	357.	365.	372.	379.	395.
0.020	362.	369.	376.	383.	393.
0.024	364.	371.	378.	384.	394.
0.030	332.	373.	380.	387.	396.
P_o \$/METU	1.104	1.107	1.110	1.113	1.118

TABLE 18

OPTIMAL BREAKTHROUGH TIME, τ^* (YEARS)

FOR EXPONENTIAL GROWTH MODEL

$i \backslash r$	0.06	0.08	0.10	0.12	0.15
0.000	1.79	1.75	1.70	1.68	1.62
0.010	1.86	1.81	1.77	1.72	1.66
0.020	2.09	1.81	1.77	1.72	1.66
0.024	2.13	1.97	1.77	1.72	1.66
0.030	2.20	2.00	1.89	1.79	1.68
P_o \$/METU	1.104	1.107	1.110	1.113	1.118

TABLE 19

OPTIMAL BREAKTHROUGH TIME, τ^* (YEARS)

FOR LINEAR GROWTH MODEL

i r	0.06	0.08	0.10	0.12	0.15
0.000	1.79	1.75	1.70	1.68	1.62
0.010	1.86	1.81	1.77	1.72	1.64
0.020	1.86	1.81	1.77	1.72	1.66
0.024	1.86	1.81	1.77	1.72	1.66
0.030	2.09	1.81	1.77	1.72	1.66
P_o \$/METU	1.104	1.107	1.110	1.113	1.118

TABLE 20

PRESENT WORTH OF ALL COSTS, $C(Q^*, T_1^*, L^*)$ (\$1976, \$1000)

FOR EXPONENTIAL GROWTH MODEL

r	0.06	0.08	0.10	0.12	0.15
0.000	3296.	2988.	2743.	2520.	2295.
0.010	3581.	3171.	2858.	2615.	2344.
0.020	4700.	3290.	2952.	2692.	2402.
0.024	4848.	3787.	2994.	2725.	2427.
0.030	5109.	3922.	3272.	2890.	2440.
P_0 \$/METU	1.104	1.107	1.110	1.113	1.118

TABLE 21

PRESENT WORTH OF ALL COSTS, $C(Q; T_1^*, L^*)$ (\$1976, \$1000)

FOR LINEAR GROWTH MODEL

$i \backslash r$	0.06	0.08	0.10	0.12	0.15
0.000	3296.	2988.	2743.	2520.	2295.
0.010	3571.	3164.	2853.	2611.	2318.
0.020	3692.	3260.	2930.	2675.	2390.
0.024	3740.	3299.	2961.	2700.	2410.
0.030	4701.	3356.	3008.	2738.	2439.
P_o \$/METU	1.104	1.107	1.110	1.113	1.118

depending on the growth model. The pump operating costs constitute between 35 and 46 percent of total costs for the exponential model and between 35 and 41 percent for the linear model. Tables 20 to 23 show that as r increases and i decreases, not only do the pump operating costs and total costs increase, but the ratios of pump operating costs to total costs increase as well.

6.4.10 The "Average Cost" per MBTU

Based on the optimal decision variables and costs, we can also compute a quantity which gives a measure of average costs of generating one MBTU of steam heat over the lifetime of the project. For example, let $i=0.10$ and $r=0.024$. Then for both models, $Q^*=385 \text{ m}^3/\text{hr}$, $L^*=25$ years, $T_1^*=109.23^\circ\text{C}$ and $\tau^*=1.77$ years. Total heat produced, THP, is therefore

$$\begin{aligned} \text{THP} &= 34.76Q^*c_f\rho_f(T_o - T_1^*)\tau^* + \int_{\tau^*}^{L^*} 34.76Q^*c_f\rho_f(T_o - T_1^*)g(t)dt \\ &= 34.76 \times 385 \times .92 (150 - 109.23) \left[1.77 + \int_{1.77}^{25} (0.338 e^{-0.0013t} \right. \\ &\quad \left. + 0.337 e^{-0.06177t} + 1.368 e^{-7.5386t}) dt \right] \\ &= 7,472,778 \text{ MBTU.} \end{aligned}$$

Let C_a be an average yearly cost, so that the total discounted cost if annual payments of C_a are made for L^* years would be equal to $C(Q^*, T_1^*, L^*)$:

$$C(Q^*, T_1^*, L^*) = \int_0^{L^*} C_a e^{-it} dt.$$

With $C(Q^*, T_1^*, L^*) = \$2,993,900$ and $i=0.10$, C_a equals \$326,163. The total undiscounted costs $= C_a \cdot L^* = 8,154,075$ dollars, which makes the average cost for the exponential model equal to

TABLE 22

PRESENT WORTH OF PUMP OPERATING COSTS, (\$1976,\$1000)

FOR EXPONENTIAL GROWTH MODEL

i r	0.06	0.08	0.10	0.12	0.15
0.000	1217.	1096.	998.	895.	803.
0.010	1380.	1203.	1065.	958.	838.
0.020	1936.	1319.	1158.	1034.	895.
0.024	2088.	1550.	1199.	1067.	919.
0.030	2357.	1688.	1346.	1161.	936.
P ₀ \$/METU	1.104	1.107	1.110	1.113	1.118

TABLE 23

PRESENT WORTH OF PUMP OPERATING COSTS, (\$1976,\$1000)

FOR LINEAR GROWTH MODEL

1 r	0.06	0.08	0.10	0.12	0.15
0.000	1217.	1096.	998.	895.	803.
0.010	1371.	1196.	1060.	954.	829.
0.020	1490.	1290.	1137.	1017.	883.
0.024	1533.	1323.	1167.	1042.	902.
0.030	1936.	1385.	1213.	1080.	931.
P ₀ \$/METU	1.104	1.107	1.110	1.113	1.118

$$\$8,154,075/7,472,778 \text{ MBTU} = 1.09 \text{ \$/MBTU}.$$

Similarly, for the linear model, the total undiscounted costs would be 8,070,993 dollars, yielding an average cost of 1.08 \\$/MBTU. Comparing these two values, we see that the difference between results with each growth model is not great.

We wish to emphasize that these are average values over the life of the project when $T_o = 150^\circ\text{C}$. The unit cost of steam heat would be lower at first and higher toward the end of the project. These averages would be substantially lower if the initial temperature were higher than 150°C .

6.5 Sensitivity Analysis

In this section we discuss the sensitivity of the optimal decision variables to changes in the parameters of the model. The rate of increase of the real value of energy has been assumed exponential (the exponential growth model) and the values for i and r are 0.10 and 0.024 respectively. Results for the linear model are generally similar to those of the exponential model, except that profits are lower and optimal pumping rates are higher for the linear model. The plots in Figure 5 to 8 and 10 to 14 represent profits, $\Pi[L, Q^*(L)]$, as a function of L , for different values of the parameter under consideration. The maximum of $\Pi[L, Q^*(L)]$ is of course Π^* .

6.5.1 Sensitivity to Well Life

Figure 5 presents the behavior of the profit function with respect to changes in expected well life. The well life has a great effect on both the profits and the optimal project life. The predominance of 25 year optimal lives in Tables 10 and 11 is due to the fact that the well life has been chosen as 25 years in the basic model. In fact it seems that with the exception of the case where $r=0$, the optimal lives are either

Figure 5. Present Worth of Total Profits vs Project Life for Given Well Lives

equal to the well lives or at L^+ , the point where $\Pi[L, Q^*(L)]$ reaches a plateau. As seen in Figure 5, the optimal project life when well life is below 40 years is equal to the well life. At $WL=40$, the global maximum of Π with respect to L is slightly higher than the local maximum at $WL=40$. Had the interest rates been higher than 10, the plateau of the profit curves would have been lower so that even at $WL=40$ years, L^* would be 40 years.

Note the discontinuity of the profit function at multiples of the well life in Figure 5. These discontinuities can be easily explained by the fact that new wells have to be installed at multiples of well life. For instance if $WL=10$, there is a sudden decrease in the amount of profits if $L=11$, because the entire new well cost must be paid even though only one additional year of heat is produced. For longer lives, the discontinuities become less significant, because the discount rate reduces the incremental cost of each well, and because each cost increment becomes progressively smaller compared to total costs. After 50 years, the profit function is relatively stable with respect to L , implying that the present worth of the well cost paid in full during the 51st year does not affect the present worth of profits significantly.

Note that well life has a significant effect on maximum profits. If expected well life increases from 10 years to 25 years, maximum profits increase from \$420,570 to \$560,890. If well life increases another 15 years from 25 to 40 years, maximum profits increase from \$560,890 to \$606,970. We conclude that the gains from prolonging well life (perhaps by extra maintenance expenditures) are substantial, but characterized by decreasing returns to additional "prolonging efforts."

