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A SIMULATOR FOR HIGH-DENSITY ION GUNS

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ABSTRACT

A simulator for an ion gun is used to predict the ion trajectories in the gun for a given electrode geometry. One simulator control is used for the space charge simulation of the entire ion beam. The beam is divided into cells of different areas in such a manner that each cell has the same space charge. Potential measurements are made at the boundaries of the cells where the accuracy of the space charge simulation is the greatest. A trajectory plotter is described which consists of a miniature tricycle, a voltage balancing unit (the same as used in dc millivolt recorders), a dc regulated power supply, and a number of resistors. The trajectories are found by "steering" the tricycle across the simulator. The trajectory plotter and the method of space charge simulation were checked experimentally and were found to have good accuracy.

1. INTRODUCTION

A simple accurate simulator is described here which is very useful for the evaluation of electrode geometries of high-density ion guns. The simulators described in the literature, including the more recent ones, are fairly laborious and complicated.¹

A uniform resistance paper is used in the simulator described here which is intended for an ion gun possessing two-dimensional symmetry. The method and equations, however, can also be used for a three-dimensional geometry possessing rotational symmetry, provided that the resistance paper used for this case has a conductivity which varies linearly in one direction and is uniform in the perpendicular direction.

Although the equations are derived for the acceleration of ions, they can be used with minor modifications for the acceleration of electrons as well.

The simulation of space charge and the ion trajectory plotter of the simulator is described in Sections II and III, respectively. Then in Section IV the simulation procedure is described in which space charge simulation and the trajectory plotter are used to predict the ion beam trajectory and the electric fields and equipotentials of the ion gun. Experimental checks of the simulator are given in Section V.

II. SPACE CHARGE SIMULATION

Let f be the ratio of the physical dimensions of the ion gun in the simulator to those of the actual ion gun, and let F be the ratio of the voltages used in the simulator to those of the ion gun.

$$f = L_s/L \quad \text{and} \quad F = V_s/V \quad (1)$$

Then

$$E_s = FE/f \quad (2)$$

where E_s , V_s , and L_s are the electric field, voltage and length in the simulator, respectively; E , V , and L are the electric field, voltage, and the corresponding length in the ion gun. Now

$$J_s = \sigma_s' E_s \quad (3)$$

where J_s is the current density in the simulator, σ_s' is the conductivity of the simulator, and E_s is the electric field in the simulator.

Substitution of (2) into (3) gives

$$J_s = \sigma_s' (FE/f)$$

or

$$E = (J_s/\sigma_s')(f/F) \quad (4)$$

Gauss's theorem in free space is

$$\iint E_{\perp} \cdot dS = 4\pi Q \quad (5)$$

where $\iint E_{\perp} \cdot dS$ is the integral of the normal component of the electric intensity over the surface which encloses the charge Q . The integral of the electric intensity over the top and bottom surfaces is zero since we have two-dimensional geometry.

For an area, A_s , and a thickness, t , of the conductive medium we have a space charge volume in the ion gun having an area A_s/t^2 and a thickness t/f . Substitution of (4) into (5) gives

$$\begin{aligned} \iint J_{s\perp} \cdot dS &= \sigma_s' (F/f) 4\pi Q \\ &= I_s (1/t^2) \end{aligned} \quad (6)$$

where I_s is the current through the area A_s of the simulator. This I_s simulates a charge Q in the ion gun. In the volume of interest of the ion gun, $A_g t/r^3$, there is a current density

$$J = \rho v \quad (7)$$

where v is the average velocity of the ions in the volume and ρ is the charge density. Thus

$$Q = \rho A_g t/r^3 = (J/v) (A_g t/r^3) \quad (8)$$

Let σ_s be the surface conductivity of the simulator. Then

$$\sigma_s = \sigma_s' t \quad (9)$$

Substitution of (9) and (8) into (6) gives

$$I_s = \frac{\sigma_s R 4\pi J}{f^2 v} A_s \quad (10)$$

or in practical units

$$I_s = 1.13 \times 10^{13} \sigma_s \cdot \frac{F}{f^2} \cdot \frac{J}{v} \cdot A_s \quad (11)$$

where I_s is in amperes, J is the average ion current density in amperes/cm² in the volume of the ion gun corresponding to area A_g of the simulator, v is in cm/sec, and σ_s is in mho's.

