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Abstract

Both in magnetohydrodynamic shocks and in accelerated partially ionized gas flow across a magnetic field, space charge separation occurs that establishes very large electric fields in the direction of motion. The width of the current layers associated with the acceleration is never less than the electron Larmor radius with no collisions and is broadened by electron collisions to a width solely determined by the effective resistivity. The electrons gain an energy regardless of collisions equal to the electric potential difference across the layer. For $\omega\tau < 1$, (ω = electron cyclotron frequency, τ = collision time) this potential corresponds to the change in kinetic energy of mass motion per ion. For slightly ionized gases, the additional stress of neutral ion collisions within the layer can make the electric potential and hence gain in electron energy very large for only modest changes in mass velocity. Hence ionization may occur when the change in kinetic energy of the ions is small compared to the ionization potential.

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THE M-LAYER

The simplest current layer in a plasma and magnetic field that has been treated analytically is that of Rosenbluth's[1] M-layer where a cold plasma stream is reflected from a magnetic field. By analyzing the forces exerted on each particle one can show that the strong magnetic pulse solution of Adlam and Allen[2] fall within the same physical description as do most present shock solutions.[3,4] The rate of electron ionization in these current layers can then be determined from a knowledge of their structure.

The M-layer (Fig. 1) is created by a magnetic field pushing on a cold ionized plasma with the assumption of no magnetic field in the plasma and no collisions. In the moving frame of the layer, ions and electrons are reflected elastically with a total momentum change of $2v\rho$, where v is the velocity of the layer and ρ the density of the plasma. The momentum flux $2v^2\rho$ must be balanced by the magnetic field pressure $B^2/8\pi$, and since the mass resides in the ions of mass M,

$$2MN\, v^2 = B_0^{\,2}/8\pi\ . \tag{1}$$

Rosenbluth has then shown in a self-consistent calculation of electric and magnetic fields that a current layer of thickness D

$$D = \sqrt{mc^2/8\pi N e^2} \tag{2}$$

is formed in which the primary stress on the ions is the electrostatic force of charge separation. The electrons on the other hand move in orbits parallel to the layer and across the magnetic field so that the electrostatic stress on the electrons is balanced by the Lorentz force $\frac{e}{c}\,(\overline{v_e} \times \overline{B})$ of the magnetic field. Since the ions are accelerated only by an electrostatic force, the potential difference across the layer V must correspond to the change in the ion kinetic energy.

$$V = eED = Mv_i^{\,2}/2 \tag{3}$$

$$\therefore \qquad \frac{e}{c} v_e B = M v_i^2 / 2D .$$

Assume B is an average field equal approximately to $B_o/2$. Then using (2) and rearranging, we get

$$\frac{m v_e^2}{2} = \left(\frac{M v_i^2}{2}\right)^2 \frac{4c^2 m}{2e^2 B_o^2} \; \frac{8\pi N e^2}{mc^2} \tag{4}$$

or

$$\frac{m v_e^2}{2} = 1/2 \left(\frac{M v_i^2}{2}\right). \tag{5}$$

In other words, the electron kinetic energy when the electron is within the layer must be approximately half the ion kinetic energy relative to the layer. The thickness of the layer must then be self-consistent with the fact that the electrons pick up most of the electrostatic potential of the layer. In other words, the layer is not much thicker than an electron Larmor radius when the electron has the kinetic energy of the ion. To show this, the electron Larmor radius in an average field $B = B_o/2$ is

$$a = \frac{2 m v_e c}{e B_o} = \frac{2mc}{B_o e}\left(\frac{M v_i^2}{2m}\right)^{1/2} = \left(\frac{2 M v_i^2 m c^2}{B_o^2 e^2}\right)^{1/2} . \tag{6}$$

but by (1)

$$a = \left(\frac{mc^2}{8\pi N e^2}\right)^{1/2} = D . \tag{7}$$

LARGE MAGNETIC PULSES

In the Adlam-Allen large magnetic pulses, the ions are similarly accelerated by charge separation electric fields, with the difference that the magnetic field exists ahead and within the pulse (Fig. 2) so that the trajectories must pass through the layer. The layer is still D in thickness but the electron drift velocity parallel to the layer is now reduced from the "perfect diamagnetic" M-layer of Rosenbluth because with larger magnetic field the Lorentz stress on the electrons at a given velocity is greater.

