Seismic Attenuation for Reservoir Characterization

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Contributors

Dr. Joel Walls*

Dr. M. T. Taner*

Naum Derzhi*

Dr. Gary Mavko**

Dr. Jack Dvorkin**

*Principal Contractor:

Rock Solid Images

2600 S. Gessner Suite 650

Houston, TX, 77036

**Subcontractor:

Petrophysical Consulting Inc.

730 Glenmere Way

Emerald Hills, CA, 94062

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ABSTRACT

We have developed and tested technology for a new type of direct hydrocarbon detection. The method uses inelastic rock properties to greatly enhance the sensitivity of surface seismic methods to the presence of oil and gas saturation. These methods include use of energy absorption, dispersion, and attenuation (Q) along with traditional seismic attributes like velocity, impedance, and AVO.

Our approach is to combine three elements: (1) a synthesis of the latest rock physics understanding of how rock inelasticity is related to rock type, pore fluid types, and pore microstructure, (2) synthetic seismic modeling that will help identify the relative contributions of scattering and intrinsic inelasticity to apparent Q attributes, and (3) robust algorithms that extract relative wave attenuation attributes from seismic data.

This project provides;

- Additional petrophysical insight from acquired data
- Increased understanding of rock and fluid properties
- New techniques to measure reservoir properties that are not currently available
- Provide tools to more accurately describe the reservoir and predict oil location and volumes.

These methodologies will improve the industry's ability to predict and quantify oil and gas saturation distribution, and to apply this information through geologic models to enhance reservoir simulation. We have applied for two separate patents relating to work that was completed as part of this project.

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EXPERIMENTAL

No experimental work is reported. Figures shown are from commercial well logs and seismic data contractors.

INTRODUCTION

This project has focused on developing technology to allow seismic P-wave attenuation to be employed as an independent direct hydrocarbon indicator (DHI). The project focused on three major goals;

- Develop methods to compute P-wave attenuation from standard well log data such as porosity, V_{shale} , and S_w .
- Develop methods to use the log-derived attenuation for generating synthetic seismic traces.
- Develop methods to compute attenuation-related attributes from reflection seismic data.

These goals were accomplished, and we have shown through testing with some field examples, that attenuation related attributes can be helpful in identifying producing gas formations.

EXECUTIVE SUMMARY

Seismic dispersion and attenuation as a tool for hydrocarbon exploration and exploitation has been understood in principle for many years (Knopoff, 1964; Hermont, 1969; Kuster and Toksoz, 1974). However, it is only now becoming technically viable owing to the recent tremendous advances in seismic data quality and computing (Rapoport, M. B. and Ryjkov, V. I., 1994, Rapoport, et al, 1997). Historically most attempts to use inelastic rock properties in real world geophysical applications were doomed by the inadequate state of amplitude and phase preservation in commercial seismic surveys. However, this has been changing rapidly and will continue to advance in the coming years.

In this project, we have shown that attenuation of seismic compressional waves can be related deterministically to such reservoir properties as porosity, V_{shale} , water saturation, and P-wave velocity. This result is based on rock physics principals that have been developed partly in this project, and partly from previous publications. The most important rock physics concept that was developed and tested during this project is that P-wave attenuation can be related to the P-wave modulus difference that exists between a partially saturated rock with homogenous fluid distribution and a partially saturated rock with “patchy” or uneven fluid distribution. This patchy attenuation model forms the basis for computations we have made of attenuation from well logs. It also allows for the all important process of fluid substitution, meaning that we can replace one fluid in the reservoir interval with another and compute the change in attenuation. The inverse of attenuation is the seismic quality factor, Q .

The second major thrust of this project was to develop some new synthetic seismic algorithms that would allow the Q curves from well logs to be used as an input along with P-wave velocity (V_p) and rock bulk density. The reason to do this was so that we could observe the *anticipated* effect of attenuation on seismic. This allows us to perform feasibility or forward modeling studies to explore questions such as “How important is Q to the seismic signature at this well location?” It also allows us to observe the effect of Q on other conventional direct hydrocarbon indicators such as amplitude and AVO (amplitude variation with offset).

The third, and most important, accomplishment of the project was the development of a suite of seismic attenuation attributes that can be applied to standard reflection seismic data sets. There are two main classes of attributes that have been developed. The first is a relative energy loss attribute called “Anomalous Absorption”. This attribute is computed from the difference between a long time window, background energy loss trend and a short-window local energy loss. This attribute is very rapid to compute and seems to compare well to computations from the well logs. A second class of attenuation attributes attempt to measure absolute (or apparent) Q from the seismic. These attributes tend to be noisier and less well correlated to logs, but they may have an important role to play as they can be used to correct the processed seismic data for the effects of local attenuation changes.

The three major developments discussed above have been applied to a set of seismic and well log data from a known gas-producing field in the Gulf of Mexico. The results are very encouraging and suggest the attenuation may be an important additional seismic hydrocarbon indicator for future explorationists. This work has resulted in the filing of two patent applications (see References).

RESULTS AND DISCUSSIONS

The results of this project will be described in detail in the following five sections. These sections are;

- 1 Attenuation from Well Log Data
- 2 Synthetic Seismograms with Q
- 3 Effects of Attenuation on Synthetics Seismograms (layer models and logs)
- 4 Attenuation from Seismic
- 5 Applications to Data.

PART 1: ATTENUATION FROM WELL LOGS

One of key goals of this project is to introduce a simple first-principle-based model for calculating attenuation in the subsurface from such inputs as porosity, mineralogy, hydrocarbon type and saturation, and pore pressure. One usage of such a model is in producing attenuation pseudo-logs in a well where other, standard, well logs curves are available. This result will be used for synthetic seismic modeling at the well and tie with real seismic. Another usage is to calculate attenuation in a pseudo-well where the original rock properties in an existing well are perturbed to reflect possible variability existing in the subsurface. Eventually, this model can be applied to an earth model to produce synthetic volumes of attenuation and assess how this

seismic attribute can serve to better characterize reservoir properties and conditions.

Below, we introduce such a model by subsequently analyzing attenuation in a partially saturated rock and fully saturated rock. The attenuation in the fully-saturated (or wet) rock serves as the attenuation background to which additional attenuation is added that is due to the partial saturation.

By comparing our model results to some of available laboratory and field data we conclude that the model is satisfactory to realistically estimate attenuation in sediment.

MODEL AT PARTIAL WATER SATURATION

Wave-induced variations of pore pressure in a partially-saturated reservoir result in oscillatory liquid flow. The viscous losses during this flow are responsible for wave attenuation. The same viscous effects determine the changes in the dynamic bulk modulus of the system versus frequency. These changes are necessarily linked to attenuation via the causality condition. We analytically quantify the frequency dependence of the bulk modulus of a partially saturated rock by assuming that saturation is patchy and then link these changes to the inverse quality factor. As a result, the P-wave attenuation is quantitatively linked to saturation and thus can serve as a saturation indicator.

Upper and Lower Elastic Limits at Partial Saturation

The frequency range of seismic radiation used for illuminating the subsurface spans four orders of magnitude, from 10^1 (seismic) to 10^4 (sonic logging) Hz. Typically, the pore-scale Biot's and squirt flow attenuation mechanisms are not engaged at these frequencies. Still, viscoelastic effects and attenuation may arise from the oscillatory liquid cross-flow between fully-liquid-saturated patches and the surrounding rock with gas. The main condition for fluid-related attenuation in the practical frequency range is the existence of such patches whose length scale is several orders of magnitude larger than the pore scale (Figure 1).

To understand physical reasons for the existence of patchy saturation, let us consider a volume of rock that consists of several sand patches with clay content slightly varying among them. These clay-content variations may have a small effect on the dry-frame elastic moduli but dramatically affect permeability (e.g., Yin, 1993) and, therefore, capillary pressure curves. Then, in a state of capillary equilibrium, this elastically homogeneous volume may have a patchy

saturation pattern (Knight et al., 1998). Indeed, if capillary pressure is the same for the adjacent patches whose capillary pressure curves are different, these patches can have very different fluid saturation. Visual proof that patches form in oil-water and air-water systems in the laboratory has been presented by Chatenever and Calhoun (1952) and Cadoret (1993). Indirect evidence that patches exist in situ has been presented by Brie et al. (1995) and Dvorkin et al. (1999).

Figure 1. Left—Macroscopic fully-saturated patches in a partially-saturated reservoir. Right – Critical length versus frequency for a patch saturated with water with the bulk modulus of 2.5 GPa and viscosity 1 cPs. The permeability of the patch is 100 mD and porosity is 0.3.

The size L of a continuous patch occupied solely by the liquid phase determines the effective bulk modulus of the partially saturated rock at a fixed global saturation and frequency. If the patch is small so that, $L < \sqrt{D/f}$ ($D = kK_w / \phi\mu$ is the diffusivity; k is the permeability; K_w is the bulk modulus of the liquid; ϕ is the porosity; and μ is the dynamic viscosity of the liquid), or the frequency is small, the wave-induced pore pressure variations between the patch and surrounding gas-saturated rock equilibrate during the wave period. The patch is relaxed. This critical size L for high-porosity high-permeability rock is plotted versus frequency f in Figure 1. It appears that the critical size may be as small as 10 cm at a frequency of 50 Hz. This length is approximately the scale of heterogeneity in the reservoir required for viscoelastic effects and attenuation to become detectable at seismic frequency.

If the patch size is smaller than L , the pressure oscillations in the patch will quickly equilibrate with the surrounding gas-saturated region. As a result, the entire partially saturated region can be treated as homogeneously saturated with a liquid-gas mixture whose bulk modulus K_F is the Reuss average of those of the liquid (K_w) and gas (K_G):

$$\frac{1}{K_F} = \frac{S_W}{K_W} + \frac{1-S_W}{K_G}, \quad (1)$$

where S_W is liquid (water) saturation.

Then the bulk modulus of the partially saturated region K_{Sat0} is determined by Gassmann's equation

$$K_{Sat0} = K_S \frac{\phi K_{Dry} - (1 + \phi) K_F K_{Dry} / K_S + K_F}{(1 - \phi) K_F + \phi K_S - K_F K_{Dry} / K_S}, \quad (2)$$

where K_{Dry} is the bulk modulus of the dry frame of the rock; K_S is the bulk modulus of the mineral phase; and ϕ is the porosity. The shear modulus of the partially saturated region G_{Sat} is the same as that of the dry-rock frame G_{Dry} .

If the patch is large, $L > \sqrt{D/f}$, or the frequency is high, the wave-induced pore pressure variations in it will not equilibrate with the surrounding dry region during the wave period. The patch is unrelaxed. Then the bulk modulus of the patch K_P will appear to be that of the fully liquid saturated rock:

$$K_P = K_S \frac{\phi K_{Dry} - (1 + \phi) K_W K_{Dry} / K_S + K_W}{(1 - \phi) K_W + \phi K_S - K_W K_{Dry} / K_S}. \quad (3)$$

Then the volumetric concentration of the fully saturated patches in the system is S_W and the effective bulk modulus of the partially saturated region $K_{Sat\infty}$ is determined by the constant-shear-modulus equation

$$\frac{1}{K_{Sat\infty} + (4/3)G_{Dry}} = \frac{S_W}{K_P + (4/3)G_{Dry}} + \frac{1-S_W}{K_{S_{W=0}} + (4/3)G_{Dry}}, \quad (4)$$

where $K_{S_{W=0}}$ is the bulk modulus of the gas-saturated rock given by Gassmann's equation

$$K_{S_{W=0}} = K_S \frac{\phi K_{Dry} - (1 + \phi) K_G K_{Dry} / K_S + K_G}{(1 - \phi) K_G + \phi K_S - K_G K_{Dry} / K_S}. \quad (5)$$

$K_{S_{W=0}}$ is very close to K_{Dry} due to the high compressibility of gas.

