Frequency Selective Volumes for Optical Spatial Filters

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Abstract: A new model is proposed for modeling metallic losses at optical frequencies and is used in the analysis of Frequency Selective Surfaces (FSSs) and Volumes (FSVs). Conventional methods for simulating metallic losses are also outlined and a comparison with those models is given for a patch FSS. Measured data for a slot-ring FSS are also given for model validation.

Keywords: Frequency Selective Surfaces, Frequency Selective Volumes, Optical Frequency Filters, Thermo-photovoltaic Filters.

I. Introduction

Frequency Selective Surfaces/Volumes (FSSs/FSVs) are periodic structures with frequency selective properties, and have been widely used for millimeter and microwave applications [1,2]. Example applications include band-stop and band-pass microwave filters, reflectors, radoms etc. Typically, FSSs consist of single or multiple dielectric layers with printed metallization. Making the metallization periodic allows for frequency selectivity and by further stacking these layers, broadband operation can be attained in much the same way as done for electric circuit filters.

This paper introduces a new mathematical model for the analysis of FSS at optical frequencies. An application at these frequencies is that of using FSS to filter those optical frequencies best suited for the efficient operation of photovoltaic cells. That is, the FSS is designed to only transmit the spectra that can be efficiently converted into electricity. Use of these filters is particularly important in space missions where they play a role in decreasing energy consumption and consequently reduce spacecraft mass, cost, and fuel loading.

Analysis of metallic FSSs at optical frequencies has been carried out by Wu [3] where the perfectly conducting (PEC) assumption was employed. However, at optical frequencies the metallic surfaces and coating exhibit plasma behavior and therefore an appropriate boundary condition must be enforced for their analysis and performance prediction. Also, because of through penetration, they can no longer be considered as surfaces. Instead it is appropriate and

necessary to model them as finite thickness layers. That is, at optical frequencies, we must analyze the periodic arrangement of metal layers in the form of frequency selective volumes (FSVs) rather than FSSs. Needless to mention, the finite thickness of the metal layers and their penetrable properties at optical frequencies implies use of suitable numerical and analytical models.

In this paper we introduce a mathematical model of the metal surfaces based on the electron motion equations at optical frequencies. Specifically, we give suitable expressions for the conductivity and dielectric constants across the metal thickness, and then proceed to validate the expressions using measured data.

II. Models for Material Simulations

Several models exist and have been used to model imperfect metallic structures in numerical simulations. A popular such model is based on resistive sheets. In this case, the parameter of importance is the resistivity, R , measured in Ω per square [4]. Using R , the sheet or layer boundary condition takes the form

$$\mathbf{E} = R\mathbf{J} = R\hat{\mathbf{u}} \times [\mathbf{H}^+ - \mathbf{H}^-] \quad (1)$$

where the superscripts $\pm$ refer to the magnetic field just above and below the thick metal layer and $\hat{\mathbf{u}}$ is the unit normal on the sheet. The resistivity of a metal layer of thickness τ is given by $R = 1/(\sigma\tau)$ in which σ is the conductivity of the metal. For PEC layers/surfaces, and $\sigma \rightarrow \infty$ therefore $R=0$. For realistic metal layers σ is large but finite. The sheet model is, of course, appropriate when τ is very small. For thicker layers, it is necessary to model the layer as a volumetric structure using the available dielectric constants describing the volume. Typically, we set $\epsilon = \epsilon_0\epsilon_r$, with $\epsilon_r = 1 - j\sigma/\omega\epsilon_0$ with ϵ_r being the relative permittivity, ϵ_0 is the free space permittivity and ω is the operating frequency in rad/sec. Nevertheless, the model based on $\epsilon_r = 1 - j\sigma/\omega\epsilon_0$ is not valid above 15THz. At optical frequencies, σ takes complex values. This is discussed next and results are given that validate our σ values at optical frequencies.

III. Material Modeling at the Optical Frequencies

Consider a homogeneous isotropic medium of relative dielectric constant ϵ , permeability μ and conductivity σ . The corresponding complex wave number is then given by

$$k^2 = \omega^2 \mu \epsilon = \omega^2 \mu (\epsilon_0 - j \frac{\sigma}{\omega}), \quad (2)$$

where $\epsilon = \epsilon_0 - j\sigma/\omega$ is the complex permittivity of the medium. At sufficiently low frequencies, σ is (to a good approximation) real. However, there is a general consensus [5] that this approximation for σ is valid only for wavelengths greater than $\lambda > 2 \times 10^{-3} \text{ cm}$ ($f < 15 \text{ THz}$). Classical expressions for σ are not therefore suited at frequencies greater than 15 THz. At optical frequencies (Fig.1), it is convenient to express ϵ as follows:

$$\epsilon = \epsilon_r + j\epsilon_i = (n^2 - \psi^2) + j2n\psi. \quad (3)$$

Here, n and ψ are real and refer to the standard refraction index and attenuation constant, respectively.