The equivalent momentum balance is

$$NMv_iv_s = \left(B_{max}^2 - B_{min}^2\right)/8\pi = \Delta (B)^2/8\pi \ , \tag{8}$$

where v_i is the change in ion velocity measured in the moving frame v_s of the layer. In the frame in which the layer is stationary electrostatics apply in which case Eq. (4) becomes

$$\frac{mv_e^{\ 2}}{2} = \frac{1}{4}\,\frac{Mv_i^{\ 2}}{2}\,\frac{\Delta (B)^2}{\langle B\rangle^2}\,\frac{v_i}{v_s} \ , \tag{9}$$

where $\langle B\rangle$ corresponds to an average field within the layer. The large pulse solution is no longer valid when the pulse velocity (v_s in this case) becomes twice the Alfvén velocity or when

$$v_s = 2\left(\frac{B_{min}^2}{4\pi NM}\right)^{1/2} \ , \qquad \text{or when} \qquad \frac{\Delta (B)^2}{\langle B\rangle^2} = 2 \ . \tag{10}$$

This limit approaches the perfect diamagnetic case of Rosenbluth where the electron kinetic energy in the layer is one half the change in ion kinetic energy.

The thickness of the layer measured in electron Larmor radii becomes from Eq. (6) and (8)

$$a = \frac{mv_ec}{e\langle B\rangle} = \frac{mc}{e\langle B\rangle}\left(\frac{Mv_i^{\ 2}}{4m}\,\frac{\Delta (B)^2}{\langle B\rangle^2}\,\frac{v_i}{v_s}\right)^{1/2} \tag{11}$$

$$= (1/2)D\,\frac{\Delta (B)^2}{\langle B\rangle^2}\,\frac{v_i}{v_s} \ .$$

Therefore in the limit of the strong shock solution where $v_i \rightarrow v_s$ and $\Delta (B)^2 \simeq \langle B\rangle^2$ we recover the results of the diamagnetic case where the layer is an electron Larmor radius thick. For weaker magnetic pulses the layer becomes a number of Larmor radii thick according to Eq. (11).

Despite the fact that the orbits of the two solutions are not continuously connected, it is nevertheless evident that the layer thickness in terms of electron Larmor radii and the electron energy within the layer are connected between the two solutions.

COLLISIONLESS SHOCKS

Present theory of collisionless shocks, particularly Gardner et al.,[3] have predicted oscillating solutions of the form of a series of waves following behind the principle wave or shock front. The general description of these waves is of the form of a series of the strong pulses already discussed. This picture of a hydromagnetic shock has recently been verified in machine calculations of Auer, Hurowitz and Kilb[4] in which a series of strong pulses of increasing spacing are observed for Mach number less than 2 and a random wave field composed of such pulses for Mach number greater than 2.

A unifying feature of all these current layer descriptions is that of charge separation electric fields in which the change of kinetic energy of the ions is equal to the electrostatic potential across the shock.

This description applies to the collisionless case, but as soon as collisions are included the layer structure can be expected to change. In particular, as soon as an average of one collision per electron occurs within the layer, we would expect the layer to roughly double in thickness to two electron Larmor radii in dimension. In order to calculate this probability, we need to know the average path length of an electron within the layer.

By (3) the velocity of an electron within the layer is

$$v_e = \frac{c}{e} \frac{M v_i^2}{2D\langle B\rangle} \qquad (12)$$

and the time t spent within the layer must be the same as the ion transit time (D/v_i) in order to maintain charge neutrality. (The charge separation is very small.) The electron path length ℓ then becomes

$$\ell = \frac{D v_e}{v_i} = \frac{c}{e} \frac{M v_i}{2\langle B\rangle} \qquad (13)$$

$$= 1/2 \text{ ion Larmor radius.}$$

This result could have been predicted on the basis that Lorentz force impulse on the electrons must correspond to that required to accelerate an ion by v_i. In a magnetic field B, this requires that an equal charge be displaced a distance equal to the ion Larmor radius across the magnetic field. It is evident that this displacement is independent of the thickness of the layer, provided only that the ions are accelerated by an electrostatic field.

