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9-ANGIE – A TWO-DIMENSIONAL, MULTIGROUP, NEUTRON-DIFFUSION-THEORY REACTOR CODE FOR THE IBM 709 OR 7090

Stuart P. Stone

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Stuart P. Stone

Lawrence Radiation Laboratory, University of California
Livermore, California

ABSTRACT

The 9-ANGIE, one of a series of reactor neutronic programs[1] for an IBM 709 or 7090 data processing system, solves the time-independent, multigroup, neutron diffusion equation for one to eighteen energy groups applied to a rectangular mesh superimposed on either an x-y or an r-z plane. It is characterized by its generalities in region description, boundary conditions, etc., without sacrificing simplicity of input preparation and ease of machine operation. The notation, the style, and the format have been held as consistent as possible with 9-ZOOM,[2] the companion one-dimensional program.

I. INTRODUCTION

The 9-ANGIE code requires a nominal 32K memory computer, an on-line card reader and printer and uses a minimum of six logical tape units, three each on Channels A and B. Several important features of the program are:

1. The capacity to handle up to 18 energy groups, 20 to 40 materials (depending on the number of groups), 300 to 500 problem regions, and up to a maximum of 2000 mesh points in a rectangular grid network of the x-y or r-z plane.

2. The transfer of neutrons is permitted between an energy group and the two immediately below and the one immediately above.

3. An Alternating Direction Implicit (ADI or Peaceman-Rachford) iteration technique is used. A power extrapolation technique using Chebyshev polynomials is used to improve the power iteration convergence.

4. Energy-dependent extrapolation length boundary conditions are applied at all outer boundaries and at the boundaries of selected interior regions for some or all energy groups.

5. Dependence of the number of neutrons per fission upon the energy of the neutron causing the fission may be considered.

6. A load routine is used which takes decimal input cards in flexible addressable format language.

7. An output (on tape for an off-line printer) which includes

a. a listing of all input data in a completely self-identified format;

b. a monitoring of the problem as it is being run;

c. spatial and energy distribution of the neutron flux and several forms of spatial power and absorption distributions in an edit which is automatically performed upon completion of the problem. An edit may be forced for any iteration cycle that has been placed on a problem dump tape.

8. The program will generate a starting source guess from input fluxes specified groupwise or from a set of fluxes obtained from a suitable problem previously run.

The equations solved by 9-ANGIE and the numerical methods employed are described. A description of the input required and the output obtained is given. A discussion of the program logic and of the operational procedure is presented and the input to a sample problem is provided. On request, the output of this problem will be furnished, along with a make tape binary card deck and a complete SAP listing.

II. THE DIFFUSION EQUATION SOLVED BY 9-ANGIE

The diffusion equation solved, written in differential-integral form, is

$$-\nabla \cdot D(E,r)\,\nabla\phi(E,r) + {}_{R}\Sigma(E,r)\,\phi(E,r)$$

$$= \int_0^\infty dE'\,\phi(E',r)\left[{}_{s}\Sigma(E',r)\,G(E,E') + \frac{f(E)}{\lambda}\,g(E')\,{}_{f}\Sigma(E',r)\right], \qquad (1)$$

which is approximated by the multi-energy-group equation

$$-D^i\nabla^2\phi^i(r) + {}_{R}\Sigma^i(r)\,\phi^i(r) = (\mathrm{Sum})_{j\neq i}\,{}_{s}\Sigma^{i,j}(r)\,\phi^j(r)$$

$$+ \frac{F^i}{\lambda}(\mathrm{Sum})_j(c_1 + c_2 c^j)\,{}_{f}\Sigma^j(r)\,\phi^j(r). \qquad (2)$$

where

1) i,j are energy group indices. (3a)

2) $\phi(E,r)$ is the scalar neutron flux. (3b)

3) D is the diffusion coefficient

$$D^i(r)\,\phi^i(r) = \int_{E^{i-1}}^{E^i} D(E,r)\,\phi(E,r)\,dE = \frac{\phi^i(r)}{3\left({}_{tr}\Sigma^i(r)\right)} \quad . \tag{3c}$$

4) ${}_s\Sigma^{i,j}(r)\,\phi^j(r)$ is the rate at which neutrons scatter from energy group j to energy group i

$$= \int_{E^{i-1}}^{E^i} dE \int_{E^{j-1}}^{E^j} dE'\, G(E,E')\,\phi(E',r)\,{}_s\Sigma(E',r) \tag{3d}$$

with $G(E,E')$ a scattering kernel.

5) F^i is the number of neutrons per fission created in group i

$$= \int_{E^{i-1}}^{E^i} f(E)\,dE \, . \tag{3e}$$

6) g(E) is a function correlating the total neutrons per fission with the energy of the neutron causing the fission.
The assumed form of g(E) is $(c_1 + c_2 c^j)$.

7) ${}_f\Sigma(E,r)$, ${}_c\Sigma(E,r)$, ${}_{tr}\Sigma(E,r)$, ${}_s\Sigma(E,r)$ are macroscopic cross sections for fission (f), capture (c) (i.e., absorption minus fission), transport (tr), and scatter (s).

$$\Sigma^i(r)\,\phi^i(r) = \int_{E^{i-1}}^{E^i} \Sigma(E,r)\,\phi(E,r)\,dE \tag{3f}$$

8) ${}_R\Sigma^i(r)\,\phi^i(r)$ is the rate at which neutrons are removed from the ith group.

$${}_R\Sigma^i = {}_f\Sigma^i + {}_c\Sigma^i + (\text{Sum})_{j\neq i}\ {}_s\Sigma^{j,i} + (B^i)^2\,D^i \, . \tag{3g}$$

where $(B^i)^2$ is the appropriate "buckling" factor to compensate for neutron leakage in the z direction in x-y problems.

$$(B^i)^2 = \left(\frac{\pi}{L_z + 2d/_{tr}\Sigma^i}\right)^2 \qquad (3h)$$

with L_z the physical dimension and d the appropriate extrapolation distance.

9) λ is the eigenvalue.

10) ∇^2 is the Laplacian operator

$$= \frac{d^2}{dx^2} + \frac{a}{x}\frac{d}{dx} + \frac{d^2}{dy^2} \qquad \begin{matrix} a = 0 & \text{x-y geometry} \\ a = 1 & \text{r-z geometry} \end{matrix} . \qquad (3i)$$

In 9-ANGIE a rectangular grid network composed of horizontal and vertical line segments is imposed. The spacing in each coordinate is arbitrary. We shall define a zone as that area bounded by any two consecutive line segments in each of the two-space dimensions. A region is one or more zones in which the diffusion coefficient and the macroscopic cross sections are constant (for each energy group). A mesh point is the intersection of two line segments.

The macroscopic cross sections are computed by the code from the neutronic data input which is per material (ℓ) per group (i) and a set of relative density factors (V) given per region (r).

$^{\ell}\sigma^i$ is the microscopic cross section in barns for the appropriate processes.

$^{\ell}\mu^{i,j}$ is a coefficient of transfer from energy group j to energy group i.

$^{\ell}N$ is the atomic density.

$^{\ell,r}V$ is the relative density factor for material (ℓ) in region (r).

Thus,

$$^{r}_{x}\Sigma^i = (\text{Sum})_{\ell}\left(^{\ell}N\right)\left(^{\ell,r}V\right)\left(^{\ell}_{x}\sigma^i\right) \qquad \text{for } x = c, f, tr. \qquad (3j)$$

$$^{r}_{s}\Sigma^{i,j} = (\text{Sum})_{\ell}\left(^{\ell}N\right)\left(^{\ell,r}V\right)\left(^{\ell}\mu^{i,j}\right)\left(^{\ell}_{s}\sigma^j\right) . \qquad (3k)$$

It should be noted that this generalized input format permits the entry of neutronic data in microscopic form for a given isotope or in macroscopic form for various mixtures of isotopes, making proper choices for N and V.

The two methods may be mixed, in which case macroscopic cross sections become additive. Equations (3d) and (3k) define the transfer of neutrons between the energy groups. These transfers are restricted to values of $i = j+1$, $j-1$ and $j-2$.

III. DIFFERENCE EQUATIONS

9-ANGIE finds a discrete numerical approximation to the multigroup time-independent neutron diffusion problem corresponding to the largest (in modulus) eigenvalue (λ_o) of Eq. (2). The problem is composed of a series of regions and interfaces.

The method of obtaining the set of algebraic difference equations is to expand the flux by Taylor's series around a mesh point $P_{x,y}$. These expansions are solved for the second derivatives which are substituted into the differential equation itself. The derivations are given in detail in Appendix A.

We shall distinguish between two types of mesh points:

1. NP or normal interior diffusion points. Each of the four zones common to an NP is a diffusion zone in which Eq. (2) applies. The usual boundary conditions, i.e., continuity of neutron flux (ϕ) and neutron current ($D\nabla\phi$), are applied.

2. NODP or nondiffusion points. One or more of the four common zones is NOT a diffusion zone. All exterior boundary points are NODP by definition. In addition, 9-ANGIE permits selected interior regions to be nondiffusion regions (frequently called rod regions).[3] An NOD region or zone is one in which the neutron flux is not defined and at the boundary of the NOD region the following condition is applied:

$$D^i \frac{\partial \phi^i / \partial n}{\phi^i} \Bigg|_\gamma = -\delta^i_\gamma \,, \tag{4}$$

where δ^i_γ is a positive constant with the derivative being taken perpendicular to the boundary γ in the direction of the NOD region.

The set of algebraic difference equations may be generally written

$$E^i_L \phi^i_L + E^i_R \phi^i_R + E^i_T \phi^i_T + E^i_B \phi^i_B - (E^i_L + E^i_R + E^i_T + E^i_B + R^i)\phi^i = V^i \,. \tag{5}$$

where

1) L, R, T, B are a left, right, top, bottom notation.
2) The source term V^i is

$$V^i = F^i W^* + S^{i,i+1} \phi^{i+1} + T^{i,i+2} \phi^{i+2} + X^{i,i-1} \phi^{i-1} \qquad (6)$$

with W^* the input power (to a given iteration cycle) properly weighted and rescaled.

The iteration cycle independent coefficients, E, R, S, T, X, Q, are modified for the boundary conditions and special cases (see Appendix A) so that a general iteration procedure may be used.

The solution is by an alternating direction implicit[4] or Peaceman-Rachford iteration procedure.[5] Two complete sets of line relaxation mesh sweeps are performed for each group, starting with i = I (highest group) and progressing down the energy index to i = 1. W^* remains fixed for a power or outer iteration; however, the most recently calculated flux values are used in V^i where available.

The initial power cycle source is obtained from the input fluxes which may be specified groupwise (flat spatial per group) or which may be taken from a suitable previously solved problem. The input power to the first power cycle is

$$W^{*,1}_{x,y} = \frac{(\text{Sum})_i \; Q^i_{x,y} \; \phi^{i,1}_{x,y}}{KR^0} \; , \qquad (7)$$

where $\phi^{i,1}_{x,y}$ are the starting fluxes and KR^0 an initial eigenvalue guess.

