Azmithual anisotropy at RHIC: the first and fourth harmonics

Anisotropic flow, an anisotropy of the particle azimuthal distribution in momentum space with respect to the reaction plane, is a sensitive tool in the quest for the quark-gluon plasma and the understanding of bulk properties of the system created in ultrarelativistic nuclear collisions [1]. It is commonly studied by measuring the Fourier harmonics ($v_n$) of this distribution [2]. Elliptic flow, $v_2$, is well studied at RHIC [3–5] and is thought to reflect conditions from the early time of the collision. Directed flow, $v_1$, was discovered almost 20 years ago [6] and has been extensively studied and reviewed at lower beam energies [7]. At RHIC energies directed flow in
the central rapidity region reflects important features of the system evolution from its initial conditions. \( v_1 \) is predicted to be small near midrapidity with almost no dependence on pseudorapidity. However, it could exhibit a characteristic "wiggle" [8], depending on the baryon stopping and production mechanisms as well as strong space-momentum correlations in the system's evolution. A similar rapidity dependence of directed flow could develop due to a change in the matter compressibility if a quark-gluon plasma is formed [9, 10]. It results in the so-called third flow component [9] or "anti flow" [10] component in the expansion of the matter. This expansion direction is opposite to the normal directed flow. \( v_1 \) has not previously been reported at RHIC.

The importance of the higher harmonics in understanding the initial configuration and the system evolution has been emphasized [11]. Recently, Kolb [12] reported that the magnitude and even the sign of \( v_4 \) are more sensitive than \( v_2 \) to initial conditions in the hydrodynamic calculations. Those higher harmonics reflect the details of the initial configuration geometry. Besides one early measurement at the AGS [13], reports of higher harmonics have not previously been published.

**Experiment**—The data come from the reaction Au + Au at \( \sqrt{s_{NN}} = 200 \) GeV. The STAR detector [14] main time projection chamber (TPC [15]) and two forward TPCs (FTPC [16]) were used in the analysis. For the higher harmonics 2 million events in the main TPC were analyzed. For the first harmonic analysis there were 70 thousand events available which included the FTPCs.

In this analysis the main TPC covered pseudorapidity (\( \eta \)) from 1.0 to 1.8, while two FTPCs covered 4.2 to 4.4 and 4.1 to 4.2. The low transverse momentum (\( p_t \)) cutoff was 0.15 GeV/c. In the present work all charged particles were analyzed, regardless of their particle type. The centrality definition in this paper is the same as used previously by STAR [17]. The errors presented in the figures are statistical.

**Analysis**—The difficulties in studying directed flow are that the signal is small and the non-flow contribution to the two-particle azimuthal correlations can be comparable or even larger than the correlations due to flow. To suppress the non-flow effects the current analysis uses the knowledge about the reaction plane derived from the large elliptic flow. One method for eliminating the non-flow contribution in a case when the reaction plane is known was proposed in [2]. It was noted that while the correlations of the components of the (first harmonic) flow vectors in the reaction plane contain both flow and non-flow contributions, the correlations of the components perpendicular to the reaction plane contain only non-flow contributions. Then the difference yields the flow contribution. Correlating the azimuthal angles of two particles (\( \phi_a, \phi_b \)), and using the event plane determined by elliptic flow (\( \Psi_2 \)) one gets:

\[
\langle \cos(\phi_a - \Psi_2) \cos(\phi_b - \Psi_2) \rangle = \langle \cos(\phi_a + \phi_b - 2\Psi_2) \rangle \approx v_{1,a} v_{1,b} \langle \cos(2(\Psi_2 - \Psi_{RP})) \rangle,
\]

where \( \Psi_{RP} \) is the azimuthal angle of the reaction plane. If only one particle is used to determine the second harmonic event plane this expression reduces to

\[
\langle \cos(\phi_a + \phi_b - 2\phi_2) \rangle = \langle \cos(4\phi - 4\Psi_2) \rangle = v_4 N/2,
\]

where \( N \) is the total number of particles used to determine the second harmonic event plane. This expression should be compared to Eq. (2). Results obtained with this method we designate by \( v_4 \{EP_2\} \). The analysis for \( v_4 \) was also done with three-particle cumulants [20] by measuring \( \langle \cos(2\phi_a + 2\phi_b - 4\phi_2) \rangle \).