COLLISION BROADENING OF A STRONG SHOCK LAYER

The condition for doubling the thickness of a layer one electron Larmor radius thick by scattering requires one collision in the path l.

$$l = \lambda = \frac{1}{N\sigma} = \frac{c}{e}\,\frac{Mv_i}{2\langle B\rangle} \tag{14}$$

or
$$\sigma = \frac{2\langle B\rangle e}{NMv_i c} \ ,$$

where σ is the electron scattering cross section at the velocity v_e.

Since the electron scattering leads to the equivalent of a diffusion of magnetic flux, the condition (14) can be derived on the basis of a resistivity resulting in a magnetic skin depth 2D at a diffusion velocity v_i. In other words, this current layer cannot be localized to less than the diffusion depth x_D for a given resistivity and velocity.

To demonstrate this, the skin depth x_D in a time t is[5]

$$x_D = \left(\frac{\eta t}{4\pi}\right)^{1/2} \quad (\eta \text{ in cgs units}) \ , \tag{15}$$

or defining a diffusion velocity $x_D/t = v_i$,

$$x_D = \frac{\eta}{4\pi v_i} \ . \tag{16}$$

Using the classical resistivity[5]

$$\eta = \frac{mc^2\nu}{Ne^2} = \frac{mc^2 N\sigma v_e}{Ne^2} = \frac{mc^2\sigma v_e}{e^2} \tag{17}$$

where the electron collision frequency $\nu = N\sigma v_e$. Therefore

$$x_D = 2D = \frac{\eta}{4\pi v_i} = \frac{mc^2\sigma v_e}{4\pi e^2 v_i} \tag{18}$$

or

$$\sigma = \frac{4\pi e^2\left(\frac{mc^2}{8\pi Ne^2}\right) v_i e^2\langle B\rangle}{mc^2\, cMv_i^2} \tag{19}$$

or

$$\sigma = \frac{2e\langle B\rangle}{NMv_i} ,$$

which is identical to the condition (14) derived on a single scattering basis. The layer therefore has a minimum thickness D, provided the electron scattering cross section is less than (19), or is diffusion broadened to a thickness given by

$$x_D = \frac{\eta}{4\pi v_i} . \tag{20}$$

However, once the layer is broader than D, the electron drift velocity is no longer given by (11); namely, the electric field extends over a dimension $x_D \geq D$, so that the electron drift velocity becomes

$$v_{eD} = \frac{c}{e} \frac{Mv_i^2}{2x_D\langle B\rangle} = v_{e\,max} \frac{x_D}{D} . \tag{21}$$

However, when $x_D > D$, there necessarily must be electron collisions, which will heat the electrons. If electrons are drifting at velocity v_{eD} through essentially stationary heavy ions, then the electrons gain a velocity increment v_{eD} randomly per collision. This dynamical friction heating of the electrons is given accurately by Spitzer;[5] but, for the present required accuracy, a random walk analysis would say that after n collisions the velocity spread will be

$$v_{eth} = v_{eD}\, n^{1/2} \tag{22}$$

where v_{eth} is the electron thermal velocity, and the width of the layer

$$x_D = D\, n^{1/2} . \tag{23}$$

Therefore, by (21)

$$v_{eth} = v_{eD} \frac{x_D}{D} = v_{e\,max} . \tag{24}$$

This implies that the electrons are heated by collisions to the same energy they would have had in the layer if there were no collisions. In other words, the electrons acquire the same energy as the change in kinetic energy of the ions regardless of layer thickness provided only that the collisions are elastic and the mass ratio infinitely large. Of course, the electrons do exchange energy with the ions after M/m elastic collisions, or by (23), when the thickness of the layer is

$$x_D = D\,(M/m)^{1/2} = a_i\,. \qquad (25)$$

This also is the condition of the limit of validity of the electron drift velocity (12) because (25) defines the ion Larmor radius, since D alone (by 7) is the electron Larmor radius at the ion energy. When the layer is broadened by diffusion to a thickness greater than the ion Larmor radius, the ions are then accelerated by a magnetic field in addition to the electrostatic field of charge separation, and the primary result of layer structure becomes invalid. This limit also corresponds to the electron collision frequency equaling the electron cyclotron frequency, i. e., $\omega_{ce}\tau = 1$. To show this, the number of electron collisions in crossing the layer a_i thick by (23 and 24) is M/m and the electron cyclotron frequency is $\frac{M}{m}\omega_{ci}$. But the time to cross the layer is one ion cyclotron period, so that one electron collision occurs per electron cyclotron period.

