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TREATMENT OF NUCLEAR DATA FOR TRANSPORT PROBLEMS CONTAINING
DETAILED TEMPERATURE DISTRIBUTIONS

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T. H. Trumbull*

ABSTRACT

The problem of creating and storing cross-section information in a problem where many nuclides are subjected to highly detailed temperature distributions and/or changing temperatures is considered. An approach involving interpolation between nuclear data libraries at various reference temperatures was investigated. Reference nuclear data libraries were obtained by Doppler broadening cross sections to the desired temperatures using the NJOY¹ code system. Several interpolation schemes over various temperature intervals were studied. Interpolated values at intermediate temperatures were compared to NJOY Doppler broadened results for the same temperature. Errors relative to the Doppler broadened results were calculated in order to judge the suitability of the interpolation scheme and temperature interval. The total, elastic scattering, capture, and fission (if applicable) reactions for ²³⁸U, ²³⁵U, ⁹⁰Zr, ¹⁶O, ¹⁰B and ¹H were considered in this study, over a temperature range of 294 K to 811 K (~70 °F to ~1000 °F).

The study showed that cross sections can be interpolated within an accuracy of 0.1% over temperature intervals ranging from 111 K (200 °F) for ¹H, ¹⁰B, and ¹⁶O. Smaller intervals are required for nuclides with more complex resonance behavior. In the case of ⁹⁰Zr, a 28 K (50 °F) interval was required to reduce errors below 0.1%. Some values of the interpolated cross sections for ²³⁸U and ²³⁵U, remained greater than the target 0.1% relative error even with the 28 K (50 °F) interval, suggesting a small interval is necessary for these nuclides.

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I. INTRODUCTION

For isothermal transport problems, Doppler broadening techniques using the kernel-broadening^{1,2,3} approach and Maxwellian free-gas distribution of target nuclide velocities allow for the convenient generation of point-wise, linear-linear interpolable, cross-section sets at a desired temperature, prior to run time. This is the case for such popular codes as MCNP⁴. In instances where nuclides exist in a problem at many unique temperatures and/or the temperature is changing, e.g., a detailed temperature feedback model, cross-section information for many nuclides at many different temperatures may be required.

In these circumstances, it is possible to create separate Doppler broadened cross-section libraries at every possible temperature of the nuclides in the problem. However, this approach may prove problematic for cases involving more than a few nuclides and temperatures because of memory storage concerns. An additional complication to this approach is that the final temperature distribution may not be known *a priori*, as is the case in a temperature feedback problem.

An alternative to creating large sets of pre-Doppler broadened cross sections is to employ a method of approximating the cross section at various intermediate temperatures, given a set of cross-section libraries Doppler broadened at a small number of reference temperatures. The reference temperatures should be chosen to span the anticipated temperature range for a particular nuclide in the problem at some predetermined interval, e.g., 300 K to 800 K in 100 K steps. The accuracy of the interpolation is dependent not only on the size of the interval but the interpolation method used, the temperature range for the problem, and the behavior of the cross section in the resolved resonance region.

Nuclides having “narrow resonances” tend to be more affected by Doppler broadening treatment and are therefore of more interest. Narrow resonances are characterized by a

large Doppler width, Γ_D , relative to the total line width, Γ , where $\Gamma_D = \left(\frac{4E_0 kT}{A} \right)^{1/2}$,

and

A = the atomic weight ratio of the of the target mass to the projectile mass,

k = Boltzmann’s constant,

T = the target nuclei temperature in K,

E_0 = the energy of the resonance.

II. CALCULATING CROSS SECTIONS AT INTERMEDIATE TEMPERATURES USING INTERPOLATION

The well-known Doppler broadening equation in terms of energy for a Maxwellian free-gas target nuclei distribution can be derived²

$$\sqrt{E}\sigma(E,T) = \frac{1}{2} \left(\frac{\alpha}{\pi E} \right)^{1/2} \int_0^\infty dE_r \sqrt{E_r} \sigma(E_r, 0) \cdot \left\{ e^{\left[-\alpha(\sqrt{E} - \sqrt{E_r})^2 \right]} - e^{\left[-\alpha(\sqrt{E} + \sqrt{E_r})^2 \right]} \right\}, \quad (1)$$

where

$$\alpha = \frac{A}{kT},$$

and

A = the atomic weight ratio of the of the target mass to the projectile mass,

k = Boltzmann’s constant,

T = the target nuclei temperature in K,

E = energy of the projectile in electron volts (eV),

E_r = the relative energy of the neutron as “seen” by the target nuclei,

$\sigma(E, T)$ = Doppler broadened cross section at energy E for target nuclei temperature T ,

$\sigma(E_r, 0)$ = cross section at energy E_r for target nuclei temperature of 0 K.