6.5.2 Sensitivity to Aquifer Porosity and Permeability

Figures 6 and 7 deal with significant geothermal parameters, namely porosity ϕ , and intrinsic permeability k . Figure 6 summarizes the effect of the uncertainty in the value of porosity. If porosity is 10 percent instead of 20 percent (base case), the optimal profit is only \$493,380 instead of \$560,890, a decrease of 12%. Note, however, that the optimal pumping rate is not at all sensitive to porosity in this range. If porosity is 30 percent, the maximum profit is \$628,190 dollars, an increase of 12%. This time, though, the optimal decision variable changes slightly: the pumping rate is reduced from 380 m³/hr to 370 m³/hr. Nevertheless, it is clear that the model is remarkably robust in determining the optimal pumping rate over a wide range of porosities. This is an important result, because it suggests major expenditures to accurately determine porosity in order to compute the "correct" pumping rate would probably not be warranted.

On the other hand, the effect of uncertainties in intrinsic permeability is greater than that for porosity. As seen in Figure 6, reduction in permeability from 200 millidarcies (base case) to 150 millidarcies reduces the optimal profits from \$560,890 to \$241,620, a reduction of 57 percent, and optimal pumping rate from 380 m³/hr to 310 m³/hr, a reduction of about 18 percent. An increase in permeability to 250 increases the profits to \$859,300, an increase of 53 percent, while Q^* increases 13 percent to 430 m³/hr. The permeability governs pumping costs. As permeability increases, the drawdown in the production well decreases, thereby requiring smaller pumps and less energy for pumping the same amount of flow. As a result, the optimal profits are increased. We conclude that unlike porosity, expenditures for accurate information on permeability may be very important, for profits and design flow rate are indeed sensitive to values of permeability.

Figure 6. Present Worth of Total Profits vs Project Life for Given Porosities

Figure 7. Present Worth of Total Profits vs Project Life for Given Permeabilities

6.5.3 Sensitivity to Initial Aquifer Equilibrium Temperature

Among the physical geothermal parameters, the one with the greatest impact on profits and the optimal decision variable, is the initial equilibrium temperature, T_o . This is the temperature at which the geothermal aquifer fluid and the aquifer matrix are in thermal equilibrium. In Figure 8, profits have been plotted as function of project life for $T_o=150^\circ\text{C}$ (our base case), 160°C , 170°C , and 180°C . As seen from this figure, optimal profits are increased with temperature in a nonlinear manner. Optimal profits and flow rates as functions of temperature can be tabulated as follows:

$T_o (^\circ\text{C})$	$\Pi^*(\$)$	$Q^*(\text{m}^3/\text{hour})$	$L^*(\text{years})$
150	560,890	380	50
160	1,982,890	875	150
170	5,007,890	2400	75
180	10,478,610	4900	50

In general, as the equilibrium temperature increases, it is optimal to extract energy at a higher rate and terminate the project in a shorter period. The only exception to this trend is when the temperature is 150°C . Since the profit is rather low at this temperature, the well cost has a severe impact on the profits, thus making it imperative to terminate the project at $L=25$. Even in this case, the total discounted profits if the project is terminated at $L=25$ are not much higher than total profits had the project life been say, 200 years (\$560,890 vs. \$559,510).

Figure 9 gives a plot of the optimal flow rate as a function of project life. As seen from the plot, the δ constraint of 6 degrees does

Figure 8. Present Worth of Total Profits vs Project Life for Given Initial Temperatures

Figure 9. Optimal Pumping Rates vs Project Life for Given Initial Temperatures

not have any effect on the pumping rate when $T_0=150$ or 160°C , because the optimal flow rates are so low that even at $L=250$ years, the constraint is not violated when Q has reached its plateau level Q' . Note however that at the higher temperatures (when Q' is higher), the δ requirement forces $Q^*(L)$ to decrease as L increases. The "reduction point" occurs at 150 years when $T_0=170^\circ\text{C}$ and 55 years when $T_0=180^\circ\text{C}$.

It is interesting that δ does not have any effect on Q^* or Π^* (that is, we would have obtained the same Q^* and Π^* even if δ had been zero. However $Q^*(L)$ would have been larger for larger lifetimes). For both of these temperatures, the profits have peaked at lifetimes shorter than the "reduction point." The explanation for this is that when $T_0=170^\circ\text{C}$ for instance, the plateau flow rate is $Q'=2400\text{ m}^3/\text{hr}$ and at $L=75$ years, the term containing L in the revenue function (equation 21) is negligible, so that the effect of increase in L is minimal. However, the cost function is still increasing in L (since L is not large enough) and therefore as L increases, the profit, which is the difference of these two terms decreases after $L=75$ years.

6.5.4 Sensitivity to Economic Parameters

(1) Cost of Electricity

Figures 10 to 14 deal with economic parameters. The economic parameter that most influences profits and optimal flow rate is the present day cost of electricity for pumping.¹ Figure 10 shows that if electricity could be obtained at a lower cost, profits, optimal flow rate, and economic lifetime would increase. Since pumping energy cost constitutes a major portion of the total costs, this result is hardly surprising.

¹Recall that electric power costs are assumed to rise with time at the same rate as the value of energy, namely, according to the particular growth model.

Figure 10. Present Worth of Total Profits vs Project Life for Given Electricity Costs

However, the magnitude of the increase in profits is interesting. The total discounted optimal profits increase by 43% when electricity cost decreases from 3¢/kwh (base case) to 2.5¢/kwh and by 130% when it decreases to 2¢/kwh. It is interesting to observe that the optimal pumping rate increases by only 10.5 percent from 380 m³/hr to 420 m³/hr when electricity cost decreases from 3¢/kwh to 2.5¢/kwh, while it increases by 142% to 920 m³/hr when electricity cost is decreased to 2¢/kwh. We conclude that pump operating cost plays a major role in the economics of geothermal energy production.

(ii) Royalty

The behavior of profits with respect to changes in royalty are more uniform. Figure 11 shows optimal profits decrease by 37% when royalty is increased from 10% (base case) to 15% while it increases by 38% when royalty is decreased to 5%. Although these changes in royalty cause significant changes in profits, they hardly affect Q^* , as Figure 11 shows. We conclude that the amount of royalty paid is of more concern to the investor than to the design engineer.

(iii) Well Cost

The effect of changes in well costs for each doublet is summarized in Figure 12. An increase of \$200,000 over well cost of \$600,000 (base case) reduces optimal profits by 52% while the same amount of decrease in well cost increases the profits by 57%. Although these changes in royalty and well cost do not significantly alter the optimal pumping rate, it is clear that well cost plays a major role in determining the economic viability of this type (non-electric) of geothermal project.

At this point it is appropriate to make a comment about the long

Figure 11. Present Worth of Total Profits vs Project Life for Given Royalties

Figure 12. Present Worth of Total Profits vs Project Life for Given Well Costs

economic lives in some of the cases presented in Figures 10, 11 and 12. In none of these cases is the optimal profit much higher than the profits at 25 years. For example, if $WC = \$400,000$, the present worth of profits at 25 years is \$854,840 while the profit at the optimal life of 160 years is \$879,850.

(iv) Land Rent and Salaries

Figure 13 deals with changes in land rents and salaries. In our basic model this total amounts to 54,000\$/year. An increase of this total annual cost to \$70,000 decreases the optimal profit by 26% while an increase to \$100,000 decreases the profits by 75%. However, the optimal pumping rate and economic life remain constant at 380 m³/hr and 25 years, respectively, in the face of these changes.

(v) Well Maintenance Costs

The effect of the annual well maintenance cost is the least among the main economic parameters. An increase of 50% in these costs reduces optimal profits by only 5% while doubling these costs reduces profits by less than 10%. Again the optimal pumping rate and economic lifetime remain unchanged. This is an interesting result when considered together with the effect of expected well life on profits. In section 6.5.1 we saw that there were significant gains in profits if well life could be prolonged. Here we see that this approach to well life prolongation -- active well maintenance -- is not very costly. Although we present no relationship between well maintenance expenditures and well life, it is reasonable to speculate that a good well maintenance program could be fairly cost beneficial. However, recalling that gains in profits were increasing at a decreasing rate as well life increased, we can also see that there would

Figure 13. Present Worth of Total Profits vs Project Life for Given Totals of Rent and Salaries

Figure 14. Present Worth of Total Profits vs Project Life for Given Annual Well Maintenance Costs

be some "optimal" maintenance expenditure, beyond which expenditure would outweigh costs. We conclude that potential gains in profits may justify further investigation in this area of "optimal well maintenance."