The actual sweep difference equations are (p is a power cycle index):

For Horizontal (Row) Sweep

$$- E^i_L \phi^{i,p}_L - E^i_R \phi^{i,p}_R + \left(E^i_L + E^i_R + R^i\right) \phi^{i,p} = V^{i,p}$$
$$+ E^i_T \left(\phi^{i,p}_T - \phi^{i,p}\right) + E^i_B \left(\phi^{i,p}_B - \phi^{i,p}\right) \qquad (8a)$$

For Vertical (Column) Sweep

$$- E^i_T \phi^{i,p+(1/2)}_T - E^i_B \phi^{i,p+(1/2)}_B + \left(E^i_T + E^i_B + R^i\right) \phi^{i,p+(1/2)} = V^{i,p}$$
$$+ E^i_L \left(\phi^{i,p+(1/2)}_L - \phi^{i,p+(1/2)}\right) + E^i_R \left(\phi^{i,p+(1/2)}_R - \phi^{i,p+(1/2)}\right) \qquad (8b)$$

It is readily seen that both of these equations are of the form

$$- A_{t+1} \phi_{t+1} - A_{t-1} \phi_{t-1} + A_t \phi_t = Z_t . \tag{9}$$

If it is postulated that there exists a Γ_{t+1} and a Δ_{t+1} such that

$$\phi_t = \Gamma_{t+1} \phi_{t+1} + \Delta_{t+1} . \tag{10}$$

one may easily find

$$\Gamma_{t+1} + \frac{A_{t+1}}{A_t - A_{t-1} \Gamma_t} \tag{11}$$

$$\Delta_{t+1} = \frac{Z_t + A_{t-1} \Delta_t}{A_t - A_{t-1} \Gamma_t} .$$

Applying these relations to the 9-ANGIE sweep equations, one may write

Horizontal Sweep

$$\Gamma^{i,p+(1/2)}_{x+1,y} = \frac{E^i_R}{HD^{i,p+(1/2)}_{x,y}} \tag{12a}$$

$$\Delta^{i,p+(1/2)}_{x+1,y} = \frac{E^i_T \left(\phi^{i,p}_T - \phi^{i,p}\right) + E^i_B \left(\phi^{i,p}_B - \phi^{i,p}\right) + V^{i,p} + E^i_L \Delta^{i,p+(1/2)}_{x,y}}{HD^{i,p+(1/2)}_{x,y}} \tag{12b}$$

$$HD^{i,p+(1/2)}_{x,y} = E^i_L \left(1 - \Gamma^{i,p+(1/2)}_{x,y}\right) + E^i_R + R^i \tag{12c}$$

$$\phi^{i,p+(1/2)}_{x,y} = \Gamma^{i,p+(1/2)}_{x+1,y} \phi^{i,p+(1/2)}_{x+1,y} + \Delta^{i,p+(1/2)}_{x+1,y} \tag{13}$$

It is easily seen that

1) $HD \equiv 0$ for a point within an NOD region (14a)

2) $\Gamma_{0,y} = \Delta_{0,y} = \Gamma_{X+1,y} = 0$ (From handling of boundary conditions) (14b)

3) As a rule $\Delta_{X+1,y} \neq 0$ (14c)

Vertical Sweep

$$\Gamma^{i,p+1}_{x,y+1} = \frac{E^i_T}{VD^{i,p+1}_{x,y}} \tag{15a}$$

$$\Delta^{i,p+1}_{x,y+1} = \frac{E^i_L\left(\phi^{i,p+(1/2)}_L - \phi^{i,p+(1/2)}\right) + E^i_R\left(\phi^{i,p+(1/2)}_R - \phi^{i,p+(1/2)}\right) + V^{i,p} + E^i_B \Delta^{i,p+1}_{x,y}}{VD^{i,p+1}_{x,y}} \tag{15b}$$

$$VD^{i,p+1}_{x,y} = E^i_B (1 - \Gamma^{i,p+1}_{x,y}) + E^i_T + R^i \tag{15c}$$

$$\phi^{i,p+1}_{x,y} = \Gamma^{i,p+1}_{x,y+1}\ \phi^{i,p+1}_{x,y+1} + \Delta^{i,p+1}_{x,y+1} \tag{16}$$

with

1) $VD \equiv 0$ for a point within an NOD region. (17a)

2) $\Gamma_{x,0} = \Delta_{x,0} = \Gamma^i_{x,X+1} = 0$ (17b)

3) As a rule $\Delta^i_{x,Y+1} \neq 0$. (17c)

The procedure is to start at $x = 0$, $y = 0$, sweeping out row $x = 0$ computing Γ^{p+1} and $\Delta^{p+(1/2)}$. A reverse sweep obtains $\phi^{p+(1/2)}$. All rows are systematically swept. The complete vertical sweep follows again starting at $x = 0$, $y = 0$.

Note that $V^{i,P}$ is a term mixed in p and p+1. The ϕ^{i+1} and ϕ^{i+2} are the most recent fluxes (p+1), while the ϕ^{i-1} are "old" fluxes (p). It is seen that this set of difference equations thus are solved by repeated iterations on the power function $W^{*,P}$, the input power to the pth iteration. One may express the matrix problem[3,6] as

$$M(W^{*,P}) = \lambda^P W^P , \tag{18}$$

where M is a linear transformation (frequently called the "power matrix") and W^P is the resulting output power of the pth cycle which is calculated from the fluxes (ϕ^{p+1}) resulting from the operation $M(W^{*,P})$.

$$W^P_{x,y} = (\text{Sum})_i\ Q^i_{x,y}\ \phi^{i,p+1}_{x,y} \tag{19}$$

9-ANGIE uses the same variant of Stiefel's "Best Strategy Method"[7] as 9-ZOOM and the reader is referred to UCRL-5682[2] for a detailed discussion of the technique. Only the actual form used in 9-ANGIE will be given here. The input power to the first cycle has already been given in Eq. (7).

At the end of each power cycle, 9-ANGIE computes and prints out:

1) Rayleigh Quotient $KR^P (p \geq 1) = (W^{*,P}, W^P)/(W^{*,P}, W^{*,P})$ (20a)

2) Modified Rayleigh Quotient $KMR^P (p \geq 1) = (W^P, W^P)/(W^{*,P}, W^P)$ (20b)

3) $KMIN^P (p \geq 2) = (MINIMUM)_{x,y} (W^P_{x,y})/(W^{*,P}_{x,y})$ (20c)

4) $KMAX^P (p \geq 2) = (MAXIMUM)_{x,y} (W^P_{x,y})/(W^{*,P}_{x,y})$ (20d)

Experience has shown that it is advantageous to delay the calculation of KMIN and KMAX until $p = 2$. 9-ANGIE uses KR^1 as a normalization factor to obtain

$$W^{*,2} = W^1/KR^1 , \tag{21}$$

which is the input power to the second iteration cycle.

Beginning with $p = 2$, the rescale of W^P becomes

$$W^{*,p+1} (p \geq 2) = \frac{4}{(\mu_p)(f)(KMIN^P)} (W^P) - \frac{2}{\mu_p} (W^{*,P}) - \frac{1}{(\mu_p)(\xi_{p-1})} (W^{*,p-1}) . \tag{22}$$

The function μ_p is an expression involving the ξ_p, ξ_{p-1}, etc., which is a rational function for the ratio of successive Chebyshev polynomials. The cycle independent input constant, f, together with KMIN, which is an underestimate for the eigenvalue, guarantees a convergent extrapolation scheme for the power. If $f = 0$, the scheme reduces to

$$W^{*,p+1} = (W^P)/KR^P . \tag{23}$$

The convergence criterion, tested during each p cycle ($p \geq 2$), is

$$\frac{KMAX^P - KMIN^P}{2KR^P} \leq \epsilon P . \tag{24}$$

IV. GENERAL CODE DESCRIPTION

The 9-ANGIE Code runs from an ANGIE Master Instruction Tape (same for all problems) which contains all the subcodes together with the necessary bootstrap and control routines. This procedure avoids any card handling in production by machine operators. Only problem inputs are handled in card form during preproduction time.

The 9-ANGIE Code requires an IBM 709 or 7090 with a nominal 32K memory, two channels (A and B) of tape units, an on-line card reader and an on-line printer. In addition, off-line printing facilities for BCD output are required. 9-ANGIE does not use (a) drums, (b) on-line card punch unit, and (c) CRT (cathode-ray tube). The card reader board should be one that reads the first 72 columns of a card with columns 73-80 ignored, a so-called right-hand board. The 9-ANGIE make tape deck is in row binary format.

To prepare an instruction tape, ready a blank Tape A1 (Channel A, logical unit no. 1), ready the ANGIE Tape Preparation Deck in the card reader, press Clear, and Load Cards. The instruction tape will be generated, rewound and the computer will stop at octal location 77515. It is recommended that the File Protect Ring be removed and the tape used for many problems. If a tape check is encountered during the writing of Tape A1, an on-line comment will indicate it. A new blank should be mounted and the process repeated.

The 9-ANGIE Code may be broken into four logical subcodes: (1) Lister, (2) Generator, (3) Main Code, and (4) Edit. There are two edits: one is an Automatic Edit (performed upon problem convergence) and the other is a Forced Edit of any desired retained problem dump. They produce identical information but use slightly different machine and tape logic.

The Lister performs four distinct functions. The first is to read those decimal cards that constitute a problem input into the computer processing the input data into a convenient form. As they are read, all cards are tested for illegitimate characters. If one is detected, images of the last two cards, one of which contains the illegitimate character, are printed on-line.

The second function of the Lister is to make a series of checks on the input data and provide where possible a detailed listing of all input quantities. This listing is written on Tape B9 which is used only as an output tape (at LRL) for subsequent printing on an off-line printer. The first series of checks are on five key quantities which are tested against code limits. These are:

1) I, the number of groups, must be ≤ 18.

2) X, the number of zones in the x dimension, and Y, the number of zones in the y dimension, must be: a) $3 \leq X \leq 100$
b) $3 \leq Y \leq 100$
c) $(X+1)(Y+1) \leq 2000$.

3) L, the number of materials, must be: a) $L \leq 40$
b) $(L)(7I+2) \leq 2560$.

4) R, the number of regions, must be: a) $R \leq 500$
b) $(R)(L) \leq 6000$.

If these limits are exceeded, the Lister cannot continue (no-off line listing possible) and an on-line comment indicates an error in these limits. If no error occurs, the Lister proceeds with further tests and, independent of subsequent errors, will write the listing on Tape B9. Included in the second series of tests are:

1) There are X number of mesh dimensions, x_x, each nonzero and positive.

2) There are Y number of mesh dimensions, y_y, each nonzero and positive.

3) The region matrix description is uniquely defined. This includes checking that each region called for is assigned a number less than R and that a correct number of rows are specified.

4) A check is made to assure that a nonzero macroscopic transport cross section exists for all zones for all groups. A divide check in the Generator Code results if this is not true. This test includes NOD regions, thus a transport cross section (though meaningless) must be included in the input.

5) Each material used is checked for (a) a fixed point identification number which must agree with a separate listing of the materials and (b) a floating point nonzero number for the atom density, N.

6) If nondiffusion regions are included, the δ^i's must be positive (or zero) and the regions (if more than one) given in a certain order.

Certain types of overdefinition are permitted which are useful in setting up series of problems. These are: 1) that not all regions specified in the input must be used in the matrix problem, and 2) that not all materials must be used (the V's of a given material may be zero for all regions). The Lister

will indicate both on-line and off-line, frequently in detail, whether or not errors are found.

The off-line listing includes a region matrix picture. The region number assigned to each zone is pictorially presented. Also included is the calculation of the volume of each zone and the sums of all the zone volumes in the various regions.

If no errors are detected, the Lister (as its third function) writes the problem input on Tape A2 (a blank tape which becomes the problem dump tape) in an arrangement that is convenient for future use. The first file consists of I+2 records; the first record contains problem constants and general information, the second record is the region matrix in expanded form. the final I records are the cross sections and the transfer coefficients arranged by energy level.

The last function of the Lister is to provide the start of an initial power cycle dump. The Lister first writes a record of code constants and then goes to an auxiliary routine which will write I records of starting fluxes. These are either spatially flat with each level's value specified in the input or are taken from a previously-run-problem's dump tape mounted on Tape A6. A check is made to assure that the Tape A6 is from a problem of correct size and read and write tape checking is made in all the Lister tape operations. An end of file and a nine-word identification record, called dummy, is written following the flux records. This dummy record, also written after the dump of each power cycle in the Main Code, is the identifying record used by the search routines to determine the last dump existing on Tape A2.

Upon completion of the Lister, the Tape A2 contains sufficient information for the problem to be continued, either immediately or at a later time. The Lister operation takes from 1 to 5 minutes depending on the size and complexity of the problem.