The nature of the convoluted integral of equation (1) does not suggest a simple interpolation scheme to calculate $\sigma(E, T')$ at some intermediate temperature, T' , given Doppler broadened cross sections, $\sigma(E, T_1)$ and $\sigma(E, T_2)$. For this study, five possible interpolation schemes were investigated:

“lin-lin,”

$$\sigma(E, T') = \sigma(E, T_1) + [\sigma(E, T_2) - \sigma(E, T_1)] \left[\frac{T' - T_1}{T_2 - T_1} \right], \quad (2)$$

“log-log,”

$$\ln(\sigma(E, T')) = \ln(\sigma(E, T_1)) + [\ln(\sigma(E, T_2)) - \ln(\sigma(E, T_1))] \left[\frac{\ln(T') - \ln(T_1)}{\ln(T_2) - \ln(T_1)} \right], \quad (3)$$

“sqrt-lin,”

$$\sigma(E, T') = \sigma(E, T_1) + [\sigma(E, T_2) - \sigma(E, T_1)] \left[\frac{\sqrt{T'} - \sqrt{T_1}}{\sqrt{T_2} - \sqrt{T_1}} \right], \quad (4)$$

“lin-log,”

$$\ln(\sigma(E, T')) = \ln(\sigma(E, T_1)) + [\ln(\sigma(E, T_2)) - \ln(\sigma(E, T_1))] \left[\frac{T' - T_1}{T_2 - T_1} \right], \quad (5)$$

and “sqrt-log,”

$$\ln(\sigma(E, T')) = \ln(\sigma(E, T_1)) + [\ln(\sigma(E, T_2)) - \ln(\sigma(E, T_1))] \left[\frac{\sqrt{T'} - \sqrt{T_1}}{\sqrt{T_2} - \sqrt{T_1}} \right]. \quad (6)$$

II.A Generating the interpolated cross sections

A temperature range of 294 K to 811 K was chosen for this study. Doppler-broadened cross section libraries were generated over this range using the NJOY¹ code system at the following temperatures: 294 K, 367 K, 478 K, 505 K, 519 K, 533 K, 547 K, 561 K, 589 K, 700 K, and 811 K. The ENDF/B files⁵ were downloaded from the National Nuclear Data Center website. Cross sections were reconstructed using a tolerance of 0.1% in the RECONR module and thinned using an error tolerance of 0.1% in the BROADR module. The reaction types examined for this study included total (mt=1), elastic scattering (mt=2), radiative capture (mt=102) and fission (mt=18), if applicable, where the “mt” numbers designate the ENDF/B reaction type⁵.

For each nuclide in the study, values of the cross sections at each temperature were mapped to a union energy mesh using linear interpolation when necessary. Five interpolation intervals of decreasing magnitude were investigated using a target temperature of 533 K: 294 K to 811 K, 367 K to 700 K, 478 K to 589 K, 505 K to 561 K, and 519 K to 547 K. For every value on the union energy mesh, an interpolated cross section, $\sigma(E, T')$, was generated at 533 K, using the reference data sets and each interpolation law (equations (2) – (6)). The resulting value of $\sigma(E, T')$ was then compared to the NJOY Doppler broadened value at 533 K and a relative error calculated.

III. RESULTS

Because Doppler broadening is a resonance phenomenon, it is expected that the nuclides with significant resonance behavior will be most affected by the interpolation treatment suggested – specifically, those with large numbers of narrow resonances.

Figure 1 through Figure 6 show the cross section as a function of energy for the nuclides examined in this study.

The comparatively large resolved resonance region and large numbers of narrow resonances in ^{238}U singles this nuclide out as the most challenging to accurately represent using an interpolation method. Other nuclide cross sections that may be difficult to accurately represent include ^{235}U , and to a lesser extent, ^{90}Zr and ^{16}O .