If the size of the patch is fixed and the frequency is changing, the effective bulk modulus of partially saturated rock will vary between its low-frequency limit K_{Sat0} and high-frequency limit $K_{Sat\infty}$. The effective compressional modulus M_{Sat} will vary between M_{Sat0} and $M_{Sat\infty}$:

$$K_{Sat0} + \frac{4}{3}G_{Dry} = M_{Sat0} \leq M_{Sat} \leq M_{Sat\infty} = K_{Sat\infty} + \frac{4}{3}G_{Dry}. \quad (6)$$

The same approach can be used to describe the low- and high-frequency behavior of a patchy system if only P-wave data are available. The P-only fluid substitution equation of Mavko et al. (1995) is essentially Gassmann's equation where the bulk moduli of the rock and the solid phase are replaced by their compressional moduli. The compressional modulus is the product of the bulk density and P-wave velocity squared. Specifically, the low-frequency compressional modulus M_{Sat0} is

$$M_{Sat0} = M_S \frac{\phi M_{Dry} - (1 + \phi) K_F M_{Dry} / M_S + K_F}{(1 - \phi) K_F + \phi M_S - K_F M_{Dry} / M_S}, \quad (7)$$

where M_{Dry} is the compressional modulus of the dry frame of the rock; and M_S is the compressional modulus of the mineral phase.

The high-frequency limit $M_{Sat\infty}$ is given by Eqn. 4 which we re-write below as

$$\frac{1}{M_{Sat\infty}} = \frac{S_W}{M_P} + \frac{1 - S_W}{M_{S_{W=0}}}, \quad (8)$$

where

$$M_P = M_S \frac{\phi M_{Dry} - (1 + \phi) K_W M_{Dry} / M_S + K_W}{(1 - \phi) M_W + \phi M_S - K_W M_{Dry} / M_S}, \quad (9)$$

and

$$M_{S_{W=0}} = M_S \frac{\phi M_{Dry} - (1 + \phi) K_G M_{Dry} / M_S + K_G}{(1 - \phi) K_G + \phi M_S - K_G M_{Dry} / M_S}. \quad (10)$$

$M_{S_{W=0}}$ is very close to M_{Dry} due to the high compressibility of gas.

An example of the low- and high-frequency bounds for the compressional modulus of a partially saturated rock is given in Figure 2. The difference between M_{Sat0} and $M_{Sat\infty}$ is large at partial saturation, especially in the low-gas-saturation domain.

It appears in Figure 2 that the curve described by Eqn. 8 is close to a straight line that connects the zero-saturation and full-saturation points. This means that $M_{Sat\infty}$ can be approximately modeled as a result of using a fictitious fluid whose bulk modulus is a linear combination of those of gas and water,

$$K_{FP} = S_W K_W + (1 - S_W) K_G, \quad (11)$$

in the fluid substitution Eqns. 2 or 7. Eqn. 9 does not describe a real physical entity resulting

from mixing two pore fluid phases. It is simply a convenient approximate (and physically deceptive) way of conducting patchy fluid substitution.

In Figure 2, left, the range of water saturation is between 0 and 1. In reality, there is always irreducible water saturation in a gas reservoir. This simply means that patchy saturation pattern cannot exist at $S_w < S_{wIrr}$ and the deviation of the patchy saturation curve from the homogeneous saturation curve starts at some finite water saturation (Figure 2, right).

Consider a porous system where water saturation is at its minimum value S_{wIrr} . Let us add some water to this system to arrive at the global water saturation $S_w > S_{wIrr}$. Let us also assume that all additional water is concentrated in fully saturated patches that start to appear as soon as S_w exceeds S_{wIrr} . Then the volumetric concentration f_p of these patches in the partially saturated system is

$$f_p = \frac{S_w - S_{wIrr}}{1 - S_{wIrr}}. \quad (12)$$

Figure 2. Left—The compressional modulus versus saturation in sand of porosity 0.35, where the dry-frame compressional modulus is 7 GPa. The compressional modulus of the mineral phase is 100 GPa, and those of water and gas are 2.5 and 0.1 GPa, respectively. The three curves have been calculated using Eqns 6, 7, and 9, as indicated on the curves. The irreducible water saturation is zero. Right – Same with irreducible water saturation 0.3. Equations used are indicated on the curves.

The bulk modulus of the fully saturated patch is given by Eqn. 3. The bulk modulus $K_{S_w = S_{wIrr}}$ of the surrounding rock at $S_w = S_{wIrr}$ is given by Gassmann's equation

$$K_{S_w = S_{wIrr}} = K_s \frac{\phi K_{Dry} - (1 + \phi) K_{Flrr} K_{Dry} / K_s + K_{Flrr}}{(1 - \phi) K_{Flrr} + \phi K_s - K_{Flrr} K_{Dry} / K_s}, \quad (13)$$

where K_{Flrr} is the bulk modulus of the homogeneous water/gas mixture at $S_w = S_{wIrr}$:

$$\frac{1}{K_{Flrr}} = \frac{S_{Wlrr}}{K_W} + \frac{1 - S_{Wlrr}}{K_G}. \quad (14)$$

Then the bulk modulus of the system with patchy saturation at $S_W > S_{Wlrr}$ is given by the constant-shear-modulus equation

$$\begin{aligned} \frac{1}{K_{Satlrr\infty} + (4/3)G_{Dry}} &= \frac{f_P}{K_P + (4/3)G_{Dry}} + \frac{1 - f_P}{K_{S_W=S_{Wlrr}} + (4/3)G_{Dry}} \\ &= \frac{(S_W - S_{Wlrr}) / (1 - S_{Wlrr})}{K_P + (4/3)G_{Dry}} + \frac{(1 - S_W) / (1 - S_{Wlrr})}{K_{S_W=S_{Wlrr}} + (4/3)G_{Dry}}. \end{aligned} \quad (15)$$

The bulk modulus of the system at $S_W < S_{Wlrr}$ is given by Eqn. 2.

The same approach can be used to calculate the compressional modulus of the system by the approximate P-only fluid substitution. The compressional modulus at $S_W < S_{Wlrr}$ is given by Eqn.

7. For $S_W > S_{Wlrr}$, the compressional modulus $M_{Satlrr\infty}$ in the system with patchy saturation is given by Eqn. (15) which we re-write here as

$$\frac{1}{M_{Satlrr\infty}} = \frac{(S_W - S_{Wlrr}) / (1 - S_{Wlrr})}{M_P} + \frac{(1 - S_W) / (1 - S_{Wlrr})}{M_{S_W=S_{Wlrr}}}, \quad (16)$$

where M_P comes from Eqn. 9 and $M_{S_W=S_{Wlrr}}$ is

$$M_{S_W=S_{Wlrr}} = M_S \frac{\phi M_{Dry} - (1 + \phi) K_{Flrr} M_{Dry} / M_S + K_{Flrr}}{(1 - \phi) K_{Flrr} + \phi M_S - K_{Flrr} M_{Dry} / M_S}, \quad (17)$$

with K_{Flrr} given by Eqn. 14.

The resulting compressional modulus versus saturation curve shown in Figure 2, right, can be approximated by a line resulting from substituting a fictitious fluid mix into Gassmann's equation or the P-only fluid substitution equation for the compressional modulus. The bulk modulus of this fictitious mix is given by Eqn. 1 for $S_W < S_{Wlrr}$ and by

$$K_{FPlrr} = \frac{S_W - S_{Wlrr}}{1 - S_{Wlrr}} K_W + \frac{1 - S_W}{1 - S_{Wlrr}} K_{Flrr} \quad (18)$$

for $S_W > S_{Wlrr}$, where K_{Flrr} is the bulk modulus of the homogeneous gas/water mix at $S_W = S_{Wlrr}$, as given by Eqn. 14.

Modulus-Frequency Dispersion and Attenuation

The difference between the elastic moduli calculated using the homogeneous and patchy

saturation equations is, in essence, the difference between the low-frequency and high-frequency values of the elastic moduli. The effect of the apparent elastic moduli increase with increasing frequency is called the modulus-frequency dispersion (or the velocity-frequency dispersion, if we consider the elastic-wave velocity instead of the elastic moduli). Let us reiterate that here we examine the modulus-frequency dispersion in the practical frequency range that includes seismic and sonic frequencies. As a result, we do not consider the ultrasonic frequency dispersion effects related to the microscopic squirt flow and the Biot mechanisms.

The transition between the lower (homogeneous saturation) curve and the upper (patchy) curve in Figure 2 is the transition between the low and high frequency. The corresponding elastic modulus dispersion is necessarily accompanied by absorption, according to the Kramer-Kronig relation for linear viscoelastic systems. To quantify this absorption, one needs to identify the frequency variation range (e.g., the low-frequency limit is at 10 Hz and the high-frequency limit is at 10 kHz) and also to know *how* the elastic modulus evolves as it changes between its low-frequency and its high-frequency values.

Two basic viscoelastic models give examples of linking attenuation to modulus-frequency dispersion. According to the *standard linear solid* (SLS) model, the elastic modulus M is related to frequency f as

$$M(f) = \frac{M_0 M_\infty [1 + (f / f_{CR})^2]}{M_\infty + M_0 (f / f_{CR})^2}, \quad (19)$$

where M_0 is the low-frequency limit; M_∞ is the high-frequency limit; and f_{CR} is the critical frequency at which the transition occurs from the low-frequency to the high-frequency limit.

The corresponding inverse quality factor is

$$\frac{1}{Q} = \frac{(M_\infty - M_0)(f / f_{CR})}{\sqrt{M_0 M_\infty} [1 + (f / f_{CR})^2]}. \quad (20)$$

The maximum attenuation occurs at $f = f_{CR}$ and is

$$\left(\frac{1}{Q}\right)_{\max} = \frac{M_\infty - M_0}{2\sqrt{M_0 M_\infty}}. \quad (21)$$

In the example shown in Figure 2, right, M_0 is determined by the lower curve and is about 8 GPa at water saturation 0.7. The patchy saturation curve drawn according to Eqn. 16 determines M_∞ which is about 10 GPa at water saturation 0.7. By selecting $f_{CR} = 500$ Hz and using Eqns.

19 and 20, we can determine the shape of the modulus-porosity and attenuation-frequency curves (Figure 3, left). The maximum inverse quality factor is about 0.11.

The *constant* (or *nearly constant*) Q (CQ) model assumes that the quality factor is constant within a frequency range. Then the inverse quality factor is

$$\frac{1}{Q} = \frac{\pi}{\log(f_1 / f_0)} \frac{M_1 - M_0}{2M_0}, \quad (22)$$

where M_0 is the modulus at frequency f_0 and M_1 is the modulus at frequency f_1 . It follows from Eqn. 22 that the modulus changes proportionally to the logarithm of frequency, i.e.,

$$M = M_0 \left(\frac{2}{\pi Q} \log \frac{f}{f_0} + 1 \right). \quad (23)$$

Figure 3. The compressional modulus and inverse quality factor versus frequency for the low-frequency modulus value 8 GPa and high-frequency modulus value 10 GPa. Left – SLS model. Right – CQ model.

By selecting from Figure 2, right, $M_0 = 8$ GPa and $M_1 = 10$ GPa, and assuming that $f_0 = 1$ Hz and $f_1 = 10$ kHz, we find, according to Eqn. 22, that the inverse quality factor is about 0.098. Fortunately, this value is not very sensitive to the selection of the low and high frequency values. For example, if we selected $f_0 = 10$ Hz and $f_1 = 20$ kHz, the resulting inverse quality factor would be about 0.12.

It is also encouraging that the inverse quality values determined from the two viscoelastic models (SLS and CQ) are about the same.