The ANGIE Generator Code takes the input information which has been written on Tape A2 and computes the power cycle independent coefficients which are needed in the Main Code. Normally, two identical tapes are written, Tapes B3 and A3; however, if Sense Switch 3 is depressed, then only Tape B3 is written. The five records written per energy level, starting with I, are:

1) Record I	$T^{i,i+2}_{x,y}$	for all x,y
2) Record II	$S^{i,i+1}_{x,y}$	for all x,y
3) Record III	$X^{i,i-1}_{x,y}$	for all x,y
4) Record IV	E^i_L, E^i_R, E^i_T, E^i_B, R^i	for all x,y (4ER record)
5) Record V	$Q^i_{x,y}$	for all x,y

The Main Code performs power iterations until (a) the problem converges (Eq. 24) or (b) the problem is force-terminated by Sense Switch 1 being depressed. The first power cycle requires a search of Tape B3 (for the Q's) together with the reading of the initial fluxes from Tape A2 in order to compute the $W^{*,1}$. Before the first power iteration is actually started the initial dump started by the Lister is modified by the addition of $W^{*,0}$ (a record of zeros) and $W^{*,1}$.

The Main Code uses Tapes A3 and B3 on alternate cycles. Tape B5 is used as a temporary working flux tape. The Code essentially brings all information needed for an energy group sweep into memory, performs the sweep and proceeds to the next group. The Code makes use of multiple block storage and much of the Generator Tape data is buffered. Upon completion of a power cycle, the code constants (for that pth cycle), the fluxes and the two sets of power, W^{*p-1} and $W^{*,p}$, are written on Tape A2 in a series of records followed by an end of file and the dummy record. Dumps at selected intervals may be permanently retained on the Tape A2. If a dump is not saved, it is destroyed by the writing of the succeeding power cycle dump. All search routines identify the tape being searched and the power cycle dumps which it passes in the search.

The Automatic Edit, performed upon problem convergence, first writes a duplicate of the last dump on Tape B5 before rewinding the Tape A2. It then uses Tape A3 as a working tape on which to expand fission and absorption cross sections needed in the Edit. This information is obtained from the input data in the first file of Tape A2.

The Forced Edit first expands the fission and absorption cross section on Tape B5 and then proceeds to search the Dump Tape A2 for the last dump or for a specified dump entered via the console keys.

In both cases the Edit is then written on Tape B9. The Edit runs from 11 pages for the smallest problem to nearly 200 for a maximum problem. The time to perform the edit varies with the size of the problem and is strongly affected by the length of the Tape A2 to be searched or to be rewound.

The Edit equations are detailed in Appendix B. Identical information is obtained from the Automatic Edit and from the Forced Edit.

Appendix C contains a series of flow diagrams for various parts of the 9-ANGIE Code. These are not given in great detail in most cases and are intended to provide the logic used and to show the basic flow of data during the various subcodes and routines.

Normally, a minimum of three logical tape units are needed on each of the A and B Channels. The tape units, their logical numbers, and their various functions are:

Channel	Logical Number	Function
A	1	ANGIE Master Instruction Tape
A	2	Problem Dump Tape
A	3	Generator Tape
A	6	Flux Copy Tape (A2 of previous problem)
B	3	Generator Tape
B	5	Working Tape
B	9	Used for off-line output only

In summary, the various tape units required by the various subcodes of 9-ANGIE are:

Make Tape : Tape A1
Lister : Tapes A1, A2, B9, and A6 (optional)
Main Generator: Tapes A1, A2, A3, and B3 (A3 is optional via SS 3)
Code : Tapes A1, A2, A3, B3, B5, and B9 (A3 is optional via SS 3)
Main Auto Edit : Tapes A1, A2, A3, B5, and B9 (A3 is required here)
Forced Edit : Tapes A1, A2, B5, and B9

V. 9-ANGIE INPUT PREPARATION

The 9-ANGIE input consists of a number of cards with the data in a flexible addressable format. These cards are read, translated, and the data is stored in the memory by the CML load routine, one of the two LRL-developed subroutines in 9-ANGIE. Further details of this subroutine will be given before describing the input itself.

As used hereafter, a word consists of a prefix and a number. The memory locations to be used and the type of translating operation to be used are obtained from interpretation of various types of words. Those used in 9-ANGIE and their meaning are:

LXXXX is interpreted as: to load into location XXXX and consecutive locations the words which follow (properly interpreted) until a new LXXXX is specified. XXXX is the absolute decimal location in the 709. The position of LXXXX on a card is arbitrary and more than one item of addressed information may be on a single card or more than one physical card may be loaded by a single LXXXX.

An means load n as a fixed point number into the address portion of a memory location (the remainder of the location is undisturbed).

Cy means load zero into y consecutive locations.

±m means load m as a floating point number (±) into a memory location.

±mSy means load m as a floating point number (±) into y consecutive memory locations.

The prefix and the number must be on the same card; however, any number of blanks may be left between words. CML uses many other types of words; an X, Y, or H prefix will cause a Lister program stop and a Z acts as a card reader end of file return and will cause the Lister not to read all of an input into the computer.

The following Hollerith characters may be used:

+	12 or (8) (3)
-	11 or (8) (4)
.	(12) (8) (3)

A, C, L, S are normal Hollerith characters.

In the above n must always be an integer with no decimal point permitted. The A, C, L, and S's are not preceded by a sign. The floating point number may take several forms: all of which are acceptable.

+m =	−m =
+231000	−.000376
+231000.00	−.376E-3
+2.31E5	−3.76E-4
+.231E6	

In the above EX is considered part of the number. The maximum number of decimal digits is ten, exclusive of sign and decimal point.

The second LRL-developed subroutine is the STF off-line print routine used to edit information onto Tape B9. It has been modified for BCD output in a form for printing on an SC-5000 printer; it is also acceptable to IBM off-line printers.

Since the Lister reads all input cards before working with the data, the order of the input information is optional except for the on-line heading card which must be the first input card. This card is not read by CML and it is retained in memory in card image form.

Any Hollerith character may be used in the heading card and all 72 columns of the card are available with the following restriction. The problem name used in the Generator, the Main Code and the Edits has been confined to two BCD words. These are taken from the first 12 columns of the heading card. The entire card is printed in on-line comments and in the Lister off-line print.

The input data thus may be packed or arranged in any desired order following the heading card. The input is detailed on the following pages (in consecutive memory order) and several variations in format are given for the sample problem outlined in Appendix D.

Decimal Location	Input Form	Lister Heading	Comments
121	AI	IMAX	I = number of energy groups.
122	AL	LMAX	L = number of materials.
123	AR	RMAX	R = number of regions.
124	AX (Z)	XMAX	X = number of "zones" in x coordinate.
125	AY (R)	YMAX	Y = number of "zones" in y coordinate.

(continued)

Decimal Location	Input Form	Lister Heading	Comments
126	Aα	ISTAR	$\alpha = I^* =$ ISTAR = highest energy group with NOD properties. ISTAR = 0 signifies no NOD regions.
127	Aβ	IBAR	$\beta = \bar{I} =$ IBAR = partial sum energy group index in edits.
128	+f	F	Convergence factor $0 \leq f \leq 1$. If omitted, f = 0 used (see page 13).
129	+A	XY (RZ)	Geometry factor (0 for x-y, 1 for r-z).
130	+H	H	The z dimension in x-y geometry.
131	+K	K	The input KR^0 guess.
132	+ϵ	EP	The convergence criterion (Eq. 24).
133	AD	DP INT	The dump interval for "saved" code dumps.

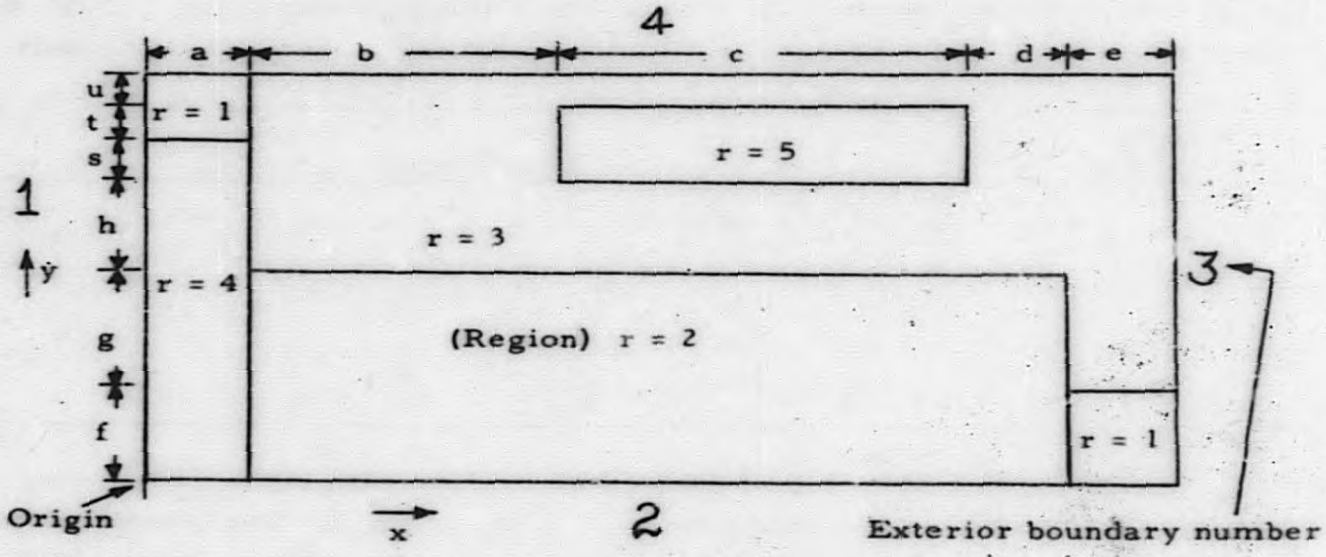
Decimal Location	Input Form	Comments and Lister Heading
143 ... 160	$+F^I+F^{I-1}+ \ldots F^1$	FBLOCK – The fission spectrum (see page 7).
161 ... 180	$+c_1+c_2+c^I+ \ldots c^1$	C1C2 CBLOCK – The $(c_1 + c_2c^1)$ (see Appendix A, Q's).
181 ... 199	$+\phi^I+\phi^{I-1}+ \ldots \phi^1$	Input flux values for spatially flat starting fluxes. If L181 = 0, Tape A6 is searched automatically.
200 ... 300	$+x_1+x_2+ \ldots x_X$	Zone dimensions (mesh intervals) in x (z) coordinate.
302 ... 402	$+y_1+y_2+ \ldots y_Y$	Zone dimensions (mesh intervals) in y (r) coordinate.
403 ... 406	$+\delta_1+\delta_2+\delta_3+\delta_4$	BOUNDARY DELTAS, if the origin is the lower left corner, the order of the 4 exterior boundary deltas is Left, Bottom, Right, Top.

(continued)

Decimal Location	Input Form	Comments and Lister Heading
407 ... 806	$AI^{*1}AR^{*1}+\delta_{I^{*1}}+\ldots\ \delta_1 AI^{*2}AR^{*2}+\ \ldots$	The NOD region specification and the various delta's. I^* of Region R^* followed by I^* number of deltas. If more than one such region must be arranged in order of decreasing I^*'s. $I^{*1}\geq I^{*2}\geq \ldots$
807 ... 6806	$+^{1,1}V+^{2,1}V+\ \ldots\ ^{\ell,r}V+\ \ldots$ $L,1V+^{1,2}V+\ \ldots\ L,RV$	VBLOCK, the relative density factors given for all materials ($\ell = 1 \ldots L$) for each region (r) $r = 1 \ldots r = R$.
10763 ... 10802	$An_1An_2 \ldots An_L$	The material identification integers corresponding to those specified in the material cross section data. The sequence is arbitrary but must agree with the cross section order and there must be a total of L numbers, each nonzero.
10803 ... 12802	The matrix description row by row starting at the origin.	
12803 ... 12902	The row repeat, must end in a zero.	

The row description and the row repeat entries completely describe the region matrix picture. Each row of the matrix is described starting at the left hand boundary (x = 1 zone to x = X zone) and proceeds outward from the y = 1 row to the y = Y row. Each zone is assigned a region number (r) corresponding to the rth set of V's (relative density factors). The row repeat card is used to specify the number of consecutive repeats of a given row.

