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EXPERIMENTAL INVESTIGATION OF THE STRUCTURAL SCATTERING DUE TO IMPEDANCE DISCONTINUITIES ON A CYLINDRICAL SHELL

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ABSTRACT

Experimental, numerical, and analytical work has shown that the response of a shell to a distributed force wave possesses unique characteristics which are dependent on the nature of structure attached to the shell. Specific characteristics which influence the response are the distribution of the discontinuities around the circumference (periodic/a-periodic), the impedance of the discontinuities relative to that of the shell, and the type of impedance (mass or stiffness). An experimental investigation into the harmonic scattering behavior of a shell due to mass discontinuities is presented in this paper. Knowledge of the key structural characteristics which influence scattering and their behavior will allow for a diagnostic tool when assessing the structural response of more complex cylindrical structures.

Changes in structural response on a mode number basis as a result of the change in mass discontinuities are explored. Spatial Fourier analysis of both the input force and output response is used to provide a measure of the structural response matrix on a mode-by-mode basis. A distributed input force is simulated by applying forces around the circumference of the shell, then subsequently weighting the phase and amplitude of the measured transfer function according to the input force wave desired. This technique was applied to a shell in its unmodified condition, with periodic masses attached, and with a-periodic masses attached. Test results indicate that some harmonic scattering is present in the clean structure (no attached mass), likely due to non-ideal boundary conditions and the weld seam of the pipe used. Test results for the structure with periodically distributed masses indicate that response harmonics equal to the sum of input the harmonic number and an integer multiple of the number of discontinuities were excited. The structure with a-periodically distributed masses appears to scatter energy from the force harmonic into all response harmonics.

INTRODUCTION

Traditional shell theory predicts structural mode shapes of a shell with a sinusoidal distribution of displacement amplitudes. Due to the orthogonal nature of these mode shapes, the response of the structure to a traveling force wave with sinusoidal content at a given harmonic is due solely to the response of the mode shape with harmonic content of the same order. Structural complexities in many real cylindrical structures lead to the response of modes at harmonics other than the input harmonic. Structural scattering as discussed in this paper refers to the phenomena in which a force wave at a given harmonic scatters into the response of modes with different harmonics.

Figure 1 shows direction of the axial (u), circumferential (v), and radial (w) deformations at the midsurface of the shell at radius R and angular location θ . Also shown in Figure 1 is the shape of the mode containing the fourth harmonic ($n_r = 4$).

Figure 1: Cylindrical Shell Coordinate System, Deformation Directions, and $n_r = 4$ Mode Shape

An approximation of the mode shapes of a shell with free-free boundary conditions and no axial variation (*i.e.* *axial harmonic* = 0) is given by¹:

$$\begin{aligned} u(\theta, \omega) &= 0 \\ v(\theta, \omega) &= -\frac{D}{n_r} \cdot \sin(n_r \theta) \cdot \cos(\omega t) \\ w(\theta, \omega) &= D \cdot \cos(n_r \theta) \cdot \cos(\omega t) \end{aligned} \quad (1)$$

where $n_r = 2, 3, 4, \dots$

This equation is valid for non-breathing, non beam bending modes. For these modes, the radial and circumferential deformations are proportional. The analyses presented in this paper focus on the response of these low-order, radial-dominated modes in the radial direction to a force in the same direction. However, significant energy could be present in the breathing ($n_r = 0$) and the beam-bending modes ($n_r = 1$) due to scattering. In this study, only the coupling between radial forces and radial modes is assessed. Direct and indirect coupling between modes and forces in other directions may have a significant influence on the scattering characteristics of the shell.

Introduction of mass discontinuities to the shell yield complex mode shapes, which may be characterized by the summation of several harmonic components. These modes are no longer orthogonal in the presence of discontinuities, yielding harmonic content across various modal orders. As a result, a purely sinusoidal forcing function can excite several modes of the structure.

Finite element analyses of cylindrical structures have shown that the response of a structure with Z periodic impedance discontinuities will contain harmonic orders n_r when excited by a force of harmonic order n_f according to the following equation²:

$$n_r = n_f \pm kZ, \quad (2)$$

where $k = 1, 2, 3, \dots$

Finite element analyses also have shown that when a-periodic discontinuities are present in the cylindrical structure, modes of all harmonic orders are excited by a given force wave. This paper describes an experimental demonstration of the response characteristics of a cylindrical shell with periodic and a-periodic mass discontinuities to a harmonic force wave.