COLLISION BROADENING OF A WEAK SHOCK LAYER

For a weak shock or magnetic pulse where the thickness D is (by Eq. 11) $2[B^2/\Delta(B)^2]\,(v_s/v_i)$ electron Larmor radii thick, the cross section for scattering to broaden the layer and the diffusion heating are the same as for the strong shock case — as it must be from the laws of irreversible magnetic diffusion.

To demonstrate this result in terms of collisions and random walk, assume as in the strong shock case, a cross section σ such that in n collisions the layer is broadened by diffusion to a thickness D. We wish to show that the electrons will be heated to an energy corresponding to the ion kinetic energy and that the resistivity corresponds to that required to give a skin depth D at a diffusion velocity v_s, namely, the rate at which magnetic flux is being compressed.

By Eq. (22), the electron thermal velocity after n collisions is increased by $n^{1/2}$. Therefore, the Larmor radius of the scattered electrons becomes (by Eq. 11)

$$a_{eth} = n^{1/2} a_{eo} = (1/2) n^{1/2}\, D\, \frac{\Delta(B)^2}{\langle B\rangle^2}\, \frac{v_i}{v_s}\,. \qquad (26)$$

The random walk diffusion of n steps a_{eth} long becomes

$$x_D = n^{1/2} a_{eth} = nD/2 \frac{\Delta(B)^2}{\langle B\rangle^2} \frac{v_i}{v_s} , \tag{27}$$

or for $x_D = D$

$$n = 2 \frac{\langle B\rangle^2}{\Delta(B)^2} \frac{v_s}{v_i} .$$

The final electron thermal energy by Eq. (22) and (9) becomes

$$\frac{m v_{eth}^2}{2} = \frac{m}{2} (n^{1/2} v_{eD})^2 \tag{28}$$

$$= (1/2) \frac{Mv_i^2}{2} \frac{v_i}{v_s} \frac{\Delta(B)^2}{\langle B\rangle^2} \frac{v_s}{v_i} \frac{\langle B\rangle^2}{\Delta(B)^2}$$

$$= (1/2) \frac{Mv_i^2}{2} ,$$

in agre ment with the strong shock case Eq. (24).

The resistivity corresponding to n collisions occurring within the ion traversal time of the layer D/v_s is by Eq. (17)

$$\eta = \frac{mc^2}{Ne^2} \frac{n v_s}{D} . \tag{29}$$

giving a diffusion thickness at the velocity v_s the shock speed of

$$x_D = \frac{\eta\beta}{4\pi v_s} = \frac{n\beta mc^2}{4\pi Ne^2 D} \frac{v_s}{v_i} = 2n\beta D , \tag{30}$$

where β is the usual ratio of particle pressure to magnetic field pressure $8\pi NkT/B^2$. For the modification of the usual diffusion equation (Eq. 16) for low particle pressure, see Rosenbluth and Kaufman.[6]

From Eq. (28)

$$\beta = \frac{Nmv_{eth}^2}{2} \frac{8\pi}{\langle B\rangle^2} \tag{31}$$

$$= 1/4 \; NMv_i^2 / \frac{\langle B\rangle^2}{8\pi} .$$

or by Eq. (8)

$$= \frac{1}{4} \frac{\Delta(B)^2}{8\pi} \frac{v_i}{v_s} \frac{8\pi}{\langle B \rangle^2} .$$

Therefore, Eq. (30) becomes

$$x_D = \frac{1}{2} n \frac{\Delta(B)^2}{\langle B \rangle^2} \frac{v_i}{v_s} D = D , \tag{32}$$

thus confirming the resistive broadening of the current layer.

