Summary

Duality is a non-trivial property of QCD that reflects the transition from the hadronic to the quark-gluon world

Spin/Flavor/Nuclear Dependence of Duality needs to be established experimentally

- Well on our way to map out for

  F_2, F_2^A, F_L, g_1

- Single quark scattering is, on average, all that matters
- Good (or bad?) news for application of factorization theorems?
- Promise to access (very) high x region

More experiments underway/planned

- Neutron
- Polarized Structure Functions
- Fragmentation
- Neutrino Scattering

Duality Violations obscure comparison with Lattice QCD through the Structure Function moments

Fragmentation

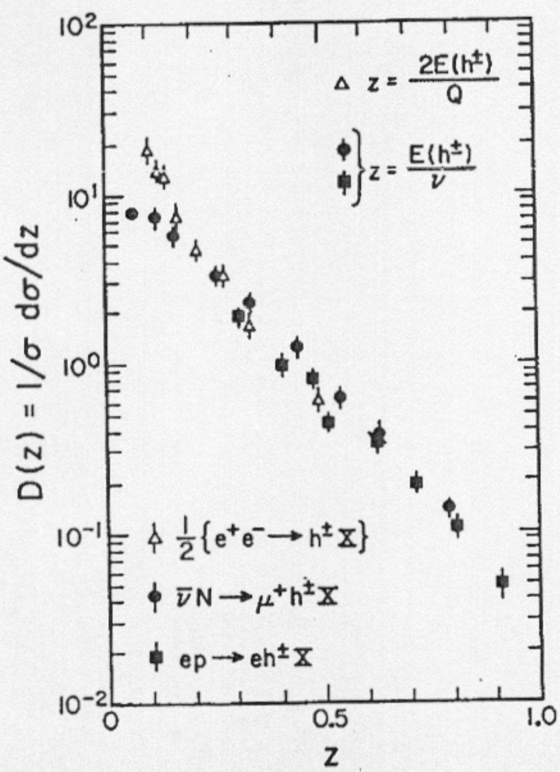
agmentation functions parametrize our lack of understanding of the **ɔn-perturbative hadron formation** process.

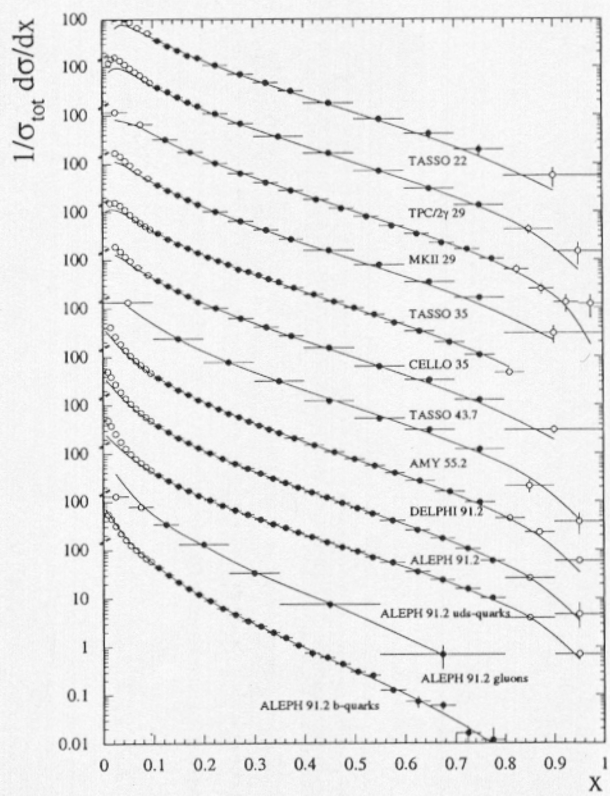
$$\frac{d\sigma}{dz}(ep \to hX) = \frac{\sum_q e_q^2 q(x) D_q^h(z)}{\sum_q e_q^2 q(x)} \quad \text{where } z \equiv E_h/E_q = E_h/\nu$$

'e should write: $\boxed{D(z, p_t, Q^2)}$ ... have observed these features:

- z-**scaling** : Fragmentation functions depend almost entirely on energy fraction $z = E_h/E_q$, with only a **weak logarithmic** Q^2**-dependence** (scaling violations) → z-scaling independent of process and energy.

$$Q^2 \frac{\delta}{\delta Q^2} D_i^h(x, Q^2) = \sum_j \int_x^1 \frac{dz}{z} P_{ji}(z, \alpha(Q^2)) D_j^h(x/z, Q^2)$$





- **limited** p_T : Transverse momentum of hadrons relative to q momentum is non-zero, but small ... typically $\langle p_T \rangle \approx 300$ MeV.

Scaling in Electropion Production

Calogeracos, Dombey, and West, Phys. Rev. D 51, 6075 (1995)

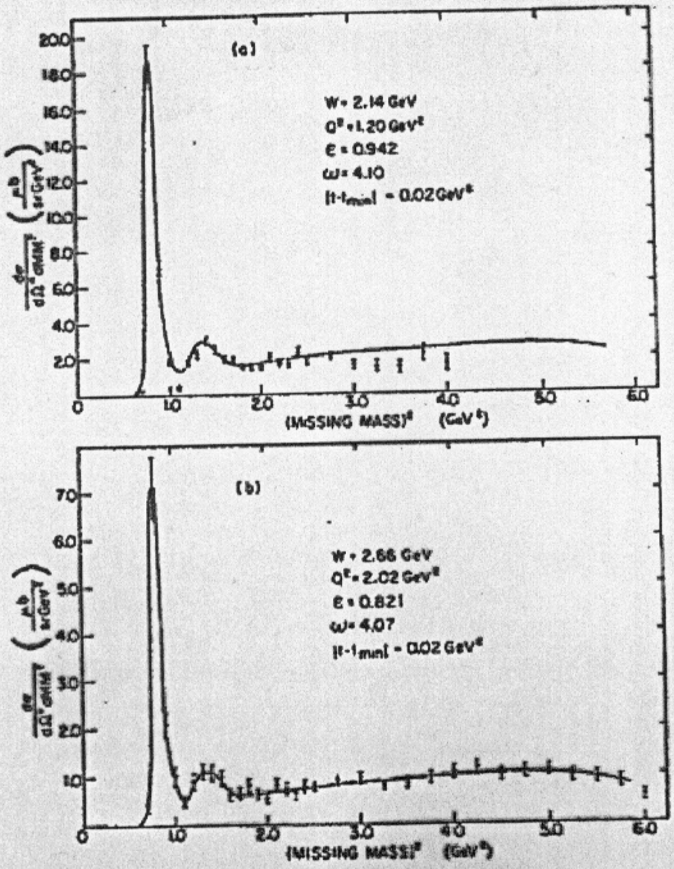
$$s^2 \frac{d^2\sigma}{dt\,dW'^2} = F(x,t,W^2)$$

Cornell : $x = 0.24$
$t = 0.02$
(end of '70's)

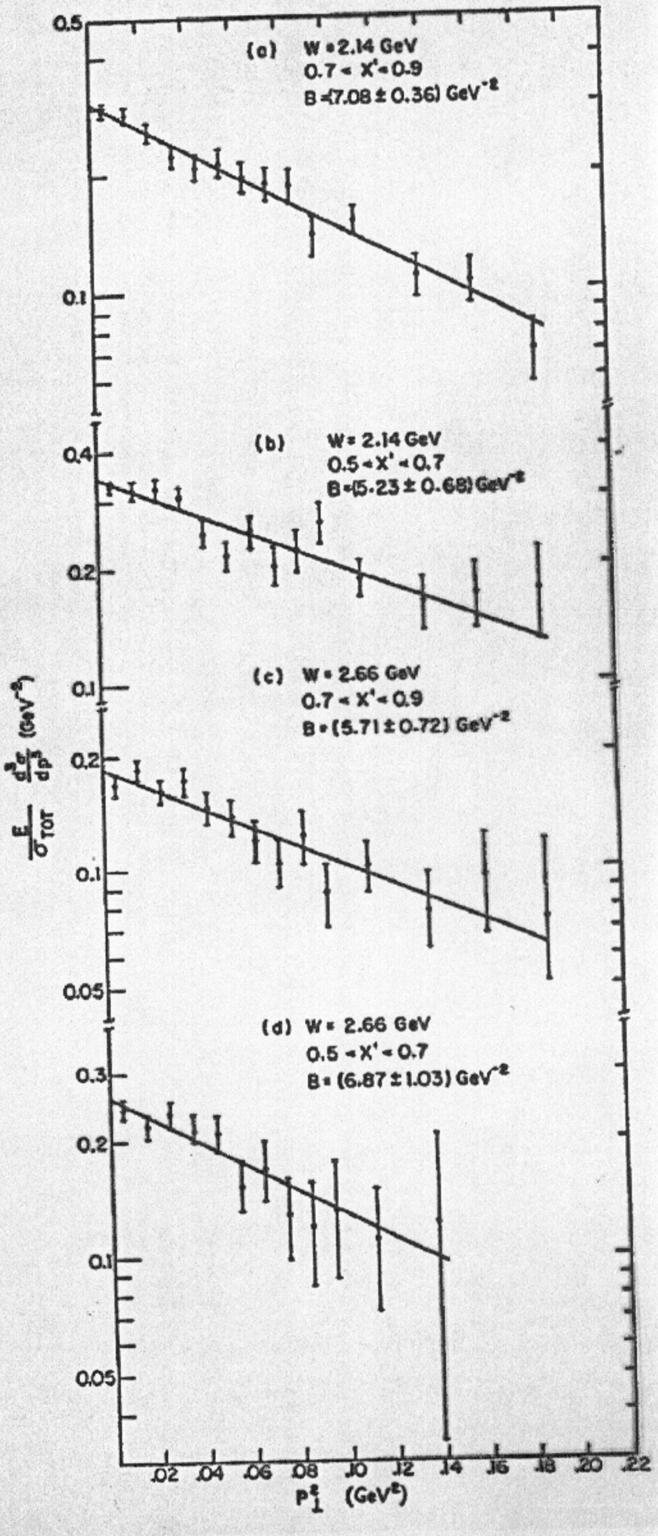
$\sqrt{s} = 2.66$ vs. 3.14

(i.e. Q^2 and W^2 different)

→ Scaling factor of 2.4



→ W'^2



Same P_T dependence

Duality in Electropion Production
Cornell data ('70's)

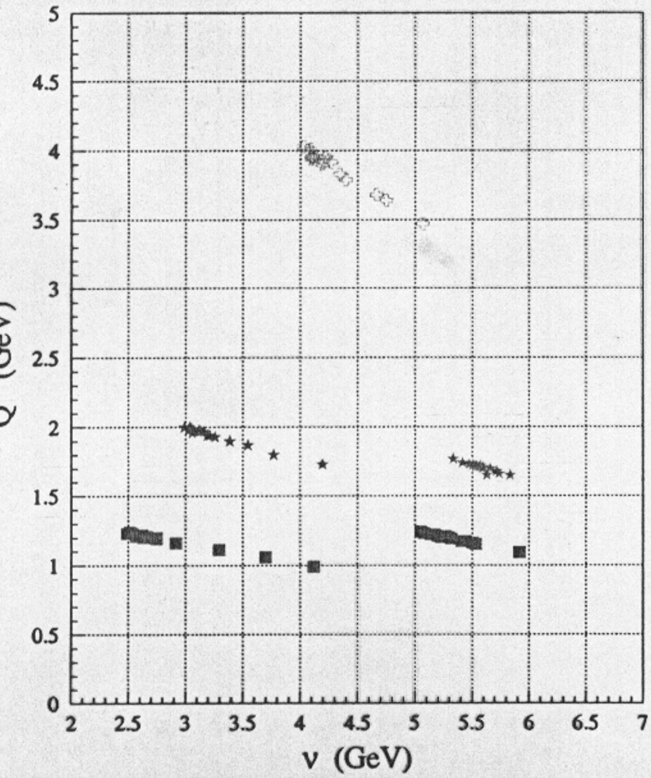
Bebek et al, Phys. Rev. Lett. 34, 75 (1975); Phys. Rev. Lett. 37, 1525 (1976); Phys. Rev. D 15, 3085 (1977)

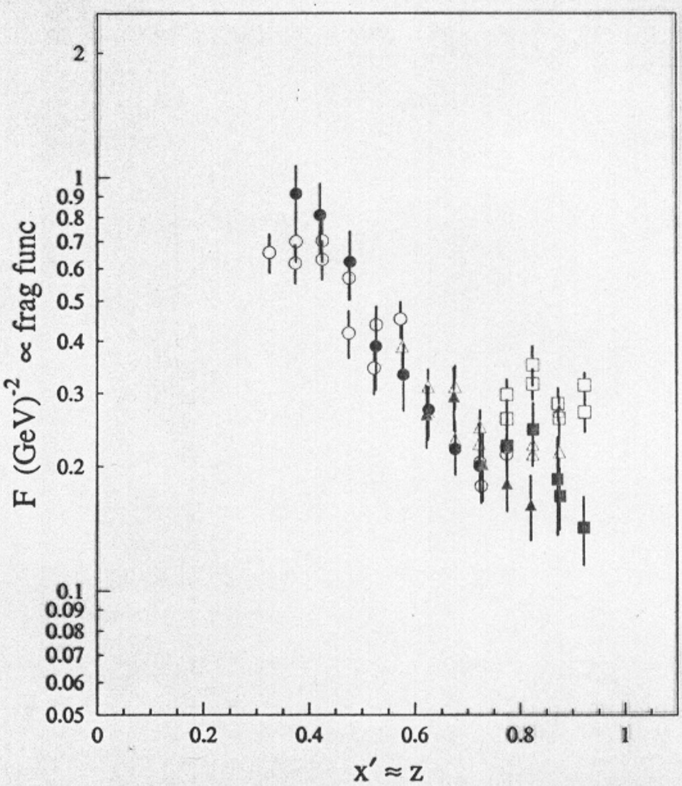
Data obtained at moderate Q^2 and ν

Typical pion momenta only ≈ 2 GeV/c!!