Example: Let a,b, etc. be the number of zones or mesh intervals between the region boundary lines as indicated in the following sketch.



Note: m = b + c + d. n = d + e. q = b + c + d + e.

The entries would be:

L10803 A4SaA2SmA1Se A4SaA2SmA3Se A4SaA3Sq A4SaA3SbA5ScA3Sn A1SaA3SbA5ScA3Sn A1SaA3Sq

L12803 AfAgAhAsAtAu+0

Note: 1) Each row's description specifies exactly X zones.

$$X = a + b + c + d + e$$

2) The sum of the row repeats specifies exactly Y rows.

$$Y = f + g + h + s + t + u$$

3) The row repeat is specified for all rows including a single row.
4) The entry into L(12803+Y) must be zero.
5) The same physical type of region occurring several times may (should) be assigned the same region number (r = 1 above).
6) No r designation can be larger than R (L123) though not all $r \leq R$ must be used. However, a set of V's must be given for all r, r = 1 ... R.
7) As used above, all lower case letters are integers (fixed point).

The 9-ANGIE Lister expands the matrix description given and lists (Tape B9) a pictorial array with each zone and its assigned region number presented.

Decimal Location	Input Form
12903 ... 15482	$An_\ell + {}^{\ell}N + {}_c\sigma^I + \ldots {}_c\sigma^1 + {}_f\sigma^I + \ldots {}_f\sigma^1 + {}_{tr}\sigma^I + \ldots {}_{tr}\sigma^1$ $+ {}_s\sigma^I + \ldots {}_s\sigma^1 + \mu^{I+1,I} + \mu^{I-1,I} + \mu^{I-2,I} + \ldots$ $+ \mu^{i+1,i} + \mu^{i-1,i} + \mu^{i-2,i} + \ldots \mu^{2,1} + \mu^{0,1} + \mu^{-1,1}$

This is the general form for material cross section data input. The cross sections are packed for all materials ($n_1 \ldots n_\ell \ldots n_L$). The identification number, n_ℓ, must agree with those entered in L10763 and following. The Lister gives each material's data in columns headed by C SIGS, F SIGS, TR SIGS, S SIGS, MU + 1, MU - 1, MU - 2, respectively, with each row being the data for an energy group ($i = I \ldots i = 1$). The material number and the N are also given. The n_ℓ's may be in any desired integer sequence, but consistence in order is required between the V's (L807), the L10763 entry and the order of the cross section input in L(12903 and following).

VI. 9-ANGIE OPERATING INSTRUCTIONS

The following descriptions for starting, restarting, and removing ANGIE problems assume that the ANGIE Master Instruction Tape has been made and is mounted on logical Tape A1. The procedures suggested are those followed at LRL, where production problems are set up during a scheduled daytime shot (usually of 1/2-hr duration) and the running of such problems is done during swing, owl, and weekend shifts by machine operators.

The normal procedure is to start a problem dump tape (A2) by listing the input and, where available, copying starting fluxes from a previous problem dump tape. The Tape A2 is then left for production operation which includes generating, running, and automatic edit (if the problem converges). Generator tapes of converged problems or problems run a maximum specified time are usually not saved. Generator tapes of interrupted problems are saved to be used in a subsequent restart of the problem. Restart procedures, which will allow continuance of most problems in trouble, will be outlined.

Before continuing, the function of the various Sense Switches (SS) will be outlined. These are also shown in the flow diagram in Appendix C.

SS 1 Function No. 1. If SS 1 is depressed (and SS 2 is up) when Tape A1 is initially read, the control routine transfers to the Forced Edit Code. SS 6 is then tested.
Function No. 2. SS 1 is tested at all convenient points to force terminate the particular ANGIE subcode in process.

SS 2 SS 2 is the first Sense Switch tested on initial reading of Tape A1. If it is depressed, the Lister Code is read in and control transferred to it. If it is up, the Lister Code is bypassed and the SS 1 (see Function No. 1) test is next.

SS 3 SS 3 controls whether one or two generator tapes are written in the Generator or used in the Main Code. If SS 3 is depressed, only Tape B3 is written or used. It is tested independently in the Generator and in the Main Code and has no function in either the Lister or the Edits.

SS 4 SS 4 determines whether the generator tapes should be written. SS 4 depressed means bypass the Generator subcode and proceed to the Main Code. It has no meaning in any other section.

SS 5 SS 5 is not used in any part of 9-ANGIE.

SS 6 SS 6 controls the search routines in the Lister, Main Code, and Forced Edit. If SS 6 is up, the search is conducted for the last dump (the dummy which follows). If SS 6 is depressed, a specific dump determined by the entry in the console keys is sought. The dump number (p) is entered as a fixed-point octal number in the address part of the keys.

To start a problem (to list), ready the input deck in the Card Reader, ready the Printer, set Sense Switches (if listing, SS 2 must be down), set Tape Selector Switches, Clear the computer, and press Load Tape. When the computer stops with a select on the Card Reader, press the Start key to read in the remaining cards. If there is an error in the input, there will be an on-line comment, a listing (if possible) will be written on Tape B9, and Tape A2 will not be started. If SS 1 is depressed during the listing operation (or initially), the code stops upon completion of the listing (and flux copy if one is made). The Tape A2 may be labeled and removed, and one may

proceed to the next operation. If SS 1 is not depressed, ANGIE proceeds to Generator, etc.

To interrupt a problem in progress, depress SS 1. This will cause the computer to stop after the next power cycle. To start a problem or to restart an interrupted problem, mount the proper tapes on the proper logical units, ready Printer, set Sense Switches, press Clear, and Load Tape.

If a problem must be removed on instantaneous notice, rewind, remove and label Tape A2 (A3, B3 if desired) along with Tape A1. If it is removed during the writing of a dump, SS 6 must be used to restart. As a rule, SS 6 is used when doubt exists over the status of the dummy record which follows the last dump or when it is desired to find a dump which is not the last dump on the tape. Otherwise, the regular start (restart) procedure will work. Problems that are in trouble due to a bad dump tape must be relisted and a flux copy made from the old Tape A2, usually with SS 6 depressed. A full-length tape will hold 25 dumps of a maximum-size ANGIE problem.

The on-line output is essentially a monitor type of output which notes the completion of the various sections of the ANGIE Code and/or various troubles. During the Main Code, the p, KMAX, KR, KMR, KMIN of each power cycle are printed on-line upon successful writing of that cycle's dump records on Tape A2. The on-line monitoring also indicates which cycle dumps are permanently saved on Tape A2. No provision (other than by a separate routine) is made for printing the listing or the edit on-line.

The following is a complete list of Program Stops (PS) in the ANGIE Code. All PS locations are octal. The suggested remedies are those frequently followed at LRL. All PS commonly occurring have on-line comments which indicate the type of error and usually a possible remedy.

Section I: There are three general stops. ONL means the following on-line comment is made.

Location 27 — A general stop resulting from an overflow or underflow. There is no on-line comment. It results from an impossible problem or a machine error. Location 22 gives the type (decrement) and the location (address).

Location 77703 — ONL – TAPE A1 BAD OR BAD READ. A tape check has occurred during the reading of the ANGIE Master Instruction Tape A1. Retry, change units or make a new tape.

Location 77515 ONL - INST TAPE MADE CONTAINS BOOT OVRFLOW LIST GEN CODE AUTO EDIT FORCED EDIT. This is the normal PS upon making an ANGIE Master Instruction Tape.

Section II: These are the PS in the 9-ANGIE Lister.

Location 40671 The normal Lister PS when SS 1 terminates 9-ANGIE at the end of the listing routine. ONL - the problem heading card plus NO ERRORS FOUND IN THIS INPUT GENERATOR INPUT IS ON TAPE A2 .

Location 41700 The common PS if errors are found in the input or if a tape check occurs in writing Tape A2. A variety of comments are made on-line which explain the cause.

ONL - TAPE BEING SEARCH NOT ANGIE TAPE refers to Tape A6 during search for starting fluxes.

ONL - TAPE A6 NOT COMPATIBLE WITH TAPE A2. The Tape A6 is from a problem of different I. or X. or Y than the problem just listed on Tape A2.

ONL - END OF TAPE ENCOUNTERED BEFORE CYCLE FOUND. In the search of Tape A6, records and/or files are not consistent with ANGIE dumps. Use SS 6 to obtain fluxes from a known dump - see ONL for cycles found during search.

ONL -DUMP CYCLE REQUESTED NOT ON TAPE. Will occur if cycle requested via SS 6 and keys is not on Tape A6. Retry for known dump.

The following PS in the Lister are infrequent and have no ONL comment.

Location 36161 Y return in CML load routine.
43355 X return in CML.
43356 H return in CML.
43361 CML error stop - retry.
43362 CML error stop - retry.
37030 Counter has exceeded limits, probably a machine error.
37643 Same as above.
41202 Tape A6 mispositioned, failed to find EOF after BSF.

Section III: These are the PS in the 9-ANGIE Generator.

Location 23632 The normal PS when 9-ANGIE is terminated after completion of the Generator. ONL - GENERATOR COMPLETED. regardless of whether the generator is terminated via SS 1 or not.

Location 23647 ONL - TAPE CHECK A2. A tape check in reading Tape A2 records.

Location 23604 ONL - TAPE CHECK A3 - TAPE CHECK B3, respec-
23612 tively. A tape check has occurred (three times) in writing Tape A3 or B3. Change tapes and/or units and regenerate.

Location 22466 No ONL. In each of these PS a counter has exceeded
22476 limits, probably as a result of a machine error - very unlikely stops.

Section IV: There are three PS in the 9-ANGIE Main Code.

Location 1661 The normal PS when SS 1 is depressed to interrupt a running problem. ONL - SENSE SWITCH ONE DOWN PROBLEM FORCED TO STOP. Save and label desired tapes, and proceed to next operation.

Location 4350 KR is zero. No ONL, an impossible problem or a machine error - an unlikely stop.

Location 2636 A common PS for all Main Code troubles. An ONL comment will state the trouble.

ONL Comment	Remarks
1) TAPE CHECK A2	A tape check in reading Tape A2. Three attempts to read have been unsuccessful. One may a) retry, b) back up one dump, c) make a new Tape A2.
2) TAPE CHECK B3 TAPE CHECK A3	Three unsuccessful attempts have been made to read a Generator tape record. Mount a new blank (if possible on a different unit) and restart with regenerating (SS 4 up).
3) TAPE CHECK B5	Three attempts have been made to write fluxes on Tape B5 unsuccessfully. Mount new blank, restart without regenerating (SS 4 down).

ONL Comment	Remarks
4) TAPE CHECK A2 (TAPE CHECK B5) MUST USE SW 6 TO RESTART START FROM LAST SAVED CYCLE	A tape check has occurred on the indicated tape during the writing of a new dump on Tape A2. The dummy on Tape A2 has been destroyed. a) If Tape B5, mount new blank and follow ONL. b) If Tape A2, retry or make new Tape A2 and restart problem.
5) NO EOF FOUND AT END OF TAPE 3 (B3)	An "end of file" was not found at the end of the Generator tape when expected. Restart with regenerating (SS 4 up).
6) NO EOF ON A2	This can occur from two causes. If this occurs during initial search of Tape A2, use SS 6 to start from known dump. If it occurs during actual running (rare), it results from no EOF after BSF, probably machine error.
7) KR NOT WITHIN LIMITS	The test $\frac{1}{2} KR^{P-1} \leq KR^{P} \leq 2KR^{P-1}$ failed. Impossible problem or machine error. (Note: the KR^{P} causing failure is printed ONL.)
8) WP OVER WSTAR EQUAL ZERO	Unlikely, usually an impossible problem.
9) TAPE BEING SEARCH NO ANGIE TAPE and END OF TAPE ENCOUNTERED BEFORE CYCLE FOUND and DUMP CYCLE REQUEST NOT ON TAPE	During initial search of Tape A2. Reasons are self-explanatory.