Attenuation and Inverse Quality Factor – Definitions and Relations

The attenuation coefficient α is defined as the exponential decay coefficient of a harmonic

wave:

$$A(x,t) = A_0 \exp[-\alpha(\omega)x] \exp[i(\omega t - kx)], \quad (24)$$

where A is the amplitude of the signal; A_0 is the initial amplitude; t is time; x is the spatial coordinate; $\omega = 2\pi f$ is the angular frequency; and k is the wavenumber. The attenuation coefficient is related to the inverse quality factor as

$$\alpha = Q^{-1} \pi f / V = \pi / (QTV) = \pi / (Q\lambda), \quad (25)$$

where V is the phase velocity; T is the period; and λ is the wavelength. By substituting Eqn. 25 into Eqn. 24 we obtain:

$$\frac{A(x,t)}{A_0} = \exp[-\frac{\pi x}{Q\lambda}] \exp[i(\omega t - kx)]. \quad (26)$$

Let us ask ourselves a question – over how many wavelengths will the amplitude decrease 10^n times? The answer comes from Eqn. 26:

$$\exp[-\frac{\pi x}{Q\lambda}] = 10^{-n} \Rightarrow \frac{x}{\lambda} = n \frac{2.3}{\pi} Q = 0.733nQ, \quad (27)$$

which means that the required number of wavelengths is $0.733nQ$.

Sometimes α is given in dB per unit length. The conversion coefficient is 8.686, i.e.,

$$\alpha[\text{dB} / \text{Length}] = 8.686 Q^{-1} \pi f / V. \quad (28)$$

Adding Together Attenuation from Separate Mechanisms

In our attempts to mathematically relate the elastic wave attenuation to reservoir properties and conditions, we will certainly have to decompose the phenomenon, in its entity, into separate mechanisms that are described by different mathematical models. This mathematical decomposition may be a reflection of a physically realizable decomposition. For example, consider a situation where elastic waves attenuate in dry sandstone due to viscoelastic clay present in the frame. This is the first mechanism of attenuation. If this sandstone is partially saturated, an additional patchy saturation dispersion mechanism may be added to the first mechanism. The question is how to mathematically add these two attenuation mechanisms together to calculate the total attenuation resulting from the superposition of the two physical situations.

One simple assumption will be that the attenuation adds geometrically. In other words, if the

first mechanism acts to reduce the initial amplitude of the signal n times, from A_0 to $A_1 = \exp(-\alpha_1 x)$, over distance x , and the second mechanism independently acts to reduce the initial amplitude m times, from A_0 to $A_2 = \exp(-\alpha_2 x)$, over the same distance, then, when acting together, the two mechanisms will reduce the initial amplitude, over the same distance, nm times to

$$A_{Sum} = nmA_0 = \frac{A_1}{A_0} \frac{A_2}{A_0} A_0 = A_0 e^{-\alpha_1 x} e^{-\alpha_2 x} = A_0 e^{-(\alpha_1 + \alpha_2) x}. \quad (29)$$

Therefore, attenuation coefficients and inverse quality factors calculated from separate mechanisms acting at the same time in the same reservoir can be added.

Attenuation at Partial Saturation – Synthetic Examples

Let us apply the patchy saturation modulus-frequency dispersion principle combined with SLS model to calculate attenuation in gas saturated samples. Our first example is for high-porosity unconsolidated sand. Specifically, we choose laboratory measurements of velocity on a hand-made mixture of Ottawa sand and kaolinite with 10% clay content (Yin, 1993). The room dry sample has porosity about 32% and bulk density of 1.8 g/cm³. The P-wave velocity measured in the air-saturated sample is plotted versus the effective pressure in Figure 4a. The corresponding compressional modulus is shown in Figure 4b.

In our attenuation calculations we assume that the irreducible water saturation is 0.1 and the bulk moduli of gas and water are 0.1 and 2.5 GPa, respectively. The inverse P-wave quality factor is plotted versus water saturation in Figure 5, left, for the effective pressure 20, 10, and 5 MPa. The corresponding attenuation coefficient in dB/m is calculated from the inverse quality factor according to Eqn. 26 and plotted versus saturation in Figure 5, right. The frequency f needed for the attenuation coefficient calculation was fixed at 5 kHz.

At a constant overburden, low effective pressure corresponds to high pore pressure. As a result, we can interpret the attenuation curves as corresponding to normally pressured gas sand (effective pressure 20 MPa); mildly overpressured sand (effective pressure 10 MPa) and strongly overpressured sand (effective pressure 5 MPa).

Figure 4. The P-wave velocity and corresponding compressional modulus versus effective pressure in a room-dry Ottawa sand with 10% kaolinite (experiments by Yin, 1993).

The first observation from Figure 5 is that using attenuation may help discriminate between large and small gas saturation – attenuation is maximum at very small gas saturation – and probably resolve the problem of “fizz” water, i.e., identify water-filled formations with only small amount of free gas present.

The second observation is that the P-wave attenuation is only weakly affected by pore pressure. It may be hard to identify overpressured formations solely on the basis of P-wave attenuation.

The second example is for low-to-medium porosity gas sandstone, similar to that described in Klimentos (1995). We choose porosity 13%; dry-rock bulk density 2.3 g/cm^3 ; irreducible water saturation 0.15; and the dry-rock P-wave velocity 2.5 km/s. The corresponding compressional modulus is 14.4 GPa. The resulting inverse P-wave quality factor is plotted versus saturation in Figure 6, left and the attenuation coefficient in Figure 6, right. For the same situation but stiffer rock (the dry rock velocity 3.5 km/s and the compressional modulus 28 GPa), the attenuation is even smaller but still noticeable.

Figure 5. Left – the inverse quality factor versus water saturation in the sand/clay mixture (described in the text) at varying effective pressure (marked on the curves). Right—the attenuation coefficient in dB/m.

As expected, the calculated attenuation in consolidated sandstone is smaller than that calculated in the soft sand. The attenuation coefficient values shown in Figure 6b are smaller than those reported by Klimentos (1995) for a gas sandstone interval at about 30% water saturation (α between 13 and 16 dB/m). One problem in matching the Klimentos (1995) measurements is that the exact parameters of his acquisition and rock properties have not been reported. In our attenuation coefficient calculations we assumed that the acquisition frequency was 5 kHz. If instead 10 kHz was selected, the attenuation coefficient values would be twice those shown in Figure 6 and closer to those reported by Klimentos (1995).

The main lesson, however, is that we probably cannot predict exact *absolute* attenuation values from a mathematical model. The value of the modeling presented here is in its variational power—the model probably cannot give us the background attenuation but instead can help us estimate attenuation *changes* due to saturation changes and other rock property and condition variations.

Figure 6. Left – the inverse quality factor versus water saturation in low-to-medium porosity gas sandstone described in the text. The upper curve is for the sandstone whose dry-rock P-wave velocity is 2.5 km/s. The lower curve is for stiffer sandstone with the dry-rock P-wave velocity 3.5 km/s. Right—the attenuation coefficient in dB/m. The red circles show the range of values calculated from well log data and reported by Klimentos (1995) for two separate wells (B and F) within the gas zone with about 30% water saturation.

More Synthetic Examples – Very Soft Sands

Let us concentrate on applying the patchy saturation approach to modeling attenuation in very soft sands. In the following synthetic example we will use the same input values as in the example shown in Figure 5 except for the compressional modulus of the dry frame. We will systematically shift its value towards zero to mimic liquefaction (or rather “gasefaction”), conditions which are probably present in severely overpressured geo-hazardous sands. Specifically, we choose three monotonically decreasing values of M_{Dry} : 2; 0.5; and 0.1 GPa. The resulting inverse quality factor is plotted versus water saturation in Figure 7, left. It exhibits a counterintuitive behavior: the softer the sand the smaller the P-wave attenuation. This result arises from the fact that the softer the sand the smaller the difference between the patchy and homogeneous elastic-modulus-versus-saturation curves (Figure 7, right). It is easy to show that for $M_{Dry} = 0$ both curves are exactly the same.

The predicted effect probably reflects some aspects of a realistic physical situation. The most important lesson, however, is that for soft rocks one has to use exact patchy saturation equation such as Eqns. 15 and 16. The fictitious fluid approach reflected in Eqns. 11 and 18 may not be used in soft rocks because it will mistakenly produce a large difference between the homogeneous and patchy saturation curves.

Figure 7. Left – the inverse quality factor versus water saturation in very soft sand. The calculation parameters are explained in the text. The curves are marked by the dry-frame compressional modulus values used and color-coded accordingly. Right—the compressional modulus versus saturation for the same example. In each color group the upper curve is for patchy and the lower is for homogeneous saturation.

3D Plots

In the following synthetic examples we investigate the dependence of the P-wave attenuation on water saturation (S_w) and the stiffness of the dry frame of the rock (M_{Dry}). In the first example we examine very soft sand by gradually reducing M_{Dry} from 3 GPa to zero. This modulus reduction may reflect an increase in pore pressure from a high to a very high value approaching the overburden. The porosity of the sand was assumed constant 0.4. The irreducible water saturation was fixed at 0.05. The bulk moduli of water and gas were fixed at 2.5 and 0.1 GPa, respectively. The compressional modulus of the mineral phase (M_s) was fixed at 100 GPa. The attenuation coefficient α is plotted versus S_w and M_{Dry} in Figure 8, left. It is maximum at low gas saturation and M_{Dry} about 0.7 GPa. Attenuation decreases as M_{Dry} increases from 0.7 GPa towards higher values and also as M_{Dry} decreases from 0.7 GPa towards smaller values.

In the second example, we examine stiffer rock with M_{Dry} varying between zero and 20 GPa. In this case we do not keep porosity constant but instead change it with changing M_{Dry} according to the critical porosity model:

$$M_{Dry} = M_S \frac{\phi}{\phi_c} \Rightarrow \phi = \phi_c \frac{M_{Dry}}{M_S}, \quad (30)$$

where ϕ_c is the critical porosity. We select $\phi_c = 0.4$ and keep all other input same as in the first example. The resulting attenuation coefficient is plotted versus S_w and M_{Dry} in Figure 8, right. It rapidly decreases with the increasing stiffness of the rock frame.

Figure 8. The inverse quality factor versus water saturation and the dry-frame compressional modulus. Left – very soft sand. Right – soft-to-stiff sandstone.

Applicability of the Model – Data and Speculations

There is evidence, albeit extremely sparse, that the patchy saturation pattern does exist and affects the elastic rock properties at well log frequencies in the 1 – 10 kHz frequency range. A classical example from Cadoret (1993) shows that the saturation pattern (scale) depends on the method of introducing gas into fully saturated rock and that this pattern, in turn, affects the velocity in the log frequency range (Figure 9, top). During this experiment, patches of air were visible in a CT-scan image of the rock.

The observed velocity dispersion is matched by P-wave attenuation which peaks at a very low gas saturation (Figure 9, bottom). Cadoret (1993) in fact matched the experimental attenuation data by using the constant Q attenuation model in combination with the velocity difference (dispersion) between the patchy and homogeneous saturation curves in Figure 9, top. These experimental data confirm the results of the patchy saturation model used to calculate attenuation.

Figure 9. Cadoret's (1993) resonant bar experiment at about 1 kHz frequency. Top – the extensional-wave velocity versus saturation at two saturation patterns resulting from two different methods of introducing gas into the porous saturated rock (marked on the curves). Apparently, the upper curve corresponds to the patchy saturation pattern while the lower curve corresponds to the homogeneous saturation pattern. Bottom – the inverse quality factor apparently resulting from the elastic moduli difference between the homogeneous and patchy saturation patterns.

Murphy (1982) observed the same effect of attenuation peaking at low gas saturation (Figure 10, top), without explicitly identifying patchy saturation. In the same experiment, noticeable shear-wave attenuation was observed in partially-saturated rock (Figure 10, bottom). The patchy saturation model introduced in this work predicts that the shear-wave attenuation is zero and, therefore, cannot explain the shear-wave attenuation apparent in the Murphy (1982) data. Other experimental data of Murphy (1982) definitely show that the extensional-wave attenuation is related to the presence of liquid in the system (Figure 11). The magnitude of the shear wave attenuation in these experiments is comparable to that of the extensional waves. How to explain and rationally model the shear-wave attenuation? One way of approaching this problem is to use the principle that shear deformation has compression and extension components and that these components do not necessarily cancel each other and still act to generate wave-induced pore fluid flow and the resulting velocity dispersion and attenuation (Mavko and Jizba, 1991). Another possibility is that shear waves reflected at interfaces produce converted P-waves who in

turn produce attenuation.