6.6 Comment about the Shape of the Profit Function

We would like to make a final comment about the shape of the profit function $\Pi(L, Q)$. As seen in Figure 3, for each L , the total cost function is a piecewise convex function of Q . It can be easily shown that the revenue function is concave in Q and therefore the profit function is a piecewise concave function of Q with discontinuities at multiples of $\bar{Q}$, the maximum flow rate from each production well. Since each segment of the profit function is concave each segment has a maximum. We can show that if Q_1 is the maximand of the i^{th} segment, then

$$Q_1 \geq Q_2 - \bar{Q} \geq Q_3 - 2\bar{Q} \geq \dots$$

In other words, the distance between the maximands and the beginning of the segments becomes progressively smaller.

Using this result we can show that for a special case (when $\alpha=0$) the line joining the maxima is also concave in Q . We think this result (which is demonstrated for our base case in Figure 15) is true for all values of α , but a formal proof has eluded us so far.

The significance of this result is that an efficient algorithm can be designed to find the value of $Q^*(L)$ for each given L . For our base case for example (see Figure 15), after finding the values of $Q_1(L)$ and $Q_2(L)$ (which can be efficiently computed), the search for $Q(L)$ terminates since $Q_1(L) > Q_2(L)$ implies that $Q^*(L) = Q_1(L)$.

Figure 15. Present Worth of Profits vs Pumping Rate for Given Project Life

7. SUMMARY AND CONCLUSIONS

This study is concerned with the economics of geothermal energy extraction for non-electric uses. We have attempted to determine the optimal rate of extraction of heat from a reservoir (by finding optimal pumping rate and injection temperature) as well as an optimal planning horizon for the project, when the real value of energy is allowed to increase with time. In addition, we have attempted to add some perspective to the existing literature on optimal resource extraction theory by conducting our analysis with specific physical production and cost functions. The production functions are hydrothermal expressions describing the relationships between the rate of geothermal brine extraction, the quality of extracted energy (water temperature) and the economic life of a field. The cost functions developed in this study are detailed expressions describing the relationships between the various costs associated with extraction of energy, and the decision variables.

In pursuing the above goals we have structured a model for optimal reservoir management which synthesizes concepts from economic theory and engineering design. The real value of energy may increase with time, and the growth is assumed to be either exponential or linear. Since the rate of increase is specified by the decision maker based on his forecast of the future, these two versions of the model cover a wide range of possibilities regarding the future increase in the market value of energy with time. The results of our analysis were presented in section 6. In this section, we

shall summarize these results and draw some conclusions.

It is generally understood that a positive discount rate deemphasizes the future in favor of the present, and we have found that to be the case in our results. Assuming shadow and actual market prices of energy are the same (as the effects of externalities have been ignored), we find that for each assumed rate of increase in the value of energy r , the present worth of maximum profits Π^* (which equals the economic value of the reservoir) decreases while the optimal pumping rate Q^* , increases as the discount rate i , increases. While it can be easily shown that the discounting process reduces the present worth of maximum profits, the effect of i on Q^* is not so obvious. We conclude that, as theory predicts, a greater emphasis on the present "tilts" the design decisions on optimal pumping rate toward more rapid heat energy extraction rates. That is, we opt for extracting heat at a higher rate, leaving less for the future when profits are heavily discounted. Furthermore, for any two growth rates r_1 and r_2 , with $r_1 < r_2$, the percentage change in both Q^* and Π^* as i is increased is greater for r_2 than for r_1 .

For each given interest rate, the maximum profit increases with r while optimal pumping rate Q^* , decreases. Since larger values of r mean energy is more valuable in the future, we conclude that as r is increased, we tend to extract heat more slowly, thus conserving a larger amount for production and sales in the future when its value is higher. Thus the interest rate and the growth rate work against each other in determining Q^* , the optimal pumping rate and Π^* , the present worth of profits.

The economic life of a reservoir L^* (the planning horizon) is

nonincreasing in i and nondecreasing in r . Thus, as i increases, future profits are discounted more heavily, and we extract a greater amount of heat per year over a period which tends to be shorter in comparison to when the discount rate is not as high. On the other hand, when energy is expected to rapidly increase in value with time, extraction of heat over a longer period of time tends to be more profitable.

Although the optimal extraction rate increases with discount rate when the value of r is small (r less than 1% for the exponential growth model and 2.4% for the linear growth model), we find that optimal economic life does not decrease in proportion as might be expected. Instead it remains constant for all values of i for these lower values of r . Accordingly, since Q^* increases with i , total energy extracted increases with i . On the other hand when r is not small, the economic life tends to decrease as i increases.

In addition to the discount and growth rates, we find that well life is an important parameter in determining the economic life of the reservoir when r is positive. This is revealed in our results that show optimal project life is equal to expected well life, except when r is large and i is small. In this latter case, the relative importance of the future, when energy value is very high, is so great that much longer economic lives are chosen. However, here the effect of increasing the discount rate i is to reduce this economic life -- since Q^* increases. We conclude that Q^* and L^* tend to move inversely as i increases.

In view of the above conclusions, we emphasize that the interpretation of L^* as the 'economic life of the reservoir' must

be understood in the context of the deterministic nature of this model. Our results state that if all parameters (in particular i and r) are known with certainty, then it is indeed optimal to pump $Q^* \text{ m}^3/\text{hr}$ of brine over L^* years. However, as the values of these economic parameters tend to change with time according to some random process, it is more appropriate to consider L^* as an expected planning period. Note that since the same values of Q^* and other design variables are associated with a wide range of values of L , the interpretation of L^* is not important for the purpose of process design.

The amount of heat extracted per unit of time is not only a function of the extraction rate but the degree to which the extracted brine is cooled in the heat exchangers. Since the effect of heat losses in transmission is neglected, this temperature is equal to the injection temperature T_i . Our results indicate that the optimal value of injection temperature T_i^* , is remarkably stable with respect to i and r and is very close to T_g , the generated steam temperature. The high costs of heat exchanger equipment resulting from the proximity of T_i to T_g are evidently offset by the value of the extra energy extracted due to this proximity. We have also shown that in the context of the Gringarten-Sauty hydrothermal model a higher T_i has no effect on prolonging the economic lifetime of the project. That is, although injecting the brine at a high temperature prolongs the duration of the (post breakthrough) time that the production temperature remains above a specified level, it is not actually profitable to do so.

In the examples we have studied in this paper, the optimal breakthrough times occur very early in the project, namely

during the first two years. The amount of heat which is extracted during this period is far less than that extracted after the breakthrough time. For example, in our base case the heat extracted before the breakthrough time is less than 12% of the total heat extracted during the life of the project. We conclude that for nonelectric uses of geothermal energy, termination of the extraction process at breakthrough time would be premature.

As we mentioned earlier, well life has a substantial effect on both the economic life of the project and maximum profits. On the other hand, reduction in profits from increasing expenditures for well maintenance costs is minimal. Although we present no relationship between well maintenance expenditures and well life, it is reasonable to speculate that a good well maintenance program might be very cost beneficial. However, gains in profits increase at a decreasing rate as well life increases, so there will probably be some optimal maintenance expenditure beyond which marginal expenditures would outweigh marginal costs.

Our results indicate that expenditures for well construction (well costs) do not have a significant effect on the optimal pumping rate but have a great impact on profits. Hence, even though engineering design is not highly sensitive to this parameter, it plays a major role in determining the economic viability of the project. As in the above paragraph, this also suggests an area of potentially productive investigation if a relationship can be generated between well capital and maintenance cost and well life.

A similar observation can be made for the importance of royalties and land rents. While both these items can significantly

affect profits, their impact on optimal pumping rate is minor. Hence, we conclude that royalties and land rents are not the obvious levers for control of extraction rate by public regulatory agencies, given that the decision has already been reached to produce a particular KGRA. However, this is not to say that rents are unimportant in influencing the profit maximizing entrepreneur's decision on the timing of production (when to commence). We leave this interesting question to a future paper.

The economic parameter that most influences profits and optimal flow rate is the present day cost of electricity for pumping assuming it escalates at the same rate as other energy values. Pumping energy constitutes a major portion of the total costs and plays a major role in the economics and engineering design of geothermal energy production. This effect is demonstrated by the fact that if the electricity cost had been 2¢/kwh instead of 3¢/kwh, the maximum profits would have been higher by 130%, the optimal pumping rate by 142% and the optimal heat exchanger area by 145%. Accordingly, these results represent a rather conservative estimate of the economic viability of geothermal (non-electric) energy extraction when the wells can be produced without extractive pumping.