III.A ^{238}U Cross Sections

Table I shows the results of the five interpolation schemes for the five temperature interpolation intervals for the total cross section of ^{238}U . Accurate representation of the strong resonance absorption characteristics of ^{238}U prove difficult using interpolation. Relative errors for the best interpolation method using the smallest interpolation temperature interval still exceed 0.2%, with an average of 0.0071% and a maximum of 0.296%. Although all the relative errors are not less than the 0.1% target, a large fraction is in this range. Depending on the desired precision of the transport calculation, this error may be inconsequential.

Results of interpolation for the other reaction rates are provided in Table II. Unlike Table I, these results are reported at the smallest temperature interval only, since the best possible results are obtained using this temperature interval. Relative errors for the fission reaction cross section are similar to those of the total cross section, however, larger maximum relative errors are observed and a greater number of errors exceed the 0.1% target, with some exceeding the 0.5% maximum value bin.

Interpolating the capture cross section results in 563 relative errors exceeding the 0.5% maximum bin value, suggesting that this reaction is not approximated well with interpolation. However, the average and maximum relative errors are comparable to the fission cross section. The interpolated elastic scattering cross section results show a greater number of points in the target range of <0.1% relative error, with maximum and average relative errors that are in a more acceptable range.

III.B ^{235}U Cross Sections

Table III shows the results for interpolating the total, capture, fission, and elastic scatter cross sections over the smallest temperature interval, 519 K to 547 K. The reduction in complexity of the cross section structure in the resolved resonance range relative to ^{238}U suggests that interpolation may yield more accurate results in ^{235}U . The total cross section average and maximum relative errors are $\leq 0.1\%$, with the one exception of the “lin-log” interpolation scheme. The interpolated fission and capture cross sections are less accurate, however, with some relative errors exceeding the 0.1% target. Maximum errors for the two reactions are $< 0.2\%$, though, and average errors are well below the 0.1% target. The elastic scattering cross section in ^{235}U is calculated well using interpolation and all relative errors are below 0.1%.

III.C ^{90}Zr Cross Sections

The ^{90}Zr cross section is dominated by scattering resonances rather than the absorption resonances of ^{238}U and ^{235}U . Table IV shows the results for interpolating the total, capture, and elastic scatter cross section for ^{90}Zr over the temperature range of 519 K to 547 K. With one notable exception, all the relative errors for the total and elastic

scattering cross section are less than 0.1%. However, one cross section point (on the lower energy “wing” of a resonance) remains at a relative error of $\sim 0.35\%$.

The interpolated capture cross sections result in higher relative errors, however, capture is a much smaller component of the total cross section than elastic scatter. Average relative errors for the interpolated capture cross sections are less than 0.1% with maximum errors of $\sim 1\%$.

III.D ^{16}O Cross Sections

The cross section structure in ^{16}O allows for greater accuracy of interpolated cross sections. The interpolation interval can be extended from 28 K (519 K to 547 K) to 111 K (478 K to 589 K) while maintaining the relative errors near the target of 0.1%.

Table V shows the results for interpolating the total, capture, and elastic scattering cross sections over a temperature range of 478 K to 589 K. At this temperature interval, it is possible to achieve average and maximum relative errors less than 0.1% using a “log-log” or “sqrt-lin” interpolation scheme for all the reactions. The capture cross section is less sensitive to the interpolation method and all methods result in relative errors of no consequence.

III.E ^{10}B Cross Sections

The few, wide resonances and large energy span covered by the smooth “ $1/v$ ” behavior of the ^{10}B cross section provides a good candidate for interpolation. As is the case with ^{16}O , the interpolation interval can be extended to 111 K while maintaining the relative errors around 0.1%.

Table VI shows the results of interpolating the total, capture, and elastic scattering cross section for ^{10}B over the temperature range of 478 K to 589 K. The total and capture cross sections are well-predicted using interpolation and relative errors are well within the 0.1% target. Relative errors calculated for the elastic scattering component of the cross section are higher but, using a “log-log” interpolation scheme, are below the target.

III.F ^1H Cross Sections

As in the case of ^{10}B , the ^1H cross section is anticipated to be a good candidate for interpolation. The smooth behavior of the cross section as a function of energy, with no resonance structure, will tend to produce more accurate interpolated cross sections. The interpolation interval can be safely extended to 111 K without exceeding the 0.1% error target.

