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Final Technical Report

During the 2001-2003 grant period, IFS scientists made notable progress in a number of research areas. In the sections that follow, we summarize the work that has been accomplished during this period.

Magnetohydrodynamics

Magnetohydrodynamics (MHD) is a vibrant area of research at IFS, engaging the efforts of a wide cross-section of the Institute's scientists. We have restricted the present section to projects that are directly relevant to tokamak research and that do not rely on advanced computation. A description of other MHD work can be found in the sections on computational physics, fundamental plasma physics, and innovative confinement concepts.

Resistive Wall Modes

The promising Advanced Tokamak (AT) scenario is economically attractive provided that a close-fitting vacuum vessel can be depended upon to substantially raise the external-kink beta-limit. This is only possible if the resistive wall mode (RWM) can somehow be stabilized. Recent DIII-D experiments have demonstrated that this mode can indeed be stabilized via a combination of strong NBI-induced toroidal plasma rotation and a sufficiently small resonant error-field.

We have developed a simple set of evolution equations that govern the development of the resistive wall mode in a large-aspect-ratio, rotating, viscous, tokamak plasma.¹ These equations take into account the nonlinear deceleration of the plasma rotation generated by mode interaction with both the resistive wall and a static error-field. Our equations have been extremely helpful in interpreting RWM data recently obtained from the DIII-D tokamak (with GA collaborator Andrea Garofalo), NSTX spherical torus (with PPPL collaborator Steve Sabbagh), and HBT-EP tokamak (with Columbia collaborator Mike Mauel). In particular, the role of the error-field in triggering plasma deceleration has been elucidated.

Neoclassical Tearing Modes

Neoclassical tearing modes (NTM) were discovered independently at the IFS and the University of Wisconsin in 1986. They are expected to be one of the key performance-limiting instabilities in a burning plasma experiment. The primary question in assessing their impact is the nature and scaling properties of the observed excitation threshold for these modes. IFS, in collaboration with UKAEA-Culham, has made important contributions to the theoretical description of the threshold,² showing that polarization currents exert a stabilizing influence only when the island is rotating at a frequency between the ion and electron drift frequencies, contrary to earlier predictions. Our results have motivated recent experiments looking at classical tearing mode growth³ and at the role of rotation on NTM stability.⁴

¹ R. Fitzpatrick, "A simple model of the resistive wall mode in tokamaks," *Phys. Plasmas* 9, 3439 (2002).

² J. W. Connor, F. L. Waelbroeck, and H. R. Wilson, "The role of polarization current in magnetic island evolution," *Phys. Plasmas* 8, 2035-2045 (2001); F. L. Waelbroeck, J. W. Connor, and H. R. Wilson, "Finite Larmor-radius theory of magnetic island evolution," *Phys. Rev. Lett.* 87, 215003-215007 (2001).

³ M. S. Chu, R. J. La Haye, M. E. Austin, L. L. Lao, E. A. Lazarus, A. Pletzer, C. Ren, E. J. Strait, T. S. Taylor, and F. L. Waelbroeck, "Study of a low β classical tearing mode in DIII-D," *Phys. Plasmas* 9, 4584-4590 (2002).

⁴ R. J. La Haye, C. C. Petty, E. J. Strait, F. L. Waelbroeck, and H. R. Wilson, "Propagation of magnetic islands in the $E \times B = 0$ frame of co-injected neutral beam driven discharges in the DIII-D tokamak," *Phys. Plasmas* 10 (9): 3644-3648 (2003).

Response to Resonant Magnetic Perturbations

The response of a confined plasma to a resonant magnetic field is of interest in many contexts. Most importantly, it influences the dynamics of wall modes and the effect of externally imposed perturbations such as feedback corrections and error fields,⁵ and the size of islands in helical configurations such as stellarators.

We have extended the theory of the response of axisymmetric plasma to a resonant magnetic perturbation to allow for the long mean-free-path effects that are expected to be important in burning plasma experiments. We find that the braking force induced by an applied magnetic perturbation vanishes when the plasma rotation frequency is such that either the electrons are at rest in the frame of the perturbation or the electric field vanishes in the frame of the perturbation ($E_r=0$). The vanishing electric field root, however, is unstable.

We have successfully benchmarked a previously published analytic formula for the error-field penetration threshold in a rotating tokamak plasma against nonlinear MHD simulations in slab geometry.⁶ The fundamental assumption underlying this formula—viz., that the response of a flowing plasma to a low-amplitude, quasi-static, resonant magnetic perturbation is governed by linear, constant- ψ layer physics—was verified over a wide range of plasma parameters, including the parameter range relevant to ohmic and NBI start-up discharges in modern-day tokamaks.

Lastly, we have shown that a magnetic island chain formed by a saturated tearing instability in a toroidal magnetic fusion device can lock to a special class of externally generated magnetic perturbation in a stabilizing phase.⁷ These perturbations—termed “designer” error-fields—could be used to control the amplitudes of tearing modes in toroidal experiments without the requirement of fast phase modulation. This type of control would be far more feasible in a reactor environment than conventional active feedback control via external magnetic perturbations.

Magnetic Reconnection

We have performed two-dimensional, nonlinear MHD simulations in order to investigate the so-called Taylor problem, in which a small-amplitude boundary perturbation is suddenly applied to a tearing-stable slab plasma equilibrium, the perturbation being such as to drive magnetic reconnection within the plasma.⁸ For numerical reasons, the investigation is restricted to the limit of large magnetic Prandtl number. Our simulation results are in excellent agreement with the analysis of Hahn and Kulsrud (modified by strong plasma viscosity). At high perturbation amplitudes, the system exhibits a phase of Sweet-Parker reconnection, as predicted by Wang and Bhattacharjee. A novel expression for the threshold perturbation amplitude required to trigger Sweet-Parker reconnection has been derived and successfully benchmarked against numerical simulations. This expression suggests that a Sweet-Parker phase is only likely to occur during forced reconnection in tokamaks when the plasma is extremely hot and the perturbation amplitude relatively large.

We have developed an improved Laplace transform theory to investigate the initial response of a stable slab plasma equilibrium enclosed by conducting walls with respect to a suddenly applied

⁵ R. Fitzpatrick and F. L. Waelbroeck, “Nonlinear dynamics of feedback modulated magnetic islands in toroidal plasmas,” *Phys. Plasmas* 7, 1 (2000).