The space charge limited current density in planar geometry is²

$$J = 2.33 \times 10^{-6} \left(\frac{m_e}{m_i} \right)^{1/2} \frac{V^{3/2}}{x^2} \text{ amp/cm}^2 \quad (12)$$

where m_e is the mass of an electron, m_i is the mass of an ion, and V is the potential at a distance x from the source electrode.

The velocity of the ion is

$$v = 1.38 \times 10^6 (V/m_i)^{1/2} \quad (13)$$

The corresponding relation for the current density in the simulator, J_s , is

$$J_s = 2.33 \times 10^{-6} \left(\frac{m_e}{m_i} \right)^{1/2} \frac{V_s^{3/2}}{x_s^2} \frac{f^2}{F^{3/2}} \quad (14)$$

where V_s is the potential in the simulator at a distance x_s from the source electrode.

We measure a potential V_{1s} at a distance x_{1s} from the ion source electrode sufficiently close so that the equipotential V_{1s} is essentially parallel to the source electrode. Substituting (13) and (14) into (11) we obtain

$$I_{1s} = \left[(0.446 \sigma_s) \left(A_{1s} / x_{1s}^2 \right) \right] V_{1s} \quad (15)$$

I_{1s} is the simulation current to the cell whose area is A_{1s} and whose average potential is V_{1s} , where the equipotential V_{1s} is at a distance x_{1s} from the source electrode.

As will be described in Section IV, the ion beam in the simulator is divided into areas of equal space charge. The simulation current to each of these areas is the same and they are all equal to I_{1s} as given by (15). Thus, the potential V_{1s} at a distance x_{1s} from the source electrode determines the space charge simulation for the entire ion beam.

III. TRAJECTORY PLOTTER

The force on an ion is equal to mv^2/R , where R is its radius of curvature, m is its mass, and v is its velocity. Then

$$eE_{\perp} = mv^2/R = 2eV/R$$

or

$$R = 2V/E_{\perp} \quad \text{cm} \quad (16)$$

where $E_{\perp}$ is the electric field perpendicular to the motion of the ion in volts/cm, and eV is the energy of the ion in electron volts.

Using (1) and (2) to convert into simulator units, Eq. (16) becomes

$$R_s = 2V_s/E_s \quad \text{cm} \quad (17)$$

where R_s is the radius of curvature in the simulator, and E_s is the electric field perpendicular to the path of the ion in the simulator in volts/cm.

The trajectory plotter is a small cart in the form of a tricycle. A picture of it is shown in Fig. 1. It has an over-all length of 5-3/4 inches. The cart has the feature that its length can be reduced by a factor of 7 by the removal of the plastic spacer. The single front wheel is rotatable and has a miniature single turn potentiometer mounted on its axle.

We have

$$\tan \theta = D/R_s \quad (18)$$

SH-17467

Fig. 1. Picture of the tricycle used in plotting trajectories.

where θ is the angle of rotation made by the single front wheel with respect to the two fixed rear wheels, and D is the distance between the fixed wheels and the movable wheel.

The radii of curvature described by the ions are normally fairly large. The geometry on the resistance paper is made sufficiently large so that the front wheel does not turn through a large angle θ while tracing the ion trajectories.

For small angles,

$$\tan \theta \approx \theta \quad (19)$$

$$\text{Let } R_q \approx D/\theta \quad (20)$$

and Δ be given by

$$\Delta = (R_q - R_s)/R_q = 1 - \theta/\tan \theta \quad (21)$$

where R_s is found by (18). Hence Δ gives the error in using (19), and is plotted in Fig. 2 as a function of R_s .

The electrode geometry on the resistance paper should be large enough, in comparison to the cart used, so that θ is always small and the desired degree of accuracy is attained. For example, if the minimum radius of curvature described by the cart is equal to $1D$, then from Fig. 2, the largest error in the radius of curvature will be 4.5%.