NOMENCLATURE

u	Axial component of shell displacement
v	Circumferential component of shell displacement
w	Radial component of shell displacement
R	Radius of shell midsurface
n	Spatial harmonic number
θ	Angular location
n_r	Harmonic order of radial shell response
ω	Circular frequency
D	Amplitude of radial deformation

t	Time
n_f	Harmonic order of radial force wave
k	Scattering harmonic multiplier
Z	Number of periodic impedance discontinuities
θ_s	Angular location of sensor s
θ_r	Angular location of force input r
H_{θ_r, θ_s}	Frequency response function of sensor at θ_s to a force input at θ_r
f	Frequency
A_{θ_s}	Frequency spectrum of acceleration at θ_s
F_{θ_r}	Frequency spectrum of force at θ_r
$H(a, b, c)$	Frequency response as a function of parameters a, b , and c
N	Maximum measurable response harmonic
A_{tw}	Traveling wave transformation matrix
A_{sw}	Standing wave transformation matrix
A	Angular orientation of periodic mass distribution

EXPERIMENTAL SETUP

A section of commercially available pipe was used for all of the experiments described in this paper. The pipe section used was a 24" long, 30" nominal O.D., 1" nominal wall thickness welded carbon steel pipe. To minimize the scattering effects of the weld seam, the pipe was machined to ensure a constant wall thickness and circular inner and outer diameters. The variation in wall thickness after machining has not yet been confirmed.

In an attempt to simulate free-free boundary conditions, the pipe was supported with its axial dimension vertical by four pressurized rubber air springs, positioned at the 45°, 135°, 225°, and 315° angular locations. The four rubber mounts were inflated with air to approximately 20psi (138kPa). At this pressure, the rigid body response of the shell resonates below 10Hz.

The primary focus of the experiments presented in this paper is on the response of low-order shell modes. The responses of the modes of interest are dominated by the radial component of the shell wall motion. Therefore, an array of radially oriented accelerometers was placed around the circumference of the shell, halfway along the axial length. The accelerometers were equally spaced in increments of 10°, resulting in an array of 36 radially-oriented accelerometers. The test was designed to allow for observation of the scattering behavior between low and high order harmonics. The sensor array chosen may resolve from unaliased response from $n_r = -17$ to $n_r = +18$. The number of accelerometers used was limited by the ability to attach test masses while maintaining uniform sensor distribution. Accelerometers were located by first measuring the outer circumference of the shell with a pi tape to an accuracy of 1×10^{-3} in. A section of magnet was then measured and cut to the necessary accelerometer spacing. The accuracy of the sensor placement is estimated to be within ± 0.005 in (1.27×10^{-4} m). Figure 2 shows the experimental setup with the four mounts supporting the cylinder vertically and the accelerometers mounted radially.

Figure 2: Side View of Experimental Setup with Attached Structure at 295° and 55° (left), and Top View of Experimental Setup with No Attached Structure (right)

Point forces were applied to the inner circumference of the shell in the radial direction with an instrumented electronic impact hammer. The responses of the 36 accelerometers and the impulse force from the hammer were acquired simultaneously in the frequency domain. Data from 15 impacts at each location were averaged to yield a data set of 36 Frequency Response Functions (FRFs), referenced to the single force input. This data was taken for force inputs at each of the 36 positions corresponding to the 36 accelerometer locations. Each of the FRFs are computed in the form:

$$H_{\theta_r, \theta_s}(f) = \frac{A_{\theta_s}^*(f) \cdot F_{\theta_r}(f)}{F_{\theta_r}^*(f) \cdot F_{\theta_r}(f)} \quad (3)$$

DATA PROCESSING

FRF data as described in equation (3) was assembled into a three-dimensional matrix of the following form:

$$H(f, \theta_r, \theta_s) = \begin{bmatrix} H_{\theta_1, \theta_1}(f) & H_{\theta_1, \theta_2}(f) & \dots & H_{\theta_1, \theta_{36}}(f) \\ H_{\theta_2, \theta_1}(f) & H_{\theta_2, \theta_2}(f) & \dots & H_{\theta_2, \theta_{36}}(f) \\ \vdots & \vdots & \ddots & \vdots \\ H_{\theta_{36}, \theta_1}(f) & H_{\theta_{36}, \theta_2}(f) & \dots & H_{\theta_{36}, \theta_{36}}(f) \end{bmatrix} \quad (4)$$

The data in this array are a function of frequency, spatial location of the response, and spatial location of the force input. Fourier analysis was applied to the two spatial dimensions of the matrix to yield a data set as a function of frequency, the spatial harmonic of the response, and the spatial harmonic of the force input.