⁶ R. Fitzpatrick, “Plasma parameter scaling of the error-field penetration threshold in tokamaks,” to appear in *Phys. Plasmas* (2003).

⁷ R. Fitzpatrick and E. Rossi, “Control of tearing modes in toroidal fusion experiments using ‘designer’ error-fields,” *Phys. Plasmas* 8, 2760 (2001).

⁸ R. Fitzpatrick, “A numerical study of forced magnetic reconnection in the viscous Taylor problem,” *Phys. Plasmas* 10 (6): 2304-2312 (2003).

wall perturbation in the Taylor problem.⁹ If the wall perturbation is switched on slowly compared to the Alfvén time, then the plasma response eventually asymptotes to that predicted by conventional asymptotic matching theory. At early times, however, there is a compressible Alfvén wave-driven contribution to the magnetic reconnection rate, which leads to a significant increase in the reconnection rate. If the wall perturbation is switched on rapidly compared to the Alfvén time, the maximum pulse-driven reconnection rate can be much larger than that predicted by conventional asymptotic matching theory.

Finally, we have employed two-dimensional nonlinear Hall-MHD simulations in order to investigate the scaling of the rate of forced magnetic reconnection in the low-collisionality Taylor problem.¹⁰ The inclusion of the Hall term in the Ohm's law for the plasma is found to greatly accelerate the rate of magnetic reconnection.

MHD Stability of Transport Barriers in Burning Plasmas

A major goal of long-pulse burning plasma experiments is to examine the MHD stability of plasmas with large fractions of bootstrap current where the current has relaxed to steady-state values. The ideal MHD issues are most serious for thin transport barriers, for which the instability driving terms (pressure and current gradients) are largest. There are theoretical and experimental results that indicate transport barrier widths will scale as a certain power of the gyro-radius, such that they become thinner (compared to the minor radius) in ignition experiments and reactors.

We have examined such cases with the GATO and TOQ codes. We found that when the steady-state bootstrap current is included (as would pertain to long-pulse burning plasmas), MHD stability is improved for thinner transport barriers. The literature contains studies that omitted the steady-state bootstrap current (appropriate for short-pulse experiments) and reached the opposite conclusion. We have devised a simplified cylindrical model (where the toroidal curvature is replaced by a poloidally constant unfavorable curvature of equivalent size) that demonstrates the steady-state current can lead to improved ideal stability with decreasing barrier width. Only low mode numbers have been examined with GATO, due to computational expense (since the short equilibrium scales of the barrier must be coupled to the longer equilibrium scales).

Burning Plasma and Energetic Particle Physics

Our main accomplishments in the investigation of energetic particle effects in fusion plasmas during the past three years are a first-principle theoretical interpretation of Alfvén Cascades, which have been seen in the JET, JT-60U, and TFTR experiments; a simulation model that explains many features of energetic particle loss in the earlier Toroidal Alfvén Eigenmode (TAE) experiments on TFTR; and a theoretical description of phase space structures and frequency sweeping phenomena produced by energetic particle-driven instabilities.

Theory of Alfvén Cascades

Alfvén Cascades are observed in tokamak discharges with either non-monotonic safety factor profile $q(r)$ or nearly flat central $q(r)$. It is important to understand these modes and their possible impact on alpha particles because the Cascades are likely to accompany internal transport barriers (ITB), which also require reversed shear profiles. The Cascades generally consist of multiple modes that emerge with frequencies below the TAE gap and whose frequencies then rise to the

⁹ R. Fitzpatrick, "Wave driven magnetic reconnection in the Taylor problem," submitted to Phys. Plasmas 10 (11): 4284-4290 (2003).

¹⁰ R. Fitzpatrick, "Scaling of forced magnetic reconnection in the Hall magnetohydrodynamic Taylor problem," Phys. Plasmas 11 (3): 937-946 (2003).

TAE gap as the minimum value of q decreases (upward chirping trend). These modes are closely related to well-known shear Alfvén waves. However, physics mechanisms had to be identified that would favor the observed upward chirping. IFS scientists have found two such mechanisms. The first mechanism is rooted in the departure from conventional MHD that occurs when the orbit size of energetic particles is larger than the mode width.¹¹ The second mechanism results from second-order toroidal effects described by MHD.¹²

Our theory for Alfvén Cascades provides a basis for an "MHD diagnostic" that is now routinely used in JET experiments with ITB's. The experimental observation that Alfvén Cascades correlate in time with ITB triggering events has been used to control ITB formation through adjustments of the power input. We have also collaborated with PPPL and JET to identify Alfvén Cascades in TFTR alpha particle experiments for which the q -profiles were flat.¹³

Simulation of Energetic Particle Loss

Since good confinement of energetic alpha particles is required for self-sustained fusion burn, the nonlinear consequences of fast particle-driven Alfvén Instabilities are important. To study this, a joint investigation was initiated between the IFS and the National Institute for Fusion Science in Japan. A near-term goal was to develop a model for TAE-induced energetic particle loss in one of the early TFTR experiments. In order to perform a relevant simulation of the observed phenomena, we developed a reduced MHD model in which the mode structure is given by linear MHD theory, and the mode amplitudes evolve self-consistently with the energetic particle distribution.¹⁴ This computationally affordable model reproduces many features of the aforementioned TFTR experiment. It was found that most of the counter-injected particles were lost to the inner wall, but the co-injected particles stayed away from the loss boundary although they would move beyond the plasma edge. The principal predictions for energetic particles are (i) preferential accumulation of the particles that move along the toroidal plasma current; (ii) a nearly flat pressure profile of the energetic particles in the plasma core; and (iii) a steep energetic particle pressure gradient supported at the low field outer edge.

The main problem with the above model is that the calculated saturation level of the perturbed fields appears to exceed experimental estimates. As losses occur at an early stage of each burst, however, we expect the pulsation rate and the stored fast-ion pressure to be relatively insensitive to the loss mechanism. This is why our present simulations reproduce them reasonably well. Another problem is that the energetic particles appear to be resistant to edge diffusive loss despite significant perturbed fields at the edge.

Alpha Particle Instability in Fusion Experiments

In preparation for the 2002 Snowmass Fusion Summer Study, IFS actively contributed to a multi-institutional assessment of alpha particle-driven Alfvénic instabilities in ITER, FIRE, and IGNITOR.¹⁵ This assessment uncovered the following points:

¹¹ H. L. Berk, D. N. Borba, B. N. Breizman, et al., "Theoretical interpretation of Alfvén cascades in a non-monotonic q -profile," *Phys. Rev. Lett.* 87, 185 (2002).

