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ABSTRACT

The objective of the present work is to study and model the interfacial structure development of air-water two-phase flow in a confined test section. Experiments of a total of 9 flow conditions in cap-turbulent and churn-turbulent flow regimes are carried out in a vertical air-water upward two-phase flow experimental loop with a test section of 200-mm in width and 10-mm in gap. Miniaturized four-sensor conductivity probes are used to measure local two-phase parameters at three different elevations for each flow condition. The bubbles captured by the probes are categorized into two groups in view of the two-group interfacial area transport equation, i.e., spherical/distorted bubbles as Group 1 and cap/churn-turbulent bubbles as Group 2. The acquired parameters are time-averaged local void fraction, interfacial velocity, bubble number frequency, interfacial area concentration, and bubble Sauter mean diameter for both groups of bubbles. Also, the line-averaged and area-averaged data are presented and discussed. The comparisons of these parameters at different elevations demonstrate the development of interfacial structure along the flow direction due to bubble interactions.

INTRODUCTION

In gas-liquid two-phase flow, the void fraction and interfacial area concentration characterize the geometric capability of the interfacial transfer of mass, momentum, and energy between the two phases. Historically, flow regime-dependent correlations have been used to determine interfacial area concentration in nuclear reactor thermal-hydraulics and safety system codes. However, most of the flow regime transition criteria were developed for static fully developed

flow. Therefore, this correlation approach does not properly account for either the dynamic nature of the change in the interfacial structure or the entrance effects of the flow. In view of this, Kocamustafaogullari and Ishii (1995) proposed a more dynamic approach, namely, the interfacial area transport equation. By mechanistically modeling the major bubble coalescence and disintegration phenomena, Wu et al. (1998) and Kim et al. (2001b) developed the one-group interfacial area transport equation, which is applicable to bubbly flow. In conjunction with the theoretical efforts, experimental study has been performed to acquire interfacial structure data in bubbly flow (Kim et al., 2001a).

In cap-turbulent, slug, and churn turbulent flow, there exist bubbles of different size and shape, such as spherical, distorted, cap, slug, and churn-turbulent bubbles. These differences in bubble size and shape lead to substantial differences in bubble transport phenomena, such as drag force, bubble interaction mechanisms, etc. In the one-group interfacial area transport equation, these differences have not been accounted for due to its one-group nature (Wu et al., 1998 and Kim et al. 2001b). Therefore, in order to describe the interfacial structure development in a more generalized two-phase flow, the one-group transport equation needs to be extended to a two-group transport equation. Meanwhile, a reliable database is indispensable to evaluate the two-group transport equation.

The present study is a continuation of the previous experimental efforts for bubbly flow (Kim et al., 2001a). Extensive experiments have been carried out in the same upward air-water flow loop to establish a reliable database, as well as to gain a better understanding of the interfacial structure development for cap-turbulent and churn-turbulent flow. A database in 9 flow conditions at cap-turbulent and churn-

turbulent flow regimes is obtained by the miniaturized four-sensor conductivity probes, which is the state-of-the-art for local interfacial measurements (Kim et al., 2000). The measured parameters are local void fraction, interfacial velocity, and bubble number frequency for both Group 1 and Group 2 bubbles at three different elevations. Here, Group 1 bubbles include spherical and distorted bubbles while Group 2 bubbles are cap-turbulent and churn-turbulent bubbles. Then, the local interfacial area concentration and bubble Sauter mean diameter can also be obtained for both groups of bubbles based on the above measured parameters. The comparisons of these parameters at different elevations demonstrate the development of interfacial structure along the flow direction. The underlying physical mechanisms of this development are also discussed in detail.

NOMENCLATURE

a_i	interfacial area concentration (1/m)
$D_{d, \max}$	maximum distorted bubble limit (m)
D_h	hydraulic diameter of test section (m)
D_{sm}	Sauter mean diameter of bubbles (m)
f_b	bubble number frequency (no./s)
g	gravitational acceleration (m/s^2)
j	volumetric flux (superficial velocity) (m/s)
v	velocity (m/s)
x	coordinate in width direction
y	coordinate in gap direction
z	coordinate in vertical direction

Greek symbols

α	void fraction
σ	surface tension (N/m)
$\Delta\rho$	density difference between two phases (Kg/m^3)

Subscripts

1	Group 1 bubbles
2	Group 2 bubbles
f	liquid
g	gas or bubbles
0	gas property at standard condition

Mathematical symbols

$\langle \rangle$	area averaging
$\langle \langle \rangle \rangle$	void-weighted area averaging

EXPERIMENTAL FACILITY

The experimental facility includes a bubble injection unit, test section, upper plenum, water reservoir, water and air delivery systems, and instrumentation. The loop is operated under atmospheric pressure. The de-mineralized water is supplied by a centrifugal pump while the air is delivered by an

air compressor. The two-phase mixture exits the test section and enters the upper plenum for separation, and the water and mists are guided back into a water reservoir for recirculation. For a more detailed description about the test facility, readers are encouraged to refer to the previous paper (Kim et al., 2001a).