Analyze in terms of fragmentation function $F(x') \sim D_u^\pi(z)$ [The author
conclude that F has no Q^2 dependence, and only weak W^2 dependence]

Concentrate on the resonance region: $1.3 < W'^2 < 3.1$ GeV^2





□	$1.3 < W'^2 < 1.9$	Open: $Q^2 < 2.2$ Closed: $Q^2 > 2.2$
△	$1.9 < W'^2 < 2.5$	
○	$2.5 < W'^2 < 3.1$	

O.k. within ≈20%, also for "expected" Q^2 evolution...

Quark-Hadron Duality in Electron Scattering

Factorization Proof →

Core Quark-Gluon Calculation + **Universal Function**

Examples of **Universal Functions**

Inclusive Scattering	**Parton Distribution Functions**
Semi-Inclusive Scattering	**Fragmentation Functions**
Exclusive Scattering	Generalized Parton Distributions

ross sections **scale** if they follow the energy-momentum dependence of
ie **core quark-gluon calculation**

Question

When do cross sections scale, and how do we experimentally "prove" factorization

Things may not be as they seem...

Deep inelastic scattering	Scaling
	Quark-Hadron Duality
	Duality in F_2, F_L, and F_T
	Duality in Nuclei
	Duality in A_1^p and g_1
Semi-Inclusive scattering	Fragmentation
	Low-Energy Factorization?
	Meson Duality/Scaling

The Origins of Quark-Hadron Duality - II

How does the square of the sum become the sum of the squares?

Close and Isgur, Phys. Lett. B509, 81 (2001)

Semi-Inclusive Hadroproduction

Destructive interference leads to factorization and duality

$$F(\gamma N \to \pi X)(x, z) = \sum_{N^*, N^{*\prime}} F_{\gamma^* N \to N^*}(Q^2, W^2)\, \mathcal{D}_{N^* \to N^{*\prime} \pi}(W^2, W'^2)$$

$$\sim \sum_q e_q^2\, q(x)\, D_{q \to \pi}(z)$$

with
- $F_{\gamma N \to N^*}$ — N^* contribution to structure function F
- $\mathcal{D}_{N^* \to N^{*\prime} \pi}$ — representing the decay $N^* \to N^{*\prime} \pi$
- $D_{q \to \pi}$ — $q \to \pi$ fragmentation function

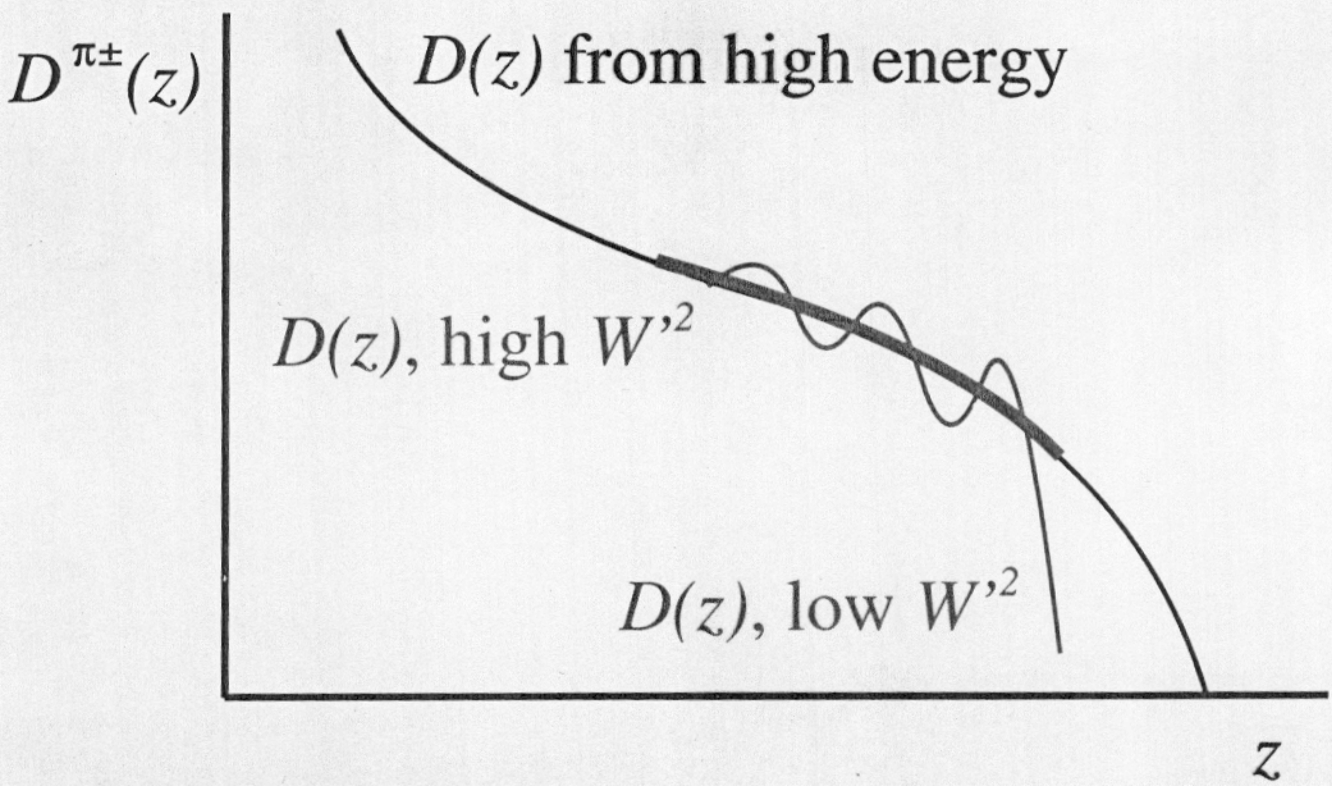
$SU(6)$ and $SU(3) \times SU(2)$ Multiplet Contributions to $\pi^{\pm}$ Photoproductior

W'	$p(\gamma, \pi^+)W'$	$p(\gamma, \pi^-)W'$	$n(\gamma, \pi^+)W'$	$n(\gamma, \pi^-)W'$
56;8	100	0	0	25
56;10	32	24	96	8
70;^{2}8	64	0	0	16
70;^{4}8	16	0	0	4
70;^{2}10	4	3	12	1
Total	216	27	108	54

Predictions

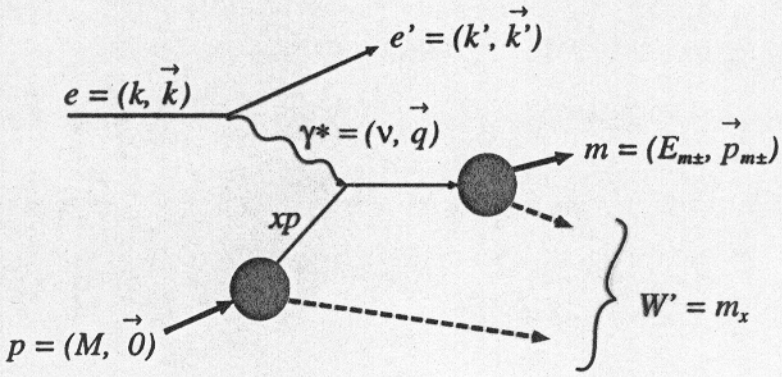
- **Duality obtained by end of Second Resonance Region**
- **Factorization and Approximate Duality for $Q^2, W^2 \leq 3$ GeV^2**

Duality for Fragmentation Functions



Duality in Meson Electroproduction

Afanasev, Carlson, and Wahlquist, Phys. Rev. D 62, 074011 (2000)



(e,e′) $$W^2 = M_p^2 + Q^2\left(\frac{1}{x} - 1\right)$$

or M_m small, collinear with $\vec{\gamma}$, and $\frac{Q^2}{\nu^2} << 1$

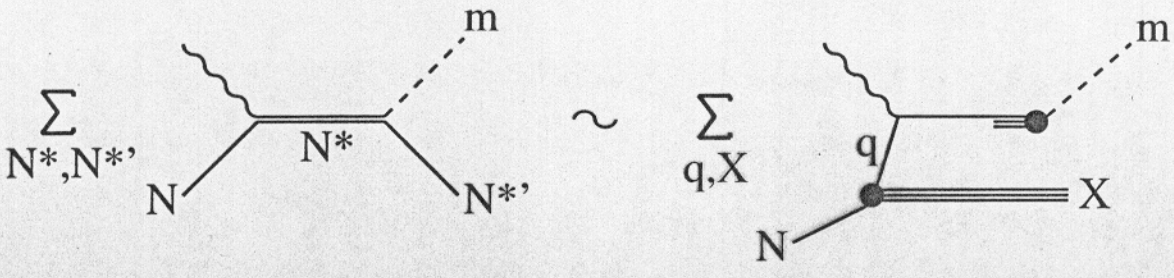
(e,e′m) $$W'^2 \approx M_p^2 + Q^2\left(\frac{1}{x} - 1\right)(1 - z)$$

$z = E_m/\nu$

actorization gives (LO QCD)

$$\frac{d\sigma}{dz} \sim \sum_i e_i^2 \left[q_i(x, Q^2) D_{q_i}^m(z, Q^2) + \bar{q}_i(x, Q^2) D_{\bar{q}_i}^m(z, Q^2)\right]$$

hadronic basis: excitation of N^* resonances and subsequent ecays into mesons and lower-lying resonances $N^{*\prime}$



Requires non-trivial cancellations of Decay Angular Distributions

The Origins of Quark-Hadron Duality - I

How does the square of the sum become the sum of the squares?

Close and Isgur, Phys. Lett. B509, 81 (2001)

$F_1 \sim \sigma_{1/2} + \sigma_{3/2}$ $\quad g_1 \sim \sigma_{1/2} - \sigma_{3/2}$

$\underline{56} + \underline{70} \rightarrow Closure$

Relative Photoproduction Strengths of **56,0$^+$** and **70,1$^-$** Multiplets

$SU(6)$:	$[56,0^+]^2 8$	$[56,0^+]^4 10$	$[70,1^-]^2 8$	$[70,1^-]^4 8$	$[70,1^-]^2 10$
F_1^p	9	8	9	0	1
F_1^n	4	8	1	4	1
g_1^p	9	-4	9	0	1
g_1^n	4	-4	1	-2	1
W			~ 1.53	~ 1.7	~ 1.7

Relative Longitudinal Production Strengths

$SU(6)$:	$[56,0^+]^2 8$	$[56,0^+]^4 10$	$[70,1^-]^2 8$	$[70,1^-]^4 8$	$[70,1^-]^2 10$
F_L^p	1	0	1	0	1
F_L^n	0	0	1	0	1

Conclusion: destructive interference between hadronic states of fferent symmetries critical feature of duality

Predictions

Second Resonance Region has too much strength for proton

Third Resonance Region has more impact for neutron

Breakdown in Duality for $Q^2 \leq 0.5$: both electric and magnetic contribute comparably