¹² B. N. Breizman, H. L. Berk, M. S. Pekker, et al., "Alfvén eigenmodes in shear reversed plasma," *Proceedings of the 19th IAEA Fusion Energy Conference*, Paper TH/4-3 (2002).

¹³ R. Nazikian, H. L. Berk, S. E. Sharapov, et al., "New interpretation of alpha-particle-driven instabilities in deuterium-tritium experiments on the Tokamak Fusion Test Reactor," *Phys. Rev. Lett.*, 91 (12): Art. No. 125003 (2003).

¹⁴ Y. Todo, H. L. Berk and B. N. Breizman, "Simulation of intermittent beam ion loss in a Tokamak Fusion Test Reactor experiment," *Phys. Plasmas* 10 (7): 2888-2902 (2003).

¹⁵ N. N. Gorelenkov, H. L. Berk, R. Budny, W. W. Heidbrink, et al., "Study of thermonuclear Alfvén instabilities," *Nuclear Fusion* 43 (7): 594-603 (2003).

1. At a given plasma beta (often determined by MHD limits), the alpha particle pressure scales with temperature as $T^{6/5}$ over a considerable temperature range. Hence the alpha drive for Alfvén instabilities increases at higher temperatures. This suggests operating at the highest density and lowest temperature that are compatible with other constraints.
2. The energetic particle beta in the nominal operation scenarios is below the anomalous loss threshold in previous experiments. However, alpha-driven instabilities may be possible in ITER advanced scenarios that have a plasma temperature of ~ 30 keV.
3. With regard to proposed experiments, IGNITOR is expected to be free from Alfvén instability. By means of a global analysis with the NOVA code, FIRE was also predicted to be stable, despite violating local stability criteria. The nominal operating point of ITER, by contrast, was found to be slightly unstable from global NOVA code calculations. In addition, it was found that the 1 MeV neutral beams planned for ITER can produce an instability drive that is roughly half the size of the alpha particle drive.

Nonlinear Frequency Sweeping

Frequency sweeping is often a signature of nonlinear structures in phase space. There is evidence that such nonlinear structures emerge from energetic particle-driven modes, including TAE and fishbone modes. Accounting for fast frequency sweeping is an important issue related to alpha particle confinement in burning plasma experiments. We have shown that the nonlinear evolution of fishbone oscillations is dominated by the MHD nonlinearity associated with the Alfvén continuum resonance.¹⁶ We studied numerically the consequences of this MHD nonlinearity when the energetic particle drive can be described by linear theory, and found that it changes the dynamics of unstable perturbations, but does not stop their growth. We conclude from this that the kinetic nonlinearity also needs to be accounted for. In another investigation, a Fokker-Planck code simulating the bounce-averaged limit showed a rapid termination of frequency sweeping for certain initial conditions.¹⁷ We explained this rapid termination in terms of adiabatic theory without collisions. There are relevant cases, however, where this theory breaks down and a more sophisticated theory needs to be developed.

Novel Control Techniques in Burning Plasmas

It is widely recognized that plasma rotation can stabilize MHD resistive wall modes, but obtaining the needed rotation with acceptable power input in a reactor is very difficult. There is also a serious feasibility issue for internal transport barriers in burning plasmas. Specifically, the heating power in a burning plasma will increase as the confinement improves, implying a high likelihood for disruptions.

This is accomplished by applying small, non-axisymmetric magnetic perturbations that induce stochasticity in high-energy trapped alpha particles (but not in bulk plasma particles). We have examined this concept with a Monte-Carlo code.¹⁸ With tailored magnetic perturbations ($\sim 1\%$ of the toroidal field), nearly all trapped alpha particles are completely lost, with little effect on bulk particles. Heat loads and erosion on the first wall are acceptable for a liquid wall, but not a solid wall. For a low-aspect-ratio case ($A \sim 2$), calculations indicate that trapped alphas become de-trapped after passing the separatrix and could be deposited in a liquid-wall divertor. A carefully chosen spectrum of perturbations could cause alpha particles to be removed from inside a transport barrier and thermalized within the ambient plasma. We predict that this concept is

¹⁶ A. Odblom, B. N. Breizman, S. E. Sharapov, T. C. Hender, and V.P. Pavukhov, "Nonlinear magnetohydrodynamical effects in precessional fishbone oscillations," *Phys. Plasmas* 9, 155 (2002).

¹⁷ D. Yu. Eremin and H. L. Berk, "Frequency sweeping of phase space structures," *Phys. Plasmas* 9, 772 (2002).

¹⁸ M. Kotschenruther, L. Zheng, and T. Rognien, "Improved plasma performance through innovative boundary control techniques," in *Proceedings of the 19th IAEA Fusion Energy Conference* (Lyon, 2002). Paper IC/P-04.

feasible with solid walls in the proposed ignition devices. Our calculations of the torque imply that the rotation speeds needed to wall-stabilize $n > 1$ modes (using ARIES Advanced Tokamak estimates) can be achieved.

Turbulent Transport

IFS turbulent transport research has become increasingly collaborative. For example, our most significant achievements during the last grant period included our participation in landmark studies on electron temperature gradient turbulence^{19,20} and on zonal flow generation²¹ in collaboration with University of Maryland and Tore Supra scientists. We have also diversified recent IFS turbulent transport research into activities such as database analysis and integrated systems analysis (the study of the feed-forward and feedback pathways between transport, MHD stability, and profile control). Our research has benefited greatly from collaborations, both national and international, with groups that operate and maintain large advanced numerical tools.

Transport Driven by the Electron Temperature Gradient

ETG turbulence is believed to be the dominant cause of electron transport in tokamak experiments. Important insights concerning its properties have been gained through simulations performed with the gyrokinetic code GS2, developed at the IFS. First, ETG-driven modes were found to produce transport levels much larger than expected. It was recognized at the IFS that there were essential differences in the dynamics of the zonal flows for the ETG and ion-temperature-gradient (ITG) problems. In a collaboration led by U. of Maryland scientists using GS2, it was found that the zonal flows were weaker in the ETG case, allowing the unfettered development of "streamers" (radially extended convection cells).²⁰ Further investigations led to the development of a theory of streamer-dominated transport.²²

In a separate collaboration with the Tore Supra group, we demonstrated the presence of a clearly defined threshold in the dependence of the turbulent heat diffusivity on the electron temperature gradient.^{19,23} The dominant dependence of the inferred critical temperature gradient is proportional to the magnetic shear, as expected from the robust electron Landau damping on thermal electrons. The data are in good agreement with the ETG transport model that we developed. This model is now a standard component in the CRONOS integrated system code at Tore Supra.