The response of the shell may be represented either as the summation of a series of orthogonal standing waves, or as the summation of a series of waves traveling in opposite directions. The data may be expressed in terms of traveling wave components as follows:

$$H(f, \theta_r, \theta_s) = \frac{1}{2N} \sum_{n=0}^N [H(f, \theta_r, n) \cdot e^{-jn\theta} + H(f, \theta_r, -n) \cdot e^{+jn\theta}] \quad (5)$$

where N is the maximum measurable harmonic. This may be rearranged in matrix form as follows:

$$H(f, \theta_r, \theta_s) = H(f, \theta_r, n) \cdot A_{tw}, \quad (6)$$

where A_{tw} is the traveling wave transformation matrix:

$$A_{tw} = \frac{1}{2N} \begin{bmatrix} e^{-j(N-1)\theta_1} & e^{-j(N-1)\theta_2} & \dots & e^{-j(N-1)\theta_{36}} \\ e^{-j(N-2)\theta_1} & e^{-j(N-2)\theta_2} & \dots & e^{-j(N-2)\theta_{36}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{+j(N-1)\theta_1} & e^{+j(N-1)\theta_2} & \dots & e^{+j(N-1)\theta_{36}} \\ e^{+jN\theta_1} & e^{+jN\theta_2} & \dots & e^{+jN\theta_{36}} \end{bmatrix} \quad (7)$$

To compute the spatial harmonic content of the response in terms of standing waves, the standing wave transformation matrix shown in equation (8) may be used.

$$A_{sw} = \begin{bmatrix} \sin\{(N-1)\theta_1\} & \sin\{(N-1)\theta_2\} & \dots & \sin\{(N-1)\theta_{36}\} \\ \sin\{(N-2)\theta_1\} & \sin\{(N-2)\theta_2\} & \dots & \sin\{(N-2)\theta_{36}\} \\ \vdots & \vdots & \ddots & \vdots \\ \cos\{(N-1)\theta_1\} & \cos\{(N-1)\theta_2\} & \dots & \cos\{(N-1)\theta_{36}\} \\ \cos\{(N)\theta_1\} & \cos\{(N)\theta_2\} & \dots & \cos\{(N)\theta_{36}\} \end{bmatrix} \quad (8)$$

To transform the force input dimension to the spatial domain, the complex conjugate-transpose of the matrices in equations (7) and (8) are used. Equation (9) demonstrates the application of the traveling wave matrix to obtain data as a function of frequency, traveling wave response harmonic, and traveling wave force harmonic.

$$H(f, n_f, n_r) = (A_{tw}^{-1})^* \cdot H(f, \theta_r, \theta_s) \cdot A_{tw}^{-1} \quad (9)$$

Other forms of the data may be obtained such as standing wave response harmonic as a function of traveling wave input by applying the appropriate transformation matrices to the data set. In the analysis presented in this paper, the response of standing wave harmonics were found to traveling force waves by replacing A_{tw}^{-1} with A_{sw}^{-1} .

The results were not found to be sensitive to the analysis approach. However, distinctive characteristics of a structure with scattering mechanisms present may be more easily when analyzing data with both analysis methods.

RESULTS

Effect of Adding Mass Discontinuities

The response of an ideal shell may be characterized by two orthogonal modes at the same frequency. These modes have no preferred orientation relative to the angular coordinate of the shell. The response of these modes will be positioned with the same orientation as the input force wave. Therefore, the response of an ideal shell to a traveling wave is a traveling wave of the same harmonic.

When periodic masses were added to the structure, it was found that the mode at the response harmonic equal to the number of masses was split into two distinct modes at different

frequencies. These two modes have preferred orientations relative to the distribution of attached structure. These orientations are such that the attached masses are either at the node or anti-node of the mode. When attached masses were added, the resonant frequency of the mode oriented with its anti-nodes aligned with the masses is found to be lower than the frequency of its corresponding orthogonal mode. This is expected as the modal natural frequency is proportional to the square root of the modal stiffness divided by the modal mass.