Duality in magnetic strength down to $Q^2 = 0$

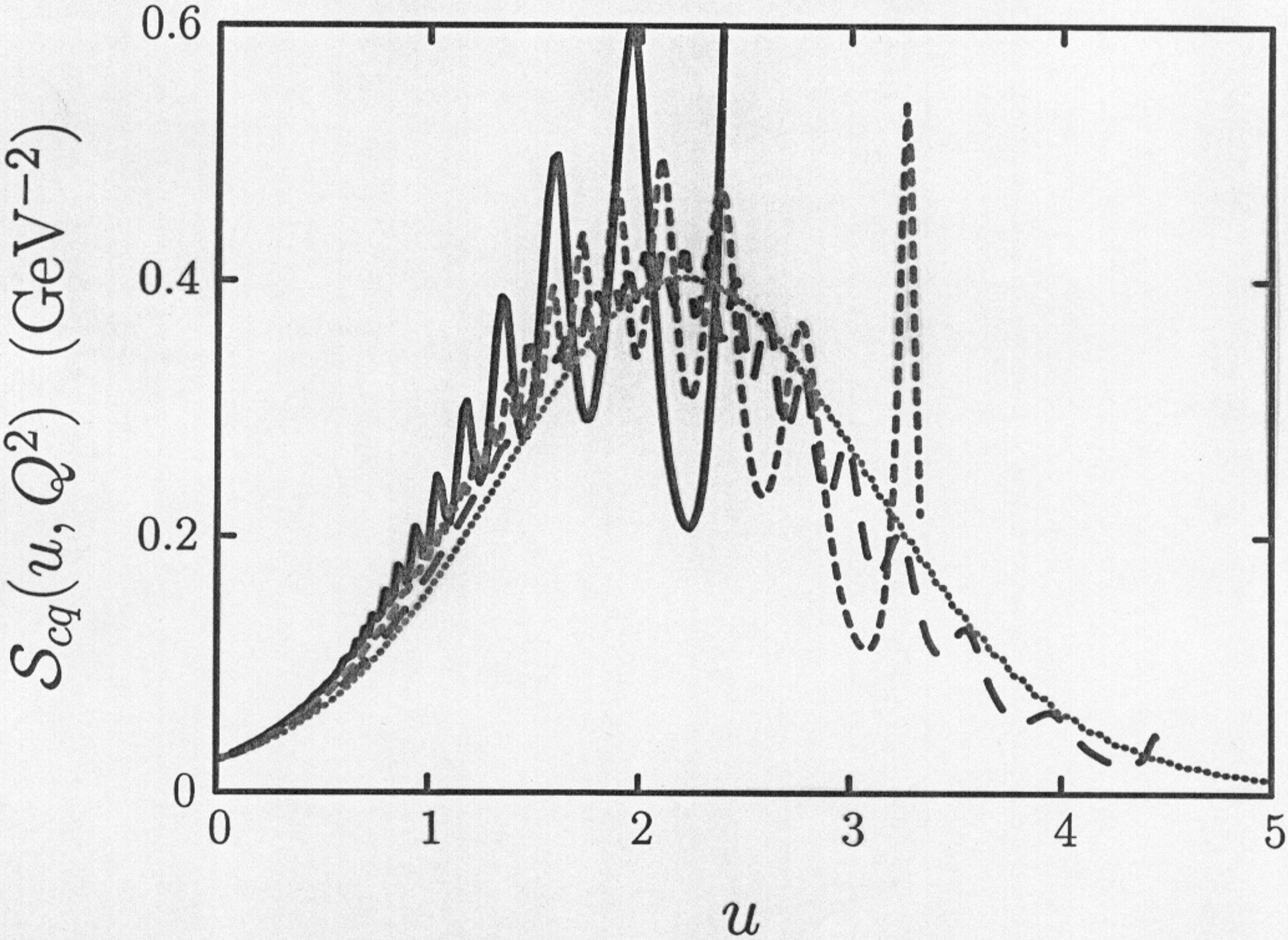
Investigation of Duality

Isgur, Jeschonnek, Melnitchouk, Van Orden, Phys. Rev. D 64, 054005 (2001)

Investigation of Quark-Hadron Duality in a Model for Large N_c QCD

Scalar qq states only, one q heavy

[**Relativity** , $N_c \rightarrow \infty$ (**valence only**), δ-functions $\rightarrow$ Breit Wigner]

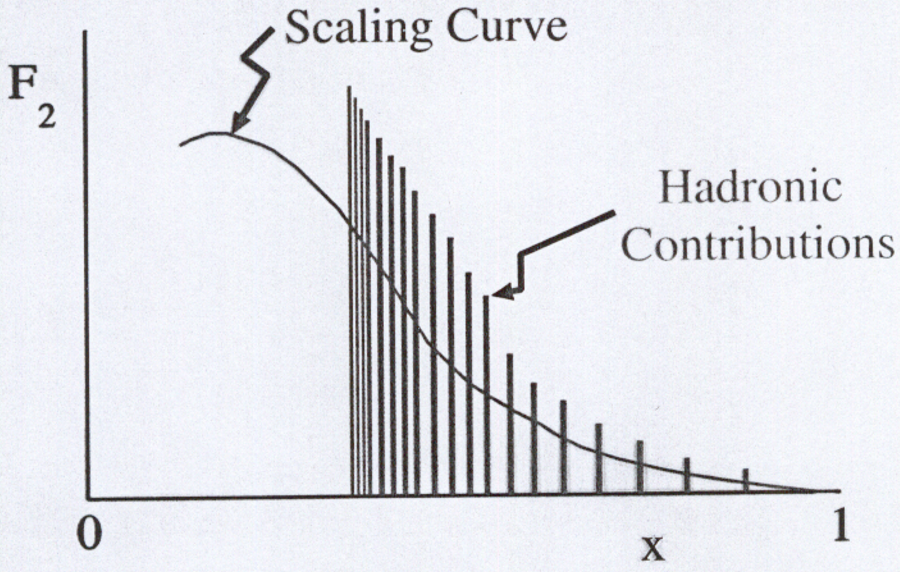


Curves are for Q^2 = 0.5, 1, 2, and 5 $(\mathrm{GeV/c})^2$

u is a Nachtmann-type scaling variable rescaled in terms of the quark mass ratio M/m

Bloom-Gilman Duality and the Quark-Parton Model

N. Isgur : N_c ---> ∞ : Infinitely narrow resonances $q\bar{q}$



- Distinction between Resonance Region and Scaling Region is spurious
- Bloom-Gilman Duality must be invoked even in the Bjorken Scaling Region ---> Bjorken Duality

In Naive Quark Model

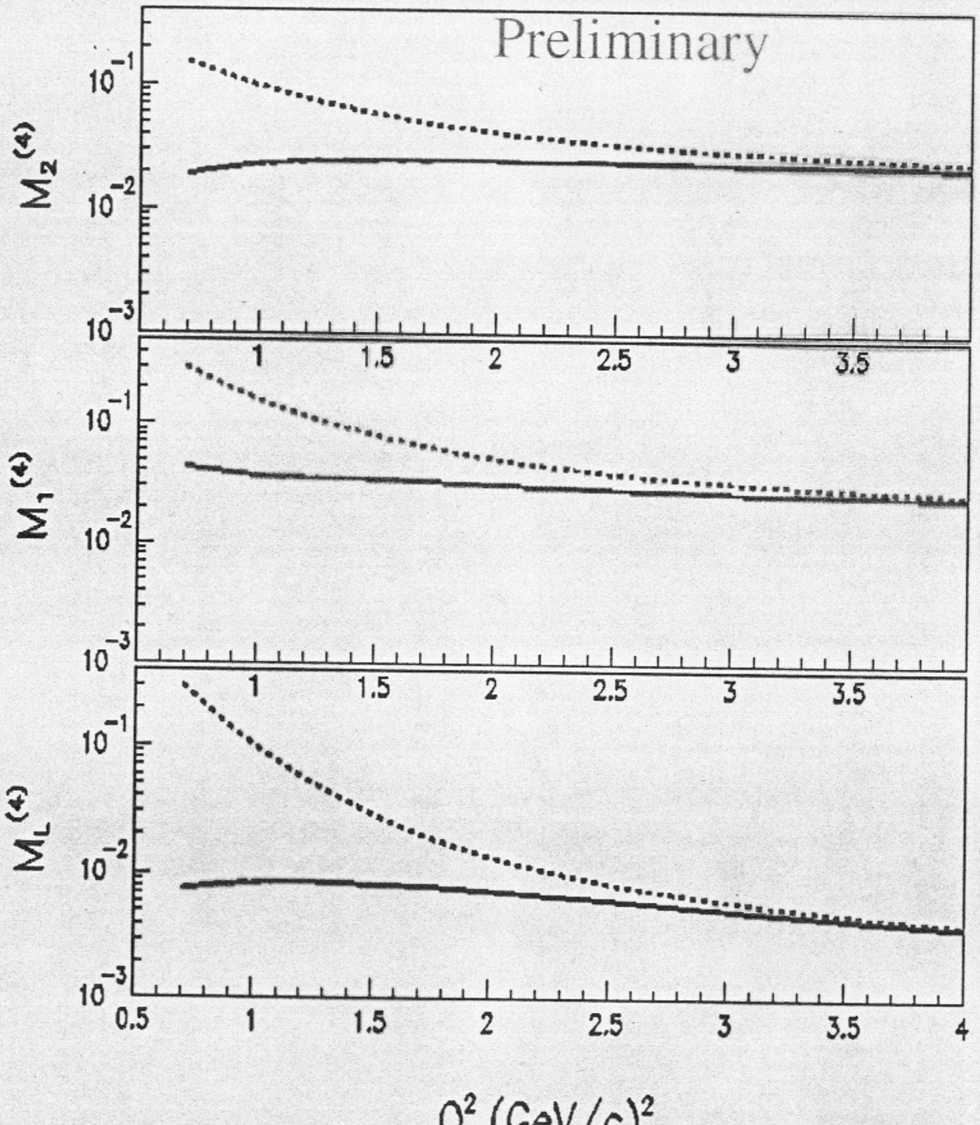
How many resonances does one need to average over to obtain a close to complete set of states to mimic closure?

F. Close : 56 and 70 states o.k. for closure (by end of second resonance region the summed contributions are close to the parton model values for proton and neutron)

Can use spin/flavor selectivity in $\vec{e}$-$\vec{N}$ scattering, $\vec{N}(\vec{e}, e'm)$ meson electroproduction, etc., to investigate duality in detail!

Sensitive Test: Proton - Neutron

n = 4 Moments of F_2, F_1 and F_L



Neglecting elastics, n = 4 moments have only a small Q^2 dependence as well.

<u>Momentum sum rule</u>

$$M_L^{(n)} = \alpha_s(Q^2)\left\{ \frac{4M_2^{(n)}}{3(n+1)} + \frac{2c\int dx\, xG(x,Q^2)}{(n+1)(n+2)} \right\}$$

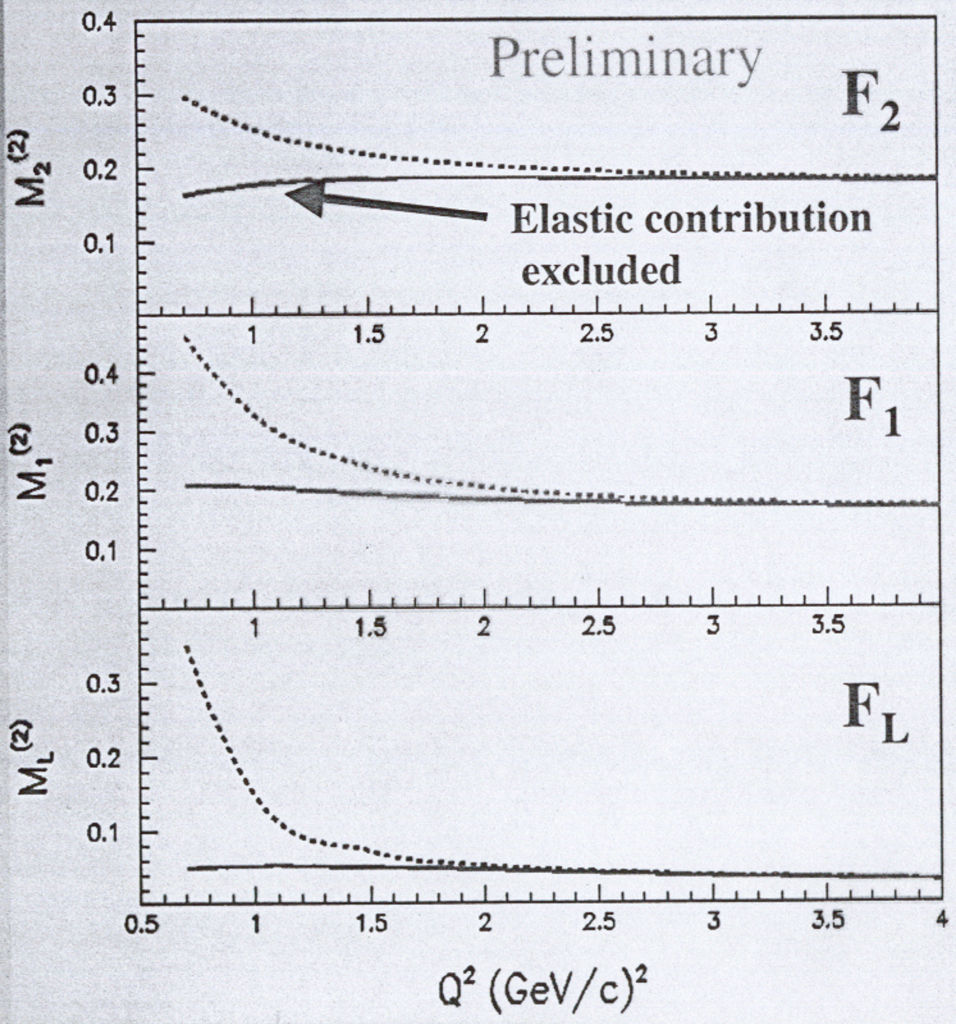
Gluon distributions!

This is only at leading twist and neglecting TM effects.