Transport Driven by the Ion Temperature Gradient

Important differences between ITG and ETG occur in (i) the effect of sheared flows, (ii) the effect of nonlinear coupling, and the (iii) the conditions for the development of an electromagnetic component. University of Maryland scientists, using the GS2 code, examined the mechanism for the generation of zonal flows and their stability. The conclusion of this work is that the radial streamers generated by the ITG instability are unstable to secondary Kelvin-

¹⁹ G. T. Hoang, C. Bourdelle, X. Garbet, G. Giruzzi, T. Aniel, M. Ottaviani, W. Horton, P. Zhu, and R. V. Budny, "Experimental determination of critical threshold in electron transport on Tore Supra," *Phys. Rev. Lett.* **87**, 125001 (2001).

²⁰ W. Dorland, F. Jenko, and M. Kotschenreuther, "Electron temperature gradient turbulence," *Phys. Rev. Lett.* **85**, 5579-5582 (2000).

²¹ B. N. Rogers, W. Dorland, and M. Kotschenreuther, "Generation and stability of zonal flows in ion-temperature gradient mode turbulence," *Phys. Rev. Lett.* **85**, 5336-5339 (2000).

²² F. Jenko, W. Dorland, M. Kotschenreuther, and B. Rogers, "Electron temperature gradient driven turbulence," *Phys. Plasmas* **7**, 1904-1910 (2000).

²³ G.T. Hoang, W. Horton, C. Bourdelle, B. Hu, X. Garbet, and M. Ottaviani, "Analysis of the critical electron temperature gradient in Tore Supra," *Phys. Plasmas* **10** (2), 405-412 (2003).

Helmholtz instabilities. The interaction of the secondary instabilities with the streamers leads to the generation of zonal flows. Near the marginal stability of the ITG modes, moderate shearing rates are sufficient to stabilize the system, leading to the phenomenon of the Dimits up-shift.

In another investigation, a relatively recent fluid closure devised to describe Landau damping²⁴ was compared to the well-known Hammett-Perkins closure and to the results of gyrokinetic simulations.²⁵ It was found that the new closure avoids the over-estimation of transport that is known to affect the Hammett-Perkins model.

Transport Modeling

Integrated physics and nonlinear dynamics simulations are needed to model the complex interaction between the effects of turbulence and MHD on evolving profiles in burning plasma. One of the challenges is the modeling of the combined physics of the edge H-mode and the internal transport barrier (ITB). We have developed models for the ITB by coupling the equilibrium sheared flows to the full neoclassical transport matrix in the parallel-architecture code TBD.²⁶ In collaboration with General Atomics, we used the ratio of the E_r -shear to the magnetic shear to parameterize the stabilizing effect of rotation and obtained good agreement with observations of barrier formation and evolution.²⁷

In collaboration with the Lehigh University group, simulation modules containing the combined neoclassical matrix elements and the ITG turbulent growth rates were applied to the problem of understanding two particular JET discharges that had been presented as a challenge to the transport community. The predictive code was subsequently applied to the analysis of the ITB formation in DIII-D shot 84682.²⁸ In another study, we simulated a classic high-field burning plasma with the BALDUR transport code, taking into account the symbiotic relation between the current penetration transient and the reversed magnetic shear transport barrier.²⁹ The simulation produced ignition at 4 seconds but was limited in time by kink mode activity at $q_{min}=1$. The kink mode starts well off axis at q_{min} and soaks into the core of the burning plasma, terminating the fusion reactions after about one second.

Computational Physics

Numerical computation is playing an increasingly large role in IFS research. In this section we present an overview that illustrates the diverse nature of IFS computational projects, from the use of legacy codes to explore new ideas, to the development and application of relatively new computational techniques.

²⁴ H. Sugama, T. H. Watanabe, and W. Horton, "Collisionless kinetic-fluid closure and its application to the three-mode ion temperature gradient system," *Phys. Plasmas* **8**, 2617-2682 (2001).

²⁵ H. Sugama, T. H. Watanabe, and W. Horton, "Comparison between kinetic and fluid simulations of slab ITG driven turbulence," *Phys. Plasmas* **10**, 726 (2003).

²⁶ W. Horton and P. Zhu, "Transport barrier dynamics," *Phys. Plasmas* **7**, 4534-4546 (2000).

²⁷ G.M. Staebler, J.E. Kinsey, P. Zhu, G. Bateman, W. Horton, A.H. Kritz, T. Onjun, A. Pankin, and R.E. Waltz, "Drift wave-based modeling of poloidal spin-up precursor and step-wise expansion of transport barriers," in *Fusion Energy: 18th Conference Proceedings*, Sorrento, Italy, 4-10 October 2000 (International Atomic Energy Agency, Vienna, 2001), Paper TH4/1.

²⁸ P. Zhu, G. Bateman, A. Kritz, and W. Horton, "Predictive transport simulations of internal transport barriers," *Phys. Plasmas* **7**, 2898-2908 (2000).

²⁹ B. Hu, W. Horton, P. Zhu, and F. Porcelli, "Density profile control with current ramping in a transport simulation of IGNITOR," *Phys. Plasmas* **10** (4), 1015-1021 (2003); W. Horton, F. Porcelli, P. Zhu, A. Aydemir, Y. Kishimoto, and T. Tajima, "Ignitor physics assessment and confinement projections," *Nucl. Fusion* **42**, 169-179 (2002).

The objective of the AEGIS (Adaptive Eigenfunction Independent Solution) project is to develop a fast, generalized magnetohydrodynamic (MHD) stability code that is able to address kinetic effects such as plasma rotation, finite Larmor radius effects, wave-particle resonance effects, and energetic particle effects. These kinetic effects are often important in thin resonant layers, so a spatially adaptive solution procedure has been devised to efficiently ensure adequate resolution. The algorithm is designed to be highly efficient on parallel architectures. The present version of the code has been successfully benchmarked with GATO for beta-limiting and kink instabilities.