Table VII shows the results for interpolating the total, capture, and elastic scatter cross sections for ^1H , over the range of 478 K to 589 K. Relative errors for total and elastic scatter are kept below 0.1% using either a “log-log” or “sqrt-lin” interpolation scheme. Relative errors for capture are very low and insensitive to the interpolation scheme used.

IV. CONCLUSIONS

Interpolation of cross sections provides a means to perform transport calculations on problems with complex and/or changing temperatures without prior knowledge of the final temperature distribution. By Doppler broadening to a set of cross sections at a predetermined number of temperatures spanning the temperature range of the problem, cross sections can be calculated “on the fly” for any temperature in the model. However,

in materials with strong resonance behavior, such as ^{238}U , fine interpolation intervals are required to obtain a reasonable degree of accuracy. In this study, an interval of 28 K (50 °F) was not suitable to reduce the relative errors of every point in the ^{238}U cross section to less than 0.1%. Integral testing of this method is necessary to characterize whether this magnitude of relative error is significant for reactor calculations. For nuclides that do not exhibit strong resonance behavior, interpolation intervals as large as 111 K (200 °F) may be acceptable.

Of the five interpolation schemes examined, the “log-log” interpolation scheme performed the best. Although, in some cases other schemes predicted a lower maximum error, the “log-log” scheme generally resulted in lower average errors and a greater number of cross section relative errors within the 0.1% target.

An alternative to the interpolation method is the *in-situ* Doppler broadening approach. This approach is to-date untested and may be viable given the increasing computational speed of computers. This method requires only a single set of cross sections, Doppler broadened to the lowest possible temperature that the nuclide will experience in the problem. In the course of particle tracking, the cross section of the constituent nuclides for the material in which the particle currently resides are Doppler broadened to the appropriate temperature, on the fly. Upon entering a new material at a different temperature, or changing the temperature of the current material, the Doppler broadening process is repeated. Integral testing of this method is necessary to determine the speed penalty for Monte Carlo codes in particular. It may very well be the most accurate approach to solve transport problems with complex and/or changing temperature distributions.

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Table I. Results of various interpolation methods over various temperature intervals for the total cross section of ^{238}U at 533 K

Temperature interval: 294 K to 811 K						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 95499)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	60.3	2.88	19323	8125	12377	55674
Log-log	46.1	2.28	21303	10594	16760	46842
Sqrt-lin	75.1	2.78	19664	8919	13611	53305
Lin-log	35.1	3.14	19432	8156	12359	55552
Sqrt-log	34.5	2.58	19789	9033	13872	52805
Temperature interval: 367 K to 700 K						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 95499)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	26.6	1.23	29717	9432	13721	42629
Log-log	19.7	0.962	35278	12839	15007	32375
Sqrt-lin	32.2	1.18	31221	10457	13938	39883
Lin-log	17.0	1.37	29799	9484	13829	42387
Sqrt-log	15.5	1.11	31544	10648	14480	38827
Temperature Interval: 478 K to 589 K						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 95499)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	3.09	0.1363	62072	11864	16089	5474
Log-log	2.24	0.1057	70486	9259	10713	5041
Sqrt-lin	3.70	0.1306	65331	13720	12312	4136
Lin-log	2.04	0.1536	62542	12794	11765	8398
Sqrt-log	1.79	0.1239	67943	10737	10154	6665
Temperature Interval: 505 K to 561 K						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 95499)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	0.825	0.0349	86876	6750	1537	336
Log-log	0.612	0.0272	88195	5634	1620	50
Sqrt-lin	0.993	0.0335	88968	3860	1954	717
Lin-log	0.562	0.0394	84723	6342	4380	54
Sqrt-log	0.500	0.0318	86207	6236	3055	1
Temperature Interval: 519 K to 547 K						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 95499)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	0.309	0.0090	94812	679	8	0
Log-log	0.296	0.0071	95268	229	2	0
Sqrt-lin	0.332	0.0087	94423	946	130	0
Lin-log	0.252	0.0101	94794	703	2	0
Sqrt-log	0.274	0.0082	95382	116	1	0

Table II. Results for interpolating the fission, capture, and elastic scattering cross sections over the temperature interval of 519 K to 547 K for ^{238}U

