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MICROWAVE INTERFEROMETER MEASUREMENTS FOR THE
DETERMINATION OF PLASMA DENSITY PROFILES IN
CONTROLLED FUSION EXPERIMENTS

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Microwave Interferometer Measurements for the Determination of Plasma Density Profiles in Controlled Fusion Experiments*

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Summary--- A method for determining the spatial distribution in density of a plasma has been outlined in a previous paper.¹ The method involves measuring the dispersive microwave propagation coefficients² simultaneously at several frequencies and comparing the measured values with the values calculated for various distribution functions. The present paper demonstrates experimental verification of the method and describes the procedure used to determine the distribution function in a sample laboratory plasma experiment.

INTRODUCTION

The microwave propagation coefficient, β , in a low temperature plasma, contained in a low intensity magnetic field ($\omega_c \lesssim 0.01\omega$) and having a small electron-collision frequency ($\nu_c \lesssim 0.01\omega_p$), may be expressed as:

$$\beta = \frac{\omega}{c} \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \quad (1)$$

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¹ Wharton, C. B. and Slager, D. M., "Microwave Interferometer Measurements for the Determination of Average Electron Density and of Density Profiles in Plasmas," UCRL-5244, June, 1958. (Also submitted to the Physical Review for publication.)

² Wharton, C. B., "Microwave Diagnostics for Controlled Fusion Research," UCRL-4836 Rev., Sept., 1957.

where

$$\omega_p = \left(\frac{\rho e^2}{m_e \epsilon_0} \right)^{1/2}, \text{ plasma radian frequency}$$

ω = microwave radian frequency

ρ = plasma density, electrons/meter³

m_e = mass of electron, 9.11×10^{-25} kg

e = charge of electron, 1.6×10^{-19} coulomb

$\epsilon_0 = \frac{1}{36\pi} \times 10^{-19}$ farad/meter

$\omega_c = \frac{e}{m_e} B$, electron gyrofrequency

B = magnetic field density, webers/m²

The plasma frequency is then:

$$f_p = \frac{\omega_p}{2\pi} = 8.97 \times 10^3 n^{1/2} \text{ cycles/sec} \quad (2)$$

where

n = plasma density, electrons/cm³

A wave propagates when $\omega_p/\omega < 1$ and is cut off when $\omega_p/\omega > 1$. The critical cutoff density is

$$n_{co} = 1.24 \times 10^{-8} f^2 \text{ electrons/cm}^3 \quad (3)$$

In a laboratory experiment, (e.g., controlled fusion experiments) the density is ordinarily not uniform throughout the volume. The density, $n(Z)$ may be characterized by some distribution function¹ so that $\beta(Z)$ is specified. To find the total effect of inserting such a nonuniform plasma sample into a microwave beam, an integration of $\beta(Z)$ over the path length must be performed. The total phase shift due to inserting this plasma then is given by

$$\begin{aligned} \Delta\phi &= \phi_0 - \phi \\ &= \frac{\omega}{c} \left[d - \int_0^d \sqrt{1 - \frac{n(Z)}{n_{co}}} dz \right] \end{aligned} \quad (4)$$

where

d = the path length, meters.

ϕ_0 = the total phase term βd for the vacuum path (no plasma).

We have carried out the integrations for a number of commonly encountered distribution functions and plotted the normalized phase functions, ϕ' , in a previous paper.¹ Figure 5 shows some of these plots.

MEASUREMENT OF PHASE SHIFT

The phase change, $\Delta\phi$, may be measured conveniently with a microwave bridge or interferometer. The fundamental bridge, with a plasma sample in one arm, is shown in Fig. 1.

Many laboratory experiments generate plasmas of a transient nature. Figure 2 sketches the response of the bridge to a sample whose density rises to a value above n_{co} and then decays away. The decrease in amplitude of the interference "fringes" near cutoff is due to a combination of scattering and absorption. The phase shift due to plasma is

$$\Delta\phi = 2\pi N \text{ radians} \quad (5)$$

where N = the number of fringes.

Another microwave circuit which permits a rapid data reduction and eliminates the amplitude effects apparent in Fig. 2 is the fringe-shift interferometer² sketched in Fig. 3. One arm of the bridge is made a path several hundred wavelengths longer than the other arm. The frequency of the klystron is then swept back and forth a small amount (~ 50 Mc). The dispersion in the long arm is much greater than that in the short arm, causing the phase of the signal from the long path to vary more rapidly and causing the bridge to be balanced and unbalanced successively as the two signals interfere with each other. These detected interference signals are amplified and clipped and applied to the intensity input of the oscilloscope. The sawtooth voltage which modulates the klystron frequency is applied to the y-axis. A vertical row of dots thus appears, the number depending on the number of interferences. When a time base convenient to the plasma experiment is applied to the x-axis a set of horizontal straight bars or "fringes" appears. If, however, the plasma density varies during the horizontal sweep, the position of the interferences shifts, giving a skewing to the bars. The presentation of this "swept-fringe interferometer" is shown in Fig. 4. In this method of presentation the amplitude effects are clipped off. One fringe shift is defined as the condition of the top of one fringe extending to a line level with the top of the next (360°).

ANALYSIS OF A PLASMA EXPERIMENT

Measurements were made on the Little Pig² experiment simultaneously at two frequencies and the frequencies were then changed one at a time while

holding everything else fixed. Results at five frequencies in all were recorded. Cutoff occurred at 17.6 KMC, indicating a peak density, n_o , of $3.8 \times 10^{12}/\text{cm}^3$. The data presentation is shown in Fig. 4. The results of the measurements are tabulated below in Table I, and plotted in Fig. 5. The path length was measured as $d = 3.2$ in. or 8.1 cm.

The plot of a trapezoidal distribution with $b = d/6$ fits within the experimental points, and probably represents a good approximation to the true distribution. A rough check with Langmuir probes shows the profile to be flat-topped, giving further correlative evidence. The presence of the probe in the discharge, however, causes a slight change in the microwave data, indicating considerable perturbation.

TABLE I

Tabulation of Data Obtained from a Plasma Sample at Five Frequencies. Repeatability of Measuring N is $\pm 1/8$ Fringe.

f (KMC)	17.6	18.0	22.6	24.6	33
λ/d	0.200	0.206	0.164	0.151	0.112
n_o/n_{co}	1	.96	.61	.51	.285
N	4.25	3.6	1.8	1.5	1.0
ϕ'	.11	.26	.705	.774	.89

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Fig. 1. Microwave bridge circuit with plasma sample in one arm.

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Fig. 2. Bridge response to a transient plasma.

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Fig. 3. Microwave fringe-shift interferometer circuit for plasma density determination.

Fig. 4. Composite presentation of the fringe-shift interferometer. Top right is the response at 33 KMC. Bottom right 24.6 KMC. Top left 22.6 KMC. Bottom left 18 KMC. At the very top left is the microwave radiation at 2.8 KMC, the gyrofrequency, and at the very top right is the plasma arc current, for reference.

Fig. 5. Plot of normalized microwave phase coefficient for various spatial distributions, with experimental points shown for five frequencies.

END