⇒ Must remove TM effects from data to extract moment of xG…we're working on it…..

n = 2 Moments of F_2, F_1 and F_L: $M_n(Q^2) = \int_0 dx\, x^{n-2} F(x,Q^2)$

DIS: SLAC fit to F_2 and R
RES: E94-110 resonance fit



Elastic Contributions

$$F_1^{EL} = G_M^2\, \delta(x-1)$$

$$F_2^{EL} = \frac{(G_E^2 + \tau G_M^2)\delta(x-1)}{1+\tau}$$

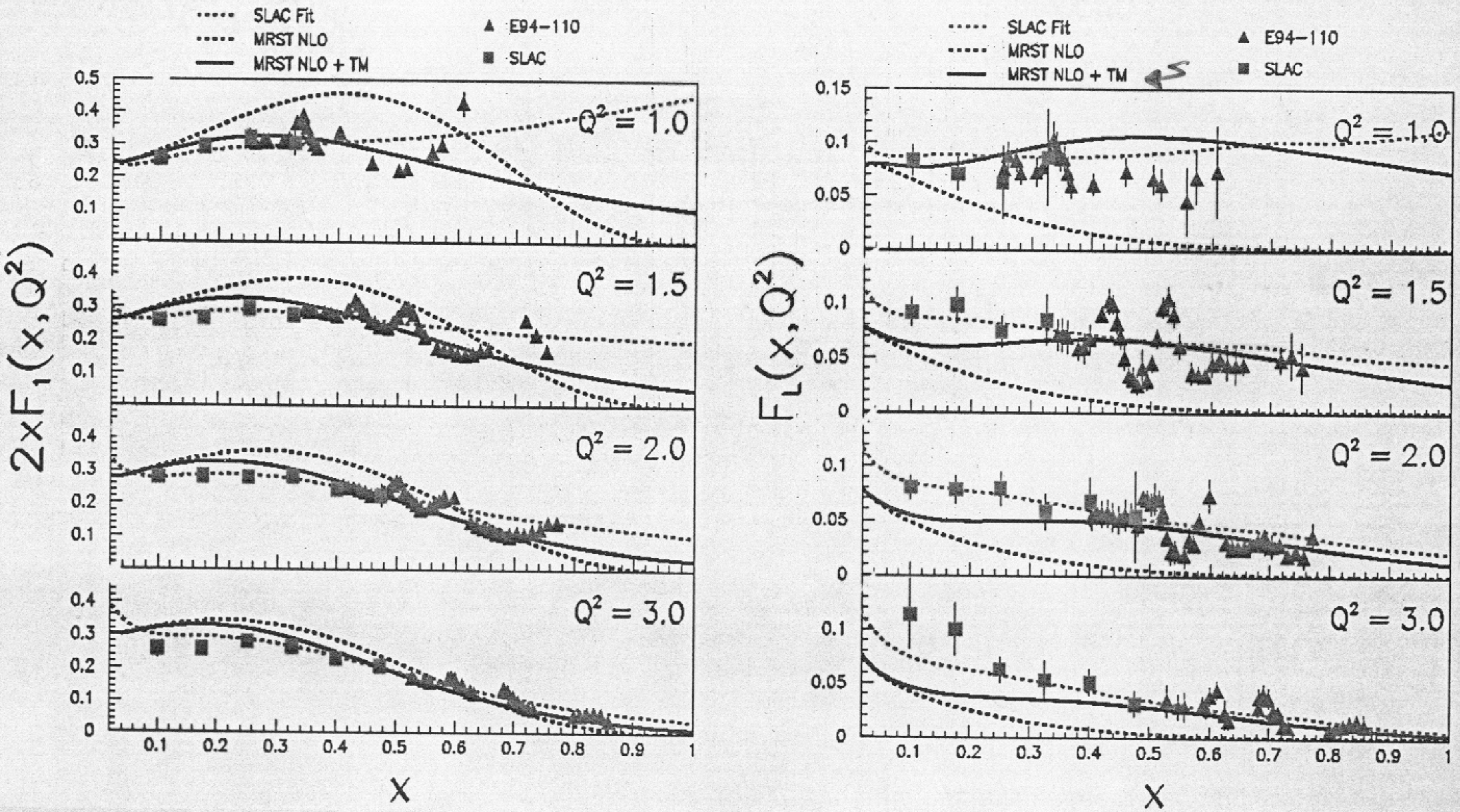
$$\tau = q^2/4M_p^2$$

$$F_L^{EL} = G_E^2\, \delta(x-1)$$

Flat Q^2 dependence → small higher twist! - not true for contributions from the elastic peak (bound quarks)

Duality in L-T Structure Functions

Add SLAC data, compare NLO and NLO + TM



Duality in QCD

Duality in the QCD picture is Scaling around the Twist-2 curve

Moments of the Structure Function

$$M_n(Q^2) = \int_0^1 dx x^{n-2} F(x, Q^2)$$

If $n = 2$, this is the duality integral!!

Operator Product Expansion

$$M_n(Q^2) = \sum_{k=1}^{\infty} \left(\frac{n M_0^2}{Q^2} \right)^{k-1} B_{nk}(Q^2)$$

with B_{nk} having logarithmic dependence

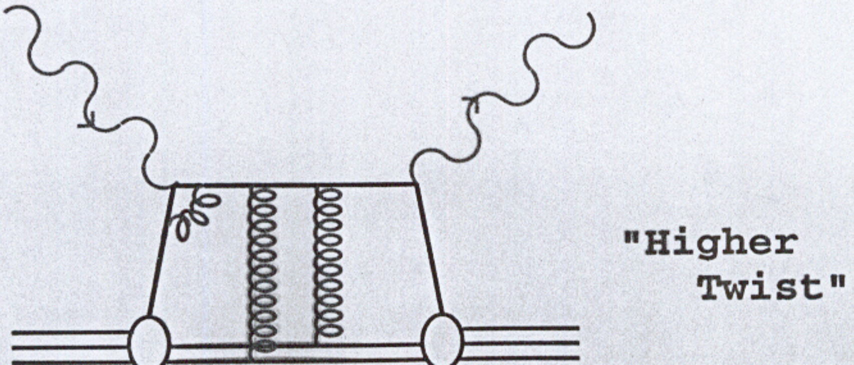
DeRujula, Georgi, Politzer (1977)

$$M_n(Q^2) = \sum_{k=0}^{\infty} E_{nk} \left(\frac{Q^2}{\mu^2} \right) M_{nk}(\mu^2) \left(\frac{1}{Q^2} \right)^k$$

where E_{nk} are dimensionless coefficients, calculated perturbatively

Ji, Unrau (1995)

The $O(\frac{1}{Q^2})$ power dependence is due to initial and final state interactions between the struck quark and target remnants



"It is fair to say that (short of the full solution of QCD) understanding and controlling the accuracy of quark-hadron duality is one of the most important and challenging problems for QCD practitioners today."

M. Shifman, Handbook of QCD, Volume 3, 1451 (2001)

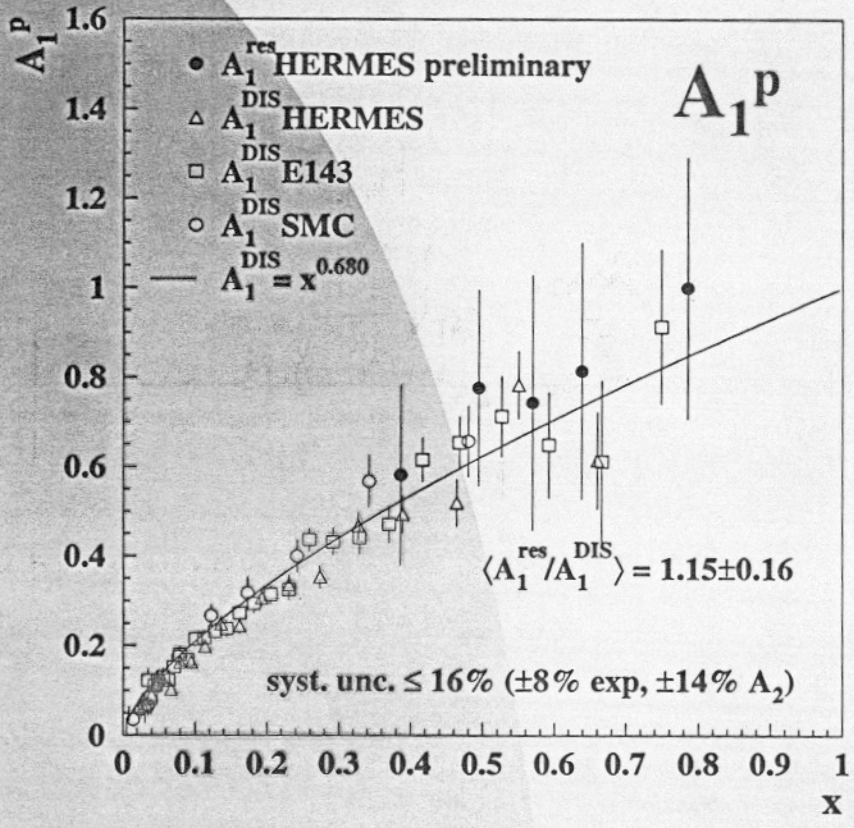
Duality violations are not easily interpretable by lattice QCD calculations!

Applications of Quark-Hadron Duality

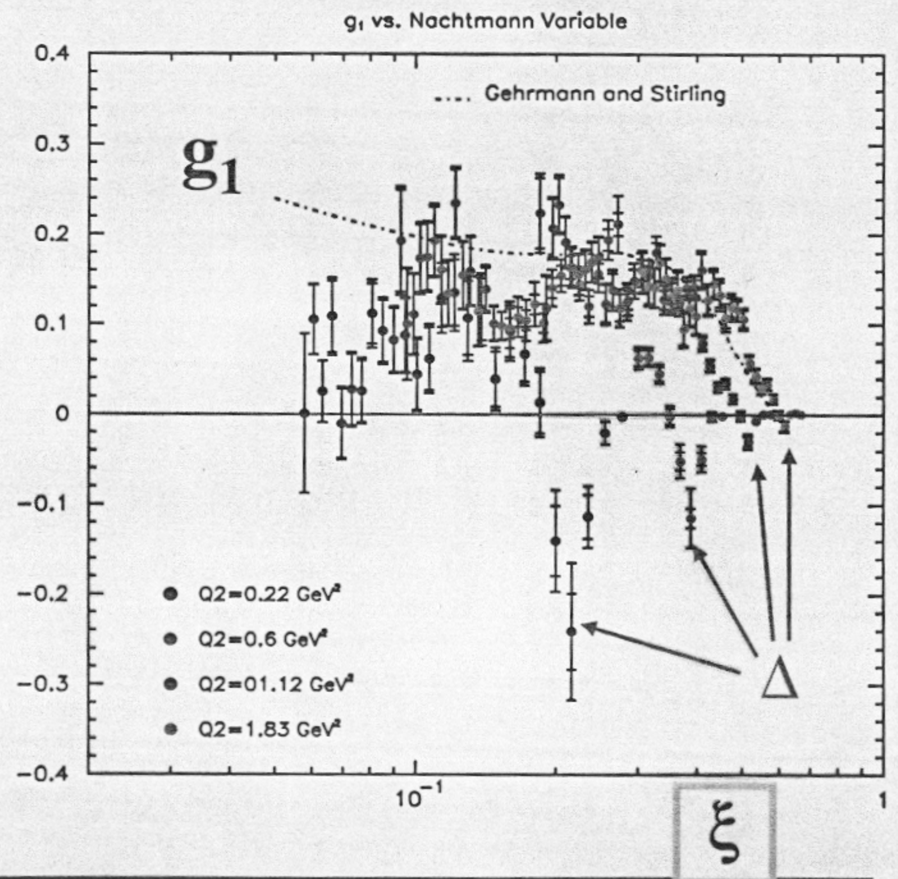
- CTEQ currently planning to use duality for large x parton distribution modeling
- Neutrino community planning to test duality
- Neutrino community using duality to predict low energy (~1 GeV) regime
  - New Bodek model successfully uses duality to extend pdf-based parameterization to the photoproduction limit successfully
- Duality provides extended access to large x regime
- Allows for direct comparison to QCD Moments
  - calculated on the lattice

... but tougher in Spin Structure Functions

HERMES



JLab Hall B

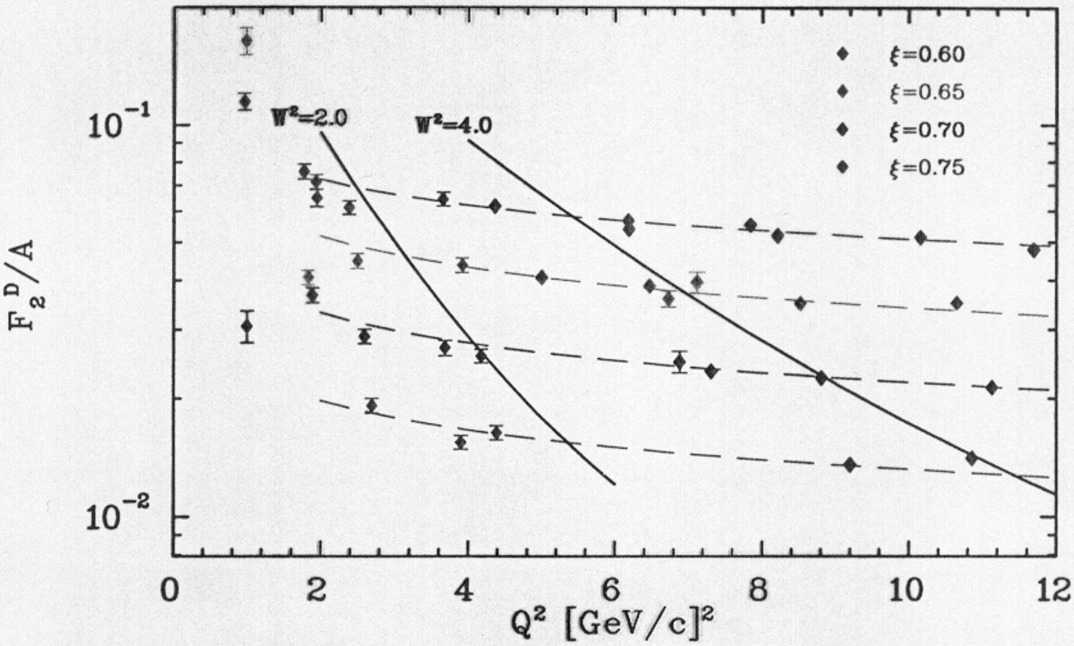


HERMES at $Q^2 > 1.6$ GeV²: agreement in A_1

CLAS: N-Δ transition region turns positive at $Q^2 = 2$!