Edge and Divertor Studies

We have proposed a new magnetic divertor configuration for tokamaks that pulls the field lines out past the toroidal field coils and into an external chamber.³⁰ This has two advantages: (a) it reduces one of the most serious engineering constraints by providing a large region outside the strong toroidal field to deal with the heat load; and (b) it significantly increases energy confinement by profoundly reducing the recycling. This novel divertor configuration could lead to the development, on a much shorter time scale than presently envisaged, of dramatically smaller ignition devices and fusion neutron sources.

Free-Boundary MHD Simulations

We have used the nonlinear, 3D resistive-MHD code called CTD, which can efficiently treat resistive walls and free boundary displacements in moderately-shaped geometries, in a number of studies including (i) studies of vertical displacement events (VDEs) in shaped tokamaks,³¹ (ii) studies of bifurcation and relaxation of radial electric fields in enhanced reversed shear tokamak plasmas,³² and (iii) the effects of liquid metal walls on equilibrium and stability in tokamaks.³³

Two-Fluid MHD Simulation

Two theoretical developments at the IFS and a related new experiment at the Fusion Research Center call for the development of a two-fluid code. The two theoretical developments are the discovery of a new closure based on relativistic two-fluid equations³⁴ and the discovery of generalized relaxed states in Hall-MHD systems.³⁵ The FRC experiment, named the Magneto-Bernoulli Experiment (MBX), is aimed at creating and studying these generalized relaxed states.³⁶ A similar experiment at the University of Maryland, named MCT, is also expected to provide information on Hall-MHD relaxation.

- ³⁰ M. Koepschevenuehler "Exhausting the plasma flux outside the TF coils", Bull. Am. Phys. Soc. 47, 185 (2002).
- ³¹ A. Y. Aydemir, "Vertical displacement events in shaped tokamaks," IFS Report #882 (2000).
- ³² K. C. Shaing, A. Y. Aydemir, R. Hiwatari, W. A. Houlberg, Y. Ogawa, and M. C. Zarnstorff, "Bifurcation and relaxation of radial electric field in enhanced reversed shear tokamak plasmas," Phys. Rev. Lett. 83, 3840 (1999).
- ³³ A. Y. Aydemir, "Effects of liquid metal walls on equilibrium and stability in tokamaks," Phys. Plasmas 8 (7), 3411-3418 (2001).
- ³⁴ R. D. Hazeltine and S. M. Mahajan, "Fluid description of relativistic, magnetized plasma," Ap. J. 567, 1 (2002).
- ³⁵ Z. Yoshida, S. M. Mahajan, S. Ohaki, M. Iqbal, and N. Shatashvili, "Beltrami fields in plasmas: High-confinement mode boundary layer and high beta equilibria," Phys. Plasmas 8, 2125 (2001); P. D. Mithani, D. O. Gomez, and S. M. Mahajan, "Dynamo action in Hall Magnetohydrodynamics," Ap. J. Lett. 567, L61 (2002); S. M. Mahajan, R. Mikiaszewski, K. I. Nikoifskaya, and N. L. Shatashvili, "Dynamical creation of channels for the particle escape in the solar corona," to appear in Ap. J. (2002); S. M. Mahajan and R. D. Hazeltine, "Sheared-flow generalization of the Harris sheaf," Phys. Plasmas 7, 1287 (2000); Z. Yoshida and S. M. Mahajan, "Variational principles and self-organization in two-fluid plasmas," Phys. Rev. Lett. 88, 095001 (2002).
- ³⁶ H. J. Quevedo, R. Bengtson, S. M. Mahajan, and P. M. Valanju, "Sheared magnetofluids and Bernoulli confinement," Bull. Am. Phys. Soc. 46, 275 (2001).

Numerical simulation of the MBX and MCT experiments requires finding the equilibria in both the initial states and the postulated final relaxed states. The analytic results for cylindrical geometry need to be extended, first to toroidal geometry and then to arbitrary geometry. We will then study the approach to equilibrium by evolving the plasma with suitable dissipative terms. The magnetofluid equations that we will use in the initial stage of this work are derived from a thermodynamic closure to the fluid equations and are somewhat simpler than the relativistic two-fluid description but are of the same general form.

Neutrals

The existing 3-D neutral transport code NUT performs calculations of neutral penetration into plasmas of arbitrary shape much faster than Monte Carlo methods (< 0.1 sec on a desktop computer). NUT owes its speed to the use of exact analytic solutions that are valid up to the scale lengths over which the plasma parameters and the corresponding neutral mean-free-paths vary. The calculation speed scales as a high power (4 to 6) of the grid spacing. NUT has been used successfully in the analysis of spatially resolved H_α data from TEXT and TFTR. A current version, NUT-0 (<http://www.frc.utexas.edu/pv/Nut0Manual/index.html>), was reviewed and approved as an NTCC module.

Fundamental Theory

Novel Fluid Closures

We have developed a novel relativistically covariant fluid closure that is valid for all collisionality regimes.³⁷ A computer code is being developed in order to solve a set of extended-MHD equations that use this new closure. Radiation resistance is calculated using the relativistic reaction prescription first introduced by Landau and Lifschitz, and recently shown to be exact by Rohrlich. We have found that radiative reaction on the fluid, although involving relatively high-order moments of the distribution, can be fully described without introducing additional variables.

Relaxed States in Hall MHD

IFS theoretical investigations have revealed the existence of equilibrium states of a two-fluid plasma that differ qualitatively from those accessible to conventional MHD plasmas. These new equilibrium states, called double-Beltrami states, are the end product of a relaxation governed by the strong nonlinear interaction between the sheared velocity field and the magnetic field. The relaxation process is conjectured to take place under the dual constraints of the constancy of the standard magnetic helicity (electron helicity when electron inertia is neglected) and the generalized ion helicity. The prediction of these double-Beltrami states formed the basis for the design of the Magnetic Bernoulli Experiment (MBX) at the Fusion Research Center. Supported by the DOE/OFES program in innovative approaches to fusion energy science, MBX has begun experimental operation (<http://www.frc.utexas.edu/pv/MBX/index.html>). The stability theory for strongly sheared plasmas is beset with physical as well as mathematical problems (e.g., non-hermiticity of the operators). We have applied variational methods and modal analysis to study the stability of Beltrami and double-Beltrami flows. The perturbations are an interesting combination of the Kelvin-Helmholtz and Alfvénic motions, which oppose or assist one another.³⁸

³⁷ R. D. Hazeltine and S. M. Mahajan, "Generalization of collisional fluid theory to long mean-free-path and relativistic motion," *Phys. Plasmas* 9, 3341 (2002).