Substituting (18) and (19) into (17), we have

$$\theta = DE_{s1}/2V_s \quad (22)$$

Now

$$E_{s1} = \Delta V_s/d \quad (23)$$

where ΔV_s is the voltage difference between the two probes of the cart, and d is the distance between the probes. Then

$$\Delta V_s = V_s(2d/D)\theta \quad (24)$$

A voltage V_f is applied across the miniature potentiometer. We then have

$$V_f \theta/\theta_f = v - v_0 \quad (25)$$

where θ_f is the maximum angle through which the potentiometer can turn, v_0 is the output voltage at zero degrees, and v_f is the output voltage when the angle of rotation is θ .

Substituting (25) into (24) we have

$$\begin{aligned} \Delta V_g &= V_g (2\epsilon\theta_f / DV_f) (v = v_0) \\ &= (V_g/V_0) (v = v_0) \end{aligned} \quad (26)$$

where

$$V_0 = DV_F / 2\epsilon\theta_f \quad (27)$$

Let

$$V_g/V_0 = 1$$

then

$$V_F/V_0 = 2\epsilon\theta_f/D \quad (28)$$

A voltage balancing unit automatically keeps the ratio of V_F to V_g the same, so that Eq. (28) holds. The wheel of the cart is rotated so that $v = v_0$ balances ΔV_g , as indicated in Eq. (26). The circuit diagram for the trajectory plotter is shown in Fig. 3.

IV. SIMULATION PROCEDURE

All dimensions of the ion gun are scaled in the ratio, f , as defined by Eq. (3). Electrode shapes are cut from aluminum foil and pinned onto the uniform resistance paper. A convenient potential ratio, F , is chosen, as also defined by (1). The potentials are then applied to the aluminum foil electrodes.

Equal areas of emission are marked off on the anode, the ion source electrode. The boundaries of the streams of ions coming from the marked areas of the anode are traced by the trajectory plotter. The boundary width of the cells used is defined by these trajectories.

The length of each cell, l , is

$$l = C \sqrt{V_g} \quad (29)$$

where V_g here is the average potential in volts of each cell, and C is an arbitrary constant.

Lengths are marked off on the ion trajectories proportional to $\sqrt{V_g}$. The ratio of the velocity of the ion in a cell, v , to the length of each cell is a constant, since v is proportional to $\sqrt{V_g}$.

The trajectories as plotted represent the boundaries of ion streams of equal current. The space charge per unit volume is proportional to the ion current density divided by the average ion velocity. Each cell, then, as we defined it, has the same space charge since the current through each cell is the same and the length of each cell is proportional to the average ion velocity in the cell. The injection current then to each cell, I_s , is a constant, and is equal to the current, I_{12} , as given by (15).

The simulation current is injected with needle probes. The circuit for it is shown in Fig. 4. The resistance of the variable resistors is made large in comparison with the impedance of the resistance paper in order to minimize interaction between probes.

The first set of trajectories and cell lengths are based upon the space charge free configuration, if no other information is available. Needle current injection probes are then placed symmetrically in each cell. One injector is used if the cell is basically square. Two or three injectors are used if the cell is basically rectangular. The variable resistors shown in Fig. 4 are adjusted so that each injector/probe has the proper fraction of I_s .

Each injector simulates a square area of space charge in the simulator. The potential at the boundary of a uniform cylinder of space charge does not change if all of the space charge is concentrated on the axis of the cylinder. The square area used in the simulator approximates a circle, the cross sectional area of a cylinder. The potential measurements are made only at the boundaries to obtain maximum accuracy of the space charge simulation.

The potential of a cell, V_{12} , at a distance x_{12} from the source electrode, is then measured. The second cell away from the source electrode is normally satisfactory. The cells should be small enough so that the equipotential at the second cell is essentially parallel to the source electrode. The shape of the cells next to the source electrode will be little, if at all, changed during an investigation of an electrode geometry.

The bracketed term in Eq. (15) is a constant. I_s , equal to I_{12} , is then found by adjusting it until it is equal to the bracketed term of (15) times V_{12} . I_s is adjusted to all the probes by varying the output voltage of the power supply shown in Fig. 4.

New trajectories are measured and new cells are drawn. The injectors are then moved to symmetrical positions in the new cells. The actual beam path is found from the trajectories obtained in the final iteration. In the procedure few iterations are needed, and a satisfactory solution is often obtained on the second iteration.