Section V: These are the PS in the Automatic Edit.

Location 16341 Normal PS upon completion of Auto Edit. ONL – EDIT OF ANGIE PROBLEM COMPLETED LABEL AND SAVE TAPE A2 GO TO NEXT PROBLEM.

Location 15346 ONL – NO EOF ON A2. Tape A2 misposition – an unlikely stop. One can do a Forced or Special Edit of last good dump on Tape A2 .

Location 15401 No ONL. Again, stops resulting from Tape A2 being mis-
15403 positioned unlikely; do Forced Edit.

Note: There are no tape check stops in the Auto Edit and the Auto Edit destroyed the Generator records on Tape A3.

Section VI: These are the PS in the Forced Edit.

Location 36522 ONL – END OF TAPE ENCOUNTERED BEFORE CYCLE FOUND

36554 DUMP CYCLE REQUESTED NOT ON TAPE

36562 TAPE BEING SEARCHED NOT ANGIE TAPE

36625 No ONL – no EOF after BSF

The above four stops result during the search of Tape A2 and are for the same causes as the corresponding stops in the Lister and the Main Code.

Location 16300 The normal PS upon completion of the Forced or Special Edit.

Location 15674 No ONL. A tape check has occurred in the writing of Tape B5. Change tape and retry.

As one might expect, experience at LRL has shown that roughly 90% of the tape check stops result during writing.

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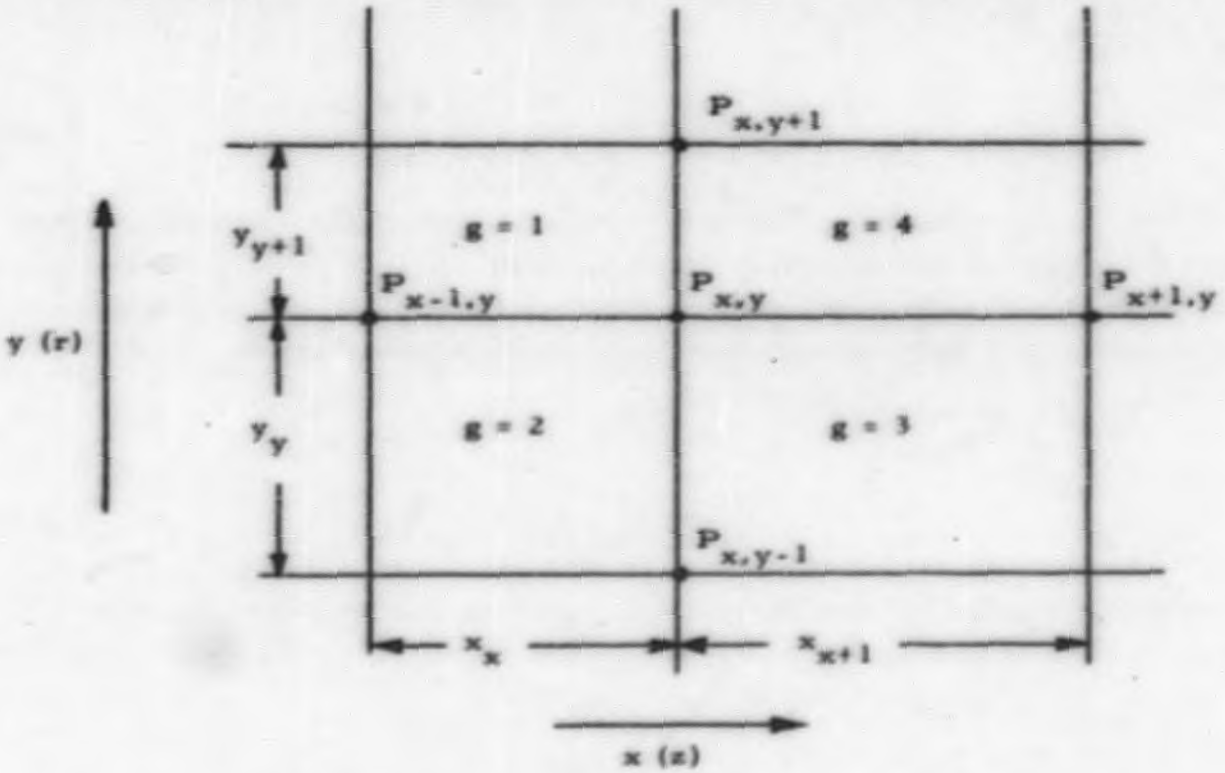
APPENDIX A

DERIVATION OF THE DIFFERENCE EQUATIONS

In this appendix, the general procedure for development of the 9-ANGIE difference equations will be made. We shall consider a general mesh point, $P_{x,y}$, illustrated in the following diagram. The energy index, i, and spatial index for $P_{x,y}$ will be dropped except where needed for clarification.



For the case in which $P_{x,y}$ is an NP, or normal interior diffusion point (with common zones g = 1, 2, 3, 4), the following derivation may be made. There are eight possible Taylor's series expansions of the flux, ϕ, in terms of the four neighboring points.

For g = 1, 4

$$\phi_{g(x,y+1)} = \phi_g + (y_{y+1}) \frac{\partial \phi_g}{\partial y} + \frac{1}{2} (y_{y+1})^2 \frac{\partial^2 \phi_g}{\partial y^2} + \ldots \qquad (25)$$

For $g = 2, 3$

$$\phi_{g(x,y-1)} = \phi_g - (y_y)\frac{\partial \phi_g}{\partial y} + \frac{1}{2}(y_y)^2 \frac{\partial^2 \phi_g}{\partial y^2} - \ldots \quad (26)$$

For $g = 1, 2$

$$\phi_{g(x-1,y)} = \phi_g - (x_x)\frac{\partial \phi_g}{\partial x} + \frac{1}{2}(x_x)^2 \frac{\partial^2 \phi_g}{\partial x^2} - \ldots \quad (27)$$

For $g = 3, 4$

$$\phi_{g(x+1,y)} = \phi_g + (x_{x+1})\frac{\partial \phi_g}{\partial x} + \frac{1}{2}(x_{x+1})^2 \frac{\partial^2 \phi_g}{\partial x^2} + \ldots \quad (28)$$

If one drops the higher order derivatives, the above expressions may be solved for the second derivative terms. The diffusion equation (Eq. 2) may be written for each zone. One multiplies each by β_g, a constant thus far arbitrary and undetermined, and sums over the four zones.

$$(\text{Sum})_g (\beta_g) \left\{ D_g^i \nabla^2 \phi_g^i - {}_R\Sigma_g^i \phi_g^i + (\text{Sum})_{j \neq i} {}_s\Sigma_g^{i,j} \phi_g^j + \frac{F^i}{\lambda} (\text{Sum})_j \left[(c_1 + c_2 c^j) {}_f\Sigma_g^j \phi_g^j \right] \right\} = 0 \quad (29)$$

Substituting the second derivative, applying the boundary condition of continuity of neutron flux and neutron current, and collecting terms, one has:

$$\begin{aligned} & E_T \phi_{x,y+1} + E_B \phi_{x,y-1} + E_L \phi_{x-1,y} + E_R \phi_{x+1,y} \\ & - \left[E_T + E_B + E_L + E_R + 1/2(\text{Sum})_g (\beta_g)({}_R\Sigma_g) \right] \phi_{x,y} \\ & + \left(\frac{a\beta_1 D_1}{2r_y} - \frac{\beta_1 D_1}{y_{y+1}} + \frac{a\beta_2 D_1}{2r_y} + \frac{\beta_2 D_1}{y_y} \right) \frac{\partial \phi_1}{\partial y} + \left(\frac{\beta_1 D_1}{x_x} - \frac{\beta_4 D_1}{x_{x+1}} \right) \frac{\partial \phi_1}{\partial x} \\ & + \left(\frac{a\beta_4 D_4}{2r_y} - \frac{\beta_4 D_4}{y_{y+1}} + \frac{a\beta_3 D_4}{2r_y} + \frac{\beta_3 D_4}{y_y} \right) \frac{\partial \phi_4}{\partial y} + \left(\frac{\beta_2 D_2}{x_x} - \frac{\beta_3 D_2}{x_{x+1}} \right) \frac{\partial \phi_2}{\partial x} \\ & + \frac{F^i}{\lambda} (\text{Sum})_j \left(Q_{x,y}^j \phi_{x,y}^j \right) + T_{x,y}^{i,i+2} \phi_{x,y}^{i,i+2} + S_{x,y}^{i,i+1} \phi_{x,y}^{i,i+1} \\ & + X_{x,y}^{i,i-1} \phi_{x,y}^{i,i-1} = 0 \end{aligned} \quad (30)$$

One desires the first derivative terms to vanish. A necessary condition is that the determinate of the β_g's vanish, a fact that can be readily verified.

The following definitions have been made, together with the assignment of an L (left), R (right), T (top), B (bottom) notation to some coefficients:

$$E_T = \frac{\beta_1 D_1 + \beta_4 D_4}{(y_{y+1})^2} \tag{31}$$

$$E_B = \frac{\beta_2 D_2 + \beta_3 D_3}{(y_y)^2} \tag{32}$$

$$E_L = \frac{\beta_1 D_1 + \beta_2 D_2}{(x_x)^2} \tag{33}$$

$$E_R = \frac{\beta_3 D_3 + \beta_4 D_4}{(x_{x+1})^2} \tag{34}$$

$$Q^i_{x,y} = \frac{1}{2}\,(\text{Sum})_g \left[(\beta_g)_f \Sigma^i_g (c_1 + c_2 c^i)\right] \tag{35}$$

$$T^{i,i+2}_{x,y} = \frac{1}{2}\,(\text{Sum})_g \left[(\beta_g)_s \Sigma^{i,i+2}_g\right] \tag{36}$$

$$S^{i,i+1}_{x,y} = \frac{1}{2}\,(\text{Sum})_g \left[(\beta_g)_s \Sigma^{i,i+1}_g\right] \tag{37}$$

$$X^{i,i-1}_{x,y} = \frac{1}{2}\,(\text{Sum})_g \left[(\beta_g)_s \Sigma^{i,i-1}_g\right] . \tag{38}$$

We shall choose $\beta_4 = (x_{x+1})\,(y_{y+1})$.

Then
$$\beta_1 = (x_x)\,(y_{y+1}) .$$
$$\beta_2 = F(y)\,(x_x)\,(y_y) .$$
$$\beta_3 = F(y)\,(x_{x+1})\,(y_y) . \tag{39}$$

where
$$F(y) = \frac{r_y - (a/2)\,(y_{y+1})}{r_y + (a/2)\,(y_y)} .$$

The above development assumes $P_{x,y}$ is an interior diffusion point. In order to include the boundary conditions, both the exterior ones and the interior nondiffusion ones, a series of special cases will be examined.

Recall that for a nondiffusion (NOD) region (1) the neutron flux is not defined, and (2)

$$D^i \left.\frac{\partial \phi^i}{\partial n}\right|_{\rightarrow \gamma} = -\delta^i \phi^i \tag{40}$$

at all points of the boundary γ.