Among the physical geothermal parameters, the one with the greatest impact on economic viability (profits) and the optimal decision variables, is the initial equilibrium temperature T_0 . Both the profits and the optimal pumping rate greatly increase as the temperature increases. For instance, an increase of 10°C in the initial temperature of 150°C (which has been chosen as our base case)

increases profits by 253% and the optimal pumping rate by 130%. Since the economic worth of a reservoir is equal to the present worth of maximum profits, we conclude that the initial equilibrium temperature is a major consideration in assigning an economic value to a particular reservoir.

In contrast to the initial temperature, uncertainties in porosity of aquifer does not affect the optimal pumping rate or profits significantly. In fact, within the range of 10 to 30 percent porosity, the optimal pumping rate remains virtually constant. This suggests that major expenditures to accurately determine porosity may not be warranted for the purpose of computing the optimal pumping rate. On the other hand, intrinsic permeability has a relatively greater impact on the economics and design of a geothermal facility. When permeability is higher, the drawdown in the production well is not as great, so smaller pumps and less energy for pumping (the same amount of flow) is required. As a result, both the optimal profits and optimal pumping rate increase. We conclude that unlike porosity, expenditures for accurate information on permeability may be important, for profits and pumping rates are indeed sensitive to values of permeability.

As a final conclusion, we recall that the hydrothermal model of Gringarten and Sauty, on which our economic model is based, assumes a horizontally infinite aquifer. Out of concern over the effect of this assumption on our optimal decision variables, we investigated the case where the aquifer is finite (2×2 kilometers). To this end, an equation showing the temperature decay vs. time for

this aquifer was developed by Chin Fu Tsang of Lawrence Berkeley Laboratory and our computer program was accordingly modified to accommodate this case. Preliminary results indicate that the design decision variables (pumping rate and heat exchanger area) are not significantly different in the finite and infinite reservoir cases. We note that an accurate determination of the relationship between reservoir size and the decision variables requires further study.

As a final comment we note that the computer program developed for this study can be readily utilized by other users. The geothermal and economic data are inputs to the program and the cost subroutine can be easily modified by different users to accommodate the particular costs involved in the exploitation of each individual field. The computer program, the cost subroutine and instructions for use of the program are given in the appendix.

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APPENDIX

OPTIMAL GEOTHERMAL EXTRACTION MODEL

JUNE 17, 1977

```

0001      DIMENSION SM(10,15),PEC(10)
0002      DIMENSION ATIME(10,10)
0003      DIMENSION G(10,10,10)
0004      DIMENSION EA(10,10),QP(10,10),ALI(10),DLT(10),AENSI(20,20)
0005      DIMENSION AINT(10),R(10),AQ(10,10),AA(10,10),ATI(10,10)
0006      DIMENSION ALIF(10,10),TOW(10,10),PRO(10,10),ATHET(10,10)
0007      DIMENSION AT1(10,10),AT2(10,10),ACOST(10,10),ACON(10,10)
0008      DIMENSION AFUM(10,10)
0009      COMMON QS,PK,D,A,B,DRCON,EFFV,EFFH,C1,C2,EXA,SPGRF,CST10
0010      COMMON CST25,EXCOF,CRF10,CRF25,AKM,Z,AM,E,COST,PUCST,E1,E2
0011      COMMON S1,S2,S3,S4,SLV10,SLV25,EE,ENCST,E5,CRFN,WCST,WMC
0012      COMMON CHCST

```

INPUT DATA FOR PROGRAM

TABLE COMMENTS

```
0013      READ (5,609) ((SM(I,J),J=1,15),I=1,8)
```

GEOLOGICAL AND AQUIFER DATA

```
0014      READ(5,11)  HCAPR,PHI,SPGRF,TO,TS,SMK
0015      READ(5,13)  H,D,RW,A,B
```

TEMPERATURE DEPLETION EQUATION COEFFICIENTS

0016 READ(5,14) PHIC,PHI1,PHI2,GAMA0,GAMA1,GAMA2

INTEREST RATES

```
0017      READ (5,15) M, (AINT(I), I=1, M)
```

ENERGY GROWTH RATES

```
0018      READ (5,15) N, (R(J), J=1, N)
```

COST EQUATION CONSTANTS

0019 READ(5,16) AM,C1,C2,UU,PK,CM,PP,AKM,Z

EXOGENOUS PARAMETERS

```
0020      READ(5,17) QMIN,QMAX,QINC,ALMIN,ALMAX,ALINC,QBAR
```

SALVAGE VALUES, PUMP EFFICIENCY, TEMPERATURE LIMIT

```
0021      READ(5,707) S1,S2,S3,S4,DELTA,EFFV,EFFH
```

ECONOMIC PARAMETERS

0022 READ(5,621) WCST,WMC,OHCST,RENT,SALRY,ROYLT

WELL LIFE, GROWTH TYPE (LIN CB LOG)

0023 READ(5,39) WLIFE,K

```
C          CALCULATE HEAT CAPACITY OF FLUID, DENSITY OF FLUID,
C          AND VISCOSITY OF FLUID.
C
0024      HCAFF=1.008*SPGRF
0025      GAMAF=62.427*SPGRF
0026      FMU=.209/100./ (2.1482* (TO-8.435+ (8078.4+ (TO-8.435) **2)
*          ** .5) -120.0)
C
C          "CALL" TO ROUTINE TO PRINT ALL INPUT DATA
C
0027      GOTO 1001
0028      1000 CONTINUE
C
C          DETERMINE LIMIT FOR LOOPING
C
0029      DKM=(ALMAX-ALMIN)/ALINC
0030      KML=IFIX(DKM)+1
0031      IF (ALMIN.EQ.0.0) KML=KML-1
0032      AK=4.88*UU/(1000.0*HCAFF)
C
C          COMPUTE HEAT CAPACITY OF AQUIFER
C
0033      HCAPA=(1.0-PHI)*HCAPR+PHI*HCAFF
C
C          COMPUTE BETA
C
0034      BETA=6.2832*H*D**2.0*HCAPA/(HCAFF*52560.0)
C
C          COMPUTE AQUIFER CONDUCTIVITY
C
0035      CKK=0.000000000011653
0036      CK=CKK*SMK*GAMAF/FMU
C
C          COMPUTE CONSTANT IN DRAWDOWN
C
0037      RATIO=D/RW
0038      DRCCN=ALOG (RATIO) / (6.2832*CK*H)
C
C          COMPUTE CHI
C
0039      CHIO=PHIO/(6.0*BETA)
0040      CHI1=PHI1/(6.0*BETA)
0041      CHI2=PHI2/(6.0*BETA)
C
C          BEGIN LOOPING FOR EACH DISCOUNT RATE
C
0042      DO 22 I=1,M
C
C          DETERMINE CAPITAL RECOVERY FACTOR FOR 10-YEAR
C          LIFETIME EQUIPMENT, 25-YEAR LIFETIME EQUIPMENT,
C          AND THE WELL.
C
0043      CON10=(1.0+AIINT(I))**10.0
0044      CON25=(1.0+AIINT(I))**25.0
```



```

0045      CCNN=(1.0+AINI(I))*WLIFE
0046      CRF10=AINI(I)*CON10/(CON10-1.0)+CM
0047      CRFN=AINI(I)*CONN/(CONN-1.0)+CM
0048      CRF25=AINI(I)*CON25/(CON25-1.0)+CM
0049      PO=0.1876*CRF25+PP
0050      PPO(I)=PO

      C
      C      BEGIN LOOP FOR EACH ENERGY RATE
      C
0051      DO 33 J=1,N
0052      DO 933 KK=1,8
0053      933 G(I,J,KK)=0.0
0054      WRITE(6,89) AINI(I),R(J)
0055      WRITE(6,40)
0056      ALFA=AINI(I)-R(J)

      C
      C      SET INITIAL PROJECT LIFE
      C
0057      AL=ALMIN
0058      IF(ALMIN.EQ.0.0) AL=ALINC
0059      QP(I,J)=0.0

      C
      C      BEGIN LOOP FOR EACH PROJECT LIFETIME
      C
0060      DO 84 J1=1,KML
0061      CSCCN=0.0
0062      CSPU=0.0
0063      CSENT=0.0

      C
      C      INITIALIZE PUMPING RATE TO MINIMUM
      C
0064      Q=QMIN

      C
      C      COMPUTE SMALLEST MULTIPLES OF 10, 25, WL CONTAINING L
      C
0065      ZZ=(AL-.001)/10.0
0066      WW=(AL-.001)/25.0
0067      YY=(AL-.001)/WLIFE
0068      KK=IFIX(ZZ)
0069      LL=IFIX(WW)
0070      MM=IFIX(YY)

      C
      C      SET PROFITS TO MINIMUM VALUE ($0.00)
      C
0071      PPO(I,J)=0.0

      C
      C      INITIALIZE ARRAY CF DATA FOR OUTPUT
      C
0072      AQ(I,J)=0.0
0073      AA(I,J)=0.0
0074      ATI(I,J)=0.0
0075      AIIF(I,J)=AL
0076      TOW(I,J)=0.0
0077      ACCST(I,J)=0.0