The air-water two-phase mixture is generated by a specially designed bubble injection apparatus. Six porous sparger units are included, three on each side of the duct-width, to generate bubbles of near-uniform size. These sparger units are arranged in equal spacing relative to each other in order to minimize artificial bubble interactions at the outlet of the bubble injection apparatus. The total liquid flow rate is achieved by two liquid injections: one through the six sparger units of the bubble injection apparatus to shear bubbles off from the sparger tube surfaces and another through the three auxiliary liquid lines located on one side at the bottom of the bubble injection apparatus.

The test section of this facility is 200-mm in width and 10-mm in gap. It is made of clear acrylic material and has great advantages for flow visualization due to the following facts: the motion of the air-bubbles is close to two-dimensional; the influence of the overlap of the bubbles on the image analysis is minimized, and the image distortion due to curvature effect is eliminated. Six local measurement ports are installed in the gap-direction along the test section. The z/D_h values from Port 1 to Port 6 are 8.0, 34.8, 61.5, 88.2, 115.0, and 141.7, respectively. In the present experiments, local measurements are performed at Port 2, Port 4, and Port 6.

MEASUREMENT TECHNIQUE

For the measurement of air volumetric flow rate, several rotameters together with a precise pressure gauge and a thermometer at the outlet of the rotameters are used. For the liquid, rotameters and vortex flow meters are installed to measure the flow rate. Also, a differential pressure (DP) cell is installed between the inlet and the exit of the test section to measure the pressure drop in the test section.

The conductivity probe, first proposed by Neal and Bankoff (1963), is based on the fundamental differences in electric conductivity between the two phases, i.e., water and air in this study. It has been one of the most widely used measurement techniques in gas-liquid two-phase flow experiments due to its capability of measuring local two-phase parameters (Kataoka et al., 1986 and Kim et al., 2000). The characteristic rise/fall of the impedance signals (voltage signals) between the sensors and common ground can be obtained as the bubbles passing through the exposed tips of the probe sensors. Based on these signals, the local time-averaged void fraction, interfacial velocity, and bubble number frequency can be calculated. Furthermore, the local time-averaged interfacial area concentration can be obtained.

Kataoka et al. (1986) formulated a mathematical expression to determine the local time-averaged interfacial area

concentration for both the double-sensor and four-sensor probes. Recently, Kim et al. (2000) miniaturized the four-sensor conductivity probe. The miniaturized four-sensor conductivity probe consists of one common sensor, which is in the front of the probe, and three rear sensors. One of the three rear sensors is arranged in such a way that it is just beside the common sensor, i.e., this rear sensor and the common sensor can be considered as a double-sensor probe that measures interfacial velocity in the flow direction since the line between the tips of these two sensors are almost parallel with the flow. The cross-sectional measurement area of the miniaturized four-sensor probe is usually on the order of 0.3-mm². This can effectively minimize both the number of missing bubbles and deformation of passing bubble interfaces.

The four-sensor probe can identify the encountered bubbles and separate the corresponding local measurement parameters into two groups based on the bubble chord length information (Kim et al., 2000). In the present study, spherical and distorted bubbles are categorized as Group 1, and cap and churn-turbulent bubbles are categorized as Group 2. In identifying bubble types, the maximum distorted bubble limit, $D_{d, \max}$, is used as the criterion of group boundary. The bubbles with a chord length smaller than $D_{d, \max}$ are categorized as Group 1 bubbles. Here $D_{d, \max}$ is given by Kocamustafaogullari et al. (1984) as

$$D_{d, \max} = 4\sqrt{\sigma/g\Delta\rho} \quad (1)$$

which is approximately 10-mm for air-water flow under atmospheric pressure condition. It should be noted, however, this approach to distinguish bubble groups could result in errors when the probe encounters the edges of a cap bubble. Nevertheless, the benchmark experiments with image analyses in the bubbly flow and slug flow showed that the differences between the two measurement methods were within $\pm 10\%$ (Kim et al., 2000).