The diffusion heating of the electrons to a temperature equal to the change in the ion kinetic energy is evident on the basis of conservation of energy. Whenever a step function of magnetic field diffuses into a conductor, provided the velocity is less than that of light, the process is irreversible and the irreversible work done becomes $p\Delta V$. Since the change in magnetic pressure is $\Delta B^2/8\pi$, this must be the irreversible work dissipated per unit volume which, in turn, is just the change in kinetic energy of the mass density.

SUMMARY OF THE LAYER STRUCTURE

For very low resistivity, electrons are accelerated in the charge separation field (α electron Larmor radii thick) to a kinetic energy equal to $1/\alpha$ of the change in kinetic energy of the ions. α is the ratio of final magnetic energy density to the change in ion kinetic energy density. At higher resistivity the layer broadens by resistive diffusion, and the electrons reach the same temperature by joule heating as the change in kinetic energy of the ions. When the layer becomes broader than an ion Larmor radius, both the charge separation field and the temperature reached in joule heating become less.

IONIZATION

It has been pointed out by Alfvén[7] that partially ionized plasma flow through a magnetic field is strongly stabilized in velocity precisely at the value where the kinetic energy of motion corresponds to the ionization potential of the neutrals. Let us consider that a current layer corresponding to the above change in velocity is formed, and then ask under what conditions the probability of ionization within the layer is high.

Let M = mass of deuteron. Then

$$\frac{Mv_i^2}{2} = 15 \text{ ev} \quad \text{(includes molecular breakup)}$$

$$v_i = 3.9 \times 10^6 \text{ cm/sec}$$

$$\sigma_{\text{electron-neutral}} = 6 \times 10^{-15} \text{ cm}^2$$

$$\eta = 0.05\, T_e^{-3/2} + \frac{0.0015\,(1 - f)}{f} \text{ ohm cm}$$

where f is fractional ionization.

Then the thickness of the layer becomes by substituting in (20)

$$x_D = \frac{0.05\, T^{-3/2} + \frac{0.0015\,(1 - f)}{f}}{4\pi \times 3.9 \times 10^6 \times 10^{-9}} \tag{33}$$

$$= T_e^{-3/2} + \frac{0.03\,(1 - f)}{f} .$$

Provided x_D is less than an ion Larmor radius a_i, then $T_e = E_i = 15$ ev, in which case for $f = 50\%$ ionized $x_D = 0.05$ cm, provided $\beta \cong 1$ and $x_D \leq a_i$.

The density at which there exists 50% probability for ionization of a neutral during its traversal of the layer is defined when the ionization time equals the traversal time. This time is

$$\frac{1}{N\sigma v_{eth}} = \frac{x_D}{v_i} = \frac{0.05}{3.9 \times 10^6} = 1.3 \times 10^{-8} \text{ sec}$$

where σv_{eth} for ionization at 15 ev temperature $\simeq 2 \times 10^{-8}$ per sec.

$$\therefore \quad N_e = 4 \times 10^{15} \text{ electrons/cc} .$$

For $\beta = 1$, B = 1500 gauss.

Since the layer thickness is proportional to $(1 - f)/f$ and the ionization rate is proportional to f, the ionization probability remains roughly constant. Therefore, 100 microns pressure of initial gas density should result in a high enough ionization probability in a current layer so that the rate of change of ionized mass should stabilize the velocity.

HIGH NEUTRAL DENSITY

If the neutral density is high enough so that a number of neutral ion collisions occur within the layer thickness, then the electric field stress needed to change the velocity of the ions by a given amount becomes greater. In other words, if we determine a given velocity change in the fluid flow by a given potential between two electrodes across a given magnetic field, then the electric field stress per ion required to change the velocity becomes greater as the effective mass per ion increases. Ion-neutral collisions essentially couple the mass of the neutrals to the electric charge of the ion, so that the electric field stress must equal the ion-neutral collisional stress. This collisional stress is a function of the ion slip, the mean relative velocity between ions and neutrals. If the ions undergo n_i neutral collisions in traversing the layer of fractional ionization f, the effective stress τ will be increased by

$$\tau \cong \tau_0 \left(1 + n_i \frac{M_0}{M_i}\right) \text{ if } n_i << \frac{1}{f} \frac{M_0}{M_i} \qquad (34)$$

and

$$\tau \cong \tau_0 \frac{1}{f} \frac{M_0}{M_i} \qquad \text{if } n_i >> \frac{1}{f} \frac{M_0}{M_i} \qquad (35)$$

where the collisionless stress $\tau_0 = NM_i v_i v_s$, and M_0 and M_i are the neutral and ion atom masses, respectively.