The experimentally measured response of the shell with 3 periodic masses to a $n_f = +3$ traveling wave was decomposed into $\sin(3\theta - \alpha)$ and $\cos(3\theta - \alpha)$ components; α represents the angular orientation of the periodic mass distribution. Figure 3 shows that the spectral shape of each of these responses resembles that of a single degree of freedom system, indicating that the two resonance frequencies correspond to modes with different preferred orientations. At resonance frequencies, the response of the shell is dominated by a standing wave field, despite the traveling wave input. Away from resonance frequencies, the amplitude of the standing wave responses is nearly equal, indicating predominantly traveling wave response.

Figure 3: Response of the $n_r = 3$ Standing Wave Components to a $n_f = +3$ Traveling Force Wave with 3 Periodic Masses

Standing vs. Traveling Wave Response

The tendency of the cylinder response harmonics to take preferred orientations causes standing wave response at the harmonic corresponding to the number of periodically attached structures. When the data is decomposed in the traveling wave coordinates, a standing wave is represented when the magnitude of the positive and negative traveling waves are equal. Figure 4 shows the experimentally measured response of the $n_r = \pm 2$ to a traveling force wave at $n_f = +2$. At the resonance frequency of the mode at harmonic $n = 2$, there is little difference (0.7dB) in magnitude between the positive and negative traveling waves, indicating a standing wave pattern at this frequency. At frequencies away from the resonant frequencies, there is a large difference in magnitude between the traveling wave components, indicating a primarily traveling wave pattern at these frequencies. The presence of standing wave patterns at some resonant frequencies may cause the

response of modes at harmonics other than those described by the relation in equation (2).

Figure 4: Response of the $n_r = \pm 2$ Components to a $n_f = +2$ Traveling Force Wave

Shell Response without Attached Structure

To allow for an assessment of the relative change in the scattered response, a set of measurements was first performed on the shell in its un-modified condition. Figure 5 shows the average amplitude of each response harmonic over the entire measurement frequency range (0-3200Hz) to an input force wave at the $n = +2$ harmonic. The response is dominated by the spatial harmonic of the same order as the input harmonic. In this representation, no significant scattering trends are seen in the data.

Figure 5: Average Frequency Response Magnitude ($f = 0-3200\text{Hz}$) of Response Harmonics -15 Through 15 for a Traveling Force at $n_f = +2$ ($e^{j2\theta}$)

Figure 6 demonstrates the frequency dependant response of each of the harmonics to the same traveling wave at $n_f = +2$. Data presented in this and following similar figures was processed at a frequency resolution of 10Hz in order to demonstrate the energy modes with split resonance frequencies. As expected, the response is dominated across the majority of the frequency range by the harmonic equal to that of the force input. However, at the resonance frequencies of the shell,

increased response is seen at the response harmonic corresponding to the mode at that frequency. Additionally, there appears to be increased response across the frequency band in the $n_r = -2$ harmonic. At the resonance frequency of the $n = 2$ mode, the magnitude of the frequency response of the negative traveling wave is approximately equal to the magnitude of the positive traveling wave (Figure 4). At all other frequencies, the primary response ($n_r = +2$) is larger than the back scattered response ($n_r = -2$). These unexpected features may be explained by scattering mechanisms present in the system in its un-modified form. Had the material of the structure been homogeneous and the boundary conditions were ideal, the response of n_r for all mode orders other than the forcing harmonic would have been expected to be 0. For example, the mounts may provide some stiffness in the radial direction, yielding a periodic boundary condition at one end of the shell. Additionally, the weld seam of the pipe may contribute to the scattering characteristics.

Figure 6: Frequency Response Magnitude for a Traveling Force at $n = +2$

Shell Response with Periodically Attached Masses

Three masses, each weighing 17.6 ± 0.2 lbs (7.7 ± 0.09 kg), were placed periodically around the outer circumference of the shell at the 55° , 175° , and 295° locations. The total mass added to the shell is approximately 10% of the total shell mass. This yields an acceleration of each mass of 82dB (~ 28 less than direct response of clean shell at resonance). The mass discontinuities were made out of spare pieces steel weighing approximately 8.8lbs (4kg). The use of a single piece yielded poor test results; therefore 17.6lb masses were used. Figure 7 shows the average Frequency Response magnitude of each response harmonic over the entire measurement frequency range where a distinct pattern resulting from the periodically attached masses may be seen. The response harmonic with the largest magnitude is still equal to the input force wave harmonic. However, a pattern of increased response at intervals of integral multiples of 3 from the primary response may be seen at approximately 15 to 20 dB below the primary response. This result is predicted by equation (2).