Fission						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 95499)			
			$\leq 0.1\%$	$0.1\% - 0.2\%$	$0.2\% - 0.5\%$	$\geq 0.5\%$
Lin-lin	1.68	0.0049	94462	644	342	51
Log-log	1.34	0.0038	94854	505	118	22
Sqrt-lin	1.77	0.0054	94111	892	425	71
Lin-log	1.25	0.0046	94578	800	100	21
Sqrt-log	1.29	0.0041	94842	555	80	22
Capture						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 95499)			
			$\leq 0.1\%$	$0.1\% - 0.2\%$	$0.2\% - 0.5\%$	$\geq 0.5\%$
Lin-lin	1.75	0.040	88038	4879	2019	563
Log-log	1.95	0.032	91466	3003	464	563
Sqrt-lin	1.70	0.042	85644	6130	3160	563
Lin-log	2.05	0.045	85413	9188	334	564
Sqrt-log	2.00	0.038	90464	4132	340	563
Elastic Scatter						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 95499)			
			$\leq 0.1\%$	$0.1\% - 0.2\%$	$0.2\% - 0.5\%$	$\geq 0.5\%$
Lin-lin	0.349	0.0085	94539	681	279	0
Log-log	0.314	0.0065	95167	324	8	0
Sqrt-lin	0.402	0.0085	94160	898	441	0
Lin-log	0.266	0.0094	94322	1163	14	0
Sqrt-log	0.290	0.0076	95038	454	7	0

Table III. Results for interpolating the total, fission, capture, and elastic scattering cross sections over the temperature interval of 519 K to 547 K for ^{235}U

Total						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 74549)			
			≤ 0.1%	0.1% - 0.2%	0.2% - 0.5%	≥ 0.5%
Lin-lin	0.094	0.0096	74549	0	0	0
Log-log	0.087	0.0063	74549	0	0	0
Sqrt-lin	0.082	0.0076	74549	0	0	0
Lin-log	0.112	0.0101	74546	3	0	0
Sqrt-log	0.100	0.0081	74549	0	0	0
Fission						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 74549)			
			≤ 0.1%	0.1% - 0.2%	0.2% - 0.5%	≥ 0.5%
Lin-lin	0.129	0.0156	74529	20	0	0
Log-log	0.137	0.0109	74537	12	0	0
Sqrt-lin	0.108	0.0123	74543	6	0	0
Lin-log	0.178	0.0175	74387	162	0	0
Sqrt-log	0.158	0.0141	74501	48	0	0
Capture						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 74549)			
			≤ 0.1%	0.1% - 0.2%	0.2% - 0.5%	≥ 0.5%
Lin-lin	0.169	0.0184	74490	59	0	0
Log-log	0.164	0.0134	74508	41	0	0
Sqrt-lin	0.155	0.0146	74527	22	0	0
Lin-log	0.193	0.0212	74083	466	0	0
Sqrt-log	0.179	0.0172	74469	80	0	0
Elastic Scatter						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 74549)			
			≤ 0.1%	0.1% - 0.2%	0.2% - 0.5%	≥ 0.5%
Lin-lin	0.0747	0.0028	74549	0	0	0
Log-log	0.0673	0.0018	74549	0	0	0
Sqrt-lin	0.0694	0.0023	74549	0	0	0
Lin-log	0.0780	0.0029	74549	0	0	0
Sqrt-log	0.0727	0.0023	74549	0	0	0

Table IV. Results for interpolating the total, capture, and elastic scattering cross sections over the temperature interval of 519 K to 547 K for ^{90}Zr

Total						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 6647)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	0.349	0.0029	6646	0	1	0
Log-log	0.358	0.0026	6646	0	1	0
Sqrt-lin	0.357	0.0027	6646	0	1	0
Lin-log	0.342	0.0031	6646	0	1	0
Sqrt-log	0.350	0.0026	6646	0	1	0
Capture						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 6647)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	1.215	0.0140	6444	147	35	21
Log-log	1.058	0.0120	6513	109	5	20
Sqrt-lin	1.263	0.0151	6383	168	75	21
Lin-log	1.114	0.0138	6462	160	6	19
Sqrt-log	1.080	0.0124	6552	70	5	20
Elastic Scatter						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 6647)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	0.352	0.0026	6646	0	1	0
Log-log	0.361	0.0024	6646	0	1	0
Sqrt-lin	0.360	0.0024	6646	0	1	0
Lin-log	0.345	0.0029	6646	0	1	0
Sqrt-log	0.353	0.0023	6646	0	1	0