EXPERIMENTAL CONDITIONS

As mentioned earlier, for each flow condition, the local measurements are performed at Ports 2, 4, and 6, which correspond to z/D_h values of 34.8, 88.2, and 141.7, respectively. At each port, measurements at a total of 36 points are made in a quarter of the cross sectional area of the test section, as shown in Fig. 1. Spot check is done to assure good symmetry of the flow before actual data acquisition takes place. To obtain good statistical results, at least 1000 Group 1 and 200 Group 2 bubbles are acquired at most measurement points (no or few Group 2 bubbles are observed in some measurement points for certain flow conditions) by adjusting the data acquisition time.

In the present study, experiments of 7 cap-turbulent and 2 churn-turbulent (Run 12 and Run 13) flow conditions have been carried out. Table 1 shows the flow conditions in detail and Fig. 2 presents them on a flow regime map. In this table, to obtain j_g values at different measurement ports, the ideal gas

law is assumed valid and the local pressure is determined by linear interpolation of the pressure drop measured by a DP cell. The flow regime investigation showed that a periodic unstable cap-turbulent and churn-turbulent region exists at low liquid flow rates. Also, in this region, noticeable local flow re-circulation occurs behind a cap or churn-turbulent bubble. This re-circulation greatly hampers the measurement accuracy of the four-sensor conductivity probe. Therefore, the current flow conditions are chosen such that the local flow re-circulation region is avoided.

It should be noted that in the previous experiments performed for bubbly flow, Run 8 is actually in cap-turbulent flow regimes (Kim et al., 2001a). However, due to the low number frequency of Group 2 bubbles as well as the available measurement technique, double-sensor conductivity probe, which is applicable to bubbly flow, was used as local instrument. In the current study, a four-sensor probe is used and the data in this flow condition is re-acquired.

Fig. 1. Measurement points at each axial measurement port.

Table 1. Experimental flow conditions

Run No	j_f [m/s]	j_{g0}^* [m/s]	Local j_g [m/s]		
			Port 2	Port 4	Port 6
8	1.89	0.39	0.29	0.32	0.36
10	0.32	0.41	0.34	0.36	0.38
11	0.95	0.47	0.37	0.40	0.43
12	2.84	1.00	0.73	0.82	0.93
13	2.84	2.01	1.43	1.61	1.84
14	0.63	0.42	0.34	0.36	0.39
15	1.89	0.58	0.44	0.48	0.53
16	1.26	0.66	0.52	0.56	0.61
17	1.40	0.52	0.41	0.44	0.48

*: This gas superficial velocity is converted to the standard condition (1 atmospheric pressure and 70°F temperature).

Fig. 2. Experimental flow conditions on a flow regime map. Thick gray line is the transition line from bubbly to slug flow identified by Mishima and Ishii (1984) for round pipe. The thick and thin black lines are the transition lines from bubbly to cap-turbulent and from cap-turbulent to churn-turbulent flow identified for the current loop (Kim et al. 2001a).

EXPERIMENTAL RESULTS

Due to the length limitation of the paper, only characteristic experimental results are given. Shown first in Fig. 3 are the local profiles of the bubble velocities at centerline (or center region) of both the width and gap directions for Group 1 and Group 2 bubbles, measured at three different locations in z -direction for Run 12. In this flow condition, j_l and j_{g0} are 2.84 and 1.00-m/s, respectively. And it is churn-turbulent flow. In general, bubble velocity distributions for both groups in y -direction at $x=90$ -mm show a quite flat profile similar to the velocity of a turbulent single-phase flow. For all the other measured flow conditions, a similar trend for bubble velocity is observed. However, at $y=1$ -mm, which is very close to the wall, the Group 2 bubble velocity has an unusual higher value than what is expected in Run 12. This phenomenon may be possibly due to measurement error. It can also be observed that Group 2 bubbles have higher velocity compared to Group 1 bubbles at each measurement location.

Bubble velocities in x -direction at $y=5$ -mm show a similar turbulent velocity profile, as shown in Fig. 3(b). However, a strong entrance effect is observed at the lowest measurement port, as shown in Fig. 3(b). This phenomenon is also observed in the other flow conditions with high liquid flow rates. However, as the flow develops, the fluctuations gradually disappear. Kim et al. (2001a) also observed this phenomenon.