Duality in Nuclei

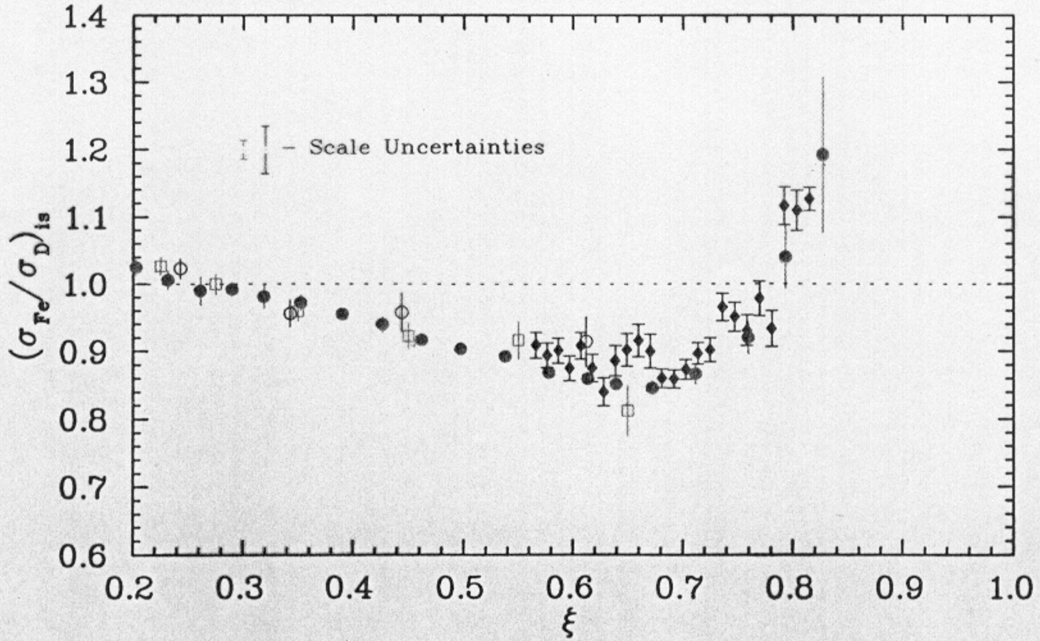
- Scaling at moderate Q^2 and W^2



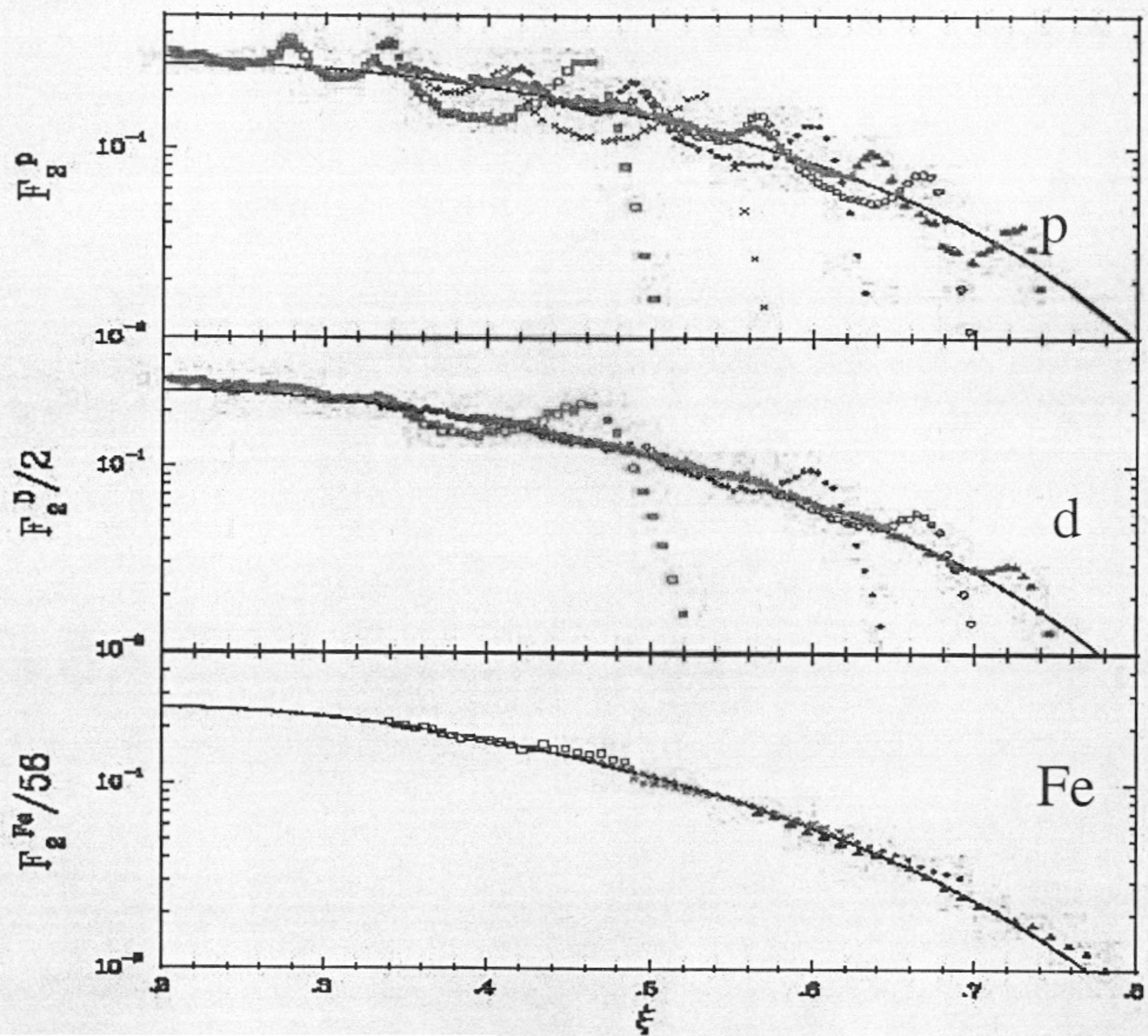
- EMC effect in the resonance region

JLab E89008 results for Fe/D $1.3 < W^2 < 2.8, Q^2 \sim 4$

SLAC E139 results in DIS region

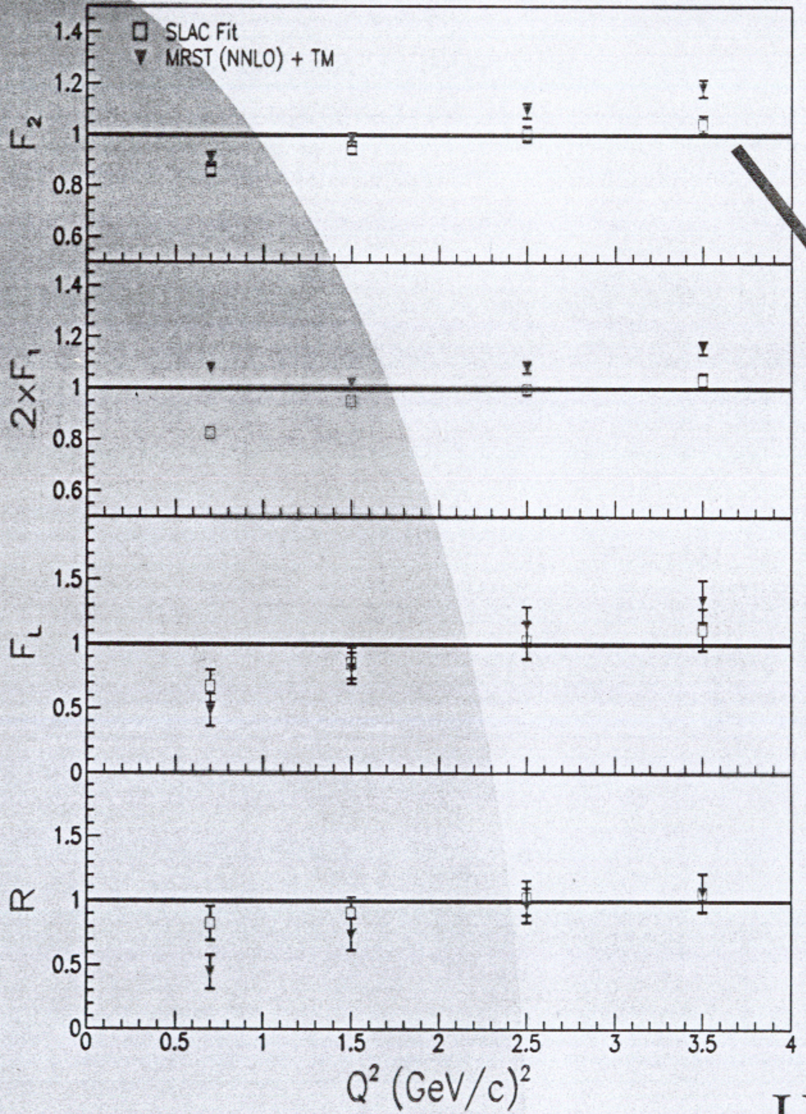


Duality "easier" in Nuclei (F_2)