Figure 10. Murphy's (1982) resonant bar experiment in the 0.5-1 kHz frequency range (frequency is indicated on the data curves). The sample is partially saturated Massilon sandstone. Top – the extensional-wave inverse quality factor versus saturation. Bottom—the shear-wave inverse quality factor versus saturation. The shear-wave attenuation is smaller than the extensional-wave attenuation but is noticeable.

Figure 11. Murphy's (1982) resonant bar experiment on partially saturated Massillon sandstone. The extensional-wave and shear-wave inverse quality factor versus frequency for three saturation values (indicated in the graph). The magnitude of the shear-wave attenuation is comparable to that of the extensional-wave attenuation.

Empirical Models

Koesoemadinata and McMechan (KOM, 2001) examined laboratory attenuation data available in the literature to statistically relate Q_p and Q_s to porosity, clay content, water saturation, and frequency. In Figure 12, left, we plot the inverse quality factor versus water saturation for the synthetic example illustrated in Figure 5. Shown is only one synthetic curve, calculated for 10 MPa effective pressure. In the same frame we plot the P- and S-wave inverse quality factor calculated from KOM using the same input parameters. The KOM Q_p^{-1} curve lies below our model curve. So does the Q_p^{-1} value calculated from Klimentos and McCann (KLM, 1990). The latter is probably irrelevant to this synthetic example because the original KLM experiments have been conducted on water-saturated consolidated sandstones at 50 MPa effective pressure and 1 MHz frequency and is displayed for reference.

In Figure 12, right, we plot the inverse quality factor versus water saturation for the synthetic example shown in Figure 6, for $V_p = 2.5$ km/s. The appropriate KOM and KLM curves are plotted as well. Neither of the calculated curves matches the values reported by Klimentos (1995).

Let us assume that KOM regression provides valid estimates for attenuation in fully water saturated rock where our model predicts zero attenuation. Then, by adding the KOM $S_w = 100\%$ Q_p^{-1} value to that given by our model for partially saturated curve, we arrive at a synthetic result that approaches the Klimentos (1995) values.

It is also encouraging that KOM prediction for Q_s^{-1} at $S_w = 100\%$ matches the Klimentos (1995) values which are about 0.05 at $S_w = 30\%$.

Figure 12. The inverse quality factor versus water saturation using various models. “US” – the patchy saturation model described in this paper; “KOM P” – KOM model for P-wave attenuation; “KOM S” – KOM model for S-wave attenuation; “KLM P” – KLM model for P-wave attenuation; “US+KOM” – combination of the patchy and KOM model where the latter is only used for 100% water saturation. Left – example illustrated by Figure 5. Right – example illustrated by Figure 6. The large red circle shows values reported by Klimentos (1995).

One conclusion and recipe for practical attenuation prediction that follow from the results shown in Figure 12, right, is to use KOM $S_w = 100\%$ values for “background” Q_p^{-1} and Q_s^{-1} calculation and add to them values given by our physics-based model.

Section Summary

In partially saturated sediment, the P-wave attenuation is largest in the “fizz-water” regions. Therefore, there is a possibility of using attenuation to seismically discriminate non-commercial gas deposits from commercial intervals.

MODEL AT FULL WATER SATURATION

In fully-saturated rock and at ultrasonic frequencies, the microscopic squirt flow induced between the stiff and soft parts of the pore space by an elastic wave is responsible for velocity–frequency dispersion and attenuation. In the seismic frequency range, it is the macroscopic cross-flow between the stiffer and softer parts of the rock. We use the latter hypothesis to introduce simple approximate equations for velocity-frequency dispersion and attenuation in a fully water saturated reservoir. The equations are based on the assumption that in heterogeneous rock and at a very low frequency, the effective elastic modulus of the fully-saturated rock can be estimated by applying a fluid substitution procedure to the averaged (upscaled) dry frame whose effective porosity is the mean porosity and the effective elastic modulus is the Backus-average (geometric mean) of the individual *dry-frame* elastic moduli of parts of the rock. At a higher frequency, the effective elastic modulus of the saturated rock is the Backus-average of the individual *fully-saturated-rock* elastic moduli of parts of the rock. The difference between the effective elastic modulus calculated separately by these two methods determines the velocity-frequency dispersion. The corresponding attenuation is calculated from this dispersion by using (e.g.) the standard linear solid attenuation model.

Upper and Lower Elastic Limits at Full Saturation

Let us assume that a heterogeneous domain of rock includes a number of homogeneous parts with porosity ϕ and the dry-frame compressional modulus M_{Dry} that vary among those parts but are constant within each individual part. Then the effective porosity ϕ_{Eff} of the domain is the arithmetic average of individual porosities:

$$\phi_{Eff} = \langle \phi \rangle, \quad (31)$$

and the effective dry-frame compressional modulus is the Backus (geometric) average of individual moduli:

$$M_{DryEff} = \langle M_{Dry}^{-1} \rangle^{-1}. \quad (32)$$

At a very low frequency and in saturated rock, the wave-induced pressure increments equilibrate between the individual parts. As a result, the effective saturated-rock compressional modulus can be calculated by applying the *P*-only fluid substitution equation (Mavko et al., 1995) to the entire domain under examination:

$$M_{SatEff0} = M_S \frac{\phi_{Eff} M_{DryEff} - (1 + \phi_{Eff}) K_F M_{DryEff} / M_S + K_F}{(1 - \phi_{Eff}) K_F + \phi_{Eff} M_S - K_F M_{DryEff} / M_S}, \quad (33)$$

where M_S is the mineral-phase compressional modulus, assumed the same for all individual

parts of the rock; and K_F is the bulk modulus of the pore fluid, also the same throughout the heterogeneous domain.

At a higher frequency, the individual parts of the domain are undrained. The saturated-rock compressional moduli of each individual part can be calculated by applying the P -only fluid substitution equation individually. Then the effective saturated-rock compressional modulus is the Backus average of the individual saturated-rock compressional moduli:

$$M_{SatEff\infty} = \left\langle \left(M_S \frac{\phi M_{Dry} - (1 + \phi) K_F M_{Dry} / M_S + K_F}{(1 - \phi) K_F + \phi M_S - K_F M_{Dry} / M_S} \right)^{-1} \right\rangle. \quad (34)$$

Consider an example where the heterogeneous domain includes two clean-sand individual parts of equal volumes. The porosities of the two parts are 0.35 and 0.30, the respective dry-frame P -wave velocities are 2.3 km/s and 2.9 km/s, and the respective dry-frame compressional moduli are 9.11 GPa and 15.6 GPa. The compressional modulus of the solid phase (quartz) is 100 GPa and the bulk modulus of the pore fluid (brine) is 2.7 GPa.

The effective porosity, according to Eqn. 1, is 0.325 and the effective dry-frame compressional modulus, according to Eqn. 2, is 11.5 GPa. The corresponding low-frequency saturated-rock compressional modulus $M_{SatEff0}$ is, according to Eqn. 3, 17.7 GPa.

The individual saturated-rock compressional moduli of the parts of the rock are 15.2 GPa and 21.7 GPa for porosity 0.35 and 0.3, respectively. Then the corresponding high-frequency saturated-rock compressional modulus $M_{SatEff\infty}$ is, according to Eqn. 4, 17.9 GPa.

The corresponding maximum inverse quality factor, according to the standard linear solid model,

$$\left(\frac{1}{Q} \right)_{\max} = \frac{M_{\infty} - M_0}{2\sqrt{M_0 M_{\infty}}}, \quad (35)$$

is 0.0056.

Inverse Quality Factor from Well Log Data

Exactly the same approach can be used if well log data are available. First, the dry-frame compressional modulus is calculated from the original log data as

$$M_{Dry} = M_S \frac{1 - (1 - \phi) M_{Sat} / M_S - \phi M_{Sat} / K_F}{1 + \phi - \phi M_S / K_F - M_{Sat} / M_S}, \quad (36)$$

where M_{Sat} is the product of the measured bulk density and the P -wave velocity squared.

Then the above-described operation is applied to calculate $M_{SatEff0}$, $M_{SatEff\infty}$, and the maximum inverse quality factor. The averaging required can be done along a moving window of

a desired length, corresponding, e.g., to a quarter-wavelength.

An example of such calculation is given in Figure 13 for a 1-km-long shaley interval. The resulting inverse quality factor appears to lie within a reasonable range between 0.01 and 0.02 which corresponds to the quality factor between 50 and 100.

Effect of Sampling in Well Log Data

An important issue is the stability of the proposed attenuation calculation method with respect to the sampling size in well log curves. Will the calculated attenuation be the same if the sampling rate is a quarter foot, a half foot, and a foot?

To address this issue we subsampled the original well log data used in Figure 13, whose sampling is a half-foot, with (a) a one-foot interval, and (b) a two-foot interval.

The resulting inverse quality factor is plotted versus depth in the fifth frame of Figure 13. It is essentially identical to that computed from the data with the original (half-foot) sampling rate.

Comparison with Empirical Model

The Koesoemadinata and McMechan (2001) model (KOM) is simply a statistical regression to most of the available attenuation data. Unfortunately, no data are available in the seismic frequency range which makes this regression no more than a guess at seismic frequency. Nevertheless, we compare our modeling results to those predicted by KOM, the latter applied to the well log data shown in Figure 13 and assuming constant (50%) clay content and 30 Hz frequency. The KOM inverse quality factor (Figure 13, fifth frame) is larger than that predicted by our model. Still, both curves lie in the same range which indirectly indicates the relevance of the rational model introduced here.

Figure 13. First three frames—well log data from a water-saturated interval (gamma ray, P -wave velocity, and bulk density). Fourth frame—calculated inverse P -wave quality factor for three sizes of running window – 75, 38, and 19 m. Fifth frame—inverse P -wave quality factor calculated from undersampled well log data (red and blue) superimposed on the curve calculated from the original data with the 38 m running window (bold gray curve). Also, in the same frame the black curve is the inverse P -wave quality factor from the Koesoemadinata and McMechan (2001) empirical model, labeled KOM.

Section Summary

Attenuation remains an elusive propagational seismic attribute that is hard to measure and theoretically model. This paper presents an attempt to model attenuation in fully saturated rock using a simple theoretical approach and first physical principles. The main purpose of modeling is not just to predict attenuation but to establish what can be said about the reservoir properties from attenuation. In other words, our aspiration is to make attenuation a reservoir characterization tool. Above, for partial saturation, we proposed that attenuation is a measure of gas saturation. Present work implies that in fully-saturated rock, *attenuation is a measure of heterogeneity*.

PART 2: SYNTHETIC SEISMOGRAMS WITH Q

Elastic vs. Viscoelastic Materials

Most seismic analysis assumes an isotropic, linear elastic Earth. Materials are **linear elastic** when the stress is proportional to strain. For elastic materials we often write

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{\alpha\alpha} + 2\mu \varepsilon_{ij} \quad 1$$

where $\sigma_{ij}, \varepsilon_{ij}$ are the individual components of stress and strain, λ, μ are the elastic constants, δ_{ij} is the Kronecker delta, and $\varepsilon_{\alpha\alpha} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$ is the volumetric strain or dilatation. For the case of *shear* stress and shear strain, this becomes:

$$\sigma_{ij} = 2\mu \varepsilon_{ij}, \quad i \neq j \quad 2$$

where μ is the shear modulus. In an elastic material, the relation between stress and strain does not depend on history or rate. No real material strictly satisfies this requirement of elasticity.