³⁸ A. Ito, Z. Yoshida, T. Tazuno, S. Ohsaki, and S. M. Mahajan, "Kelvin-Helmholtz instability in Beltrami fields," *Phys. Plasmas* 9, 4856-4862 (2002).

We applied our analysis of the double-Beltrami states to laboratory and astrophysical plasmas. In a laboratory plasma, we have provided an alternative picture of the H-mode as a self-organized state in the shear layer³⁹ and have initiated an investigation of the phenomenon of current holes in discharges with internal transport barriers. Current holes are a remarkable manifestation of perfect diamagnetism that can be interpreted as the experimental realization of a double-Beltrami state. Reflecting the principal role of the ion flow, the shielding length for the plasma is determined by the ion skin depth. Regarding astrophysical applications, we have contributed insights on the formation and heating of the solar corona,⁴⁰ the creation of flows and the solar wind, the granulation structure on the solar surface, and the coronal mass ejections in the solar atmosphere.⁴¹

Upper Bounds in Turbulence

Statistical approaches for explaining turbulence lead to the problem of closure, which has instigated various physical but *ad hoc* assumptions. An alternative approach is to manipulate the equations of motion to obtain bounds on physical quantities, such as the energy dissipation rate. We have applied a method based on this approach, the "background method" of Doering and Constantin, to incompressible MHD fluid turbulence in shear flow configurations.⁴²

Hamiltonian Chaos, Renormalization, and Transport

The equations that describe charged particle dynamics and magnetic field lines are well known to be Hamiltonian. This has led fusion scientists to make important contributions to the field of low-degree-of-freedom Hamiltonian chaos theory. An effective means of studying low-degree-of-freedom Hamiltonian systems is to analyze symplectic maps that describe Poincaré sections. Invariant tori in these symplectic maps provide transport barriers, and such maps have been used to model tokamaks and other devices. The maps most studied satisfy the twist condition corresponding to field line systems that describe tokamaks with monotonic q -profiles. In earlier work, the IFS pioneered the study of non-twist systems (reverse shear configurations) and derived the standard nontwist map. As a result of this work, the nature of the breakup of shearless tori is now understood in great detail and their resilience is widely appreciated.⁴³

Hamiltonian Description of Fields

The Vlasov equation, ideal MHD, and all other ideal fluid and kinetic plasma models are Hamiltonian field theories, i.e., infinite degree-of-freedom Hamiltonian systems. These basic models possess action functional descriptions that are infinite dimensional generalizations of Hamilton's and other variational principles of mechanics.

Hamiltonian structure facilitates stability analyses, perturbation theory, and the construction of constants of motion.

³⁹ S. M. Mahajan and Z. Yoshida, "A collisionless self-organizing model for the high-confinement (H-mode) boundary layer," *Phys. Plasmas* 7, 635 (2000).

⁴⁰ S. M. Mahajan, R. Miklaszewski, K. I. Nikol'skaya, and N. L. Shatashvili, "Beltrami fields in plasmas: High-confinement mode boundary layers and high-beta equilibria," *Phys. Plasmas* 8, 2125 (2001).

⁴¹ S. Ohsaki, N. L. Shatashvili, Z. Yoshida, and S. M. Mahajan, "Magnetofluid coupling: Eruptive events in the solar corona," *Astrophys. J.* 559, L61-65 (2001).

⁴² F. Petrelis, A. Alexakis, C. Doering, and P. Morrison, "Bounds on dissipation in magnetohydrodynamic problems in plane shear geometry," *Phys. Plasmas*, 10 (11): 4314-4323 (2003); A. Alexandros, F. Petrelis, P. J. Morrison, and C. R. Doering, "Bounds on dissipation in magnetohydrodynamic Couette and Hartmann shear flows," *Phys. Plasmas* 10 (11): 4324-4334 (2003).

⁴³ A. Apra, A. Wurm, and P. J. Morrison, "Renormalization and destruction of I_γ gamma(2) tori in the standard nontwist map," *Chaos* 13 (2): 421-433 (2003).

Statistical Mechanics of Continuum Systems

Since the early work of Onsager and Lee, attempts have been made to describe continuum systems (i.e., the partial differential equations of fluid and plasma dynamics) by techniques of statistical mechanics. In this approach, partition functions are calculated, for which modes of the continuum system may be taken to be analogous to particles, and arguments are made whereby enstrophy or other ideal invariants may serve as entropy-like quantities. Such statistical mechanics descriptions are possible because of the Hamiltonian structure described above.

Innovative Confinement Concepts

In recent years, interest has grown in expanding the scope of fusion research to allow for the exploration of innovative confinement concepts. The IFS has theoretically investigated three of these concepts—the reversed field pinch, the magnetic dipole, and the belt pinch.

Reversed Field Pinches

In nearly all reversed field pinch (RFP) experiments, the $m=1$ and $m=0$ tearing modes present in the plasma phase-lock together to form a highly peaked, strongly toroidally localized pattern in the perturbed magnetic field. This pattern, commonly known as the "slinky" pattern, can give rise to severe edge loading problems that limit the maximum achievable toroidal current. In collaboration with researchers at the University of Padua, we have developed a theory that explains virtually all salient features of the slinky pattern seen in the Reversed Field experiment (RFX).⁴⁴ The central premise of this theory is that at high ambient mode amplitude, the various tearing modes occurring in the plasma phase-lock together in a configuration that minimizes the magnitudes of the electromagnetic torques exerted at the various mode rational surfaces. The theory successfully predicts the profiles of the edge radial and toroidal magnetic fields generated by the $m=0$ and $m=1$ modes, the phase relations between the various modes, the presence of a small toroidal offset between the peaks of the $m=0$ and $m=1$ contributions to the overall slinky pattern, and the response of the pattern to externally generated $m=0$ and $m=1$ magnetic perturbations.

We have found remarkable agreement between experiment and theory, and are now confident that we understand the various factors that determine whether or not tearing modes rotate in RFP plasmas. This is an important area of RFP research, since non-rotating tearing modes have a highly deleterious effect on RFP performance.

Magnetic Dipoles

We have carried out an investigation of the stability of magnetic dipole configurations aimed at contributing to the theoretical base necessary for interpreting data from the magnetic dipole experiment being constructed at MIT, as well as data from space observations of magnetic substorms in the Earth's geotail. More generally, our investigation serves to advance our understanding of the properties of very high beta plasmas.