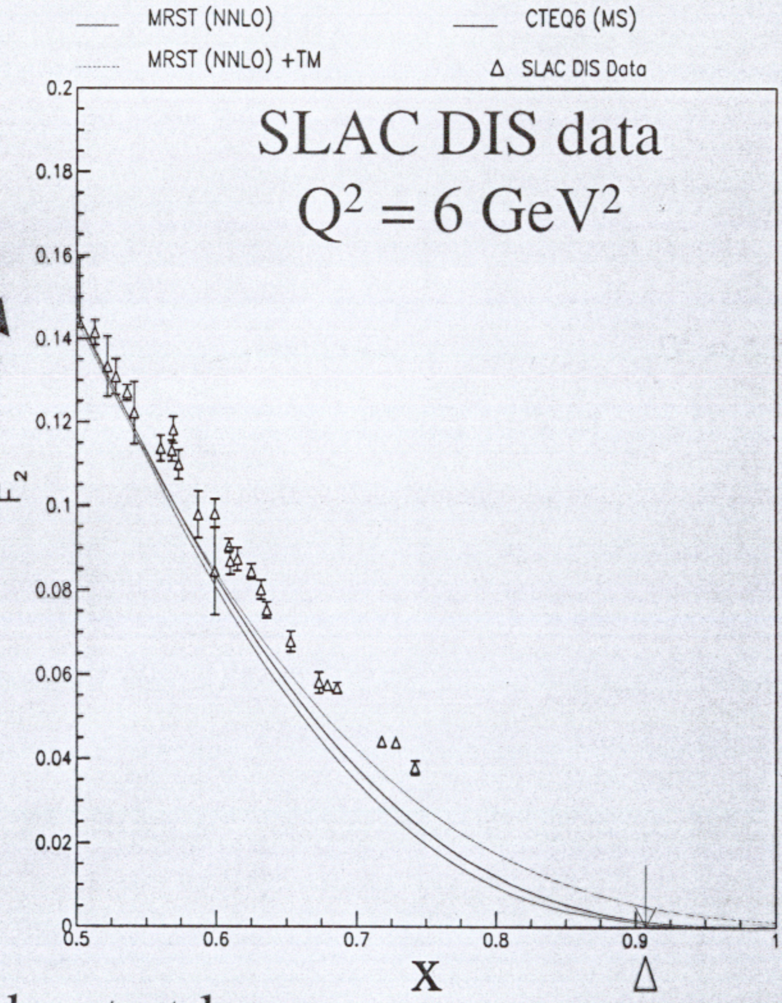


Quantification

Ratio Res / Scaling

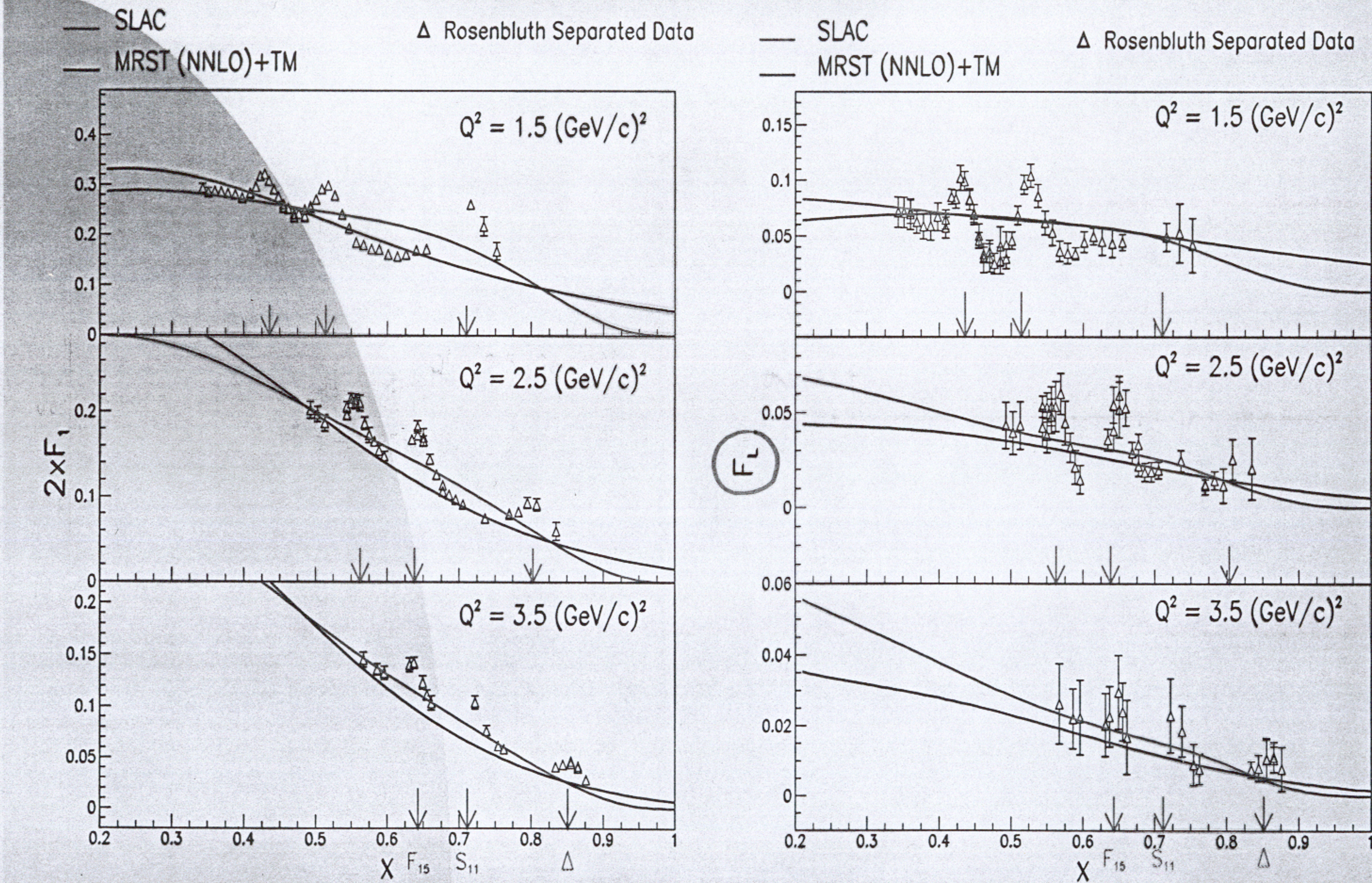


Large x Structure Functions

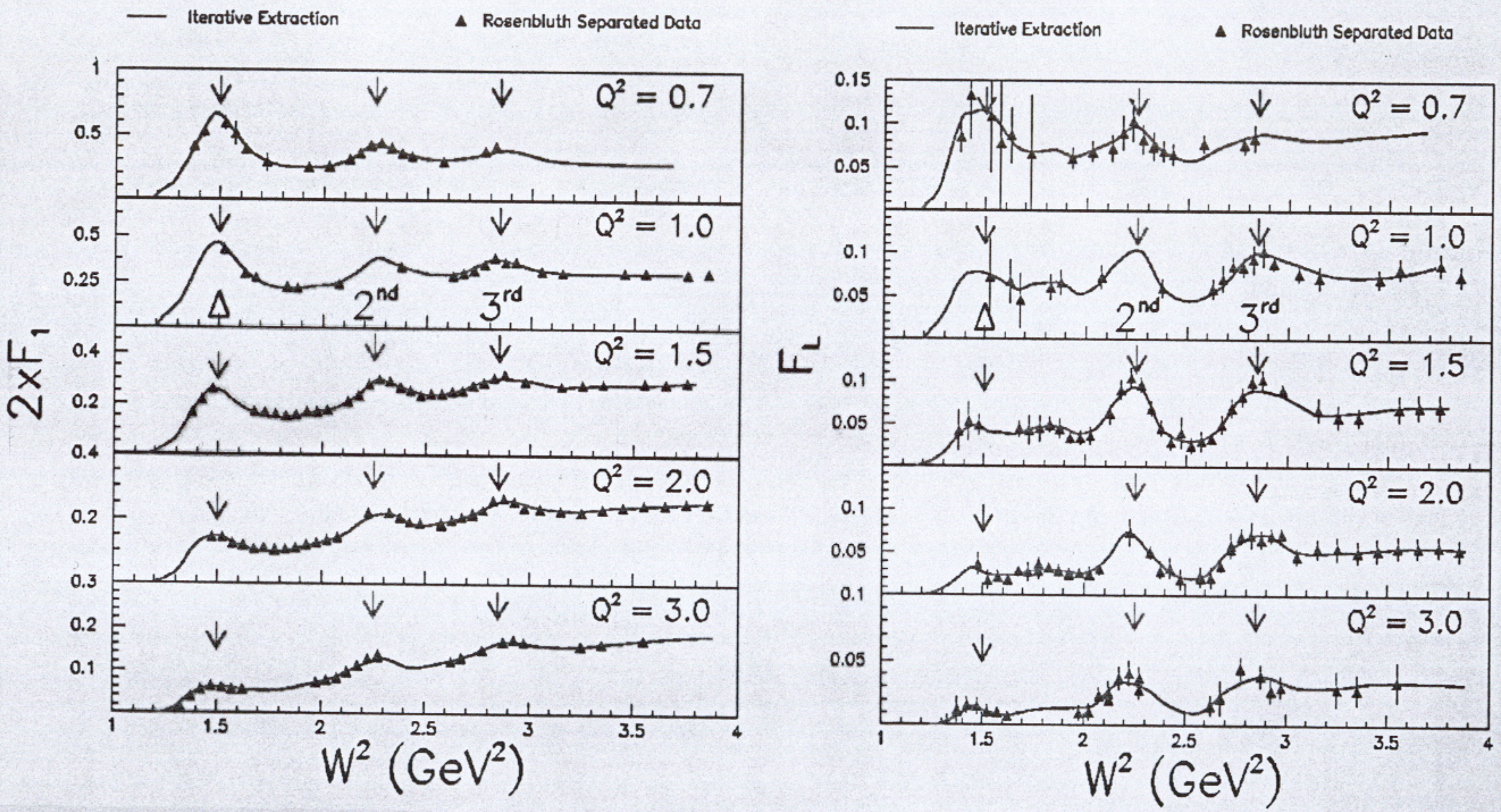


Undershoot at large x

Duality observed in *all* unpolarized structure functions



L-T Separated Structure Functions

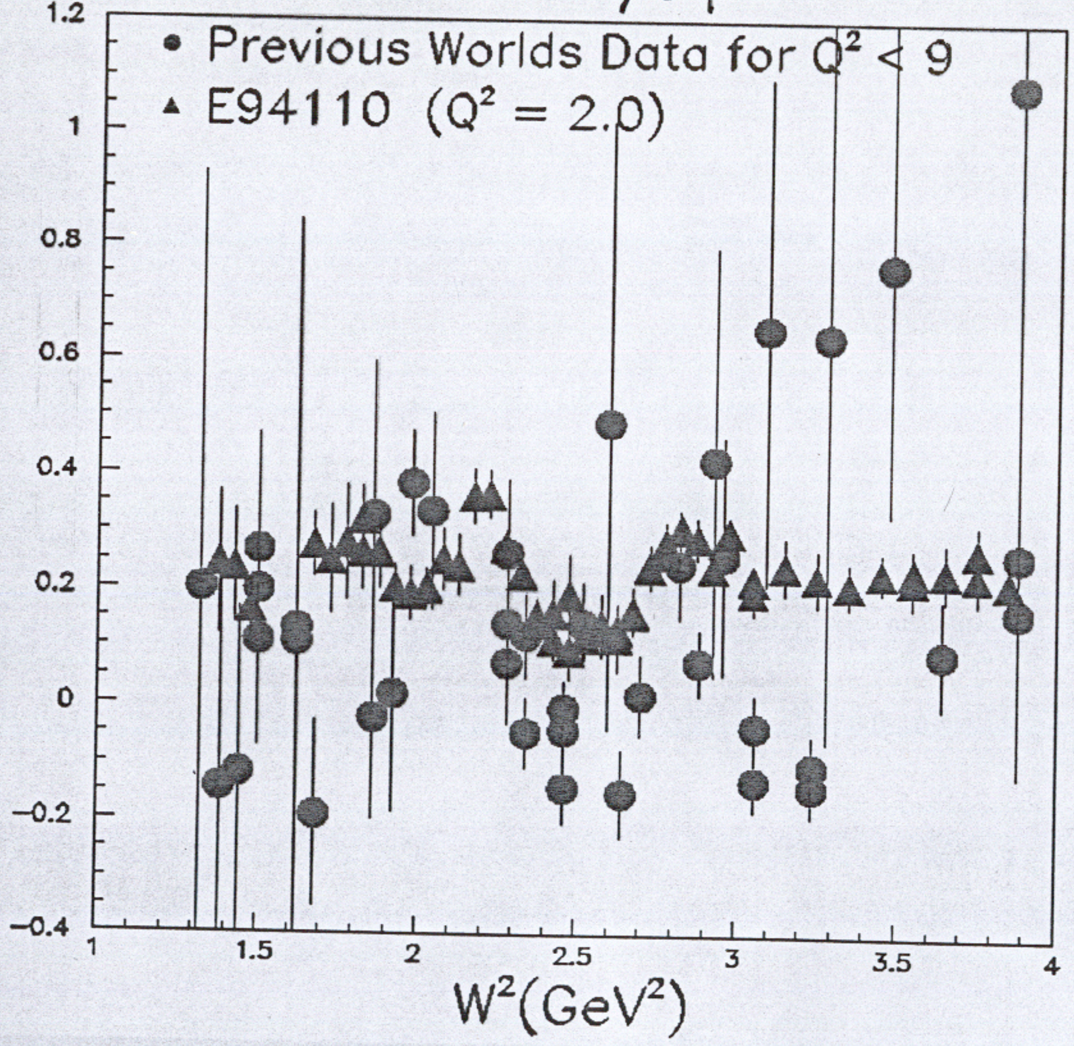


- Good agreement between methods.
- Very strong resonant behaviour in F_L!
- Evidence of different resonances contributing in different channels?

What about the other structure functions F_L, F_1?

World's L/T Separated Resonance Data (until 2002):

$R = \sigma_L / \sigma_T$

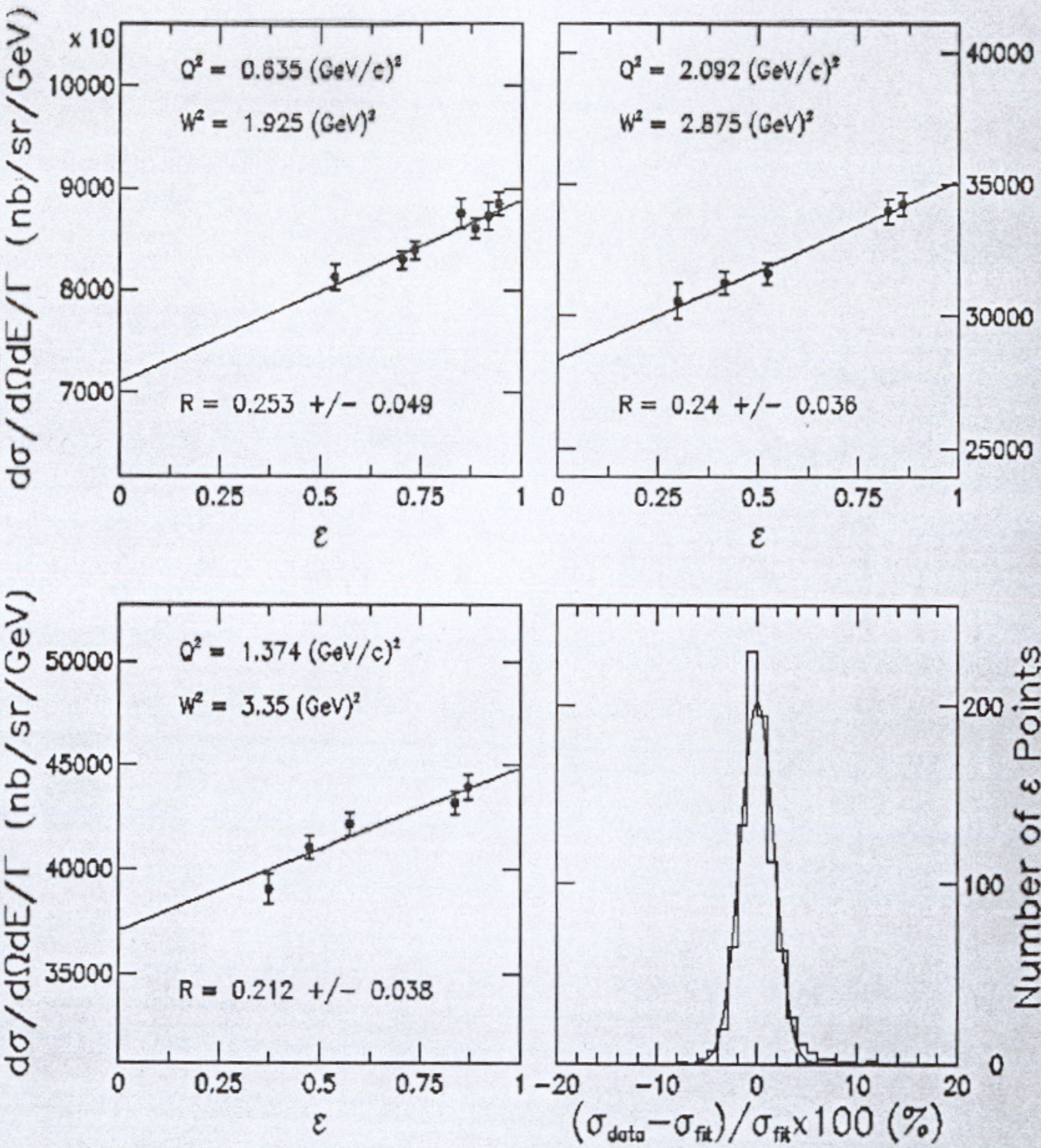


- Now able to study the Q^2 dependence of individual resonance regions!
- Clear resonant behaviour can be observed!