In contrast, **linear viscoelastic** materials also depend on stress-strain rate or history. Phenomenologically, we can adjust or “patch” the elastic stress-strain relation to include these rate effects, by adding terms to the relation that depend on the time derivatives of stress and strain. For example, shear stress and shear strain might be related using one of the following simple models:

$$\dot{\gamma}_{ij} = \frac{\dot{\gamma}_{ij}}{2\mu} + \frac{\sigma_{ij}}{2\eta}, \quad i \neq j \quad \text{Maxwell solid} \quad 3$$

$$\sigma_{ij} = 2\eta \dot{\gamma}_{ij} + 2\mu \varepsilon_{ij}, \quad i \neq j \quad \text{Voigt solid} \quad 4$$

$$\eta \dot{\gamma}_{ij} + (E_1 + E_2) \sigma_{ij} = E_2 (\eta \dot{\gamma}_{ij} + E_1 \varepsilon_{ij}) \quad \text{Standard linear Solid} \quad 5$$

where E_1 and E_2 are additional elastic moduli, and η is a material constant resembling viscosity. In the Fourier domain, the stress-strain relation for a viscoelastic material can be expressed as

$$\tilde{\sigma}(\omega) = M(\omega) \tilde{\varepsilon}(\omega) \quad 6$$

where $M(\omega)$ is the frequency-dependent complex modulus.

It is important to point out that there is an infinite number of functional forms that can

describe physically realizable viscoelastic materials. These three are just the simplest.

The **quality factor**, Q , is a measure of how dissipative the material is. The lower the Q , the larger is the dissipation. When Q becomes infinity, the material is elastic. There are several ways to express Q . One precise way is as the ratio of the imaginary and real parts of the complex modulus:

$$\frac{1}{Q} = \frac{M_I}{M_R} \quad 7$$

Alternatively, we can write Q in terms of energies:

$$\frac{1}{Q} = \frac{\Delta W}{2\pi W} \quad 8$$

where ΔW is the energy dissipated per cycle of oscillation and W is the peak strain energy during the cycle.

The Requirements of Causality

For linear viscoelastic systems, causality requires that there be a very specific relation between velocity or modulus dispersion and Q . That is, if the dispersion is completely characterized for all frequencies, then Q is known for all frequencies, and vice versa.

In the Fourier domain we can write

$$\tilde{\sigma}(\omega) = M(\omega)\tilde{\epsilon}(\omega) \quad 9$$

where $M(\omega)$ is the complex modulus. For the material to be causal, the real and imaginary parts of $M(\omega)/(i\omega)$ must be Hilbert transform pairs (Bourbié et al., 1987):

$$M_I(\omega) = \frac{\omega}{\pi} \int_{-\infty}^{+\infty} \frac{M_R(\alpha) - M_R(0)}{\alpha} \frac{d\alpha}{\alpha - \omega} \quad 10$$

$$M_R(\omega) - M_R(0) = -\frac{\omega}{\pi} \int_{-\infty}^{+\infty} \frac{M_I(\alpha)}{\alpha} \frac{d\alpha}{\alpha - \omega} \quad 11$$

where $M_R(0)$ is the real part of the modulus at zero frequency, which results because there is an instantaneous elastic response of a viscoelastic material. If we express this in terms of Q

$$Q^{-1} = \frac{M_I(\omega)\text{sgn}(\omega)}{M_R(\omega)} \quad 12$$

then we get

$$Q^{-1}(\omega) = \frac{|\omega|}{\pi M_R(\omega)} \int_{-\infty}^{+\infty} \frac{M_R(\alpha) - M_R(0)}{\alpha} \frac{d\alpha}{\alpha - \omega} \quad 13$$

and its inverse:

$$M_R(\omega) - M_R(0) = \frac{-\omega}{\pi} \int_{-\infty}^{+\infty} \frac{Q^{-1}(\alpha) M_R(\alpha)}{|\alpha|} \frac{d\alpha}{(\alpha - \omega)} \quad 14$$

From these we see the expected result that a larger attenuation generally is associated with larger dispersion. Zero attenuation requires zero velocity dispersion. These are known as the Kramers-Kronig relations.

For purposes of seismogram computation, the Kramers-Kronig relations dictate how velocity depends on Q , and also how the frequency-dependence of Q is related to the frequency-dependence of velocity.

The Kennett Algorithm

Kennett (1974, 1983) used the invariant imbedding method to recursively generate the seismic response of a layered medium by adding one layer at a time. The overall reflection and transmission matrices, $\hat{R}_D$ and $\hat{T}_D$ respectively, for downgoing waves through a stack of layers are given by the following recursion relations:

$$\hat{R}_D^{(k)} = R_D^{(k)} + T_U^{(k)} E_D^{(k)} \hat{R}_D^{(k+1)} E_D^{(k)} \left[I - R_U^{(k)} E_D^{(k)} \hat{R}_D^{(k+1)} E_D^{(k)} \right]^{-1} T_D^{(k)} \quad 15$$

$$\hat{T}_D^{(k)} = \hat{T}_D^{(k+1)} E_D^{(k)} \left[I - R_U^{(k)} E_D^{(k)} \hat{R}_D^{(k+1)} E_D^{(k)} \right]^{-1} T_D^{(k)} \quad 16$$

where $R_D^{(k)}$, $T_D^{(k)}$, $R_U^{(k)}$, and $T_U^{(k)}$ are just the single-interface downward and upward reflection and transmission matrices for the k th interface.

The Kennett algorithm is one of the best-known and most robust methods to generate normal incidence synthetic seismograms in a linear elastic layered Earth model.

Normal Incidence Seismograms with Q

We can adapt the elastic Kennett algorithm to simulate wave propagation in a viscoelastic (attenuating) Earth by replacing the velocities in each model layer with complex velocities. For example, we can perturb the velocity (or modulus), by adding a small imaginary part, such as:

$$M = M_R \left(1 - iQ^{-1} \right) \quad 17$$

which is consistent with the definition of Q:

$$\frac{1}{Q} = \frac{M_I}{M_R} \quad 18$$

For values of Q ~ 15 or greater, this can be expressed in terms of velocities as

$$V \approx V \left(1 - i \frac{Q^{-1}}{2} \right) \quad 19$$

In fact, we can incorporate Q into the generation of synthetic seismograms in a “quick and dirty” way without making any changes to the elastic Kennett code. We simply replace the real elastic velocities in the Earth model with complex (frequency-independent) velocities—that is, we just change the input model. This will yield some of the expected physical results in the computed seismogram: a decrease in wave amplitude with propagation, stretching of the wavelets, and spectral decay. For large values of Q (> 100), this will yield generally reasonable and acceptable results.

The problem is that this approximate simulation of wave propagation using frequency-independent velocities will violate causality. As we discussed earlier, causality requires a very specific relation between frequency-dependent velocity or modulus and Q, as dictated by the Kramers-Kronig relations. Hence, the more correct way to model wave propagation is to modify the Kennett algorithm to allow for causal, frequency-dependent complex velocity functions. Since the Kennett algorithm is computed in the frequency domain, it is fairly straightforward to implement Q.

When choosing a causal velocity function, there are an infinite number of choices – that is, there are an infinite number of Hilbert transform pairs that will allow the real and imaginary parts of the complex, frequency-dependent modulus to satisfy the Kramers-Kronig relation. One choice is the Standard Linear Solid, mentioned earlier. The Standard Linear Solid gives the narrowest possible frequency band of high attenuation – the most frequency-dependent Q. At the other extreme is the Constant Q model, in which there is no variation of Q with frequency.

The best choice of dispersion relation for the Earth is not known. This is because Q and velocity dispersion are difficult to measure accurately over large bandwidths. We believe that

the nearly constant Q models are reasonable choices, especially considering that seismic data are fairly narrow band.

For seismogram generation, Kennett suggested one specific relation between frequency-dependent velocity and Q, to preserve causality:

$$V_p(\omega) = V_p \left(I - i \frac{\text{sgn } \omega}{2Q(\omega)} \right) \quad 20$$

$$Q(\omega) = Q_o \left(I - \frac{\ln \left(\frac{\gamma \omega}{\omega_o} \right)}{\pi Q_o} \right) \quad 21$$

This corresponds to a slowly varying (nearly constant) Q vs. frequency function.

Another choice is given by Aki and Richards, which we implement in our modeling:

$$V_p(\omega) = V_p(\omega_o) \left(I - i \frac{\text{sgn } \omega}{2Q(\omega)} + \frac{1}{\pi Q} \ln(\omega / \omega_o) \right) \quad 22$$

$$Q(\omega) = Q_o \left(I - \frac{\ln \left(\frac{\gamma \omega}{\omega_o} \right)}{\pi Q_o} \right) \quad 23$$

This is the functional form used in generating the synthetics shown in Part 3 of this report.

Raytracing Code for Synthetic Seismograms with Q

We created a Matlab program to generate synthetic CDP gathers using ray tracing in a 1-D (layered) Earth model. The program begins by shooting a fan of rays from the surface (depth = 0). The current version shoots 25 rays, spanning from zero to 50 degrees at the surface, equally spaced in $\sin(\theta)$. Each of these rays is traced downward through the layered model: Angles are computed using Snell's law, then horizontal distance and travel times, for each ray in each layer are computed via ray tracing.

After the basic kinematics are computed by ray tracing in the layered model, the input model is converted to equal sample intervals in *time*, before generating reflectivities. The time intervals are set equal to the time interval of the input wavelet. This obviously resamples model. The ray tracing times, horizontal distance of propagation, and ray angles are all computed on the original input model, and only the reflectivities are computed after resampling to time. Resampling is done carefully -- if layers are thin, then there is first oversampling to at least one sample per layer, then velocities are averaged using Backus, and density and angles are averaged linearly back to the wavelet sample interval.

Reflectivities are computed on the time-resampled model using Zoeppritz, and the seismic trace comes from convolving the reflectivity sequence with the wavelet. Reflectivities are computed at the desired offsets by interpolating angles and times from the fan of rays. Reflectivities and transmission coefficients are computed using full Zoeppritz.

Two reflectivity sequences are computed: one with moveout and the other without. Then convolution with the wavelet creates the traces. Hence, there is no NMO stretch.

Calculation is for *precritical* only -- rays are prevented from going critical, and there are no turning rays. Also, we only compute primaries. This is standard ray tracing combined with a convolution model. We do not model spherical divergence, and there is no ray focusing or defocusing estimated.

We recommend that the wavelet be sampled at least 15 samples per wavelength, in order to ensure that the depth-to-time converted Earth model is sufficiently well sampled.

Input Parameters

<i>z</i>	Depth to BOTTOM of layers. First <i>z</i> is thickness of first layer. Can also be thought of as depth to reflectors. First <i>z</i> is depth to first reflector. Computation starts from first depth point.
<i>vp,vs,rho</i>	Interval properties corresponding to the depths
<i>deltax</i>	offset interval of desired traces. First trace is normal incidence
<i>nx</i>	number of traces to compute
<i>wvlt</i>	input wavelet
<i>deltat</i>	sample interval of input wavelet and output seismogram
<i>nshift</i>	number of samples to shift, if wavelet is two-sided.
<i>tmode</i>	= 1 (default) to calculate transmission losses; = 0 to ignore transmission losses

Outputs

<i>s</i>	array of seismograms, one per column, moveout corrected
<i>snmo</i>	array of seismograms, one per column, with moveout

ttop	2-way time to top of model
x	vector of computed offsets
ssxx	array of $\sin(\theta)$, giving angles of incidence corresponding to each seismogram.

Raytracing Examples

Figure 1 shows logs of V_p , V_s , and density from the North Sea. The model has 4006 layers. We input these logs into the raytracer, assuming a 50 Hz Ricker, sampled with 31 points, yielding a sample interval, $\Delta T=1.3$ ms. These assume an elastic earth. Figure 2 shows an excellent agreement of the near-offset trace from the gather with a normal incidence Kennett algorithm (no multiples), again in an elastic earth. Finally, Figure 3 shows gathers with and without NMO.

Figure 1. North Sea logs used for generating synthetics.

Figure 2. Comparison of normal incidence elastic traces computed using the raytracer, and the Kennett algorithm.

Figure 3a. Elastic earth gathers, assuming a 50 Hz Ricker wavelet.

Figure 3b. NMO corrected elastic earth gathers, assuming a 50 Hz Ricker wavelet.