In other words, when the number of collisions are less than that required to thermalize the ions to the neutrals within the layer (34), then the additional stress is just proportional to the few relatively large energy collisions that take place. On the other hand, if a sufficiently large number of collisions occur so as to thermalize the neutrals to the ions at each point in the layer, then the ions will behave as if they had a larger mass by just the neutral-to-ion mass ratio. At small fractional ionization, the former case is the more likely since a sufficiently high density to thermalize the ions will also result in $\omega_e \tau_e \geq 1$ and therefore small charge separation.

As an example, consider an MHD generator, where the potential is shorted in a single stage. Assume

$$N_0 = 10^{17} \text{ argon atoms/cc}$$
$$B = 10^4 \text{ gauss}$$
$$\sigma \text{ (electron neutral)} \cong 10^{-16} \text{ cm}^2$$
$$\therefore \quad \omega\tau = 10 .$$

Further assume

$$v_i = 2 \times 10^5 \text{ cm/sec}$$

$$\eta = 0.05\, T_e^{-3/2} + \left(\frac{1-f}{f}\right) 2.5 \times 10^{-5} \text{ ohms cm}$$

$$x_D = 20\, T_e^{-3/2} + \left(\frac{1-f}{f}\right) 1 \times 10^{-2} \text{ cm} .$$

Assuming that the electron temperature will be limited by the ionization potential of argon at 15 ev, then for $f = 10^{-2}$, $x_D = 1.5$ cm. For an alkali metal ion like Cs^+, the effective scattering cross section on argon is $\sigma(Cs^+ - A) \cong 10^{-15}\ \text{cm}^2$.

Therefore in moving 1.5 cm a Cs^+ ion will undergo $n_i = \frac{1.5}{N_0 \sigma} = 150$ collisions, which is enough by (34) and (35) to couple the neutral gas fraction to the ions, giving an effective mass

$$NM_i \text{ (effective)} \simeq 100 \text{ A atoms} .$$

Since

$$\frac{M_A v_i^2}{2} = 0.8 \text{ volts} ,$$

the potential of the layer, which is the integral of the stress, becomes larger by the number of neutrals per ion or 80 volts across the layer. The electron temperature will of course not reach this value due to elastic collisions, line excitation, and ionization. The number of elastic electron neutral collisions that occur in traversing the layer, assuming that the electron temperature is limited by ionization to about 10 ev, is

$$c = \frac{x_D}{v_i} N\overline{\sigma v} = 3 \times 10^4 \text{ collisions} .$$

The energy lost to neutrals by elastic collisions will be

$$w_e = c \frac{m}{M} kT_e = 4 \text{ ev} .$$

This is negligible compared to the available heat energy (namely, 80 ev per electron) and so this energy will go into ionization and excitation. Assuming no more neutral cesium is available (only argon remaining), roughly equal energy goes into excitation and ionization, so that 2-1/2 atoms of argon should be ionized in passage through the layer. In addition, the cesium should remain ionized regardless of the ambient gas temperature. Therefore there exists the possibility that sufficient nonequilibrium ionization can be maintained to effectively couple the gas to the magnetic field, yet have a low stream temperature.

INSTABILITY BROADENING

Finally, it would be incomplete not to point out that the collisionless current layer structure is not realistic from the standpoint of electron stream instabilities. The state of a high-energy monoenergetic electron stream traversing relatively cold ions is an exceedingly unstable situation – both due to plasma oscillations growth as described by Buneman[8] and Helmholtz (or velocity shear) instability as described by Northrop.[9] The effective electron collision frequency for the nonlinear amplitude growth has been shown by Buneman to be of the order of 1/5 the plasma frequency, which in most cases is high compared to the electron cyclotron frequency – resulting in a very broad current layer. However, once the layer is more than a few electron Larmor radii thick due to any collisions either cooperative or single particle, the electron velocity distribution is close to thermal and so instability growth should be limited. The result is that no current layer can probably exist less than a few electron Larmor radii thick, and in the presence of additional collisions will become broader according to the laws of magnetic field diffusion. The irreversible heat generated among the electrons will always correspond to the change in kinetic energy of the ions, provided only that $\omega\tau < 1$.