Figure 4 is the line-averaged (in y -direction) bubble velocities for both Group 1 and Group 2 bubbles for Run 12. It

can be seen that that bubbles tend to move faster at higher elevation. In this flow condition, the void-weighted area-averaged bubble velocity for Group 1 and Group 2 bubbles, i.e., $\langle\langle v_{g1} \rangle\rangle$ and $\langle\langle v_{g2} \rangle\rangle$, at z/D_h of 34.8, 88.2, and 141.7 are 3.85 and 4.35, 3.93 and 4.38, and 4.09 and 4.68-m/s, respectively. This is mainly due to the pressure effect. For Group 2 bubbles, the larger size may also lead to higher bubble velocity.

Furthermore, the line-averaged (in y -direction) void fraction, interfacial area concentration, Sauter mean diameter, and bubble number frequency for both Group 1 and Group 2 bubbles are presented in Fig. 5 to Fig. 8 for Run 12. The interfacial area concentration has a physical meaning of the interfacial area per unit mixture volume. The Sauter mean diameter is defined as

$$D_{smk} = 6\alpha_k / a_{ik}, k = 1 \text{ or } 2. \quad (2)$$

For Group 1 bubbles, the void fraction in the center region of x -direction dramatically decreases along the flow direction because Group 1 bubbles coalesce to form Group 2 bubbles in this region, as shown in Fig. 5(a). For α_2 , entrance effect can be observed again. Nevertheless, α_2 slightly increases as the flow develops. Also, it has a larger value around the center region of x -direction and relatively small value close to the wall at Port 4 and Port 6. This implies that the large bubbles tend to move to the center of the test section.

For the interfacial area concentration of Group 1 bubbles, a significant wall peak is observed as shown in Fig. 6(a). This phenomenon is related to the bubble number frequency, bubble velocity, and bubble size. The Sauter mean diameter of Group 1 bubbles is quite uniform across the width of flow channel at Port 4 and Port 6, as indicated in Fig. 7(a). While for the bubble frequency, there exist more Group 1 bubbles close to the wall region as shown in Fig. 8(a). The combination of these effects lead to the wall peak at the highest measurement port. For Group 2 bubbles, a_{i2} shows a similar profile as α_2 .

The size of Group 1 bubbles decreases as the flow evolves for Run 12, as shown in Fig. 7(a). This indicates that the bubble disintegration is dominant instead of bubble coalescence at the region close to the wall of x -direction. This can also be verified from the bubble number frequency as shown in Fig. 8(a). However, in the center region, Group 2 bubbles slightly grow along the flow direction due to the pressure effect as well as the coalescence between Group 1 and Group 2 bubbles. Meanwhile, the slow increase of the Group 2 bubbles frequency indicates the formation of Group 2 bubbles from the coalescence of Group 1 bubbles at the center region of flow channel.

The line-averaged bubble number frequency and interfacial area concentration for Runs 8 and 15 acquired at Port 6 are compared in Fig. 9. Runs 8 and 15 have the same j_l , while Run 15 has a higher j_g , as shown in Table 1. Run 8 is cap-turbulent flow while Run 15 is at the transition region from cap-turbulent to churn-turbulent flow.

In Run 15, more Group 2 bubbles exist while the number of Group 1 bubbles is slightly smaller compared to Run 8. The

(a) Center region in width direction ($x=90$ mm)

(b) Centerline in gap direction ($y=5$ mm)

Fig. 3. Bubble velocity of Group 1 and Group 2 bubbles at (a) center region in width direction ($x=90$ mm) and (b) centerline in gap direction ($y=5$ mm) for Run 12 at three elevations. The lines shown in the figures are only for the purpose of clear demonstration.

(a) Group 1 bubbles

(b) Group 2 bubbles

Fig. 4. y -direction line-averaged bubble velocity of (a) Group 1 and (b) Group 2 bubbles for Run 12 at three elevations.

(a) Group 1 bubbles

(b) Group 2 bubbles

Fig. 5. y -direction line-averaged void fraction of (a) Group 1 and (b) Group 2 bubbles for Run 12 at three elevations.

(a) Group 1 bubbles

(b) Group 2 bubbles

Fig. 6. y -direction line-averaged interfacial area concentration of (a) Group 1 and (b) Group 2 bubbles for Run 12 at three elevations.

(a) Group 1 bubbles

(b) Group 2 bubbles

Fig. 7. y -direction line-averaged Sauter mean diameter of (a) Group 1 and (b) Group 2 bubbles for Run 12 at three elevations.