Fig. 4. Wiring diagram for space charge simulator.

Fig. 5. $V_s^{3/2}$ vs D_s^2 in planar geometry. The points are experimentally determined while the solid line is theoretical.

V. EXPERIMENTAL CHECKS

A. Space Charge Simulation

In planar geometry, for space charge limited emission, the relation between the applied voltage, V , and the interelectrode spacing, θ , is

$$V^{3/2} = K_1 \theta^2 \quad (30)$$

where K_1 is a constant.

With parallel plate geometry in the simulator and space charge limited emission we then have

$$V_s^{3/2} = K_2 \theta_s^2$$

where V_s is the potential in the simulator at a distance θ_s from the source electrode, and K_2 is a constant. This was checked experimentally. With the potential of the emitter at zero the potential of the collector is taken to be one thousand. These are arbitrary units and are proportional to the applied voltage. The relationship $V_s^{3/2}$ versus θ_s^2 is plotted in Fig. 5 where the theoretical solid curve is a straight line and the dots are experimental points.

In coaxial cylindrical geometry, for space charge limited emission from the inner cylinder, we have

$$V^{3/2} = K_3 \beta_1^2 r' \quad (31)$$

or in the simulator

$$V_s^{3/2} = K_4 \beta_1^2 r'_s$$

where V is the applied voltage between the inner and outer cylinders, r' is the ratio of the radii of the outer to inner cylinders, K_3 and K_4 are constants, V_s is the equipotential at a radius r'_s times the source radius, and β_1^2 is a function of r'_s and its value is given in Ref. 3. The relationship $V_s^{3/2}$ versus $\beta_1^2 r'_s$ is plotted in Fig. 6, where a comparison of the experimental points with the theoretical curve is shown.

We have a similar relation when the outer cylinder is the emitter and the inner cylinder is the collector. The relation in the simulator is then

$$V_s^{3/2} = K_5 \beta_2^2 r'_s$$

Fig. 6. $V_b^{3/2}$ vs $\beta_1^2 r_b^2$ in cylindrical geometry. The inner cylinder is the emitter while the outer cylinder is the collector. The points are experimentally determined while the solid line is theoretical.

Fig. 7. $V_b^{3/2}$ vs $\beta_2^2 r_b^2$ in cylindrical geometry. The outer cylinder is the emitter and the inner cylinder is the collector. The points are experimentally determined while the solid line is theoretical.

where K_s is a constant, and β_z^2 is a function of r_s^1 and is given in Ref. 3. The relationship $V_s^{3/2}$ versus $\beta_z^2 r_s^1$ is plotted in Fig. 7, where the experimental and theoretical results are compared.

B. Trajectory Plotter

The trajectory plotter was checked by placing it in a uniform electric field. In such a field, an ion describes a parabola. The path of an ion injected perpendicular to a uniform electric field is given by

$$y = Cx^2 \quad (32)$$

where x is the distance the ion has traversed perpendicular to the electric field, y is the distance traversed parallel to the electric field when it has covered a distance x in the perpendicular direction, and C is a constant.

Such a motion for the cart requires the angle which the rotatable wheel makes with the front wheels to be constantly changed. This is a good check on the system. The experimental result of a trajectory is shown in Fig. 8, where the solid curve is theoretical while the dots are experimental. The experimental points were determined from two trajectory traces.

C. Uniformity of the Resistance Paper

A complete uniformity of the resistance paper is essential for the simulator to reproduce the fields accurately. Parallel geometry and cylindrical geometry were checked experimentally on the resistance paper, and the results are shown in Figs. 9 and 10. A potential of 10, in arbitrary units, was first applied in one direction and then reversed. The distance between the electrodes was divided into equal intervals. The data were then recorded only after the potentials exceeded 5.00, or very nearly so. An over-all deviation of less than $\pm 1\%$ was observed.

Fig. 8. Trajectory of an ion in a uniform electric field. The points are experimental while the solid curve is theoretical.

Fig. 9. Experimentally determined potentials in the simulator for parallel plate geometry.

MUL-9779

Fig. 10. Experimentally determined potentials in the simulator for cylindrical geometry.

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