```

```

0078      ACON(I,J)=0.0
0079      APUM(I,J)=0.0
0080      AENST(I,J)=0.0

```

```

C
C
C
C

```

```

      COMPUTE INTERMEDIATE VALUES FOR EQUATIONS DESCRIBING
      THE CCST FUNCTION CF BOTH EXPONENTIAL AND LINEAR GROWTH

```

```

0081      DIS1=-AINT(I)*AL
0082      DIS2=EXP(DIS1)
0083      DIS3=-AINT(I)*10.0*(KK+1)
0084      DIS4=EXP(DIS3)
0085      DIS5=-AINT(I)*25.0*(LL+1)
0086      DIS6=EXP(DIS5)
0087      DIS7=-AINT(I)*WLIFE*(MM+1)
0088      DIS8=EXP(DIS7)
0089      E=(1.0-DIS2)/AINT(I)
0090      E1=(DIS2-DIS4)/AINT(I)
0091      E2=(DIS2-DIS4)/AINT(I)
0092      E3=(1.0-DIS4)/AINT(I)-S2*E2
0093      E4=(1.0-EXP(-ALFA*AL))/ALFA
0094      E5=(DIS2-DIS8)/AINT(I)
0095      EE=E+(1.0-DIS2*(1.0+AINT(I)*AL))*R(J)/AINT(I)**2.0
0096      IF(K.EQ.1) EE=E4
0097      IF(K.EQ.2) GO TO 401

```

```

23
C
C
C
C
C
C
C

```

```

      DETERMINATION OF PARAMATERS FOR EXPONENTIAL CASE

```

```

      BREAKTHROUGH HAS NOT OCCURED.
      FIND EXCHANGER AREA, INJECTION TEMPERATURE, AND REVENUES.

```

```

0098      BKT=BETA/Q
0099      IF(BKT.GT.AL) GO TO 333
0100      GO TO 335
0101      333 TI=(150.7*E3*CRF10)/(AK*34.76*PO*HCAPF*EE)+TS
0102      EXA=Q*(ALOG(TO-TS)-ALOG(TI-TS))/AK
0103      EXCOF=EXA/Q
0104      THETA=34.76*PO*HCAPF*EE*(TO-TI)
0105      GO TO 207

```

```

C
C
C
C
C
C
C

```

```

      BREAKTHROUGH HAS OCCURED.
      FIND INJECTION TEMPERATURE, HEAT EXCHANGER AREA, AND
      REVENUES.

```

```

      EVALUATE TEMPERATURE DEPLETION EQUATION.

```

```

0106      335 CONTINUE
C
C
C
0107      SIGMA=EXP(-ALFA*BETA/Q)
0108      SIG1=(1.0-SIGMA)/ALFA

```

```

C
C

```

```

      COMPUTE SIGMA2

```

```

C
0109      SIG01=CHIO*Q+ALFA
0110      SIG11=CHI1*Q+ALFA
0111      SIG21=CHI2*Q+ALFA
0112      SIO11=-SIG01*BETA/Q
0113      SIO12=-SIG01*AL
0114      SI111=-SIG11*BETA/Q
0115      SI112=-SIG11*AL
0116      SI211=-SIG21*BETA/Q
0117      SI212=-SIG21*AL

C
C          TEST FOR EXPONENTIAL UNDERFLOW
C
0118      DO1=EXP(SIO11)
0119      IF(SIO12.LT.-170.0) GO TO 61
0120      DO2=EXP(SIO12)
0121      GO TO 62
0122      61      DO2=0.0
0123      62      D11=EXP(SI111)
0124      IF(SI112.LT.-170.0) GO TO 63
0125      D12=EXP(SI112)
0126      GO TO 64
0127      63      D12=0.0
0128      64      D21=EXP(SI211)
0129      IF(SI212.LT.-170.0) GO TO 65
0130      D22=EXP(SI212)
0131      GO TO 66
0132      65      D22=0.0
0133      66      SIG2=GAMAO*(DO1-DO2)/SIG01+GAMA1*(D11-D12)/SIG11
          *      +GAMA2*(D21-D22)/SIG21
0134      205     TI=(150.7*E3*CRF10)/(AK*34.76*PO*HCAPF*(SIG1+SIG2))+TS
0135      GO TO 204

C
C          DETERMINATION OF PARAMETERS FOR THE LINEAR CASE.
C
0136      401     BKT=BETA/Q
0137      IF(BKT.GT.AL) GO TO 334
0138      GO TO 336

C
C          BREAKTHROUGH HAS NOT OCCURED.
C          FIND INJECTION TEMPERATURE, HEAT EXCHANGER AREA,
C          AND REVENUES.
C
0139      334     TI=(150.7*E3*CRF10)/(AK*34.76*PO*HCAPF*EE)+TS
0140      EXA=Q*(ALOG(TO-TS)-ALOG(TI-TS))/AK
0141      EXCOF=EXA/Q
0142      THETA=34.76*PO*HCAPF*EE*(TO-TI)
0143      GO TO 207
0144      336     CONTINUE

C
C          BREAKTHROUGH HAS OCCURED. EVALUATE TEMPERATURE
C          DEPLETION EQUATION.
C
0145      SS1=-AINT(I)*BETA/Q

```

```

0146      SS2=EXP(SS1)
0147      SS3=(1.0-SS2)/AINT(I)
0148      SS4=(1.0-SS2*(1.0-SS1))/AINT(I)**2.0
0149      DB1=CHIO*Q+AINT(I)
0150      DB2=CHI1*Q+AINT(I)
0151      DB3=CHI2*Q+AINT(I)
0152      DB11=CE1*BETA/Q
0153      DB12=CE1*AL
0154      DB21=DE2*BETA/Q
0155      DB22=DB2*AL
0156      DB31=CE3*BETA/Q
0157      DB32=DB3*AL

```

C
C
C

TEST FOR EXPONENTIAL UNDERFLOW

```

0158      DB111=EXP(-DB11)
0159      IF (CB12.GT.170.0) GO TC 301
0160      DB122=EXP(-DB12)
0161      GO TO 302
0162      301 DB122=0.0
0163      302 DB211=EXP(-DB21)
0164      IF (DB22.GT.170.0) GO TC 303
0165      DB222=EXP(-DB22)
0166      GO TO 304
0167      303 DB222=0.0
0168      304 DB311=EXP(-DB31)
0169      IF (CB32.GT.170.0) GO TO 305
0170      DB322=EXP(-DB32)
0171      GO TO 306
0172      305 DB322=0.0
0173      306 CONTINUE
0174      FR1=GAMAO*(DB111-DB122)/DB1+GAMA1*(DB211-DB222)/DB2+GAMA2
*      *(CB311-DB322)/CB3
0175      FR2=GAMAO*(DB111*(1.0+CB11)-DB122*(1.0+DB12))/DB1**2.0
*      +GAMA1*(DB211*(1.0+DB21)-DB222*(1.0+DB22))/DB2**2.0
*      +GAMA2*(DB311*(1.0+CB31)-DB322*(1.0+DB32))/DB3**2.0
0176      FR3=PC*(SS3+FR1)+R(J)*(SS4+FR2)*PO
0177      TI=(150.7*E3*CRF10)/(AK*34.76*HCAFF*FR3)+TS

```

C
C
C

COMPUTE AREA OF THE HEAT EXANGER

```

0178      204 EXA=Q*(ALOG(TO-TS)-ALOG(TI-TS))/AK
0179      EXCOF=EXA/Q
0180      IF (DELTA.EQ.0.0) GO TO 32

```

C
C
C
C
C

CCOMPUTE CONDITION THAT TEMPERATURE DIFFERENCE IS
SMALLER THAN DELTA.

```

0181      CON1=CHIO*Q*AL
0182      CON2=CHI1*Q*AL
0183      CON3=CHI2*Q*AL
0184      IF (CON1.GT.170.0) GO TC 27
0185      CON11=GAMAO*EXP(-CON1)