Magnetic dipole configurations exist in planetary magnetospheres and have the attractive feature that they are stable to ideal MHD plasma instabilities even for high beta. Ideal stability is ensured by plasma compressional effects when the scale length for the plasma pressure is comparable to or greater than the radius of curvature of the magnetic field lines. The MHD model, however, is known to be inadequate for collisionless plasmas. This has motivated us to use a quadratic variational form derivable from the drift kinetic equation. We developed a procedure for

⁴⁴ R. Fitzpatrick and P. Zanca, "Phase Locking of tearing modes in the reversed field experiment," *Phys. Plasmas* 9, 2707 (2002).

evaluating stability in terms of the ratio of the orbit-averaged ion drift frequency to the ion diamagnetic frequency. This procedure allowed us to determine the stability boundaries for both MHD and drift modes.⁴⁵

We have applied our analysis to a two-dimensional analogue of the point dipole equilibrium. By examining the particle drifts, we verified MHD stability for high as well as low plasma beta. We confirmed this result by solving the ballooning eigenmode equation. We have also examined the stability of a local flux surface equilibrium in the Earth's geotail. We concluded that the Earth's geotail is unstable whenever the plasma gradient becomes sufficiently steep that the product of the curvature of the Earth's vacuum magnetic field and the plasma pressure gradient scale length exceeds the value of 0.4. Such instabilities could serve as a trigger for magnetic substorms when the geotail is compressed and stressed during periods of enhanced solar wind activity.⁴⁶

At the outer flux surfaces of finite-volume magnetic dipole configurations, where the pressure may be forced to decrease over a scale length shorter than the field line curvature, the criterion for compressional stabilization is violated and collisionless MHD instabilities are likely to occur. It is thus of interest to explore whether non-ideal effects, such as finite particle Larmor radius, can significantly reduce ideal MHD growth rates, thereby allowing sharper pressure gradients at the edge and correspondingly more compact dipoles. We have investigated the stability of the outer boundary of a cylindrically symmetric kinetic equilibrium, using an integral eigenmode equation valid for arbitrary Larmor radius. We find that when the plasma frequency exceeds the cyclotron frequency within the boundary layer, FLR stabilization is ineffective if the conducting wall is at a distance larger than a small fraction of a Larmor radius from the plasma edge.

An additional problem is that with large pressure gradients, particle drifts can be reversed at sufficiently high plasma beta, thereby destabilizing compressional modes. The compressional response continues to be stabilizing only if the curvature drift frequency is also larger than typical MHD growth rates. This requirement is difficult to satisfy, but may be achievable in a two-component plasma where a small population of "hot" particles support the equilibrium pressure gradient, while the principal "background" component provides a large plasma inertia.

Belt Pinch and Liquid Metal Walls

Over twenty years ago, belt pinch experiments found ideal MHD stable plasmas with beta ~ 50%, and "normal" aspect ratio, but abnormally large elongation ($\kappa < 5$). These plasmas benefited from wall stabilization with a very close fitting shell. Inspired by these experiments, we put forward the idea that using liquid metal walls may allow belt pinches to become attractive fusion power plants.⁴⁷ Liquid metal alloys containing lithium do not suffer the engineering problems of solid conductors, and could be used as a very close fitting shell under DT reactor conditions.^{48,49} We have examined several related issues.

First, we investigated the beta limits of elongated tokamaks ($2 < \kappa < 4$) with an ideal wall using the GA codes GATO and TOQ for reactor-relevant equilibria with high bootstrap fraction. We

⁴⁵ H. Vernon Wong, W. Horton, J. W. Van Dam, and C. Crabtree, "Low frequency stability of geotail plasma," *Phys. Plasmas* 8 (5), 2415-2424 (2001).

⁴⁶ W. Horton and H. V. Wong, "Stability properties of high-pressure geotail flux tubes," *J. Geophysical Research* 106, 18,803 (2001).

⁴⁷ M. Kotchenreuther et. al., "Reactor advantages of the belt pinch and liquid metal walls", in *Fusion Energy: 18th Conference Proceedings*, Sorrento, Italy, 2000 (IAEA, Vienna, 2001), Paper ICP/13.

⁴⁸ M. A. Abdou, A. Ying, N. Morley, K. Gulec, S. Smolentsv, M. Kotchenreuther, et. al., "On the exploration of innovative concepts for fusion chamber technology," *Fusion Eng. Des.* 54, 181-247 (2001).

⁴⁹ M. Z. Youssef, H. Khater, M. Kotchenreuther "Plasma conducting shells and their impact on TBR and activation in CLIFF design," *Fusion Technol.* 39, 804-809 (2001).

found that ballooning beta limits increase by a factor of ~ 2 as κ increases from 2 to 3, and by another factor of ~ 1.5 for κ up to 4, but then saturates. As κ increases, higher n external modes require wall stabilization, necessitating a progressively closer fitting shell. For elongations $\kappa \sim 3$ -4, the relevant mode numbers are too large (> 5) to calculate with GATO at reasonable expense.

Second, we have written a novel finite element, initial value code to examine resistive wall axisymmetric instabilities using the perturbed Grad-Shafranov equation. At high elongation, axisymmetric instabilities require stabilization by a passive shell and an active feedback system. For a conducting shell corresponding to 2 cm of lithium with reactor dimensions, axisymmetric mode growth rates are low enough to be actively stabilized for a wall distance ~ 10 -20 % of the minor radius. We have found positions for sensor and active loops that give stabilization, for active coils located behind a neutron shield.

Plasma Applications

Plasma Thrusters

IFS scientists have been developing a theoretical physics basis for plasma thrusters within the context of the Variable Specific Impulse Magnetoplasma Rocket (VASIMR) project. The VASIMR device has a magnetic mirror configuration and consists of three main components: a low-energy helicon plasma source, which creates cold plasma via RF-discharge; an ion cyclotron resonance heating (ICRH) section, which is used to deposit RF-power into the plasma; and a magnetic nozzle, which forms a highly directed super-Alfvénic outgoing plasma flow. IFS scientists have addressed fundamental physics issues related to all of these three components.

Helicon source: We showed that the radial density gradient in a plasma forms a potential well for the helicon modes.³⁰ The resulting mode frequency is significantly lower than that for a uniform plasma. This may explain the high efficiency of the source at low frequencies. We are currently developing a first-principle theory for helicon sources based on a self-consistent treatment of the particle balance, power balance, and RF-field structure. The problem of particle balance reduces to kinetic ion transport under the effect of the ambipolar electric field and ion-neutral collisions.³¹ The three key physics ingredients have been combined into a 1-D numerical model for a source with predominantly radial flow.³² Our theory of plasma flow inside the source provides a physical explanation for the substantial gas pressure increase and the transition of plasma profiles observed experimentally.