This discussion has been partly based upon a series of lectures on magnetohydrodynamic generators given in the Electrical Engineering Department of the University of California, Berkeley, California. The work also was partly performed under auspices of the U. S. Atomic Energy Commission.

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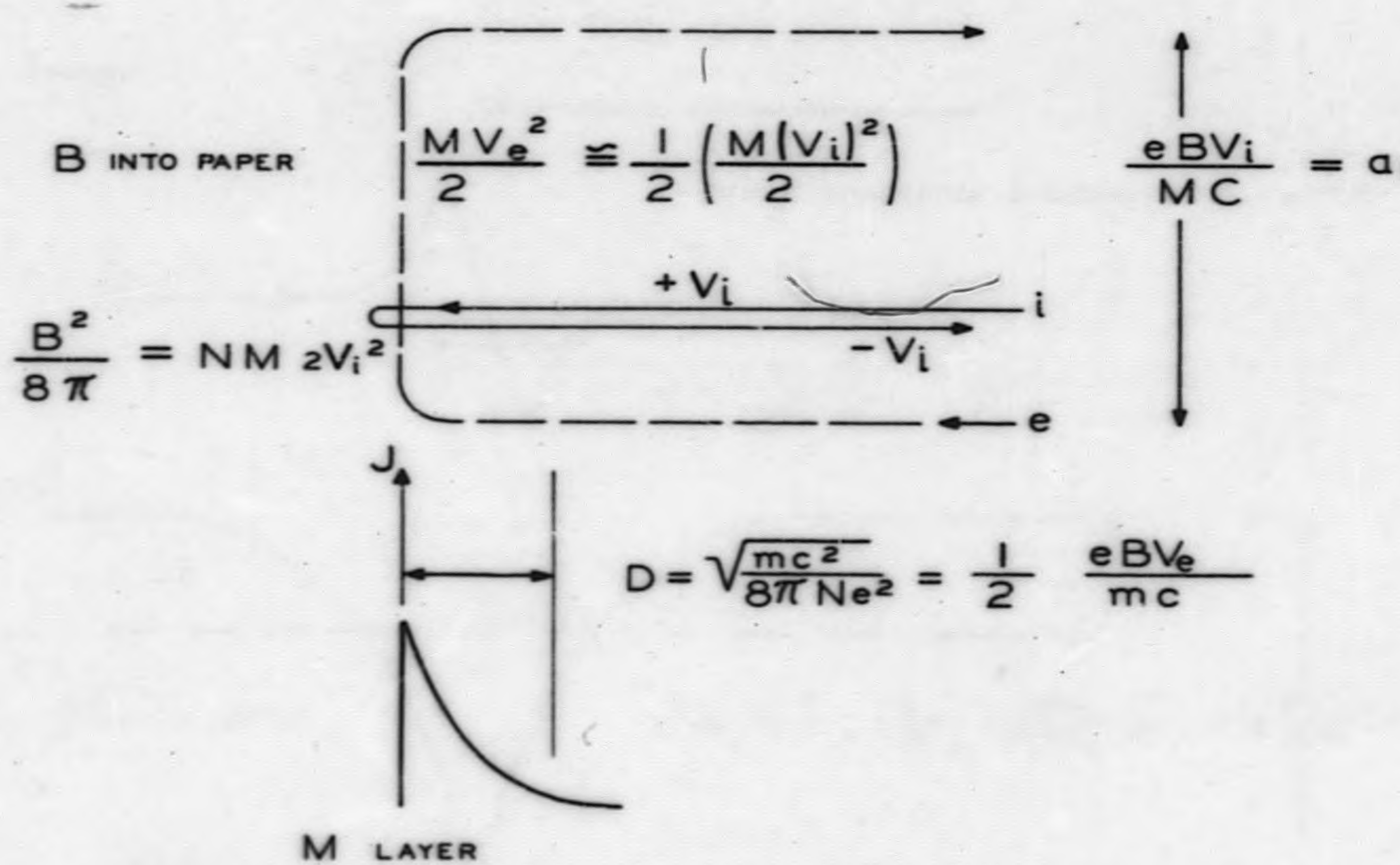

Fig. 1. M-layer.