For illustration consider that the $g = 1$ zone is an NOD zone with a δ_1^i. One makes the usual Taylor's series expansions where possible. Since there is no diffusion equation in the $g = 1$ zone, we shall postulate that $\beta_1 \equiv 0$. An equation identical to Eq. (30) (with $\beta_1 \equiv 0$) is readily obtained with the coefficients of the flux points as shown, except the flux derivative terms which now become

$$+\left(\frac{a\beta_2 D_2}{2r_y} + \frac{\beta_2 D_2}{y_y}\right)\frac{\partial \phi_2}{\partial y} - \left(\frac{\beta_4}{x_{x+1}}\right)\frac{\partial \phi_4}{\partial x}$$

$$+\left(\frac{a\beta_4 D_4}{2r_y} - \frac{\beta_4 D_4}{y_{y+1}} + \frac{a\beta_3 D_4}{2r_y} + \frac{\beta_3 D_4}{y_y}\right)\frac{\partial \phi_4}{\partial y}$$

$$+\left(\frac{\beta_2 D_2}{x_x} - \frac{\beta_3 D_2}{x_{x+1}}\right)\frac{\partial \phi_2}{\partial x} + \cdots \tag{41}$$

The application of the nondiffusion condition together with previous definitions of the β's yields ar added term to the coefficient of $\phi_{x,y}$. If one extends the above argument to all possible combinations, one finds that the coefficient of $\phi_{x,y}$ may be written as $(E_L + E_R + E_T + E_B + R)$, where

$$R = \frac{1}{2}(\text{Sum})_g \left[(\beta_g)({}_R\Sigma_g)\right] + \left[\frac{\delta_4\beta_1}{x_x} + \frac{\delta_3\beta_2}{x_x} + \frac{\delta_2\beta_3}{x_{x+1}} + \frac{\delta_1\beta_4}{x_{x+1}}\right.$$

$$+ (\delta_4\beta_3)\left(\frac{1}{y_y} + \frac{a}{2r_y}\right) + (\delta_3\beta_4)\left(\frac{1}{y_{y+1}} - \frac{a}{2r_y}\right)$$

$$\left. + (\delta_2\beta_1)\left(\frac{1}{y_{y+1}} - \frac{a}{2r_y}\right) + (\delta_1\beta_2)\left(\frac{1}{y_y} + \frac{a}{2r_y}\right)\right]. \tag{42}$$

We have imposed the following conditions:

1) $\beta \equiv 0$ for any nondiffusion zone,

2) $\delta \equiv 0$ for any diffusion zone.

Note that a $\delta = 0$ is a permissible boundary condition (symmetry). The above expression may be extended in any obvious manner to include exterior boundary points. In this case we shall reflect the first and last zone dimension to avoid division by zero. This effectively introduces a set of pseudo zones to which we assign the boundary δ's. Nondiffusion regions terminating on the boundaries thus may be easily superimposed.

The last special case must consider zones bordering the axis ($y = 0$) in cylindrical coordinates. We shall impose that $y = 0$ in r-z geometry must be a symmetry boundary. Furthermore, we shall use the condition that

$$\underset{y \to 0}{\text{Limit}} \quad \left(\frac{a}{y}\right)\left(\frac{\partial \phi}{\partial y}\right) \rightarrow a \frac{\partial^2 \phi}{\partial y^2} \ . \tag{43}$$

Following the procedure outlined above, we can again make Taylor's series expansions where possible. We may incorporate this case, provided:

1) $F(y) = 0$ for $y = 0$ in both x-y and r-z geometry

2) $$E_T \Big|_{y=0} = \frac{(1 + a)}{(y_1)^2} (\beta_1 D_1 + \beta_4 D_4) \ . \tag{44}$$

If in x-y geometry an axial boundary condition other than one of symmetry is present, it is handled like any other set of boundary points, provided programming care is taken in evaluating the term $a/2r_y$.

The preceding development permits the calculation of a general set of cycle independent coefficients for use in the flux iteration code with all special cases and boundary conditions handled by modification of said coefficients. These coefficients are computed only once during a problem and are written on tape during the Generator part of the ANGIE running procedure.

APPENDIX B

EDIT DETAILS OF 9-ANGIE

The Automatic Edit and the Special or Forced Edit are identical in the output information. All edit material is oriented in the same manner as the matrix picture in the Lister routine, i.e., the origin in the lower left hand corner.

Each page of the edit is headed by two lines which contain the following information:

1) code identification - ANGIE,
2) the problem name - two BCD words taken from the first 12 columns of the problem heading card,
3) the particular edit data on that page,
4) the page number,
5) the geometry of the problem x-y or r-z,
6) the X (or Z) of the problem,
7) the Y (R) of the problem,
8) the number of the energy group, I, of the problem,
9) the KR of the cycle, p, that is edited,
10) the cycle, p, that is edited,
11) the IBAR, the group partial sum index that was used.

In addition x_x, y_y numbering is included, first immediately below the heading lines for the x_x, and secondly, at the right hand side of the page for y_y.

The following quantities are edited in the order given:

I. NEUTRON FLUX - The neutron flux ($\phi^i_{x,y}$) is edited for all x,y for all groups starting at i = I ... i = 1. The origin, $P_{0,0}$, is designated by x = 0, y = 0.

II. ZONE VOLUME for each zone, $0 \leq x < X$, $0 \leq y < Y$;

$$= \left[(r_{y+1})^{(1+a)} - (r_y)^{(1+a)} \right] (x_{x+1}) (f_v) . \qquad (45)$$

where $f_v = H$ $\quad a = 0$

$f_v = \pi$ $\quad a = 1$.

These and all following edit quantities designate the "origin" point or zone by x = 1, y = 1.

III. TOTPOWAVDENS, total power average density for each zone, $0 \le x \le X$, $0 \le y \le Y$:

$$= (\text{Sum})_{i=1}^{i=I} \left[\left({}_f\Sigma_g^i \right) \left(\phi B_{x,y}^i \right) \right] . \tag{46}$$

where

$$\phi B_{x,y}^i = \frac{1}{4} \left(\phi_{x,y}^i + \phi_{x+1,y}^i + \phi_{x,y+1}^i + \phi_{x+1,y+1}^i \right) \quad \text{and}$$

${}_f\Sigma_g^i$ is the macroscopic fission cross section of the zone bounded by the four flux points of ϕB.

IV. PARPOWAVDENS, the partial power average density is the same as III except the upper limit of the energy sum is i = IBAR.

V. TOTPOWPTDENS, the total power point density, $0 \le x \le X$, $0 \le y \le Y$:

$$= (\text{Sum})_{i=1}^{i=I} \left[\left({}_f\Sigma_g^i \right) \left(\phi_{x,y}^i \right) \right] . \tag{47}$$

VI. PARPOWPTDENS, the partial power point density is the same as V except the upper limit of the energy sum is i = IBAR.

VII. TOTPOW ZONE, the total power per the gth zone

$$= (\text{TOTPOWAVDENS})_g \, (\text{ZONE VOLUME})_g \, (\bar{r}_v) , \tag{48}$$

where

$$\bar{r}_v = \frac{2}{\pi}, \quad a = 0$$

$$\bar{r}_v = 1, \quad a = 1 .$$

VIII. PARPOW ZONE, the partial power per zone follows logically from VII with the use of $(\text{PARPOWAVDENS})_g$.

IX. TOTABS ZONE, the total nonfissioning capture per zone.

$$= (\text{ZONE VOLUME})_g \, (\bar{r}_v) \left[(\text{Sum})_{i=1}^{i=I} \left({}_c\Sigma_g^i \right) \left(\phi B_{x,y}^i \right) \right] . \tag{49}$$

X. PARABS ZONE, the partial nonfissioning capture per zone is the same as IX except the upper limit of the energy sum is i = IBAR.

The following quantities are summed over regions (r) and listed on the final page of the edit with the appropriate headings.

Quantity Summed (over r)	Edit Heading
1) ZONE VOLUME	VOLUME
2) TOTPOW ZONE	TOTPOW
3) PARPOW ZONE	PARPOW
4) TOTABS ZONE	TOTABS
5) PARABS ZONE	PARABS

APPENDIX C

FLOW DIAGRAMS FOR THE 9-ANGIE

The set of flow diagrams for the 9-ANGIE code, shown on following pages, are not given in great detail; however, the general logic shown in these diagrams together with the SAP listing should provide sufficient information for insertion of patches, changes, etc.

The first two diagrams detail the control routine with a block diagram logic of the various subcodes emphasizing the sense switch logic. This is followed by diagrams of the Lister, Generator, Main Code and Automatic Edit. Further details of a typical search and the code dumping procedure are also given. The Forced Edit is sufficiently similar to the Automatic Edit, with the differences previously described, that a separate diagram was not included for the Forced Edit.

The following legend is given to aid in reading the flow diagrams.

1) PR – ONL — Print a comment via the on-line printer.
2) TPCK X — Tape Check on Channel X.
3) RTBXY — Read (record) Tape (binary mode), X Channel, Y unit no.
4) RTBNTXY — Read Tape Binary Non-Transmitting, X Channel, Y unit no. (method of skipping a record)
5) WBCDB9 — Write output in BCD mode on Tape B9.
6) REWXY — Rewind tape, X Channel, Y unit no.
7) WEFXY — Write end of file on tape, X Channel, Y unit no.
8) BSFXY — Backspace file, X Channel, Y unit no.
9) BSRXY — Backspace a record, X Channel, Y unit no.
10) Cd, Tp, Ld, Rd, SS have the usual card, tape, load, read and sense switch meaning used throughout the report.

Control Routine and General Logic I

Control Routine and General Logic II

From control routine
Begin
Rd 1st cd (image)
CML subroutine rds all input cds
CML error
PR - ONL (images, etc.)
6
Errors
Test limits I, L, R, X, Y
PR - ONL
6
No errors
WBCD B9
Errcnt + 1 → Errcnt
Error
Test
1) δ's
2) x_x
3) y_y
4) V's
5) A, N (each f) etc.
WBCD B9
No errors
Through?
No
Yes
Errcnt +1 Errcnt
Error
Expand and test region matrix
No errors
WBCD B9
Compute zone and region volumes
WBCD B9
10
6
PR - ONL Errors, no tp 2A, etc.
PR - ONL Tape check 2A
11
No
Yes
10
Errcnt = 0?
Yes
WTB A2 (constants)
TPCK (A)
No
WTB A2 region matrix
TPCK (A)
No
Yes
Yes
Combine (ℓμ^ij)(ℓσ^j) and rearrange
WTB A2 (ith level)
TPCK A
No
Yes
11
No
all i?
Yes
WEF A2
PR - ONL tape A2 made
12
11
Yes
i2
WTB A2 p=0 constants
TPCK (A)
No
φ^I = 0?
Yes
No
WTB A2 flat φ^i's
11
Yes
TPCK (A)
No
all i?
Yes
No
WEF A2
WTB A2 (dummy)
WEF A2
11
Yes
TPCK (A)
No
SS 14
No
Return to control routine
Yes
6
REW all tapes
HTR
PR - ONL if A6 "size" wrong
6
Do search Tp A6 test size, etc. See separate search detail
RTB A6 1 group φ's
TPCK A
No
WTB A2 1 group φ's
TPCK A
No
all i?
No
Yes
Yes
BSR A6
3 tries?
No
Yes
BSR A2
3 tries
No
Yes
11
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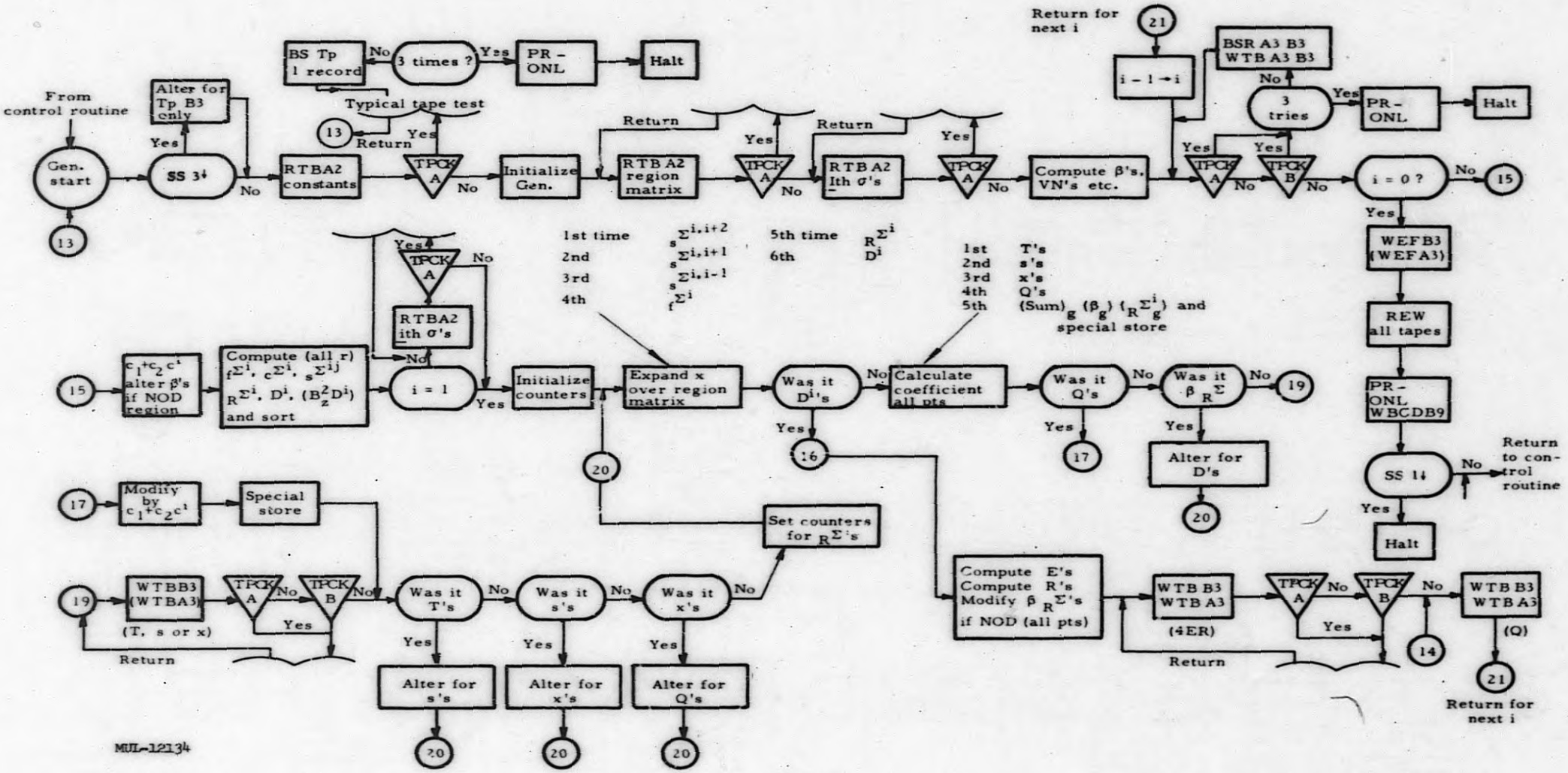
Lister