Finite Q is implemented in the code by means of a causal earth attenuation filter. Causality requires that this filter be minimum phase. Hence, we design a series of minimum phase filters whose power responses approximate a frequency-independent Q . We implement the filter just before convolution. At each reflector, we replace the single reflectivity spike with a series of spikes corresponding to the coefficients of the causal earth filter (scaled by the reflectivity). The filter simulates the entire attenuation along the two-way travel path from the surface to that reflector.

Figure 4 shows a comparison of normal incidence traces computed using the Kennett algorithm and the raytracer, with $Q=50$. We see an excellent agreement of amplitude and phase.

Figure 4. Comparison of normal incidence traces from the Kennett algorithm and the raytracer, $Q=50$.

Figure 5 shows comparisons of synthetic gathers in an elastic earth and an earth having $Q=50$, both computed using the raytracer. We see the expected effects of attenuation: reduced amplitude and frequency (particularly at the farther offsets). We also see additional phase delays, which are the result of velocity dispersion, which must accompany attenuation in a causal earth.

Figure 5. Synthetic gather in an elastic earth (top), and in an earth with $Q=50$ (bottom).

PART 3: EFFECT OF ATTENUATION ON SYNTHETIC SEISMIC

Comparison of Synthetic Seismic Programs

In this section we examine results from two synthetic seismic programs: Osiris seismic modeling system provided by Odegaard and a synthetic seismic program, developed by Rock Solid Images (RSI), implementing the Kennett method for normal incidence.

Osiris

Osiris is the commercial seismic modeling program developed and marketed by Odegaard AS of Copenhagen, Denmark. This program implements the reflectivity method for seismic media representing a stack of visco-elastic transversely isotropic homogeneous layers. It provides a variety of source models and receiver geometries as well as several velocity dispersion models. This program computes the full elastic wave field, containing all of the multiples, AVO effects, reflection losses, refracted, inhomogeneous and converted waves and other physical phenomena describing the interaction of a seismic wave with "cake-layered" earth model. In other words, the synthetic seismic field is almost as complex as the real seismic measurement.

RSI's Kennett Code

The RSI's Kennett code is an implementation of the Kennett method of seismic modeling for the special case of normal incidence plane seismic waves in a stack of homogeneous plane visco-elastic layers. The computed synthetic field contains all of the effects occurring at the normal incidence (i.e., multiples, transmission losses, etc.), but nothing else. There are no converted waves, AVO, head waves, etc. The resulting wave field is even simpler than zero-offset real seismic: no spherical diversion, inhomogeneous waves, etc.

Ideally any synthetic computation program needs to be calibrated by comparing the synthetics it produced with a controlled laboratory experiment. A cheaper alternative would be to compare the output of two independent, significantly different, synthetic programs. If they match, this comparison provides a means of cross-validation for both. For the match to occur, we need to pose a comparable problem to both programs. We computed and matched synthetic seismic traces from the five-layer model with both the Kennett code and Osiris synthetics program.

Both Osiris and Kennett code provide full elastic wave solution for their respective models, and should produce similar synthetic traces (zero-offset trace for Osiris). However, they employ different propagation and source models. In the Kennett code the propagation mode is 1-D,

which is the same as exciting plane waves only. In Osiris, the seismic wave is excited by a point source (there are other options, but no plane wave source) and the propagation model is true 3-D (2-D optionally). These differences bring about a significant difference in the zero-offset traces.

Spherical diversion. The 3-D propagation mode in Osiris causes the amplitude decrease with time as $1/t$. To match the Osiris zero-offset synthetic trace with the Kennett synthetic trace the former needs to be multiplied by t .

Inhomogeneous waves. The point source, modeled in Osiris, in addition to the conventional spherical P-wave, generates inhomogeneous waves, exponentially decaying with depth. One of them has P-wave particle motion, but travels with the S-wave velocity. It is reflected at the first boundary as conventional P-wave and is subsequently recorded at zero offset with PS-wave arrival time, thoroughly confusing the picture. To achieve a clean match between the two synthetics we had to make the top layer so thick, that this wave arrived after the last reflection of interest.

Since we had no control over the velocity dispersion implementation in Osiris, we turned the dispersion off in both programs. We provided identical wavelets to both programs, and considered the z-component of the particle velocity output in Osiris. The reflection arrival time in Kennett code is fully defined by the thickness of the top level. In Osiris it is defined by both this thickness and also source and receiver depths, which cannot be zero. To achieve a complete match, we had to effectively reduce the thickness of the top layer for Kennett code by the sum of the source and receiver depths in Osiris.

Though we were only interested in the zero-offset trace, we had to compute at least one more non-zero offset trace in Osiris, or else computational instabilities completely distorted the seismic fields. After taking all of the above measures we reduced the difference between the synthetic seismic traces obtained from each program to the level of 0.1% of each reflection.

Synthetic Models

Achieving virtually identical synthetic seismic traces from these different programs serves as cross-validation for both. The subsequent experiments have been performed with the Kennett normal incidence code because: We have access to the source code, which allowed us to easily control computational parameters and integrate the synthetics computations with our graphical and I/O systems. This code allows to perform computations and displays on a PC in MatLab or Octave environment, which is faster and more convenient. The normal incidence model allows us to exclude from the synthetic traces some of the physical effects that take place in 3-D models (like inhomogeneous waves) but have no relevance to the topic of our investigation, attenuation effects on seismic reflection and transmission.

Five-layer Model

We started investigating the effects of attenuation on synthetic seismic with a simple model with five thick layers, presented in Figure 1.

Figure 1. Model with five thick layers.

The thickness of the layers was adjusted so as to avoid any interference between primary reflections and intrabed multiples. The synthetic was computed twice with the same elastic parameters but with different Q values. The first synthetic was computed with Q values shown in Figure 1 (model Q). For the second computation we set the Q values in all layers to 300 (high Q). The synthetic traces obtained in this case are equivalent to the results of conventional zero-offset synthetic seismic modeling with multiples. Comparing these synthetic traces allows us to separate the acoustic contrast effects from the attenuation effects. The wavelet used in these computations and its spectrum is presented in figure two.

Figure 2. Wavelet and its spectrum. The central frequency is 30 Hz.

The synthetic seismic traces computed with high Q and model Q (as shown in Figure 1) are presented in figure 3. The amplitude effect of attenuation can be seen by comparing the reflection amplitude of model Q synthetics (black curve) with the high Q synthetics (pink curve). Attenuation obviously reduces the amplitudes of seismic reflections.

Figure 3. Synthetic seismic traces computed with model Q and high Q.

However most of the methods designed to estimate seismic attenuation from seismic traces are based on the way attenuation affects seismic spectra. Namely, higher frequencies in the seismic wave experience higher attenuation than the lower ones, thus shifting the dominant frequency of seismic reflections.

Figure 4. Effect of attenuation on individual reflections.

Figure 4 represents individual reflections from each of the four reflectors in the model, computed with model Q and high Q. All reflections have been equalized by their RMS value. This graph shows how the apparent frequency decreased after the seismic wave passed through the wet sand layer. To make this frequency shift more apparent, we computed the spectra of all reflections, which are represented in figure 5. The spectrum shifts to lower frequencies when the seismic wave passes through layers with model Q values, and stays unchanged when the Q values are high. The dominant frequency of the latest reflection is around 20 Hz, while the top reflection has the dominant frequency of the original wavelet equal to 30 Hz.

Figure 5. Reflection spectra.

Wedge Model

Experiments with the five-layer model demonstrated that the synthetic seismic exhibits the expected effects of attenuation on the seismic waves. In the next series of experiments we wanted to investigate the synthetic seismic sensitivity to the sand thickness. For this purpose we computed the synthetic seismic traces in a series of five-layer models with the sand thickness decreasing from 450 m to zero. At the same time the thickness of the Shale 3 layer increased from 660 m to 1110 m to keep the arrival time of reflection 4 constant. We investigated the amplitude and frequency effects of varying sand thickness with model Q and high Q values in the model.

Figure 6. Schematic representation of the series of 1-D models with decreasing sand thickness.

The plots in Figure 7 represent synthetic seismic traces displayed on the background of color-coded instantaneous amplitude. The amplitude of the bottom reflection is of greatest interest here. The amplitude of the bottom reflection in synthetic computed with high Q remains constant regardless of sand thickness and is practically unaffected by the tuning effects in the sand layer. On the other hand, the decrease in the sand thickness when synthetic is computed with model Q causes significant increase in the amplitude of the bottom reflection. Even when the sand thickness falls below tuning, and reflections 2 and r merge, the amplitude of the bottom reflection continues to increase with the sand thickness decrease, demonstrating high sensitivity of the synthetic seismic to the changes in the attenuation along the ray path.

Figure 7. Synthetic traces and the Instantaneous Amplitude.

The frequency effects of changing sand thickness can be observed in the plots presented in figure 8. Here the same synthetic traces are plotted on the background of color-coded dominant frequency. The tuning effects can be clearly seen on the dominant frequency of reflections 2 and 3. These effects seem to be the same when synthetic is computed with model Q and high Q values. The dominant frequency of the bottom reflection, however, is unaffected by tuning, and only responds to changes in the sand thickness when synthetic seismic is computed with model Q values.

Figure 8. Synthetic traces and the Dominant Frequency.

Well 1 In-situ Case

Subsequent synthetic seismic computations were performed on an earth models built from well log measurements obtained in a deep water well in the western GOM (well 1). The well log measurements had been subjected to the standard MOSS™ procedures in order to obtain the most accurate set of elastic parameters. The Q values have been predicted through a combination of theoretical and empirical methods. The synthetic traces were subsequently compared with the seismic values extracted from the migrated seismic stack along the well bore path.

Figure 9 represents the wavelet and its spectrum used in the synthetic seismic experiments with earth models based on well 1. This wavelet was selected after the spectral analysis of the migrated stack, with which the synthetic seismic traces were to be compared.

Figure 9. Wavelet used to compute synthetics in well based earth models. The dominant frequency is equal to 20 Hz.

Building an earth model for synthetic seismic computations from the well measurements is a non-trivial task. First of all, there is uncertainty in the shallow part of the model. To correctly match synthetic and seismic we have to know the velocity and Q distribution from the seismic datum surface all the way to the bottom of the well. The velocity can be estimated from the check shot information, but there is no reliable way to estimate the attenuation in the shallow part of the well, which had not been logged.

This problem had been resolved in the following way: The seismic wavelet had been estimated at the time location of the beginning of the logged interval. We assumed the earth model above this depth to consist of one homogenous non-absorptive ($Q=300$) layer. The synthetic trace had been shifted and scaled in order to match the time and amplitude of reflection from the top of the logged interval with the corresponding reflection on the seismic trace (check shot information had been used to identify that reflection and time shift).

In this way, we could study the effect of only the known part of the well on the synthetic trace and compare it with the effect of real subsurface properties in the same depth range on the real seismic data. This solution is adequate for zero-offset synthetic studies, but additional efforts

will be needed for offset synthetics. The well log contained 8600 samples at 0.5 ft increments. The synthetic seismic program considers each well log sample as a measurement at the top (or bottom) of a separate geologic layer. The computation time and memory requirements are proportional to the number of layers, and can become quite large when the number of layers exceeds 1000. On the other hand, the seismic wave, having apparent frequency of 20 Hz, provides depth resolution of 15 ft at best. In other words, the seismic wave effectively averages the models' characteristics in a sliding window of the above size. Therefore, there is no need to retain in the earth model built for synthetic computations the sampling interval of the original well measurements.

Instead we built a smoothed model with larger sampling intervals d equal to 5 ft, which reduced the number of samples in the model to 800. The elastic parameters at each depth h of the smoothed model were computed as Backus average of the elastic parameters from the original well log computed in the depth range between $h-d/2$ and $h+d/2$. Most of the other parameters in the smoothed model (porosity, S_w , etc.) were computed as algebraic average of those parameters in the original well in the same depth range. The Q values were computed as harmonic average.

Figure 10. Comparing the original well log with the smoothed model.

In figure 10 we can see graphs of original and resampled/smoothed values of Q_p (top plot) and RHO_B (bottom plot). The original well log values are plotted in pink color. V_p plots look similar to RHO_B . These plots illustrate the smoothing effect. The resampled model still contains more details than the normal incidence seismic wave is sensitive too. To illustrate this fact figure 11 display the resampled model compared with the well log upscaled to seismic resolution.