MUL-12600

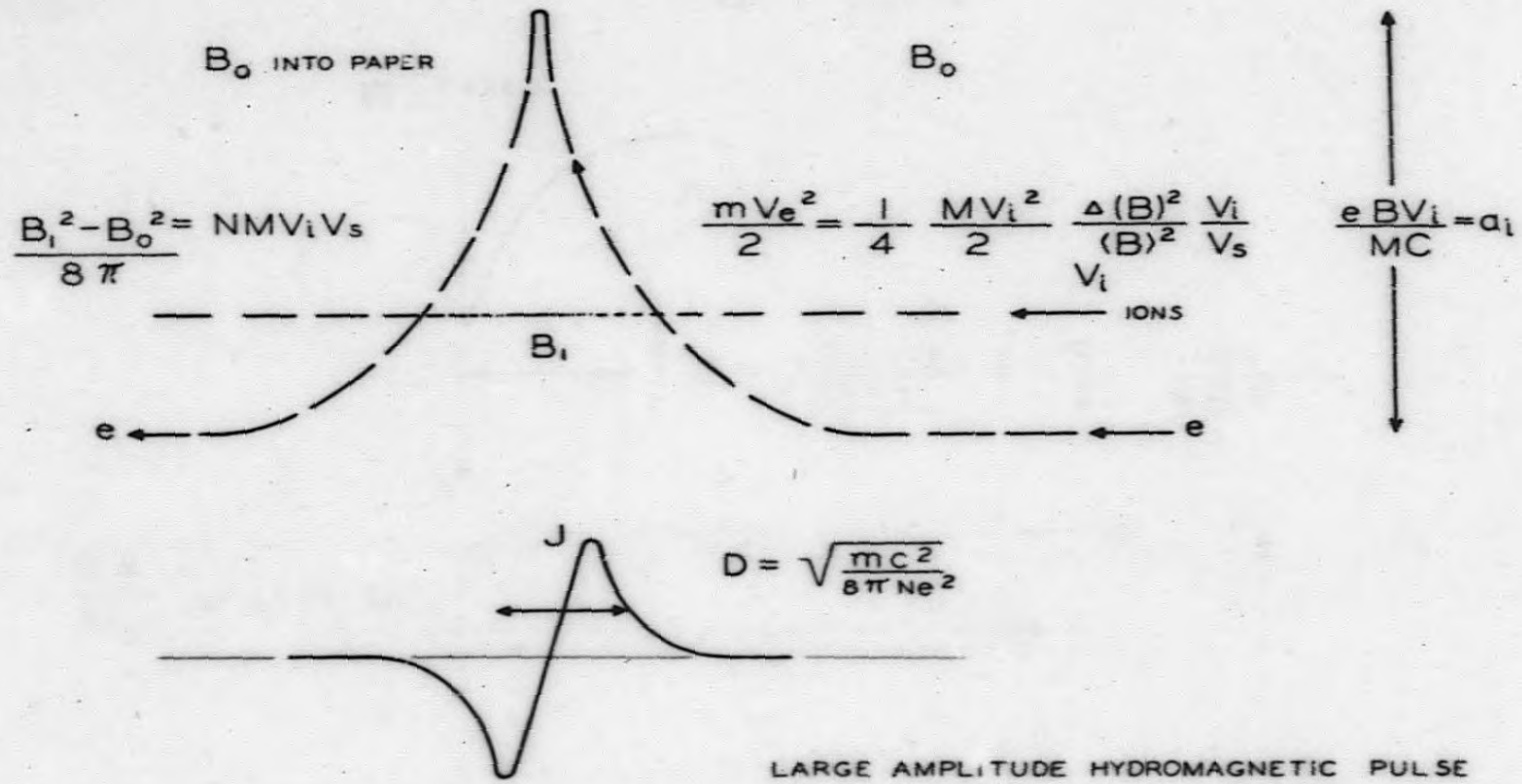

Fig. 2. Large amplitude hydromagnetic pulse.

MUL-12601