**\( v_1 \) results**—Fig. 1 shows the results in comparison to the lower beam energy data of NA49 [21]. The NA40 data are also replotted so as to be at the same distance from beam rapidity [24] as the STAR results. The RHIC \( v_1(\eta) \) results differ greatly from the unshifted SPS data in that they are flat near midrapidity and only become significant at the highest rapidities measured. However, when plotted in the projectile frame relative to their respective beam rapidities, they look similar. It should be noted that at the SPS energies of 40A GeV and 158A GeV [21], this \( y-y_{beam} \) scaling does not work, but \( y/y_{beam} \) scaling does. In the pseudorapidity region \( |\eta| < 1.2 \), \( v_1(\eta) \) is approximately flat with a slope of \( (-0.25 \pm 0.27(stat)) \)\% per unit of pseudorapidity, which is consistent with predictions [8–10].

Note that the sign of \( v_1 \) is undetermined because \( v_1 \) enters as the square in Eq. (2). We have plotted \( v_1 \) in the positive hemisphere going negative toward beam rapidity as it does at the lower beam energy. In the NA49 analysis [21] the sign of \( v_1 \) had been determined by defining \( v_1 \) for protons near beam rapidity to be positive for peripheral collisions. On the other hand, since the measured correlation of Eq. (2) is positive, we can conclude that
we have measured the sign of $v_2$ to be positive. While the absolute values of $v_2$ at RHIC are well determined [3-5] this is the first direct indication that the elliptic flow at RHIC is in-plane.

$v_4$ results— The results as a function of $p_t$ are shown in Fig. 2 for minimum bias collisions (0—80% centrality). Shown for $v_4$ are both the analysis relative to the second harmonic event plane, $v_4\{EP_2\}$, and the three-particle cumulant, $v_4\{3\}$. Both methods determine the sign of $v_4$ to be positive. As a function of $p_t$, $v_4$ rises more slowly from the origin than $v_2$, but does flatten out at high $p_t$ like $v_2$. The $v_0(p_t)$ values are consistent with zero. The hydrodynamic calculations of Kolb [12] for pions from $b = 7$ fm collisions agree very well with our measured $v_4$ for charged particles for centrality 20 to 30%. However, he calculates $v_0$ to be $-1.2\%$ at 2 GeV/c, while we observe in Fig. 2 for minimum bias data that it is essentially zero. It also appears to be zero in our data for all the individual centralities. Ollitrault has proposed [22] for the higher harmonics that $v_0$ might be proportional to $v_2^\eta/2$ if the $\phi$ distribution is a smooth, slowly varying function of $\cos(2\phi)$. In order to test the applicability of this scaling we also plotted $v_0^2$ and $v_0^3$ in the figure as dashed lines. The proportionality constant has been taken to be 1.2 in order to fit the $v_4$ data.

Kolb [12] points out that for $v_2 > 10\%$, which occurs at high $p_t$, and no other harmonics, the azimuthal distribution is not elliptic, but becomes “peanut” shaped. He calculates the amount of $v_4$ (which looks like a four-leaf clover) needed to eliminate this waist. Our values of $v_4$ as a function of $p_t$ are about a factor of two larger than needed to just eliminate this waist.

The results for $v_4$ as a function of pseudorapidity are approximately flat in the acceptance of the main TPC ($|\eta|<1.2$) with an average value of $(0.44 \pm 0.02)\%$. However, in the FTPCs ($2.7<|\eta|<4.0$) the average value is $(0.06 \pm 0.07)\%$, consistent with zero, with a two sigma upper limit of 0.2%. Consistent with the first observation by PHOBOS [3], at $\eta = 3$ for minimum bias collisions we observe $v_2 = (3.66 \pm 0.10)\%$, which is a factor 1.8 smaller than at midrapidity. Thus $v_4$ seems to fall off faster at high rapidity than $v_2$. This faster fall off at high pseudorapidity is also consistent with $v_4$ scaling like $v_2^3$.