Table V. Results for interpolating the total, capture, and elastic scattering cross sections over the temperature interval of 478 K to 589 K for ^{16}O

Total						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 2785)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	0.1352	0.0072	2655	130	0	0
Log-log	0.0787	0.0024	2785	0	0	0
Sqrt-lin	0.0480	0.0013	2785	0	0	0
Lin-log	0.2705	0.0146	2616	39	130	0
Sqrt-log	0.1350	0.0068	2669	116	0	0
Capture						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 2785)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	0.0421	< 0.0001	2785	0	0	0
Log-log	0.0443	0.0001	2785	0	0	0
Sqrt-lin	0.0432	< 0.0001	2785	0	0	0
Lin-log	0.0421	0.0001	2785	0	0	0
Sqrt-log	0.0432	0.0001	2785	0	0	0
Elastic Scatter						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 2785)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	0.1352	0.0072	2655	130	0	0
Log-log	0.0787	0.0024	2785	0	0	0
Sqrt-lin	0.0480	0.0013	2785	0	0	0
Lin-log	0.2706	0.0146	2616	39	130	0
Sqrt-log	0.1351	0.0068	2669	116	0	0

Table VI. Results for interpolating the total, capture, and elastic scattering cross sections over the temperature interval of 478 K to 589 K for ^{10}B

Total						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 1035)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	0.0001	< 0.0001	1035	0	0	0
Log-log	0.0001	< 0.0001	1035	0	0	0
Sqrt-lin	0.0001	< 0.0001	1035	0	0	0
Lin-log	0.0001	< 0.0001	1035	0	0	0
Sqrt-log	0.0001	< 0.0001	1035	0	0	0
Capture						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 1035)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	0.0001	< 0.0001	1035	0	0	0
Log-log	0.0002	0.0001	1035	0	0	0
Sqrt-lin	0.0001	< 0.0001	1035	0	0	0
Lin-log	0.0002	0.0001	1035	0	0	0
Sqrt-log	0.0002	0.0001	1035	0	0	0
Elastic Scatter						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 1035)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	0.1369	0.0218	889	146	0	0
Log-log	0.0754	0.0079	1035	0	0	0
Sqrt-lin	0.0415	0.0042	1035	0	0	0
Lin-log	0.2702	0.0435	852	38	145	0
Sqrt-log	0.1357	0.0209	902	133	0	0

Table VII. Results for interpolating the total, capture, and elastic scattering cross sections over the temperature interval of 478 K to 589 K for ^1H

Total						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 686)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	0.1342	0.0462	471	215	0	0
Log-log	0.0752	0.0121	686	0	0	0
Sqrt-lin	0.0413	0.0062	686	0	0	0
Lin-log	0.2671	0.0928	432	39	215	0
Sqrt-log	0.1328	0.0443	485	201	0	0
Capture						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 686)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	0.0001	< 0.0001	686	0	0	0
Log-log	0.0001	< 0.0001	686	0	0	0
Sqrt-lin	0.0001	< 0.0001	686	0	0	0
Lin-log	0.0001	< 0.0001	686	0	0	0
Sqrt-log	0.0001	< 0.0001	686	0	0	0
Elastic Scatter						
Method	Max. Rel. Error (%)	Avg. Rel. Error (%)	Number of point-wise cross sections with Rel. Errors of (total = 686)			
			$\leq 0.1\%$	0.1% - 0.2%	0.2% - 0.5%	$\geq 0.5\%$
Lin-lin	0.1356	0.0467	470	216	0	0
Log-log	0.0755	0.0117	686	0	0	0
Sqrt-lin	0.0416	0.0063	686	0	0	0
Lin-log	0.2714	0.0942	431	39	216	0
Sqrt-log	0.1357	0.0452	482	204	0	0

Figure 1. Total microscopic cross section for ^{238}U Doppler broadened to 294 K.

Figure 2. Total microscopic cross section for ^{235}U Doppler broadened to 294 K.

Figure 3. Total microscopic cross section for ^{90}Zr Doppler broadened to 294 K.

Figure 4. Total microscopic cross section for ^{16}O Doppler broadened to 294 K.

Figure 5. Total microscopic cross section for ^{10}B Doppler broadened to 294 K.

Figure 6. Total microscopic cross section for ^1H Doppler broadened to 294 K.