From control routine
Gen. start
13
Alter for Tp B3 only
Yes
SS 34
No
BS Tp 1 record
No
3 times ?
Yes
PR - ONL
Halt
Typical tape test
13
Return
Yes
RTBA2 constants
TPCK A
No
Initialize Gen.
Return
RTB A2 region matrix
Yes
TPCK A
No
Return
RTB A2 Ith σ's
Yes
TPCK A
No
Compute β's, VN's etc.
Return for next i
21
i - 1 → i
BSR A3 B3
WTB A3 B3
No
3 tries
Yes
PR - ONL
Halt
Yes
TPCK A
No
Yes
TPCK B
No
i = 0 ?
No
15
Yes
WEFB3 (WEFA3)
REW all tapes
PR - ONL WBCDB9
SS 14
No
Return to control routine
Yes
Halt
1st time
2nd
3rd
4th
$_s\Sigma^{i,i+2}$
$_s\Sigma^{i,i+1}$
$_s\Sigma^{i,i-1}$
$_f\Sigma^i$
5th time
6th
$_R\Sigma^i$
D^i
1st T's
2nd s's
3rd x's
4th Q's
5th $(\text{Sum})_g (\beta_g)(_R\Sigma^i_g)$ and special store
Yes
TPCK A
No
RTBA2 ith σ's
No
i = 1
Yes
Compute (all r) $_f\Sigma^i$, $_c\Sigma^i$, $_s\Sigma^{ij}$ $_R\Sigma^i$, D^i, $(B_z^2 D^i)$ and sort
15
$c_1+c_2c^i$ alter β's if NOD region
Initialize counters
Expand x over region matrix
Was it D^i's
No
Yes
16
Calculate coefficient all pts
Was it Q's
No
Yes
17
Was it $\beta\ _R\Sigma$
No
19
Yes
Alter for D's
20
20
Set counters for $_R\Sigma$'s
17
Modify by $c_1+c_2c^i$
Special store
19
WTBB3 (WTBA3)
(T, s or x)
Return
TPCK A
No
TPCK B
No
Yes
Was it T's
No
Was it s's
No
Was it x's
No
Yes
Alter for s's
Yes
Alter for x's
Yes
Alter for Q's
20
20
20
Compute E's
Compute R's
Modify β $_R\Sigma$'s
if NOD (all pts)
WTB B3 WTB A3
(4ER)
Return
TPCK A
No
Yes
TPCK B
No
14
WTB B3 WTB A3
(Q)
21
Return for next i
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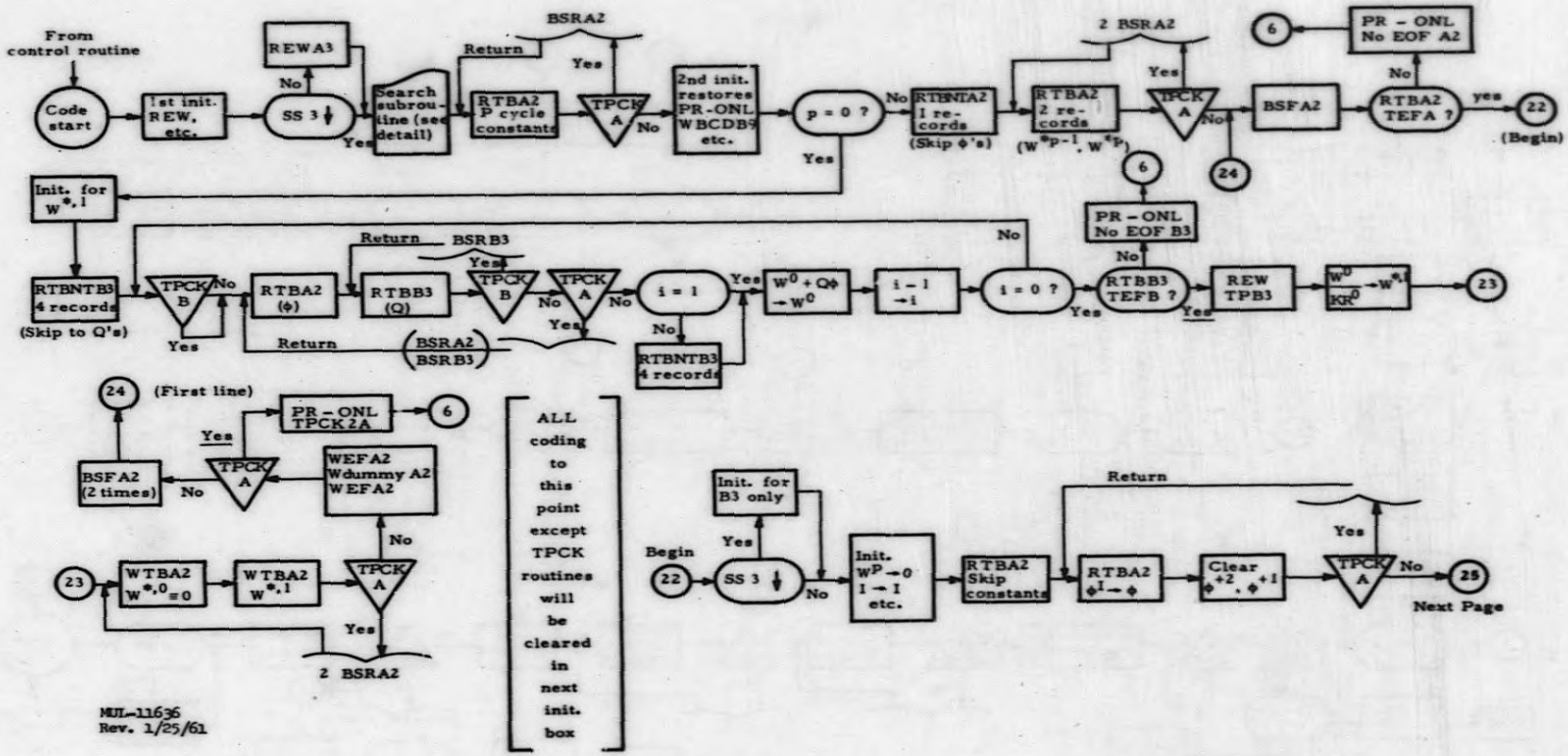
Generator