ICRH: We developed a self-consistent nonlinear model for RF-power absorption in the ion cyclotron frequency range by a steady-state plasma flow,³³ generalizing the well-known linear magnetic beach problem of Stix. We verified the theoretical predictions by 1-D fluid-type simulations of the RF-field structure and wave energy conversion into ion flow energy.

Magnetic nozzle: The importance of our contributions to the understanding of plasma detachment in VASIMR was recently highlighted by the project leader and also by the VASIMR peer review panel. The work of the entire VASIMR team, which includes scientists from the IFS, was recognized with the Star Award for excellence in 2000.

³⁰ B. N. Breizman and A. V. Arefiev, "Radially localized helicon modes in non-uniform plasma," *Phys. Rev. Lett.* **84**, 3863-3866 (2000).

³¹ B. N. Breizman and A. V. Arefiev, "Ion kinetics in a magnetized plasma source," *Phys. Plasmas* **9**, 1015-1024 (2002).

³² A. V. Arefiev, "Theoretical studies of the VASIMR plasma propulsion concept," IFS Report #949, Ph.D. dissertation, University of Texas at Austin (2002).

³³ B. N. Breizman and A. V. Arefiev, "Single-pass ion cyclotron resonance absorption," *Phys. Plasmas* **8**, 907-915 (2001).

Light Trapping in Semiconductor Plasmas

The advent of femtosecond lasers has made it possible to study phenomena such as frequency upshifting in semiconductor, rather than gaseous, plasmas. A semiconductor plasma is comprised of electrons and holes. As long as the laser fluence is kept below their damage threshold, semiconductor plasmas have several advantages over gaseous plasmas as experimental systems: viz., automatic confinement, initial homogeneity, easy control of density over a wide range, and provision for inexpensive and reproducible table-top experiments.

In an earlier IFS paper (1999), it had been found that blue-shifting and also wave trapping of light pulses can occur in semiconductor plasmas. This prediction was made at the same time as the well-publicized discovery at Harvard University of light trapping in Bose-Einstein condensates. Light trapping in semiconductors would, however, have the advantage of inexpensive room-temperature operation; also it would have the group velocity go to zero, rather than the phase velocity as in BEC materials. An experimental paper last year by a group at MIT indeed did find slowing of light in a semiconductor crystal.

Following up on the initial prediction of light trapping in semiconductor plasmas, the IFS (in a collaboration with Georgian Academy of Sciences) found that the addition of a second pump laser pulse can lead to a cascade of further up-shifting, due to multiple reflections of the probe pulse.⁵⁴ This result of a "machine gun" of increasingly blue-shifted laser pulses is relevant to modern efforts for the development of coherent tunable radiation sources.

Dynamics of Singular Laser Beams

The behavior of laser beams with phase singularities (or wave-front dislocations) is important for research into all-optical signal processing and logic. IFS scientists (in a collaboration with Georgian Academy of Sciences and Tokyo University) studied the dynamics of singular laser beams in plasmas, rather than in optical media. It was found that singular beams carrying angular momentum have robust and predictable behavior, leading to the formation of long-lived, compact, and moderately stable filamentary bundles of ultra-high intensity electromagnetic radiation.⁵⁵ These results could prove to be important for plasma accelerators and fast ignitor schemes.

Physics of Laser-Irradiated Micro-Clusters

The IFS has established a collaboration with the Texas High-Intensity Optical Research (THOR) laser group. The objective of the collaboration is to contribute to the theory for high-pressure plasmas that result from the explosion of solid targets irradiated by very short-pulse, intense lasers. The interest in laser-cluster interaction is motivated by the results of experiments that produced fusion reactions by irradiating deuterium micro-clusters with a very short laser beam. There are two conceivable scenarios for the cluster explosion: an electrical and a thermal one. The essential difference is that the thermal scenario preserves quasi-neutrality whereas the electrical scenario does not. The choice between the two depends on the response of the electrons. An IFS model of the electric explosion shows that there is a natural trend for the electron population to have a two-component distribution function:⁵⁶ a cold core that responds to the laser field adiabatically and a halo that undergoes stochastic heating. Two-dimensional PIC simulations

⁵⁴ V. I. Berozhiani, S. M. Mahajan, and R. Miklaszewski, "Multiple reflections: Cascaded upshifting of laser pulses by semiconductors," *J. Opt. Soc. Am. B* **18**, 617-622 (2001).

⁵⁵ V. I. Berozhiani, S. M. Mahajan, Z. Yoshida, and M. Pekker, "Dynamics of self-trapped singular beams in an underdense plasma," *Phys. Rev. E* **65**, 046415-1 - 046415-6 (2002).

⁵⁶ B. N. Breizman and A. V. Arifiev, "Electron response in laser-irradiated micro-cluster," *Plasma Physics Reports*, **29** (7): 593-597 (2003).

of a single filament-like cluster confirm this. We have also found that the explosion scenario affects the symmetry of the ion expansion.

Plasma Processing

We have investigated the dynamics of AC sheaths used for etching and the filling of trenches in the plasma processing of semiconductors. We focused primarily on the low-frequency regime where the RF frequency is small compared to the ion plasma frequency and to the transit frequency for ions crossing the sheath. This regime is of interest in developing pulsed etching processes in which the electron population is allowed to decay so as to create pure ion-ion plasmas that allow negative-ion etching through sheath inversion. We have pointed out the role of the blocking capacitance in determining the transient response in such pulsed discharges.³⁷ In a separate investigation, we have also studied the role of presheath processes and shown that reducing the RF frequency below the presheath transit frequency leads to a modulation of the ion current, with a beneficial effect on the ion energy distribution function at the substrate surface.³⁸ More recently, we have discovered through numerical simulations that sheath inversion can also occur in the high-frequency regime for sufficiently high bias potential.

³⁷ F. L. Waelbroeck, "Analytic solution for low-frequency rf sheaths in pulsed discharges," *Phys. Rev. E* **65**, 66407-66418 (2002).

³⁸ N. Xiang and F. L. Waelbroeck, "Effects of presheath dynamics on radio-frequency sheaths, *J. Appl. Phys.* **93** (9): 5034-5042 (2003).