```

```

0186      GO TO 117
0187      27      CON11=0.0
0188      117     IF(CON2.GT.170.0) GOTO 28
0189      CON22=GAMA1*EXP(-CON2)
0190      GO TO 18
0191      28      CON22=0.0
0192      18      IF(CON3.GT.170.0) GO TC 29
0193      CON33=GAMA2*EXP(-CON3)
0194      GO TO 19
0195      29      CON33=0.0
0196      19      CON=CON11+CON22+CON33
0197      DEL=DELTA/(TO-TI)

C
C          IF THE TEMPERATURE HAS FALLEN BELOW THE ALLOWABLE
C          LIMIT, CONTINUE TC NEXT LIFE AND START SEARCHING.
C
0198      IF(CON.GT.DEL) GO TO 32
0199      GO TO 34
0200      32      CONTINUE

C
C          COMPUTATION OF THETA1 AND THETA2
C
0201      IF(K.EQ.2) GO TO 402

C
C          EVALUATION FOR THE EXPONENTIAL CASE.
C
0202      206     THET1=34.76*PO*HCAFF*SIG1*(TO-TI)*Q
0203      THET2=34.76*PO*HCAFF*SIG2*(TO-TI)*Q
0204      THEIA=THET1+THET2
0205      GO TO 207

C
C          EVALUATION FOR THE LINEAR CASE.
C
0206      402     THETA=Q*34.76*HCAFF*(TC-TI)*FR3
0207      207     X=Q/QBAR
0208      IF(X.EQ.1.0) GO TO 78
0209      GO TO 79

C
C          FIND COSTS FOR A SINGLE WELL OPERATING AT MAXIMUM RATE
C
0210      78      QS=QBAR
0211      CALL KOST
0212      CSCCN=COST+SLV10+SLV25
0213      CSPU=PUCST
0214      CSENT=ENCST*EE
0215      COSIS=CSCON
0216      PUMCS=CSPU
0217      EECST=CSENT
0218      GO TO 80
0219      79      CONTINUE
0220      IF(X.LT.1.0) GO TO 99
0221      II=IFIX(X)
0222      GO TO 101
0223      99      II=0

```

```

C
C      COMPUTE TOTAL COSTS.
C
0224      101  QS=Q-II*QBAR
0225          IF(CS.EQ.0.0) GO TO 81
0226          GO TO 82
0227      81   COSTS=CSCON*II
0228          PUMCS=CSPU*II
0229          EECST=CSENT*II
0230          GO TO 80
0231      82   CALL KOST
0232          COSTS=CSCON*II+COST+SLV10+SLV25
0233          PUMCS=CSPU*II+PUCST
0234          EECST=ENCST*EE+CSENT*II
0235      80   ALLCS=COSTS+(RENT+SALRY)*E
C
C      COMPUTE NET REVENUES
C
0236          REV=THETA*(1.0-BOYLT)
C
C      CCMPUTE PROFITS IN THOUSANDS OF DOLLARS.
C
0237          REV1=(REV-ALLCS)/1000.0
C
C      SCALE DATA TO 1,000-$ RANGE FOR OUTPUT
C
0238          SCCON=CSCON/1000.0
0239          COSTS=ALLCS/1000.0
0240          PUMCS=PUMCS/1000.0
0241          EECST=EECST/1000.0
C
C      ARE THE PROFITS FOR THIS PROJECT LIFE A NEW MAXIMUM?
C      IF SO, UPDATE DATA ARRAY.
C
0242          IF(REV1.GT.PRO(I,J)) GC TO 31
0243          GO TO 30
0244      31   PRO(I,J)=REV1
0245          AQ(I,J)=Q
0246          AA(I,J)=EXA
0247          ATI(I,J)=TI
0248          ALIF(I,J)=AL
0249          BKT=BETA/Q
0250          TOW(I,J)=BKT
0251          ACOST(I,J)=COSTS
0252          ACON(I,J)=SCCON
0253          APUM(I,J)=PUMCS
0254          AENST(I,J)=EECST
C
C      NEXT VALUE OF Q (PUMPING RATE)
C
0255      30   Q=Q+QINC
0256          IF(Q.GT.QMAX) GO TO 34
0257          GO TO 23
0258      34   CONTINUE