Main Code Part I

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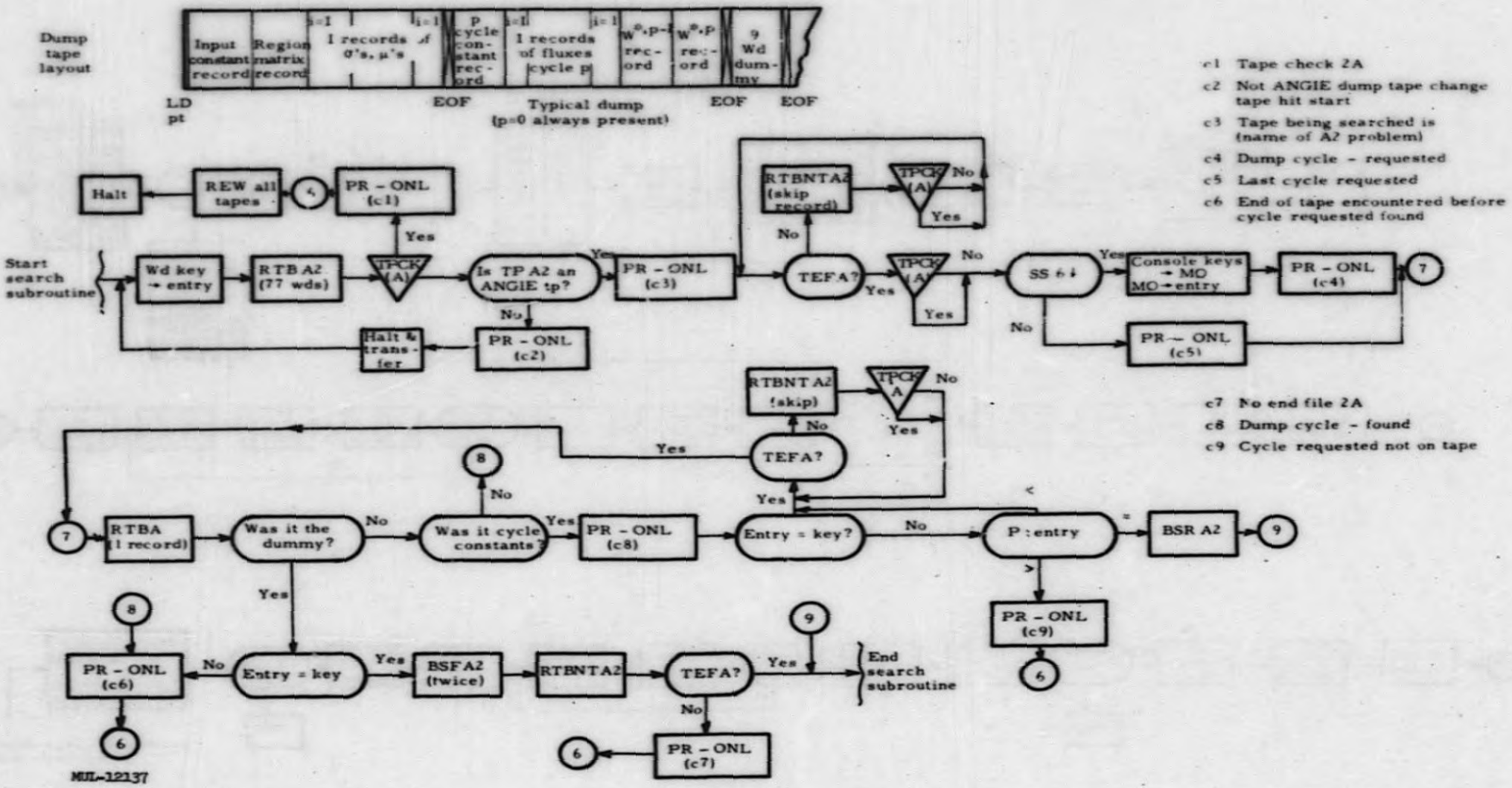
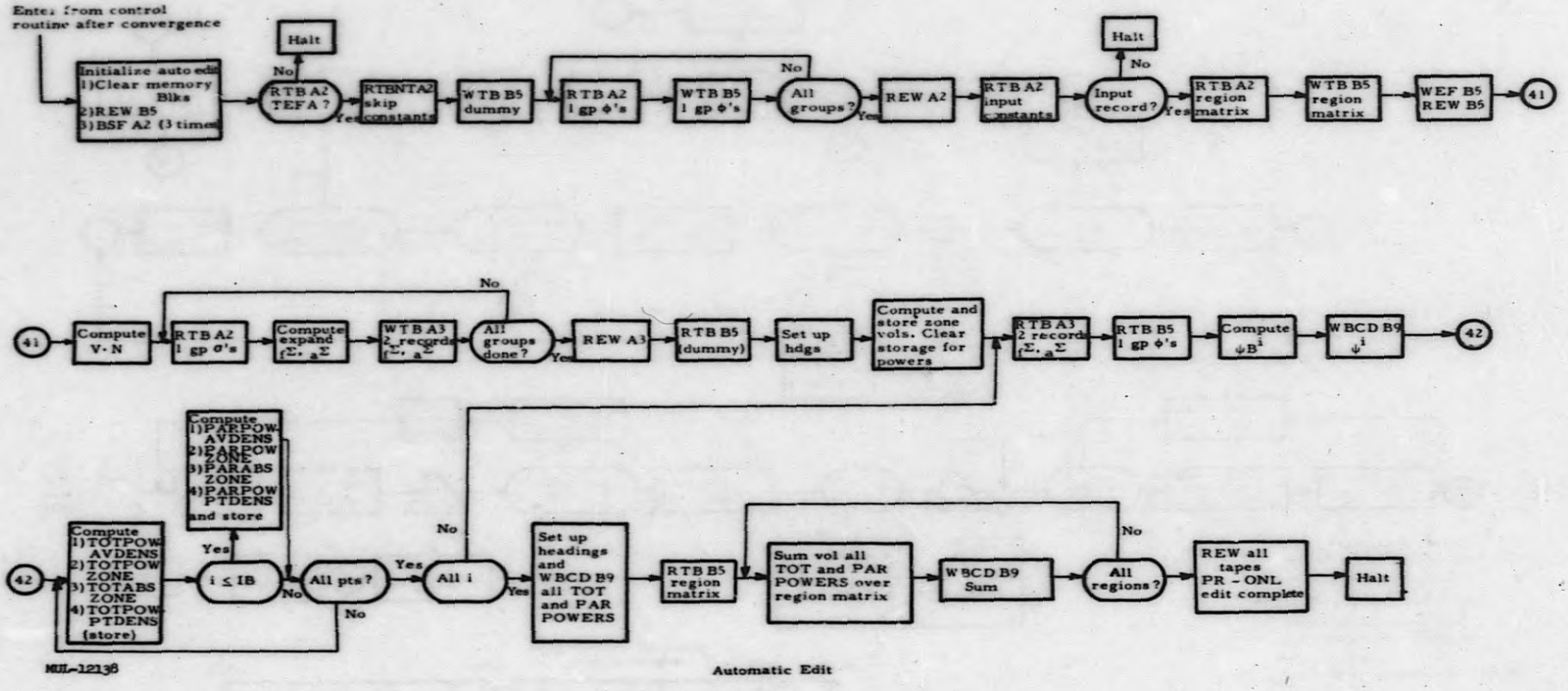
Main Code Part II

Main Code Part III



Typical Tp A2 Search Routine



MUL-12138

Automatic Edit

APPENDIX D

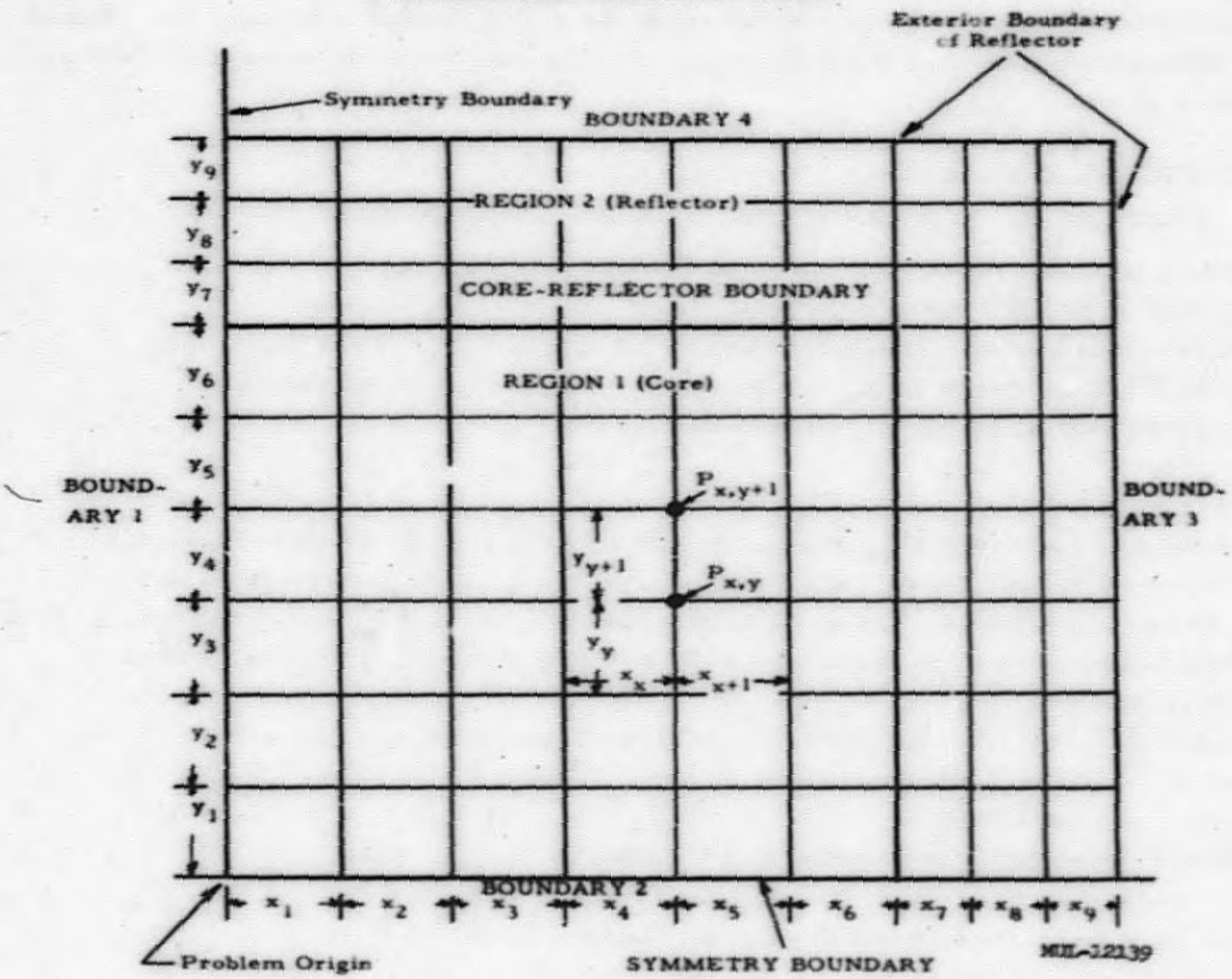
TEST PROBLEM

The sample listed below is for Experiment U-6.[8] The representation of this problem is not intended to be the best possible but is for example only. The sample input has several examples of varied word format and styles possible in a 9-ANGIE input. A layout of the mesh is shown on page 50. Representative items from the on-line monitoring and from the Automatic Edit are also given.

Sample Input

```
 TEST  BCD1  20 FEB 60  TEST FOR ENLARGED ANGIE
L121 A18 A3 A2 A9 A9 A0 +.4+0+104.9 +1+.002 A3
Li43+.478+1.261+591E-3 +.147+.33E-1 +0S13 L161+1+0S19
L181+1+2+1+1S16 L200+7.7258333S6+5.08S3 L302+7.7258333S6+5.08S3
L403+0+0+.46948S2 L807 +1+.072042+1 +0+0+1 L10763A3A7A9
L10803A1S6A2S3 A2S9 L12803A6A3+0
L12903
A3+.6842781E-4
+.114+.129+.184+.316 +.584+1.08+1.68 +2.83 +.525+9.24 +17.8+19.9+13.3
+6.7+17+34.7+62.8+101 +1.37+1.21+1.3+1.69+2.34+3.26+3.99+5.9 +10.5
+18.8+39.4+48.9+44.7+28.5+82.9+183+344+539 +2.8+3.27+4.1+5.1+6.78
+8.53+10+14+16.6+19.6+25.4+27.8+25.4 +20.2+32.5+54.7+91+138 +0S54
A7+6.842781E-5
+.02+.06+.15+.22+.4+.6+.8S2+1.7+15.4+49.6+54.7+117.2+1.7+2.3+2.2
+2.5 +2.75 +0S36 +0S18 +0S54
A9 +.08252394
+0S12 +.001S2 +.002+.004+7E-3 +.0013E1 +0S18
+1.41+1.97+3.07+3.95+4.32 +4.36S7+4.37+4.46S4+4.9
+1.49+2.09+ +4.18+4.57 +4.62S7 +4.63+4.72S4 +4.9
+0+.363+0 +0+.088+0 +0+.1371+0 +0+.1371+0 +0+.1371+0 +0+.1371+0
+0+.1371+0 +0+.1371+0 +0+.1371+0 +0+.1371+0 +0+.1371+0 +0+.1371+0
+0+.1371+0 +0+.1371+0 +0+.1371+0 +0+.1371+0 +0+.1556+0 +.2363+0+0
L130+104.902
```

709 ANGIE TEST
Problem Crit. U-6
Extracted from UCRL-5175, Table I



REFERENCES

[1] S. P. Stone and R. E. Lingenfelter, Neutron Diffusion Theory Programs and Their Application to Simple Critical Systems, Lawrence Radiation Laboratory report UCRL-5913 (1960).

[2] S. P. Stone, E. T. Collins, and S. R. Lenihan, 9-ZOOM, a One-Dimensional, Multigroup, Neutron Diffusion Theory Reactor Code for the IBM 709, Lawrence Radiation Laboratory report UCRL-5682 (1959).

[3] G. G. Bilodeau, W. R. Cadwell, J. P. Dorsey, J. G. Fairey, and R. S. Varga, PDQ – an IBM 704 Code to Solve the Two-Dimensional Few-Group Neutron-Diffusion Equations, Westinghouse Electric Corporation report WAPD-TM-70 (1957).

[4] E. L. Wachspress, R. M. Stone, and C. E. Lee, Mathematical Techniques in Two-Space Dimension Multigroup Calculations, Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, (Geneva, 1958) Vol. 16, p. 483.

[5] D. W. Peaceman and H. H. Rachford, The Numerical Solution of Parabolic and Elliptic Differential Equations, J. Soc. Indust. Appl. Math., Vol. 3, No. 1 (1955).

[6] G. Birkhoff and R. S. Varga, Reactor Criticality and Non-Negative Matrices, Westinghouse Electric Corporation report WAPD-166 (1957).

[7] E. L. Wachspress, CURE, a Generalized Two-Space-Dimension Multigroup Coding for the IBM 704, Knolls Atomic Power Laboratory report KAPL-1724 (1957).

[8] H. L. Reynolds, Critical Measurements and Calculations for Enriched-Uranium Graphite-Moderated Systems, Lawrence Radiation Laboratory report UCRL-5175 (1958).

/sh

END