Now able to extract F_2, F_1, F_L and study duality!

Rosenbluth Separations

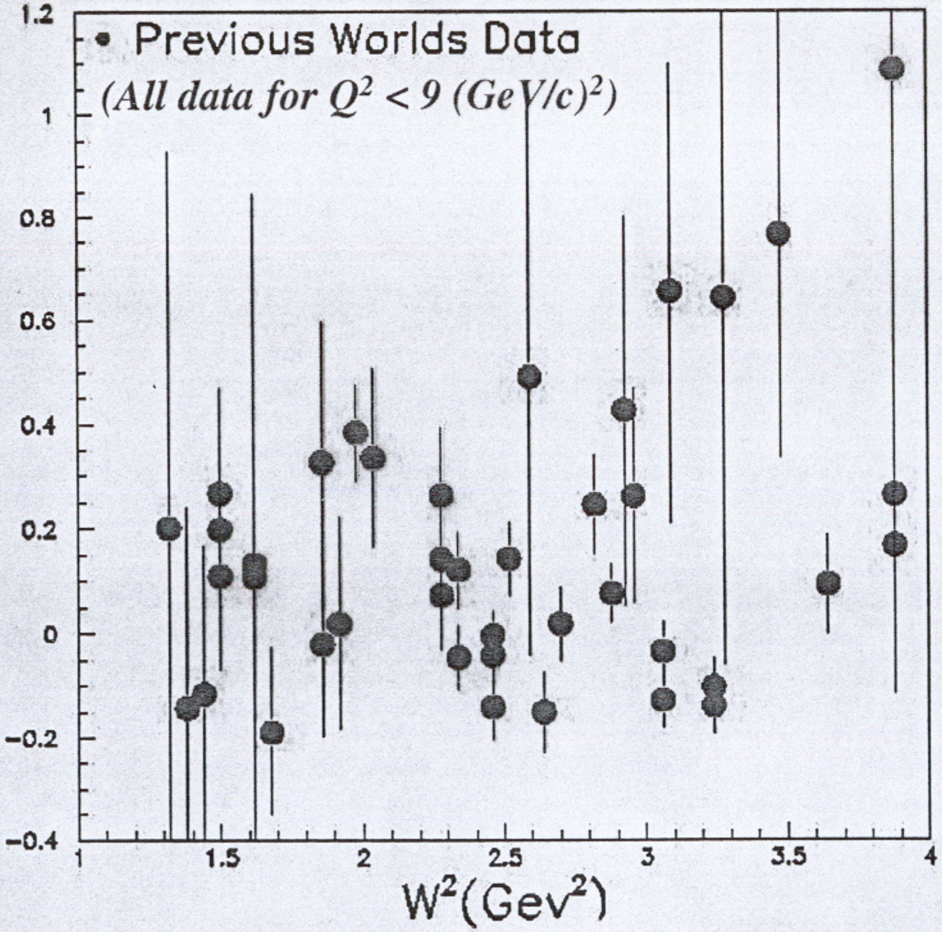
- 180 separations total (most with 4-5 ε points)
- Spread of points about the linear fits is Gaussian with σ ~ 1.6 % consistent with the estimated point-point experimental uncertainty (1.1-1.5%)
  - a systematic "tour de force"



What about the other structure functions F_L, F_1?

World's L/T Separated Resonance Data (until 2002):

$R = \sigma_L/\sigma_T$



- Not able to study the Q^2 dependence of individual resonance regions!
- No resonant behaviour can be observed!

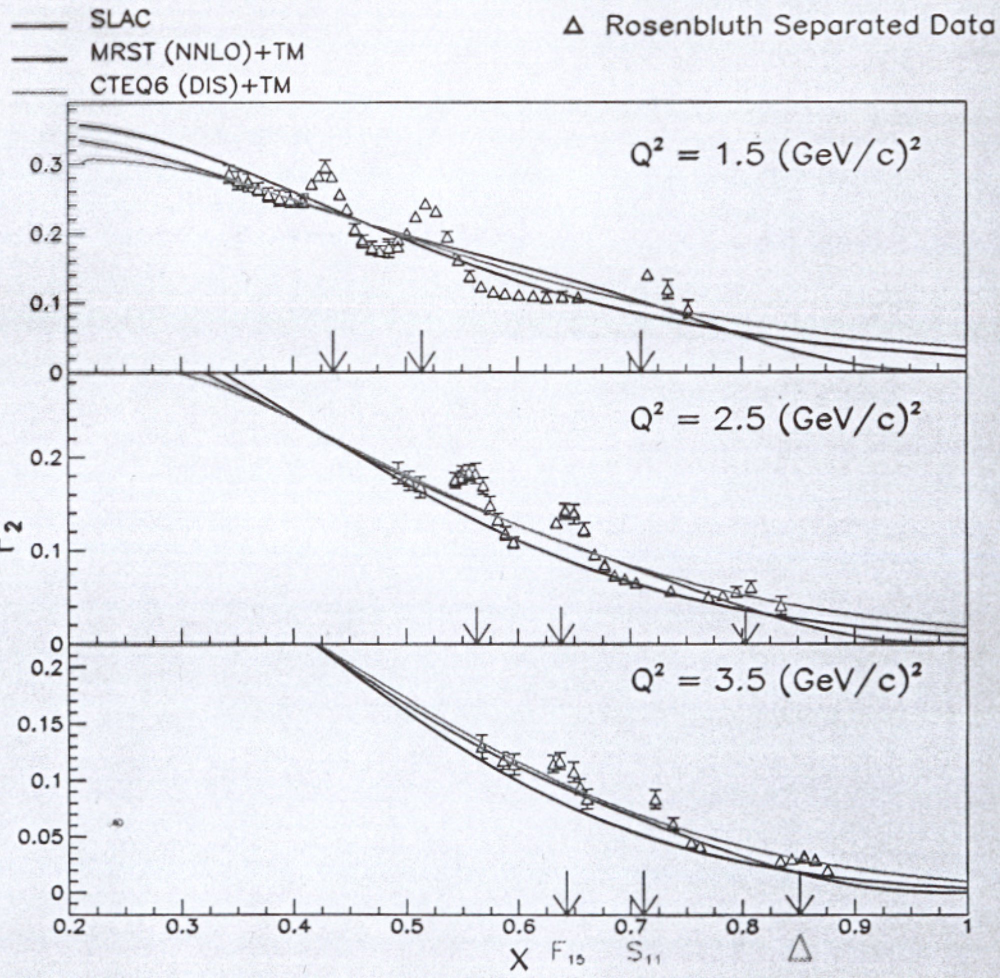
JLab E94 10: a global survey of longitudinal strength in the resonance region……

Duality in the F_2 Structure Function

$$\frac{d\sigma}{d\Omega dE'} \propto \sigma_{Mott} \Sigma e_i^2 x[q_i(x,Q^2) + \overline{q_i}(x,Q^2)]$$

First observed ~1970 by Bloom and Gilman

- Old approach I: compare in Nachtmann ξ (includes target-mass correction)
- Old approach II: F_2 not truly F_2 (assumed R)
- New approach I: Target-mass corrections in QCD-type calculation → enables comparison at same Q^2
- New approach II: Truly F_2



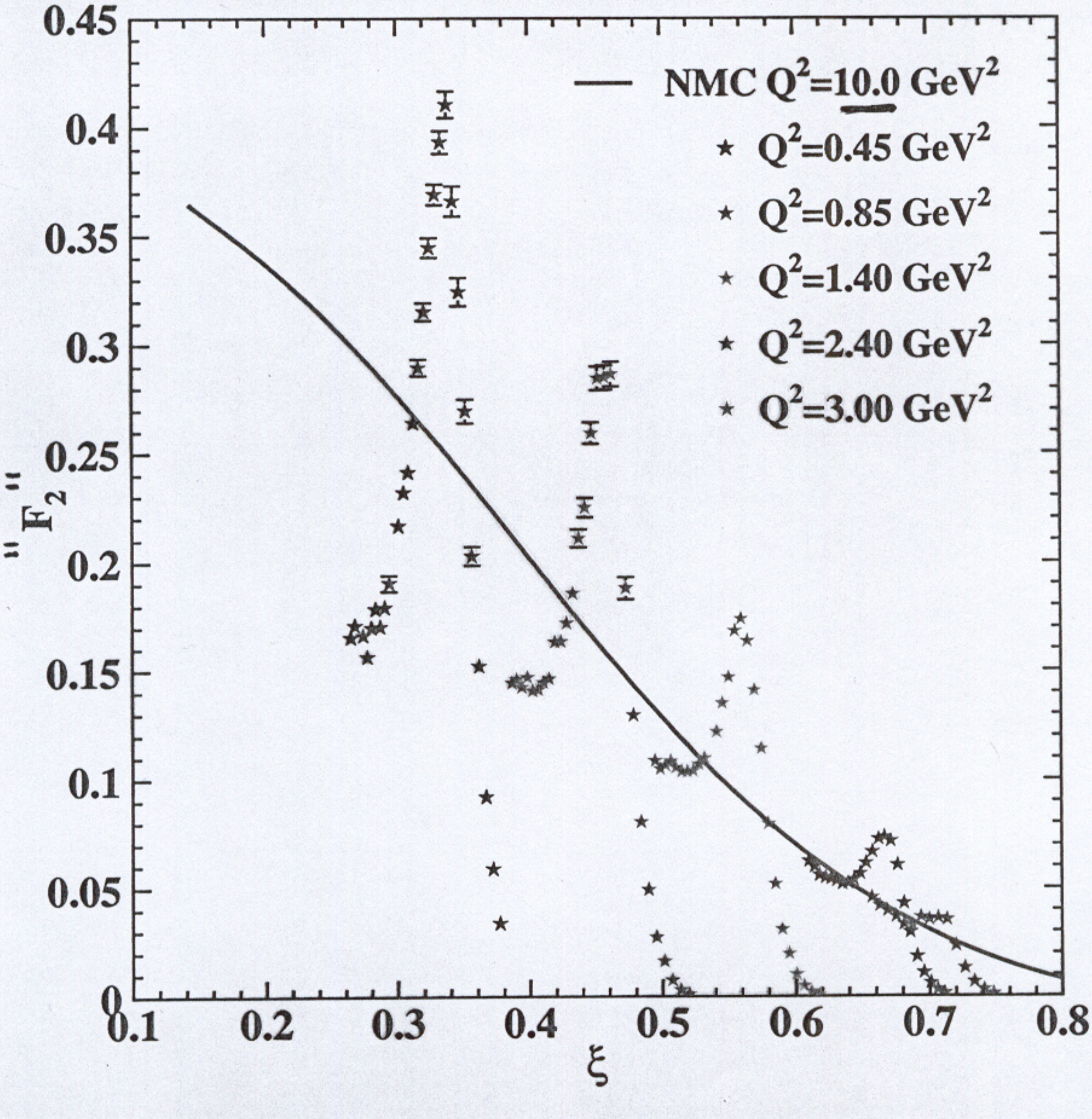
CTEQ6 (DIS) + TM …

Duality in F_2

Define $N - \Delta$ transition region as $1.2 < W^2 < 1.9$ GeV^2

JLab Hall C data (1996)