Figure 11. Comparing the original resampled model with the upscaled well.

The upscaling had been performed by adaptive Backus averaging within constant time intervals of 4 ms. The light-blue curve represents the upscaled well log and confirms the fact that the seismic wave effectively senses a much smoother medium, and therefore the resampled model retains more than enough detail.

As with the five-layer model, we computed synthetic seismic with the same acoustic parameters of the resampled earth model twice: with model Q (predicted Q values) and with high Q (a $Q_p=300$ in all layers). Comparing these synthetic traces allows us to separate the effects of the acoustic impedance contrasts, including the "extrinsic", or scattering attenuation from the effects of intrinsic attenuation of the seismic waves in the model. The plots in figure 12 provide instantaneous amplitude comparison between synthetic traces with model Q (top plot) and high Q (bottom plot). Of interest to us is the comparison between the amplitude levels of the first reflection in the model (*R1*) (topmost wet sand) and the second reflection (*R2*) (gas sand). When

the amplitudes are only defined by the acoustic impedance contrast (high Q case) the ratio between $R2$ and $R1$ is $R2/R1 \approx 2$, whereas with model Q $R2/R1 \approx 1.3$. The decrease of the relative amplitude of the gas sand reflection with model Q is caused by attenuation in the shale layer between those two sands, and has very little to do with the attenuation in the gas sand whatever its value is.

Figure 12. Instantaneous amplitude plots.

The frequency effects of attenuation are illustrated in figure 13, where the same synthetic traces are plotted on the background of color-coded dominant frequency.

Figure 13. Dominant frequency plots.

The dominant frequency content of the seismic trace is affected by both extrinsic attenuation (e.g., interfering reflections within a pack of thin layers) and intrinsic attenuation. With high Q values only the extrinsic effects take place, and cause an increase in the dominant frequency. With model Q values the intrinsic attenuation causes the decrease of the dominant frequency with time, which is the effect usually observed in real seismic.

Considering the difference that model Q values make on synthetic we can expect that the synthetic with model Q values will come considerably closer to the real seismic than the synthetic with high Q values (conventional synthetic).

Figure 14 allows us to compare the two synthetics (model Q and high Q) with the real

seismic trace. To ease the identification of various reflections on the synthetic and real seismic traces, the Vp curve is displayed at the top of the plot.

Figure 14. Comparison between synthetic and real seismic reflection amplitudes.

We can see, that though the amplitude of reflection 1 is the same for all three traces, reflection 2 on the synthetic trace with high Q (light blue curve) is much higher than the reflection amplitude on the real seismic trace (black curve), while the amplitude of reflection 2 on synthetic trace with model Q (pink) is much closer to it. This comparison confirms that including model Q values in the synthetic computations provides a better synthetic-to-seismic match. It also confirms that the predicted Q values are reasonably close to the Q values in the real rocks.

Well 1 Fluid Substitution

To study how the synthetic traces change with changing fluids, a pseudo-well was constructed from well 1 by replacing gas with brine in the gas sand. The elastic properties and Q values had been predicted in the pseudo-well via rock physics modeling. Then the synthetic traces were computed with the high Q and model Q values.

Figure 15. Synthetic computations with fluid substitution

Figure 15 presents the pseudo-well curves and synthetic traces. The in-situ curves and traces are color-coded red, the fluid-substituted ones are color-coded blue. Substituting gas with brine eliminates the attenuation and Vp anomaly associated with the gas sand, but still leaves an anomaly in bulk rock density, which produces a reflection from the sand, though weaker than the reflection from the in-situ case. This effect is visible on both high Q and model Q synthetic traces. However, the correlation coefficient between the two high Q synthetic traces (for in-situ and brine filled wells) is 0.58, while for the model Q it is 0.7. In other words, fluid substitution causes greater changes in the synthetic traces when attenuation is not included in the synthetic computations. It means that at least in this case, the seismic is less sensitive to the fluid change than it could be expected from the analysis of conventional synthetics.

PART 4: ATTENUATION FROM SEISMIC

Method 1: Anomalous Absorption Attribute

During the development of a Q computation, we have observed that the results are very noisy, when viewed closely. However, when viewed over a larger volume, they indicate some anomalous low or high Q zones. There are also several other inherent problems, which we have touched upon before (Taner et al, 2002). We know that Q is a bedding attribute, rather than an interface attribute. Therefore, our observation ability will depend on the bed thickness and the complexity of the bed interfaces. In many instances we do not seek a very accurate Q measurement. Instead a reasonable estimate such as a low, medium or high Q with defined approximate limits would be acceptable. Therefore locating possible low Q zones, rather than actually measuring Q values will become a step closer to Q estimation. In this report we give computational details of one such estimation approach.

The quality factor Q represents the transmission efficiency of the material. A high quality factor (Q) represents transmission of energy with high efficiency or small loss. Since we are seeking to detect low quality factor zones, we must detect higher energy loss areas. It is natural that the higher frequencies, while losing approximately the same percentage per cycle, lose more of their energy over the same travel distance. This results in shifting the spectral content of the seismic waves to lower frequencies, as well as narrowing their bandwidth. These two indicators will be used to detect anomalous zones. Another point is that the energy loss is continuous over travel distance, or seismic travel time. If all material were homogenous, we would expect the energy loss be of exponential form. Any variation of the quality factor will perturb this form. However, in general the overall loss trend will be smooth due to integration of the absorption effects over the travel distance. Any anomalously high absorption zone will perturb this smooth trend and will appear as a sharper rise in the rate of loss. Therefore, we try to detect the anomalous zones as a deviation from an overall trend. Let $f(t)$ represent a seismic trace and $E(t)$ represent its envelope (Taner et al, 1979). First, the seismic trace $f(t)$ is decomposed into Gabor sub-bands using the Gabor-Morlet decomposition; (Morlet et al, 1982, Koehler, 1983, Taner, 2001)

$$G(t, f) = \sum_{\tau} f(t - \tau) \cdot g(f, \tau) \quad (1)$$

where ;

$$g(f, \tau) = e^{-i2\pi f\tau} \cdot e^{-\alpha(f)\tau^2} \quad (2)$$

Since the output is complex trace in each sub-band, then

$$a(f, t) = |G(f, t)| \quad (3)$$

here $a(f, t)$ represents the joint time-amplitude spectrum of the seismic trace, and t = time, f = frequency. (See figure 1)

Next, we compute the first and the second moments of the joint time frequency spectrum at each data sample. The first moment gives the mean value of the instantaneous spectrum;

$$F_{ave}(t) = \frac{\sum_f f \cdot a(f, t)}{\sum_f a(f, t)} \quad (4)$$

and the second moment is;

$$F_{RMS}^2(t) = \frac{\sum_f f^2 \cdot a(f, t)}{\sum_f a(f, t)} \quad (5)$$

The difference of the squares the second and first moments will be equal to the variance. For a Gaussian shaped amplitude spectrum the variance is the square of the half bandwidth. In general variance is proportional the bandwidth. The second moment represents the instantaneous RMS frequency.

$$\sigma^2(t) = F_{RMS}^2(t) - F_{ave}^2(t) \quad (6)$$

The bandwidth and RMS frequency measurements are instantaneous values and may contain local noise. First, we estimate the general trend with a weighted low pass filter of the variance (instead of actual bandwidth) and the RMS frequency; Since both the bandwidth and RMS frequency decay with time, we use their product rather than the individual term. Using the product of two observations is equivalent to logical .AND. condition. The trend is computed by low pass filter and by weighting with the envelope. This will give more weight where there is high energy. We assume that the noise is in the background, generally in the relatively low amplitude zones. Let $C(t)$ represent the product of RMS frequency and the bandwidth ';

$$C(t) = 2.F_{RMS}(t).\sigma(t) \quad (7)$$

The envelope weighted output, depending on the choice of filter $W(t)$, is given by;

$$CW(t) = \frac{\sum_{\tau} E(t-\tau)C(t-\tau)W(\tau)}{\sum_{\tau} E(t-\tau)W(\tau)} \quad (8)$$

The local variation is also computed by envelope weighting, but with a relatively higher bandpass filtering; Let $CL(t)$ and $CS(t)$ represent low and high bandpass filtered outputs, respectively, then the difference between the trend and the local variation is;

$$AZ(t) = CS(t) - CL(t) \quad (9)$$

here the zones to the left of the yellow line (negative values zones of $AZ(t)$) represent areas of lower frequency and narrower bandwidth, hence they can be interpreted as the anomalous absorption zone indicators.

Figure 1: Gabor-Morlet decomposition of a seismic trace; Left most is the seismic trace, its envelope is plotted next to it. The middle group of traces represent the instantaneous amplitudes of the Gabor-Morlet sub-bands. The difference between the yellow trend line and the red local variation curve represents the possible anomalous zones.

Figure 1 schematically shows the joint time-frequency decomposition, given the general trend and the local variations.

Summary of Method 1

We have presented a simple method for the computation and detection of anomalous absorption zones. This zone indicator computation seems to be rather robust and can become a useful basis for several other computations. Since the actual Q computation is very noisy, it could be better to estimate Q values within anomalous absorption zones. A second use is to combine the anomalous absorption zone indicators with zones of low and the zones of high acoustic impedance. These hybrid attributes further focus our attention on the possible areas of interest. For example low acoustic impedance with high absorption rates will indicate possible porous sands. In carbonate zones we may look for high acoustic impedance with higher absorption rates.

Test of Method 1 on Seismic Data Set

In Figure 2 below, we show the results of computing the anomalous absorption (AA) attribute on a Gulf of Mexico 3D seismic data set. This figure shows the intersection of one of the producing wells with a time slice through the AA attribute volume. At the top of the producing horizon we see high AA (red) near the location where the wellbore intersects the time-slice. A similar result is shown in Figure 3 where the time-slice is viewed from an oblique angle.

Figure 2: Time-slice through 3D Anomalous Absorption volume. Red areas are high absorption (seismic attenuation). Well 1 is a gas producer at the level of the seismic time slice.

Figure 3: Time-slice through 3D Anomalous Absorption volume. Red areas are high absorption (seismic attenuation). Well 1 is a gas producer at the level where it intersects seismic data.

Method 2: Q Computation with Minimum Phase Operators

Introduction

Last year we experimented with several methods to estimate Q from seismic data. Spectral division, the main tool for Q computation, still proves to be the best method. We have used the Gabor-Morlet Joint Time-Frequency decomposition to estimate the local spectra. This gives fairly good results with synthetic data. The biggest problem was the choice of the band-width for Q estimation. The next most interesting method was spectral estimation by the minimum phase inverse operators. This method proved to be very flexible and robust. It was flexible, because by changing the order of the polynomial division, one could compute either the Q computation or Q compensation operator. We have decided to implement this method for computing Q from 3-D seismic data. This report will cover the technical details of Q computation with minimum phase operators. All of the methods gave inhomogeneous results (it appeared noisy, because results did

not track events), hence we had to consider some spatial consistency checking and proper smoothing.

General Assumptions and Problems:

There are a number of methods proposed for Q computation. Most of the methods give poor to fair estimates of Q. The most common problem is Q estimates are very noisy or spatially very inconsistent. Q values, being a bedding attribute, are similar to the interval velocities. Their effects are accumulative, hence they can be measured by differentiation, which sets a condition conducive for additional noise generation. Like velocities, we could estimate average, RMS or interval Q values. However, the most interesting is the inter-bed Q estimate. Here we come up against the resolution problem: “How thin a bed can we resolve?” Velocities are measured from picked events. Attenuation on the other hand, is measured from spectral signatures, as well as picked events. Therefore, we are not bound to track events and compute spectral variations over closely spaced windows. However, the closer the windows, the smaller the Q effects, and hence we get noisier estimates. It means that we have to consider some noise reduction procedures.