```

```

C
C      ROUND PROFITS TO NEAREST $10
C
0259      IFRO=PRO(I,J)*100.0+0.5
0260      PRO(I,J)=IFRO
0261      PRO(I,J)=PRO(I,J)/100.0
0262      WRITE(6,87) PRO(I,J),ALIF(I,J),AQ(I,J),AA(I,J),AII(I,J),
*      TOW(I,J),ACON(I,J),ACOST(I,J),APUM(I,J),AENST(I,J)
0263      36      CONTINUE
C
C      ARE THE PROFITS FOR THIS COMBINATION OF DISCCUNT AND
C      ENERGY RATES AT A MAXIMUM?
C      IF SO, PLACE THE DATA INTO THE ARRAY.
C
0264      IF(PRO(I,J).GT.QP(I,J)) GO TO 708
0265      GO TO 709
0266      708      QP(I,J)=PRO(I,J)
0267      G(I,J,1)=PRO(I,J)
0268      G(I,J,2)=ALIF(I,J)
0269      G(I,J,3)=AQ(I,J)
0270      G(I,J,4)=AA(I,J)
0271      G(I,J,5)=ACOST(I,J)
0272      G(I,J,6)=AENST(I,J)
0273      G(I,J,7)=AII(I,J)
0274      G(I,J,8)=TOW(I,J)
0275      709      AL=AL+ALINC
0276      84      CONTINUE
0277      33      CONTINUE
0278      22      CONTINUE
C
C      OUTPUT TABLES OF RESULTS
C
0279      DO 503 JZ=1,8
0280      WRITE(6,618)
0281      WRITE(6,607) (SM(JZ,J),J=1,15)
0282      WRITE(6,610)
0283      WRITE(6,604)
0284      WRITE(6,603) (AINT(JY),JY=1,M)
0285      WRITE(6,604)
0286      WRITE(6,611)
0287      WRITE(6,604)
0288      WRITE(6,602)
0289      DO 501 JX=1,N
0290      WRITE(6,604)
0291      IF(JZ.GT.6) GO TO 500
0292      WRITE(6,617) R(JX),(G(JY,JX,JZ),JY=1,M)
0293      GO TO 501
0294      500      WRITE(6,619) R(JX),(G(JY,JX,JZ),JY=1,M)
0295      501      CONTINUE
0296      WRITE(6,604)
0297      WRITE(6,602)
0298      WRITE(6,604)
0299      WRITE(6,612) (PPO(I),I=1,M)
0300      WRITE(6,620)

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0301      WRITE(6,604)
0302      WRITE(6,610)
0303      WRITE(6,613)
0304      IF(K.EQ.2) GO TO 505
0305      WRITE(6,605)
0306      GO TO 504
0307      505  WRITE(6,606)
0308      504  CONTINUE
0309      PH=PHI*100.0
0310      503  CONTINUE
0311      11   FORMAT(2F6.2,F6.4,3F6.2)
0312      13   FORMAT(5F6.2)
0313      14   FORMAT(6F6.4)
0314      15   FORMAT(I1,5X,8F8.4)
0315      16   FORMAT(8F7.2,F8.3)
0316      17   FORMAT(7F8.2)
0317      39   FORMAT(F6.2,3X,I1)
0318      40   FORMAT(8X,'NPV$',5X,'YES',4X,'Q*',4X,'A*',4X,'TI',4X,'TAU',
*          4X,'CSTQB',4X,'TCST',4X,'PUCST',3X,'ENCST'//)
0319      89   FORMAT('1',///,17X,'INTEREST RATE=',F6.4,2X,
*          'RATE OF INCREASE OF PRICE=',F6.4//)
0320      87   FORMAT(6X,F9.2,2X,F5.0,1X,F5.0,1X,F5.0,1X,F6.2,1X,F5.2,2X,
*          F7.1,2X,F7.1,1X,F7.1,1X,F7.1)
0321      602  FORMAT(15X,'-----',
*          '-----')
0322      603  FORMAT(15X,'I= ',F6.2,4X,F6.2,4X,F6.2,4X,F6.2,4X,
*          F6.2,5X,'I')
0323      604  FORMAT(15X,'I',8X,'I',53X,'I')
0324      605  FORMAT(26X,'P(T)=P(0)*EXP(+R*T)',/)
0325      606  FORMAT(26X,'P(T)=P(0)*(1+R*T)',/)
0326      607  FORMAT(15X,15A4,////////)
0327      609  FORMAT(15A4)
0328      610  FORMAT(15X,'-----',
*          '-----')
0329      611  FORMAT(15X,'I',2X,'R=',4X,'I',53X,'I')
0330      612  FORMAT(15X,'I P(0) ',F6.3,4X,F6.3,4X,F6.3,4X,F6.3,4X,
*          F6.3,5X,'I')
0331      613  FORMAT(////////)
0332      617  FORMAT(15X,'I',F5.3,'I',F6.0,4X,F6.0,4X,F6.0,4X,F6.0,
*          4X,F6.0,5X,'I')
0333      618  FORMAT('1',///,41X,'TABLE',////)
0334      619  FORMAT(15X,'I',F5.3,'I',F6.2,4X,F6.2,4X,F6.2,4X,F6.2,
*          4X,F6.2,5X,'I')
0335      620  FORMAT(15X,'I',1X,'$/MBTU',1X,'I',53X,'I')
0336      621  FORMAT(5F10.2,F6.4)
0337      707  FORMAT(4F5.2,3F8.2)
0338      WRITE(6,800)
0339      800  FORMAT('1')
0340      STOP

C
C          SPECIAL SECTION FOR PRINTING OUT INPUT VALUES
C
0341      1001 WRITE(6,1101)
0342      WRITE(6,1102) HCAPR,HCAPI,PHI

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```

0343      WRITE (6,1103) SPGRF,TC,TS
0344      WRITE (6,1104) SMK,GAMAF,FMU
0345      WRITE (6,1105) H,D,RW
0346      WRITE (6,1106) A,B
0347      WRITE (6,1107) PHIO,PHI1,PHI2
0348      WRITE (6,1108) GAMAO,GAMA1,GAMA2
0349      WRITE (6,1109) AM,CM,PF
0350      WRITE (6,1110) C1,C2,WCST
0351      WRITE (6,1111) UU,PK,OHCSST,RENT,SALRY,ROYLT
0352      WRITE (6,1112) AKM,Z
0353      WRITE (6,1113) S1,S2,S3
0354      WRITE (6,1114) S4
0355      WRITE (6,1115) EFFV,EFFH
0356      WRITE (6,1116) QMAX,QMIN,QINC
0357      WRITE (6,1117) QBAR,ALMAX,ALINC
0358      WRITE (6,1118) WLIFE,WMC,DELTA
0359      IF (K.EQ.1) WRITE (6,1119)
0360      IF (K.EQ.2) WRITE (6,1120)
0361      WRITE (6,1121) (AINT(I),I=1,M)
0362      WRITE (6,1122) (R(I),I=1,N)
0363      GOTO 1000
0364      1101  FORMAT ('1',43X,'PROGRAM DATA'////)
0365      1102  FORMAT (' ', 'HEAT CAP. CF ROCK',F8.2,
*           3X,'HEAT CAP. CF FLUID',F8.2,
*           3X,'POROSITY CF AQUIFER',F8.2)
0366      1103  FORMAT (' ', 'SPEC. GRAVITY OF FLUID',F8.4,
*           3X,'AQUIFER INIT. TEMP.',F8.2,
*           3X,'STEAM TEMPERATURE',F8.2/)
0367      1104  FORMAT (' ', 'INTRINSIC PERMEABILITY',F8.2,
*           3X,'FLUID UNIT WEIGHT',F8.2,
*           3X,'FLUID VISCOSITY',F11.8/)
0368      1105  FORMAT (' ', 'AQUIFER HEIGHT',F8.2,
*           3X,'WELL SEPERATION',F8.2,
*           3X,'WELL RADIUS',F8.2)
0369      1106  FORMAT (' ', 'A - HEIGHT',F8.2,
*           3X,'B - HEIGHT',F8.2/)
0370      1107  FORMAT (' ', 'PHI(1)',F8.4,
*           3X,'PHI(2)',F8.4,
*           3X,'PHI(3)',F8.4)
0371      1108  FORMAT (' ', 'GAMMA(1)',F8.4,
*           3X,'GAMMA(2)',F8.4,
*           3X,'GAMMA(3)',F8.4/)
0372      1109  FORMAT (' ', 'UNIT PIPE CLEANING COST',F8.2,
*           3X,'INSURANCE/TAX % COST',F8.2,
*           3X,'FUEL & OPERATING COSTS',F8.2)
0373      1110  FORMAT (' ', 'BOWL UNIT COST 220 GPM',F8.2,
*           3X,'BOWL UNIT COST 1100 GPM',F8.2,
*           3X,'WELL COST',F10.2)
0374      1111  FORMAT (' ', 'HEAT XFER. COEFF.',F8.2,
*           3X,'PIPE INSTILLATION',F8.2,
*           3X,'OVERHEAD COST',F8.2/
*           ' ', 'RENT',F8.2,
*           3X,'SALARIES',F9.2,
*           3X,'ROYALTIES',F8.2)

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0375      1112  FORMAT (' ', 'PUMP SERVICE COST MULT.', 'F8.2,
*          3X, 'ELECTRICITY COST', 'F9.3/)
0376      1113  FORMAT (' ', 'PUMP SALVAGE VALUE', 'F8.2,
*          3X, 'HEAT XCHG. SALVAGE', 'F8.2,
*          3X, 'PIPE SALVAGE VALUE', 'F8.2)
0377      1114  FORMAT (' ', 'WELL SALVAGE VALUE', 'F8.2/)
0378      1115  FORMAT (' ', 'VERT. PUMP EFFICIENCY', 'F8.2,
*          3X, 'HORZ. PUMP EFFICIENCY', 'F8.2)
0379      1116  FORMAT (' ', 'PUMP RATE LIMIT (MAX)', 'F8.2,
*          3X, 'PUMP RATE LIMIT (MIN)', 'F8.2,
*          3X, 'PUMP RATE INCREMENT', 'F8.2)
0380      1117  FORMAT (' ', 'MAX. WELL FLOW RATE', 'F8.2,
*          3X, 'PROJECT LIFE', 'F8.2,
*          3X, 'TIME INCREMENT', 'F8.2)
0381      1118  FORMAT (' ', 'WELL LIFE', 'F8.2,
*          3X, 'WELL MAINTENENCE', 'F8.2,
*          3X, 'XCHG. TEMP. DELTA', 'F8.2)
0382      1119  FORMAT (///, ' ', 'GROWTH IS EXPONENTIAL')
0383      1120  FORMAT (///, ' ', 'GROWTH IS LINEAR')
0384      1121  FORMAT (///, ' ', 'DISCOUNT RATES = ', 'F8.4)
0385      1122  FORMAT ( /, ' ', 'ENERGY COST RATES = ', 'F8.4)
0386      END

```

```

0001      SUBROUTINE KOST
          C
          C      THIS SUBROUTINE CCMPUTES THE DISCOUNTED COSTS FOR ONE
          C      DOUBLET (Q<=QMAX) .
          C
          C      ALL PARAMETERS USED ARE LOCATED IN COMMON STORAGE.
          C
0002      CCMON QS,PK,D,A,B,DRCCN,EFFV,EFFH,C1,C2,EXA,SPGRF,CST10
0003      COMMON CST25,EXCOF,CRF10,CRF25,AKM,Z,AM,E,COST,PUCST,E1,E2
0004      COMMON S1,S2,S3,S4,SLV10,SLV25,EE,ENCST,E5,CRFN,WCST,WMC
0005      CCMON CHCST
          C
          C      PIPE COST
          C
0006      PCST=PK*D*(0.1313*QS+1.323*QS**0.5-4.36)
          C
          C      ANNUAL PIPE CLEANING COST
          C
0007      CLCST=AM*D
          C
          C      HEAT EXCHANGER COST
          C
0008      ECST=5000.0+150.7*EXCOF*QS
          C
          C      HCRIZONTAL PUMP CCST
          C
0009      PCSTH=24.0*QS
          C
          C      HORIZONTAL MOTOR COST
          C
0010      MCSTH=0.0546*SPGRF*QS**2.0*DRCON/EFFH+1907.1
          C
          C      BCWL UNIT COST
          C
0011      BUCST=C1+(C2-C1)*QS**2.0/40000.0
          C
          C      COLUMN ASSEMBLY CCST
          C
0012      COCST=(A+4.0+QS*DRCON)*(0.1313*QS+1.323*QS**0.5-4.36)+3981.0
          C
          C      VERTICAL SHAFT COST
          C
0013      SHCST=(.001339*QS*SPGRF*(DRCON*QS+A+B)/EFFV+0.0768*(DRCON*
          * QS+A)-10.132)*(DRCON*QS+A)
          C
          C      VERTICAL MOTOR COST
          C
0014      MCSTV=0.0546*SPGRF*QS*(DRCON*QS+A+B)/EFFV+1907.1+
          * 0.35*(DRCON*QS+A)
          C
          C      CAPITAL COSTS FOR PUMPS
          C
0015      PPCST=COCST+MCSTV+SHCST+MCSTH+BUCST+PCSTH