Fig. 3 shows the centrality dependence for $p_t$-integrated $v_2$, $v_4$, and $v_6$ with respect to the second harmonic event plane and also $v_4$ from three-particle cumulants ($v_4\{3\}$). The five-particle cumulant, $v_4\{5\}$, (not shown in the figure) is consistent with both methods but the error bars are about two times larger. The $v_0$ values are close to zero for all centralities. These results are averaged over $p_t$, thus reflecting mainly the low $p_t$ region where the yield is large, and also averaged over $\eta$ for the midrapidity region accessible to the STAR TPC ($|\eta|<1.2$). To again test the applicability of $v_0^\eta/2$ scaling we have also plotted $v_2^3$ and $v_2^3$ in the figure as dotted histograms. The proportionality constant has been taken to be 1.4 to approximately fit the $v_4$ data. The larger constant here compared to that used in Fig. 2 is understood as coming from the use of the square of the
average instead of the average of the square, and because the integrated values yield weight low $p_\perp$ more, where the best factor is slightly larger. The $v_n\{EP_2\}$ values averaged over $p_\perp$ and $\eta$ ($|\eta|<1.2$), and also centrality (minimum bias, 0 – 80%), are (in percent) $v_2 = 5.18 \pm 0.005$, $v_4 = 0.44 \pm 0.009$, $v_6 = 0.043 \pm 0.037$, and $v_8 = -0.06 \pm 0.14$. Since $v_8$ is essentially zero, we place a two sigma upper limit on $v_8$ of 0.1%. Also, $v_8$ is zero, but the error is larger because the sensitivity decreases as the harmonic order increases.

**Systematic uncertainties** — In both approaches, $v_n\{EP_2\}$ and $v_4\{EP_2\}$, the non-flow effects are suppressed compared to the case where the fourth harmonic event plane is used. The remaining non-flow correlations, along with event-by-event flow fluctuations, are thought to be the major contributors to the systematic uncertainties. Background from secondary particles is expected to be less than 15%, and remaining acceptance effects are measured to be very small. All errors and limits quoted so far are statistical, and should be increased by the systematic uncertainties below.

From non-flow effects we estimate the relative systematic uncertainty in $v_4\{3\}$ to be about 20%. The largest contribution comes from situations in Eq. (3) where one particle is correlated with one of the other particles due to non-flow, and with the third particle via flow. Our estimate is based on the assumption that the entire difference in the published values $[3]$ of $v_2\{EP_2\}$ and $v_2\{4\}$ is due to non-flow effects. Comparison of $v_4\{3\}$ to $v_4\{5\}$ leads to a similar estimate for this systematic error.

From non-flow effects we estimate the relative systematic uncertainty in $v_4\{3\}$ also to be about 20%. Our estimate is based on the assumption that our two-particle correlation value of $v_1$ using only the first harmonic event plane in the FTPCs, $v_1\{EP_1\}$, of about 3% is entirely due to non-flow effects.

The other effect important for the comparison of our results to theoretical calculations is event-by-event flow fluctuations. As was discussed $[3]$, flow measurements are done by two or many particle correlations, resulting in, not $\langle v_n\rangle$, but $\langle v_n^k\rangle^{1/k}$ If flow fluctuates event-by-event, it could lead to a difference between these two quantities. Fluctuations in the initial geometry of the collision at fixed impact parameter can account for the difference between $v_2\{EP_2\}$ and $v_2\{4\}$ $[3]$, and also between $v_4\{EP_4\}$ and $v_4\{3\}$ $[23]$. Although the flow fluctuation contribution to $v_4\{3\}$ is greatly reduced, it still could lead to an effect of about a factor of 1.2 to 1.5.

**Conclusions** — We have presented the first measurement of $v_1$ at RHIC energies. $v_1(\eta)$ is found to be approximately flat in the midrapidity region, which is consistent with microscopic transport models, as well as hydrodynamical models where the flatness is associated with the development of the expansion in the direction opposite to the normal directed flow. Within errors we do not observe a wiggle in $v_1(\eta)$ at midrapidity. The pseudorapidity dependence of $v_1$ in the projectile fragmentation region is very similar to that observed at full SPS energy. We observe a positive correlation between the first and second harmonics, indicating that elliptic flow is in-plane. This is the first direct measurement at RHIC of the orientation of elliptic flow relative to the reaction plane.

We have measured $v_4$ as a function of $p_\perp$, $\eta$, and centrality. We observe that $v_4$ appears to scale approximately as $v_2^2$, as a function of $p_\perp$, $\eta$, and centrality. $v_4$, although essentially zero, is not inconsistent with scaling as $v_2^2$. This is the first measurement of higher harmonics at RHIC and it is expected that these higher harmonics will be a sensitive test of the initial configuration of the system, since they provide a Fourier analysis of the shape in momentum space which can be related back to the initial shape in configuration space. In fact, it has been emphasized that $v_4$ has a stronger potential than $v_2$ to constrain model calculations and carries valuable information on the dynamical evolution of the system.

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[24] For the STAR data the beam rapidity was taken as 5.37 and for NA49 as 2.92 in the center-of-mass frame.

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