Figure 7: Average Frequency Response Magnitude ($f=0-3200$ Hz) of Response Harmonics -15 Through 15 for a Traveling Force at $n_f = +2$ with 3 Periodic Masses

Figure 8 shows the frequency dependant response of each response harmonic to the traveling wave force excitation. Similar to the results shown in Figure 6, the response is dominated across the majority of the frequency band by the $n_r = +2$ harmonic. The frequency range over which the unexpected responses are elevated appear to be broader than seen in the case with no attached mass. Also seen in this figure is a relative increase in the broadband response of modes satisfying equation (2). For example, relative to the un-modified shell, increased response is seen in the $n_r = -1$ harmonic.

Figure 8: Frequency Response Magnitude for a Traveling Force at $n_f = +2$ with 3 Periodic Masses

To better demonstrate the effects of the addition of the periodic masses to the shell, a measure of scattering is obtained by comparing the results from the periodic case to those of the perfect shell. For the purposes of this analysis, the level of scattering is defined as the difference between the response of harmonic n_{r1} to an input force at n_f of the same harmonic and the response of another harmonic n_{r2} to the same input force. Since some scattering was present in the case with no impedance discontinuities, the difference in scattering between cases is found. Figure 9 shows the difference in scattering

between the un-modified shell and the shell with three periodic masses for a for input of $n_f = 2$. As seen in this figure, the periodically attached masses yield a distinct pattern in the spatial harmonic spectrum. The change in scattering at the excitation harmonic is shown to be nearly 0. However, the change at other harmonics is amplified by adding the periodic masses. It should be noted that as a result of the scattering from the periodically attached masses, responses in the direction opposite to that of the traveling force wave are at times larger than those in the same direction as the force wave.

Figure 9: Change in Scattering between Unmodified Shell and Shell with 3 Periodic Masses for a Force at $n_f = +2$

Figure 10 shows the difference in scattering between the unmodified shell and the shell with three periodic masses attached for a force input of $n_f = -4$. Again, strong increases in the scattering behavior are demonstrated at response harmonics consistent with equation (2).

Figure 10: Change in Scattering between the Unmodified Shell and Shell with 3 Periodic Masses for a Force at $n_f = -4$

Shell Response with A-periodically Attached Masses

Two 17lb masses were attached to the shell and were positioned at the 215° and 295° angular locations. Figure 11 shows the average FRF magnitude over the measurement frequency range to a traveling wave force at $n_f = +2$. Data

presented in this figure demonstrates that the addition of a-periodically spaced masses yields an increased response in a broad range of response harmonics. The highest scattered response is approximately 20dB below the primary response. In this figure, responses at $n_r = +9, -9, -4$, and 0 appear to be lower than other harmonics. The relationship between the spacing of the discontinuities and these response harmonics has not been investigated.

Figure 11: Average Frequency Response Magnitude ($f = 0-3200\text{Hz}$) to a Traveling Force at $n_f = +2$ with 2 A-Periodic Masses

Figure 12 shows the frequency-dependant nature of the response of the shell with a-periodic masses to a $n_f = +2$ traveling wave force. Relative to the case with periodically attached structure (Figure 8), increased response in all spatial harmonics is seen. Additionally, the majority of the scattering appears to occur at resonance frequencies. In the periodic case, significant scattering away from resonance frequencies is observed.

Figure 12: Frequency Response Magnitude to a Traveling Force at $n_f = +2$ with 2 A-periodic Masses

SUMMARY AND CONCLUSIONS

The harmonic scattering characteristics of a cylindrical shell with mass discontinuities was demonstrated with experimental data. The response of the cylindrical structure with periodically attached masses was shown to have unique identifying characteristics. Scattered response was observed at harmonics according to the relationship given in equation (2). The response of the cylindrical structure with a-periodically attached masses was shown to increase the response across a broad range of harmonics. The scattered response of the a-periodic case appeared to occur primarily at shell resonance frequencies while the scattering for the periodic case appeared to occur over a broad frequency range. Additionally, the response characteristics of a nearly ideal cylinder were shown to contain strong evidence of harmonic scattering, likely due to

the boundary conditions and weld seam of the pipe. If this, or similar experiments were continued, the differences in scattering characteristics between stiffness and mass dominated impedance discontinuities should be investigated. A cast test article and better mounting could yield clean shell results closer to those expected of an ideal shell..

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