Fig. 8. y-direction line-averaged bubble number frequency of (a) Group 1 and (b) Group 2 bubbles for Run 12 at three elevations.

Fig. 9. Line-averaged bubble number frequency of (a) Group 1 and (b) Group 2 bubbles and interfacial area concentration of (c) Group 1 and (d) Group 2 bubbles for Run 8 and Run 15 at $z/D_h = 141.7$.

(a) Group 1 bubbles

(b) Group 2 bubbles

Fig. 10. $\langle \alpha \rangle$ along z -direction for (a) Group 1 bubbles and (b) Group 2 bubbles. The lines shown in the figures are only for the purpose of clear demonstration.

(a) Group 1 bubbles

(b) Group 2 bubbles

Fig. 11. $\langle a \rangle$ along z -direction for (a) Group 1 bubbles and (b) Group 2 bubbles. The lines shown in the figures are only for the purpose of clear demonstration. Legends are the same as Fig. 10.

interfacial area concentration has the similar trend. Furthermore, the measurements show that the bubble velocities for both bubble groups are larger in Run 15. Also, $\langle D_{sm2} \rangle$ in Run 15 is 22.8-mm, larger than that in Run 8, 19.8-mm. However, $\langle D_{sm1} \rangle$ in Run 15 is 2.6-mm, slightly smaller than that in Run 8, 2.8-mm. These phenomena imply that Group 1 bubbles coalescence has significant contribution to inter-group transport of bubbles in Run 15. It is also interesting to note that the area-averaged void fractions for Runs 8 and 15 are less than 0.2, although they are already in cap-turbulent flow regime. This is mainly due to the geometry and size of the test section, as well as the unique liquid inlets in this experimental loop.

Finally, axial development of $\langle \alpha \rangle$ and $\langle a_i \rangle$ for both bubble groups are shown in Figs. 10 and 11, respectively. For $\langle \alpha_1 \rangle$, Runs 11 and 15 show a relatively large decrease along the flow direction in the cap-turbulent flow regime. This implies that the effect of the inter-group transport from Group 1 to Group 2 due to Group 1 bubble coalescence is more significant compared to the pressure effect alone. However, in Run 14, $\langle \alpha_1 \rangle$ has an increasing trend as the flow develops. This may be due to the fact that this flow condition is at bubbly to cap-turbulent flow transition region and no significant inter-group transfer occurs. Therefore, the expansion due to the pressure effect dominates. On the other hand, $\langle \alpha_2 \rangle$ increases along the axial direction in almost all the flow conditions except Run 14. This may be due to the increase of Group 2 bubble velocity.

Corresponding to the void fraction, the area-averaged interfacial area concentration for Group 1 bubbles, $\langle a_{i1} \rangle$, also show a decreasing trend along the axial direction for Runs 10, 11, 15, and 16. However, in Runs 12 and 13, $\langle a_{i1} \rangle$ increases significantly as the flow develops due to the generation of Group 1 bubbles from large bubble disintegration. $\langle a_{i2} \rangle$ exhibits similar features to the $\langle \alpha_2 \rangle$ except Run 13. In Run 13, there is a significant decrease for $\langle \alpha_2 \rangle$ while an increase is observed for $\langle a_{i2} \rangle$ at Port 4. Large increase of Group 2 bubble velocity at Port 4 might attribute to this phenomenon. In general, as the flow travels upward, more Group 2 bubbles are formed due to the inter-group transport from Group 1.

CONCLUSIONS

The experiments in a total of 9 flow conditions at cap-turbulent and churn-turbulent flow regime have been performed in a vertical air-water upward loop. The miniaturized four-sensor conductivity probes are used to measure local time-averaged interfacial velocity, void fraction, bubble number frequency, interfacial area concentration, and bubble Sauter mean diameter for both Group 1 and Group 2 bubbles. Due to the geometry of the present experimental loop, no slug flow regime is identified, instead, the cap-turbulent and churn-turbulent flow occur at smaller void fraction compared to the conventional round pipe.

The experimental results demonstrate the development of the interfacial structure due to the bubble interactions and the

pressure effect. In flow conditions of high liquid flow rate, bubble disintegration mechanism is more important than bubble coalescence in view of the bubble interactions. In relatively low liquid flow rates, bubble coalescence can be considered as the major bubble interaction mechanism in cap-turbulent flow. However, the pressure effect is always important in the present study due to the atmospheric pressure experimental condition.

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