```

C
C
C

PRESENT WORTH OF PUMP COSTS

0016

$$PUCST = PPCST * CRF10 * (E + E1 * (1.0 - S1))$$

C
C
C

ANNUAL PUMP OPERATING COSTS

0017

$$ENCST = AKM * Z * (23.80 * QS * SPGRF * (DRCON * QS + A + B) / EFFV + 152.86 * \\ * (DRCON * QS + A) + 23.80 * QS * SPGRF * DRCON * QS / EFFH)$$

C
C
C

TOTAL DISCOUNTED COST FOR ONE DOUBLET WITHOUT SALVAGE

0018

$$CST10 = COCST + MCSTV + SHCST + MCSTH + BUCST + PCSTH + ECST$$

0019

$$CST25 = PCST + OHCST$$

0020

$$COST = (CST10 * CRF10 + CST25 * CRF25 + CLCST + WMC + WCST * CRFN) * E + ENCST * EE$$

C
C
C

SALVAGE COSTS

0021

$$SLV10 = PPCST * CRF10 * E1 * (1.0 - S1) + ECST * CRF10 * E1 * (1.0 - S2)$$

0022

$$SLV25 = PCST * CRF25 * E2 * (1.0 - S3) + OHCST * CRF25 * E2 * (1.0 - S4) \\ * + WCST * CRFN * E5$$

0023

RETURN

0024

END

DATA CARD LAYOUT

Cards 1-8

Title Cards	
1	80
TITLES FOR OUTPUT TABLES	

Card 9

Geological and Aquifer Data						
1	7	13	19	25	31	37
xxx.xx	xxx.xx	x.xxxxx	xxx.xx	xxx.xx	xxx.xx	
HCAPR	PHI	SPGRF	TO	TS	SMK	

Card 10

1	7	13	19	25	31	80									
xxx.xx	xxx.xx	xxx.xx	xxx.xx	xxx.xx											
H	D	RW	A	B											

Card 11

Coefficients of the ϕ Function						
1	7	13	19	25	31	37
X.XXXXX PHI0	X.XXXXX PHI1	X.XXXXX PHI2	X.XXXXX GAMA0	X.XXXXX GAMA1	X.XXXXX GAMA2	

Card 12

Interest Rates											
1	2	7	15	23	31	39	47	55	63	71	80
X		xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx		
M		AIN(1)	AIN(2)	AIN(3)	AIN(4)	AIN(5)	AIN(6)	AIN(7)	AIN(8)		

Card 13

Growth Rates in Value of Energy										
1	2	7	15	23	31	39	47	55	63	71 80
x N	/	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	/
		R(1)	R(2)	R(3)	R(4)	R(5)	R(6)	R(7)	R(8)	

Card 14

Economic Parameters										80
1	8	15	22	29	36	43	50	57	65	
xxx.xx	xxx.xx	xxx.xx	xxx.xx	xxx.xx	xxx.xx	xxx.xx	xxx.xx	xxx.xx	xxx.xx	/
AM	C1	C2	UU	PK	CM	PP	AKM	Z		

Card 15

Grid Search Parameters							80
1	9	17	25	33	41	49	57
xxx.xx	xxx.xx	xxx.xx	xxx.xx	xxx.xx	xxx.xx	xxx.xx	/
QMIN	QMAX	QINC	ALMIN	ALMAX	ALINC	QBAR	

Card 16

Economic Parameters							80
1	6	11	16	21	29	37	45
xx.xx	xx.xx	xx.xx	xx.xx	xxxx.xx	xxxx.xx	xxxx.xx	/
S1	S2	S3	S4	DELTA	EFFV	EFFH	

Card 17

1	11	21	31	41	51	57	80
xxxxxxxx	xxxxxxxx	xxxxxxxx	xxxxxxxx	xxxxxxxx	xxxxxx	/	
WCST	WMC	OHCST	RENT	SALRY	ROYLT		

Card 18

1	7	9	10	11	80
xxx.xx	/	x	/	/	/
WLIFE		K			

GUIDE TO DATA CARD LAYOUT

CODES	DESCRIPTIONS	SYMBOLS	UNITS
HCAPR	Heat Capacity of Rock	$\rho_R c_R$	cal/cc ^o C
PHI	Porosity of Aquifer	ϕ	Fraction
SPGRF	Specific Gravity of Fluid	sp. gr.	_____
TO	Initial Equilibrium Temperature	T_o	^o C
TS	Steam Temperature	T_s	^o C
SMK	Intrinsic Permeability	k	millidarcies
H	Thickness of Aquifer	h	m
D	Doublet Separation	D	m
RW	Well Radius	r_w	m
A	Static Level of Fluid	z	m
B	Friction Losses	b	m
PHI ϕ	Coefficient of g Function	ϕ_1	_____
PHI1	Coefficient of g Function	ϕ_2	_____
PHI2	Coefficient of g Function	ϕ_3	_____
GAMA ϕ	Coefficient of g Function	γ_1	_____
GAMA1	Coefficient of g Function	γ_2	_____
GAMA2	Coefficient of g Function	γ_3	_____
M	Number of Interest Rates	—	_____
N	Number of Energy Growth Rates	—	_____
AIN(T)	Interest Rates, $I \leq M$	i	Fraction
R(J)	Rates of Growth in Value of Energy, $J \leq N$	r	Fraction

GUIDE TO DATA CARD LAYOUT (continued)

CODES	DESCRIPTIONS	SYMBOLS	UNITS
AM	Pipe Cleaning Cost	p_c	\$/m/year
C1	Cost of 50 m ³ /hr Bowl Unit	c_1	\$
C2	Cost of 250 m ³ /hr Bowl Unit	c_2	\$
UU	Overall Heat Transfer Coefficient of Fluid	$U(0)$	BTU/hr-ft ² -°F
PK	Pipe Support Multiplier	k_p	_____
CM	Miscellaneous Costs of Capital in Equation (26)	m	Fraction
PP	Constant term in Equ. (27)	_____	_____
AKM	Pump Maintenance Coefficient	k_m	_____
Z	Electricity Cost at Time Zero	R_o	\$/kwh
QMIN	Min. Pumping Rate to be Considered	Q_{min}	m ³ /hr
QMAX	Max. Pumping Rate to be Considered	Q_{max}	m ³ /hr
QINC	Incremental Increase in Pumping Rate	Q_{inc}	m ³ /hr
ALMIN	Min. Project Life to be Considered	L_{min}	years
ALMAX	Max. Project Life to be Considered	L_{max}	years
ALINC	Incremental Increase in Project Life	L_{inc}	years
QBAR	Capacity of Each Production Well	$\bar{Q}$	m ³ /hr
S1	Salvage Value of Pumps	s_1	Fraction of Remaining Payments

GUIDE TO DATA CARD LAYOUT (continued)

CODES	DESCRIPTIONS	SYMBOLS	UNITS
S2	Salvage Value of Heat Exchangers	s_2	Fraction of Remaining Payments
S3	Salvage Value of Pipes	s_3	Fraction of Remaining Payments
S4	Salvage Value of Well Assemblies	s_4	Fraction of Remaining Payments
DELTA	Min. Allowable Temperature Difference	δ	$^{\circ}\text{C}$
EFFV	Vertical Pump Efficiency	Eff_V	Fraction
EFFH	Horizontal Pump Efficiency	Eff_H	Fraction
WCST	Well Cost per Doublet	WC	\$
WMC	Annual Well Maintenance Cost	WM	\$/year/Doublet
OHCST	Well Assembly Cost per Doublet	WA	\$
RENT	Annual Land Rents	Rent	\$/year
SALRY	Annual Salaries	Salaries	\$/year
ROYLT	Royalty	η	Fraction of Gross Revenues
WLIFE	Well Life	WL	Years
K	No. Denoting Model :	_____	_____
	1 = Exponential Growth Model	_____	_____
	2 = Linear Growth Model	_____	_____