A derivation for σ based on the electron motion equation within the metal region can be found in the literature [6].

Fig. 1. Metallic structure model at optical frequencies

The dielectric constant at the desired frequencies can be found from available measured data for n and ψ [7]. The Appendix gives a quadratic interpolation of the measured data at specific frequencies as given in [7].

IV. Patch Array

To evaluate the proposed models, let us consider the commensurate array of patches as shown in Fig.2. The geometry refers to a unit cell of size $700 \times 700 \text{ nm}^2$, consisting of two metallic regions: the outer ring which is 50 nm wide, and the inner block which has an area of $400 \times 400 \text{ nm}^2$. The thickness of the unit cell is 90 nm

and the analysis was done using a finite element-boundary integral code [8]. Results based on four different models for the conducting patches are presented in Fig.3. For the thick PEC metal, the finite element method was used to model the precise thick geometry. Clearly, each of the four models gives different answers, particularly in the amount of absorption (see Fig.3) which for our case cannot be attributed to the accuracy in computing the reflection coefficient. Specifically, the resistive sheet and thick aluminum models do not provide much absorption. Also, they provide a resonant shift to shorter wavelengths as compared to the other models. The new complex σ model predicts an approximate loss of 10%, but measurements are needed for verification. This is done in the next section for a Gold slot-ring FSS printed on a multilayered substrate.

Fig. 2. (a) Array of elements and (b) unit cell (element for the patch FSS modeled as a FSV).

Fig. 3. Transmitted power relative to the incident field as computed using the different metal layer models.

Fig. 4. Transmitted/Received and Absorbed power (relative to the incident field) for the new metal layer model based on complex σ .

We give in Fig. 4 the transmitted, reflected and absorbed power results for the new model (relative to the incident field). Note that the observed power loss in the transmission coefficient is due to absorption/loss in the thick metal layer.

V. Slot Ring FSV

To validate the proposed optical model using complex σ , we considered the structure shown in Figure 5. This FSV refers to a slot-ring configuration printed on a multilayer substrate. The slot-ring is made of Gold and the slot width is $w = 110$ nm with the overall unit cell being $1\mu\text{m} \times 1\mu\text{m}$. There are three dielectric layers beneath the Gold slot-ring and these form the substrate. They consist of a Silicon Dioxide layer of thickness 450nm, a Silicon layer of thickness 670nm and a layer of Silicon Dioxide of thickness 450nm (see [9] for fabrication details). The thickness of the Gold slot-ring is assumed to be 50nm throughout the analysis. The ϵ' and ϵ'' values for the substrate are shown in Figure 5.

Figure 6 shows the measured and simulated Transmission/Reflection coefficients as well as the power absorbed within the conducting and dielectric regions. As seen, the measured and simulation curves are in very good agreement.

VI. Conclusion

The complex ' σ ' model for simulating metal layers at optical frequencies was presented. The model is based on using a complex conductivity expression instead of the usual real σ used at microwave frequencies. Differences of this new model as compared to microwave models were presented using simulations. Also measured data were given for a slot-ring Gold periodic array printed on a Silicon/Silicon Dioxide substrate to more accurately model losses in metallic layers. The measured and simulated data were found to

be in good agreement. The proposed approach can be used for modeling other materials at optical frequencies.

Fig. 5. (a) Array of elements and (b) unit cell geometry of the ring-slot FSV.

Fig. 6. Transmitted/Received/reflected and Absorbed power (relative to the incident field) for the slot-ring FSV shown in Fig. 5.

APPENDIX: Continuous Frequency Dependent Data for the ϵ' and ϵ'' Values of Gold and Aluminum (1-10 μ m)

The figures below are based on measured data given in [7]. Palik [7] gives a set of data at irregular wavelength intervals for n and ψ . These values must then be interpolated to obtain continuous curves for ϵ' and ϵ'' as a function of frequency or wavelength. A quadratic interpolation was adapted and the actual expression is the equation displayed on the figures. Figure 7 gives the corresponding ϵ' and ϵ'' curves for Gold. These curves were used to generate the transmission/reflection/absorption results given in Figure 6. We remark that ϵ' is negative for wavelengths 1-10 μ m (30-300THz), and these values were used in the finite element simulation models. We remark that this model, often referred to as the Drude model, and the given values (polynomial expression in Figure 7.) are not valid for lower (microwave) frequency, and more specifically below 15THz (20 μ m), where in fact the ϵ' values become positive as expected. Data for other materials can be obtained using the corresponding n and ψ values.

Fig.7. Actual measured data from [7] and interpolated values of (a) ϵ' and (b) ϵ'' for Gold.

Also, ϵ' and ϵ'' values for Aluminum in 1-10mm band can be found using

$$\epsilon'(x) = -54x^2 - 250x + 260, \epsilon''(x) = 59x^2 - 160x + 160. \quad (10)$$

Note that the equations given in Fig.7 and (10) are only valid in the (1-10mm) band. One should refer to data in [7] to reconstruct the corresponding equations outside of this band.

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