NMC parameterization of DIS F_2 data picked at Q^2 = 10 (GeV/c)^2



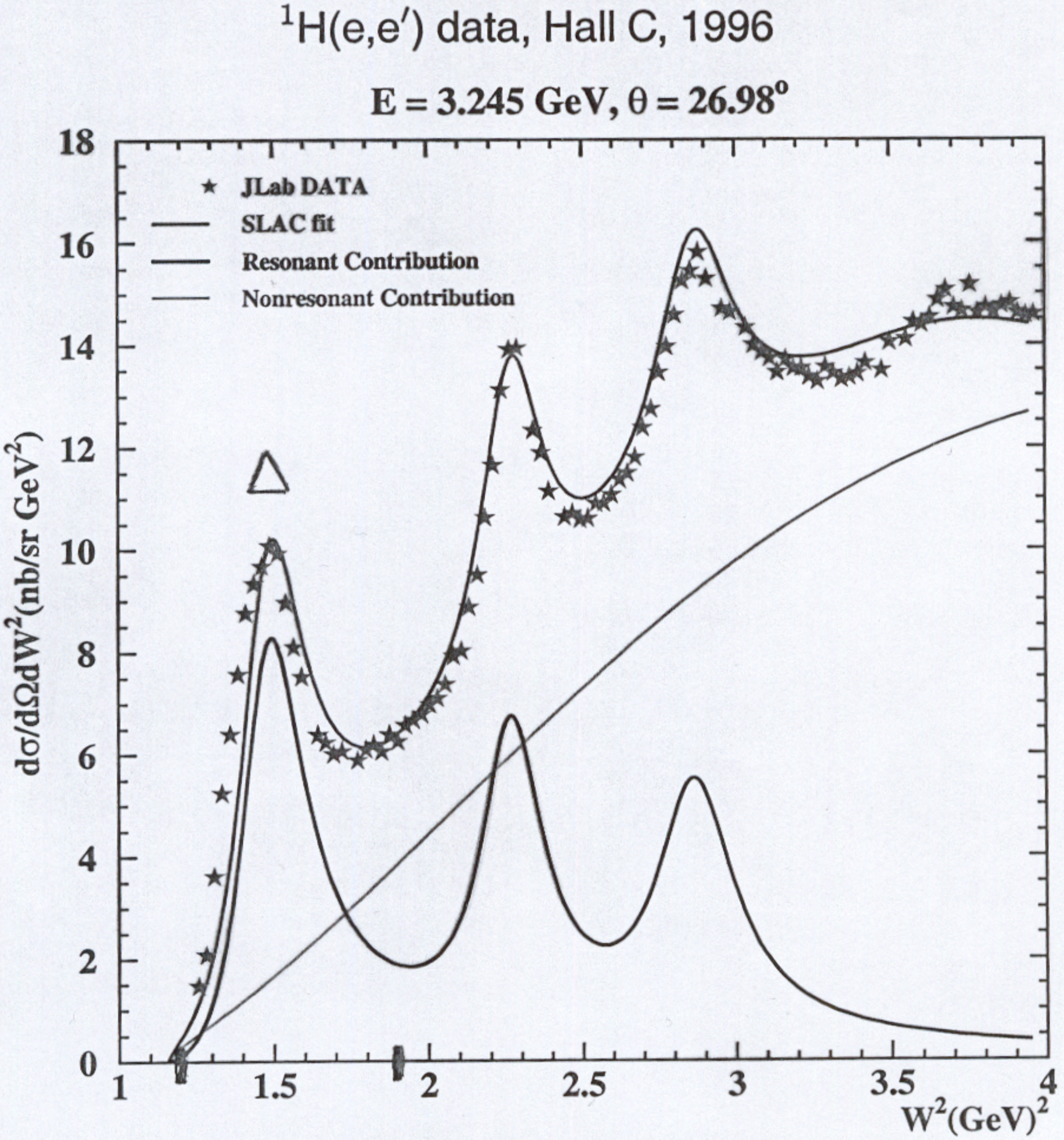
Obviously, duality does not hold on top of the peak!

However, when averaged over the chosen W^2 region, the $N - \Delta$ region mimics the DIS parameterization – Duality holds!

(Note that the QCD Q^2 evolution is not large in this region of x)

But : "F_2", "Q^2", averaging?

nclusive Nucleon Resonance Electroproduction



$$Q^2 = 4EE' sin^2(\Theta/2)$$
$$\nu = E - E'$$
$$x = Q^2/(2M\nu)$$
$$W^2 = M^2 + 2M\nu - Q^2 = M^2 + Q^2(1/x - 1)$$

Spectrum consists of Resonant Contributions

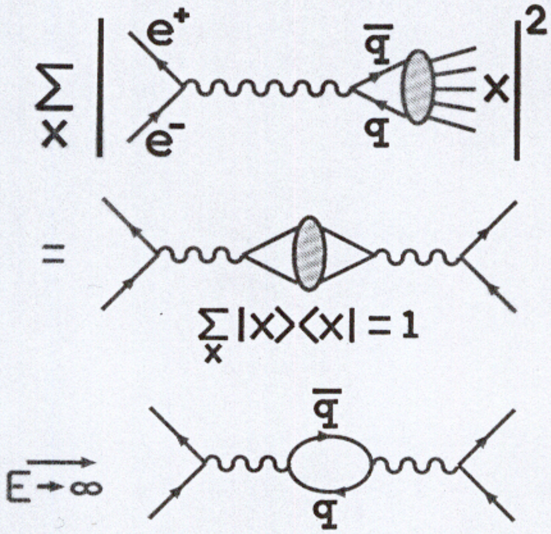
and Nonresonant Contributions

- **At high energies: interactions between quarks and gluons become weak**
  (“asymptotic freedom”)
  - ↗ efficient description of phenomena afforded in terms of quarks
- **At low energies: effects of confinement make strongly-coupled QCD highly non-perturbative**
  - ↗ collective degrees of freedom (mesons and baryons) more efficient
- Duality between quark and hadron descriptions
  - ◆ reflects relationship between *confinement* and *asymptotic freedom*
  - ◆ intimately related to nature and transition from *non-perturbative* to *perturbative* QCD

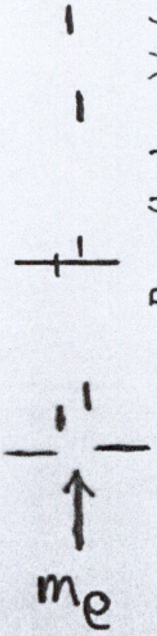
Duality defines the transition from soft to hard QCD.

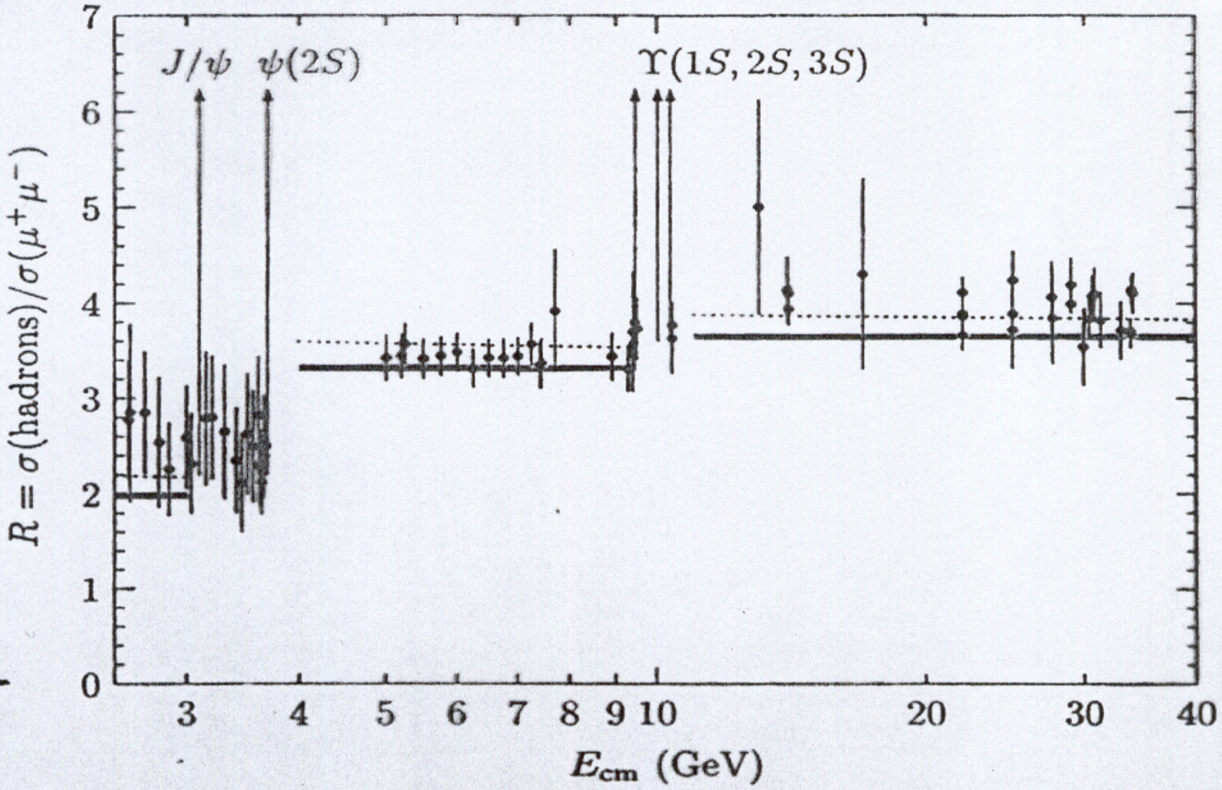
$e^+e^- \rightarrow \mathcal{HADRONS}$

CROSS SECTION ~



$$\lim_{E\to\infty} \frac{\sigma(e^+e^- \rightarrow X)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = N_c \sum_q e_q^2$$





Also seems to work locally!

Quark-Hadron Duality

complementarity between quark and hadron descriptions of observables

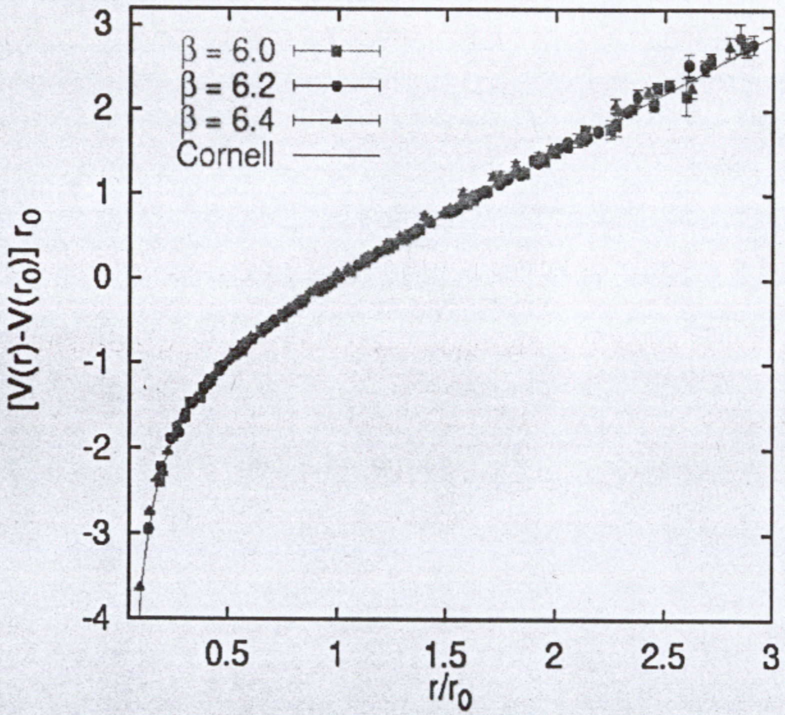
At high enough energy:

Hadronic Cross Sections averaged over appropriate energy range	=	Perturbative Quark-Gluon Theory
Σ_{hadrons}		Σ_{quarks}

Can use either set of complete basis states to describe physical phenomena

QCD and the Strong Nuclear Force

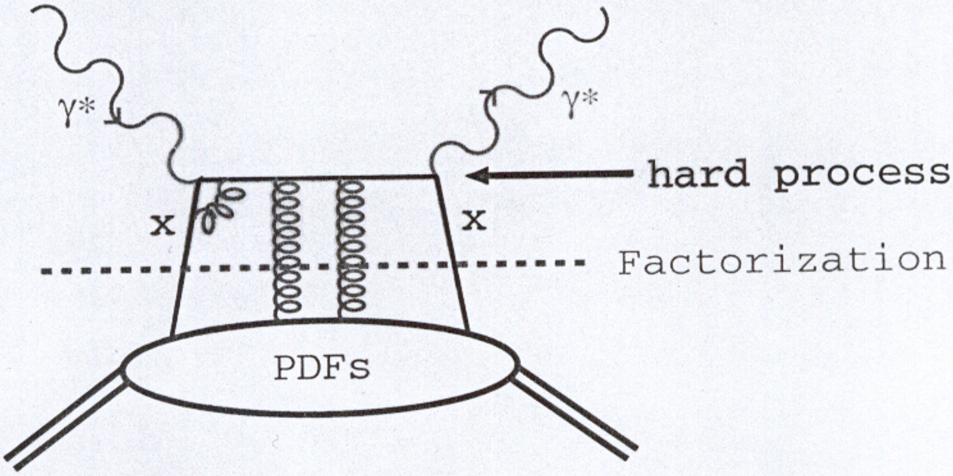
QCD has the most bizarre properties of all the forces in nature



- **Confinement:**
  - **restoring force between quarks at large distances equivalent to 10 tons, *no matter how far apart***
- **Asymptotic freedom:**
  - **quarks feel almost no strong force when closer together**

QCD in principle describes all of nuclear physics - at all distance scales - *but how does it work?*

Deep Inelastic Scattering



$$\frac{d\sigma}{d\Omega dE'} \sim \sigma_{Mott} \left(\sum_i e_i{}^2 x \left[q_i(x, Q^2) + \bar{q}_i(x, Q^2) \right] \right)$$

Bjorken Limit: $Q^2 \to \infty, \nu \to \infty$

Empirically DIS region is where **logarithmic scaling** is observed

$Q^2 > 1$ (GeV/c)^2, $W^2 > 4$ GeV^2

Even worse : Averaged over W^2 region "works" also for $W^2 < 4$ GeV^2

Quark-Hadron Duality

Nachtmann ξ Final quark "on-shell" $\to (xp + q)^2 = 0$

$$2x(pq) - Q^2 + x^2 M^2 = 0$$

Neglect of M gives $x = Q^2/2(pq)$

Keeping $x^2 M^2$ **term, with** M **the proton mass, gives**

$$\xi = \frac{2x}{1 + \sqrt{1 + 4x^2 M^2/Q^2}}$$