One of the common methods is to incorporate areal estimates. This is based on the assumption that Q does not vary abruptly from trace to trace, but it may vary from layer to layer. This assumption implies Q values should be more consistent along bedding, hence we can put some restriction or initial condition in our computation. In the present method we have included an edge preserving smoothing process prior to the Q computation. This helped to some degree. After smoothing, we have scanned the data to determine the principal slopes of events. Summing adjacent traces along the principal slopes formed average traces. First experiments were run by computing Q from the scanned average traces. Results, while improved over previous trials, still showed excessive scattering of Q values. After Q computation, we used the computed slopes as the direction of data to be included in the spatial smoothing. Results looked too blocky. The second method was to average autocorrelation functions. As in the edge preserving smoothing, we have averaged autocorrelations of the trace at the center and its eight neighbors. Computations showed considerably better stability and produced higher resolution of Q values. While there was some scatter, consistencies along bedding became discernable.

The most accurate method of Q computation is the spectral ratio. However, before the ratio is computed, we have to make sure the spectra do not have any zero or negative values. These erroneous values are the result of poorly computed autocorrelation functions. Gabor-Morlet joint time-frequency analysis (JTFA) provides us with instantaneous amplitude and phase spectra. This can be used for ratio computation. Our JTFA experiments with synthetic data did give good results. Only problem is, due to noisy background, we cannot use the full spectral band. We have

to determine the zone with best signal to noise ratio. We can determine this band in the minimum phase method, which I will describe below.

Computation

The program examines 3 cross-lines at one time. This is used in the edge preserving smoothing section routine. The main computational steps are as follows;

- 1) Read data, in the beginning accumulate at least 2 cross lines, otherwise work with 3 cross lines at a time. During the process, output one cross line and read a new one recursively.
- 2) Apply edge-preserving smoothing to one cross line,
- 3) Apply dip-scan to generate dip optimized traces and event dips,
- 4) Apply two different method of autocorrelation computation;
 - a) Autocorrelation was computed from dip optimized traces. This did not work well.
 - b) Autocorrelations of central trace and eight of its neighbors were averaged. This produced stable results.
- 5) Compute Q values between zero crossing more that 3 samples apart, (computational details will be described in the next section.)
- 6) Pick Q values along the event dips using a Gaussian weight to compute sample by sample weighted Q average.
- 7) Output Q values one cross line at a time, read new one in until all lines are done.

Mathematical Detail

Q computation is performed by synthetic division of minimum phase inverse wavelets computed over two time zones. The minimum phase wavelets do not have any zeros on the unit circle. They can be very short, hence their autocorrelation can also be very short. This will improve the time resolution.

- 1) Select a short operator length, say 60 to 80 milliseconds,
- 2) Autocorrelation window length could be about 200 to 400 milliseconds.
- 3) For each window, find the nearest zero crossing, and place the beginning of autocorrelation window to the zero crossing. This will give a stable estimate.
- 4) Compute autocorrelation at each zero crossing and compute minimum phase inverse operator from each autocorrelation by the Wiener-Levinson algorithm. (This was modified in the second method. Autocorrelations of the center trace and its eight neighbors are summed to get a better and more stable autocorrelation estimate.)

5) For each adjacent (in time) pair of operators, determine the spectral ratio computation band. This band must contain best signal to noise ratio between the two spectra involved in the computation.

Compute inverse of deeper operator and compute its Fourier transform. This will give the estimate of the deeper wavelet amplitude spectrum. The deeper wavelet has been through more of the absorptive medium than the shallower wavelet, hence it should have the narrower band width. We determine the main peak of the spectrum and the $\frac{1}{2}$ power point on either side of the main peak. Since the operator was very short, our spectrum will be smooth, reflecting mainly the seismic wavelet. Therefore $\frac{1}{2}$ maximum amplitude band limits will be best signal to noise zone for spectral ratio computation.

Unfortunately, very short minimum phase inverse operators spectra had considerable amount of power in low frequency region that produce the peaks at very low frequencies with very narrow band limits. This method was abandoned in favor of the method used after spectral division, as explained below.

6) Compute synthetic (polynomial) division between two adjacent operators, with the shallow operator on the numerator and deeper one in the denominator. The quotient will be the Q operator.

7) Compute Fourier transform of the quotient and determine its log amplitude spectrum.

8) Line fit to the log spectrum over the band width determined previously. The slope of the fitted line will be the desired Q value. The line fit should contain some weighting to prevent adverse effects of outliers. We used number of narrow band limited areas over the whole spectra and computed Q values for each zone, then used the most consistent Q value as the final result.

In subsequent tests we have observed that averaging autocorrelations along bedding is better than trace averaging. This implies that we can apply some weighted smoothing along bedding that minimize the scatter with least damage to the Q structure.

Figure 1: A portion of a cross line input to the computation program.

Figure 1 shows a portion of the line used as a test line for Q computation procedure described in this report. We have generated sections from various stages of processing to show the effects of each processing step.

The input data is first passed through the edge preserving smoothing in order to improve the signal to noise ratio and improve the lateral continuity. By improved lateral continuity we expect to compute more consistent Q values along the events.

Figure 2: Input data after edge preserving smoothing.

Edge preserving smoothing, shown in Figure 2 is a fast and effective process. It improves lateral continuity while preserving discontinuities such as faults and structural or stratigraphic

terminations. Comparing Figure 2 with Figure 1 help visualize the degree of improvement of noise suppression.

We assume that computed Q values should be more continuous along the bedding and may change more rapidly from one bed to the next in depth direction. In order to follow the bedding we need to have the bed dips. This is computed by the dip scan routine.

Figure 3: Result of dip scanning and lateral continuity enhancement.

The dip scan routine considers an odd number of traces, as requested by the user, and scans linear dips around each trace location. It uses a user specified window length, and computes continuous semblance in time for each dip. It detects the dip of maximum semblance and generates data in that window by summing traces along maximum semblance dip. The maximum semblance, corresponding dip and optimum trace are saved for the final output. Figure 3 shows the output from dip scanning process. The dips computed by this routine show the direction of bedding, which will be used in lateral smoothing of the computed Q values.

Figure 4 shows the computed Q values from averaged autocorrelation functions. We used three adjacent in-lines. Thus, for each trace in the center there were 8 neighboring traces, hence we could average 9 autocorrelation functions. Dips inline and crossline directions gave the estimated positions of the zero crossings on the neighboring traces. Hence autocorrelations were taking at relatively the same events along the structural dip directions. .

Figure 4 shows clearly that most Q values were computed within the acceptable range. This was also indicated by the stable autocorrelation printout of the program. We still have considerable amount of variable Q values, but there are definite similarities along the bedding patterns. This will be suitable to smoothing application following the bed structure. The warmer colors indicate lower Q values, and cooler colors indicate higher Q values. Dark blue colors are representing Q values around 120, and red values are around 15.

Figure 4: Q computation using the average autocorrelation functions of neighboring traces.

In order to reduce the noise and recover some continuity, we apply lateral continuity enhancing smoothing. In this process, a number of adjacent samples along the bed direction are used. If the center trace is a valid Q number, we sum all valid Q numbers weighted by a Gaussian filter and determine the Gaussian average, and replace the center value with the average.

Figure 5: Spatial averaging applied to the Q computation shown on figure 4. Smoothing along the structural dip direction is clearly discernable.

If the center value is invalid (like $Q = 300$), then we look at the number of valid and invalid samples around the point along the bed. If the number of invalid is the majority, then the point remains invalid. If valid Q values are majority, then Gaussian average will be the result. This way averaging is done in the direction of bedding and some invalid locations within valid

computations will be eliminated.

Method 2: Conclusions

We have used two stages of lateral continuity enhancement process before Q computation. For each output location, a center trace and its eight neighboring trace autocorrelations were averaged. Resulting autocorrelation functions proved to be much more stable and gave considerably more accurate and consistent results. The Q computation method used in this experiment, minimum phase inverse operator polynomial division, is proven to be an accurate and robust method. It provides spectral division results without the danger of division by zero. Computation is fast and requires minimum user input decisions.

PART 5: APPLICATIONS TO DATA

Seismic and Well Log Derived Attenuation in Gulf of Mexico

Figure 1 shows the seafloor and two interpreted horizons, the “Trim B” unconformity and the Miocene unconformity. This data focuses on a major gas discovery that has been proven by three well penetrations. We will examine one of these wells (Well #1) in detail for the purpose of computing seismic attenuation and comparing the log-derived attenuation to seismic data derived anomalous absorption.

Figure 1: Deep water (~3000 ft) Gulf of Mexico study area, including three gas wells.

Figure 2: Log data for Well #1. Curve in red is computed attenuation ($1/Q_p$).

In this well, we were provided all necessary log curves to allow for computation of porosity, Vclay, and water saturation. From these curves, we computed seismic scale attenuation ($1/Q_p$) using the patchy saturation model of Dvorkin, et al. This method was described in a previous report (Q2 Report, April 2002). These results show that zones of partial gas saturation, for example just below 3.6 km depth, exhibit a high value of attenuation. An oil saturated zone just above 3.8 km depth also shows an attenuation anomaly, but not as strong as that associated with gas. High attenuation zones below 3.9 km were caused by a low computed gas saturation condition ($<50\%$ gas saturation). This low gas saturation was later found to be erroneous. It was caused by a misinterpretation of the resistivity log. The zones should have been computed at 100% S_w , which would have eliminated the Q anomalies.

Using a seismic data visualization system, we have displayed the seismic amplitude data for the zones of interest (Figure 3).

Figure 3. Seismic amplitude data along three intersecting planes.

The data is displayed on three intersecting planes, two of which include the wellbore trajectories for wells 1, 2, and 3. We will look more closely at comparisons of the seismic data and the wellbore data.

In Figure 4 we show the seismic anomalous absorption attribute colored so that blue represents very low anomalous absorption and red indicates high anomalous absorption. On each wellbore the gas saturation curve is shown in red. Seismic data near wells 2 and 3 show high (red) values of seismic attenuation in zones that correspond closely to the gas saturated zones in each well. In well 1 we also see a zone of high attenuation at the gas saturated zone. Note that well 1 just missed a much larger zone of high seismic attenuation. This substantial anomaly would be a very attractive target for a sidetrack well at some future time.

Figure 4: Anomalous seismic absorption volume with wellbore paths.

In Figure 5 we show the numerical values of the anomalous absorption attribute extracted along the path of the wellbore. This figure demonstrates that the methods we have developed for both seismic and well log computation of Q are giving consistent results. Of course the well log attenuation is much higher resolution than the seismic anomalous absorption attribute. The results confirm that Q or attenuation is a powerful tool for detection of hydrocarbons and can help in selecting new drilling targets.

Figure 5: Comparison of attenuation from well log and seismic data.

CONCLUSIONS AND ACCOMPLISHMENTS

The primary conclusions from this work are;

- 1 Rock physics concepts can be used to compute Q from conventional well log data.
- 2 Attenuation can have a substantial impact of seismic response, and can cause unexpected changes in seismic amplitude with offset , especially at the bottom of a gas zone.
- 3 Attenuation in seismic data can correspond to gas bearing reservoirs, and can be used as a reconnaissance tool in exploration.
- 4 Attenuation should not be used exclusively, but should be combined with information from other seismic attributes, and with geologic insight.

We have accomplished substantially all of the major goals set out for the project. The following specific items have been completed;

Task 1: Develop and test empirical and theoretical rock physics relations for Q

Task 2: Determine Q from Well Logs (fluid substitution, etc.)

Task 3: Log-based Synthetic Seismic with Q

Task 4: Q from Field Seismic and Comparison to Well Log Q

We have filed two US patent applications covering various aspects of the technology that was developed during this project (see References).

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Patent Applications

- Docket No. RSI-03-02: "Method for processing seismic data to identify anomalous absorption zones", inventor M. Turhan Taner, serial no. 10/670,110, filed on Sept. 24, 2003
- Docket No. RSI-03-04: "Method for determining formation quality factor from seismic data", inventos M.Turhan Taner and Sven A. Treitel, serial no. 10/687,129, filed on Oct. 16, 2003.