The Appreciation of Stochastic Motion in Particle Accelerators

Keith Symon
University of Wisconsin-Madison, Madison, Wisconsin

and

Andrew Sessler*
Lawrence Berkeley Laboratory, Berkeley, California

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A description is given of the analytic and numerical work, performed from July 1955 through August 1956, so as to develop, and then study, the process of making intense proton beams, suitable for colliding beams. It is shown how this investigation led, in a most natural way, to the realization that stochasticity can arise in a simple Hamiltonian system. Furthermore, the criterion for the onset of stochasticity was understood, and carefully studied, in two different situations. The first situation was the proposed (and subsequently used) "stacking process" for developing an intense beam, where stochasticity occurs as additional particles are added to the intense circulating beam. The second situation occurs when one seeks to develop "stochastic accelerators" in which particles are accelerated (continuously) by a collection of radio frequency systems. It was in the last connection that the well-known criterion for stochasticity, resonance overlap, was obtained.

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I. Introduction

Some fifty years ago, when designers of particle accelerators began to make extensive numerical calculations of particle orbits in proposed machines, phenomena were often observed that would now be called "chaotic". The term used in the accelerator community was "stochastic". Such phenomena were observed both in horizontal and vertical particle motions transverse to the design orbit, and in longitudinal motion along the orbit. They occurred in certain cases when nonlinear forces were important.

Although stochastic behavior was familiar to accelerator designers, such behavior generally occurs in regions of phase space outside the stable region where accelerator design calls for particles to be. As a result there is very little mention of stochastic behavior in the accelerator literature. We are unaware of any paper devoted specifically to his topic. The only paper we have found which mentions this topic in any detail is one by the present authors written in 1956 as a Midwestern Universities Research Organization, MURA report (Internal MURA Report 130, April 16, 1956), and published also in the Proceedings of the CERN Symposium on High Energy Accelerators and Pion Physics, 1956 (Ref.1). Even that paper devotes to the topic only about half a page out of 15.

II. Longitudinal Particle Motion

With the advent of alternating gradient (AG) focusing, and the invention of fixed field alternating gradient (FFAG) focusing, it became possible, for the first time, to seriously contemplate colliding beam devices. What was required was a method of building up an intense beam by means of radio frequency manipulation of particles. Thus attention was focused upon the longitudinal motion of particles, work described in 1956.[1] Most particularly, prior to that time, attention was directed upon the small amplitude motion of particles being accelerated. Now, for the first time, attention had to be paid to large amplitude motion and, even, to the motion of particles not being accelerated.

In Ref 1, we treated the longitudinal motion of a particle under the action of the radio-frequency accelerating voltages applied across an accelerating gap. Thus we only had one accelerating gap per turn. This is a very simple problem to calculate numerically, and with suitable approximations it can also be treated analytically. We are given the (angular) frequency of revolution ω(E) as a function of the energy E (the only property of the accelerator guide field that comes into the theory), and the voltage V(t) applied to the accelerating gap. If the particle is at an accelerating gap, we add the gap voltage at that time to its energy E. We then calculate using ω(E) the time for the next arrival at the gap, and the phase of the accelerating voltage at that time. Repeating the calculations, we prepare a phase plot, energy E versus radio-frequency (rf) phase, ϕ, upon arrival at the gap, and for each return to the gap, we plot a point at the corresponding energy and phase. An example is shown in Fig. 1 (Fig 4c of Ref 1). The equations employed were simply:

$$E_{n+1} = E_n + V(t_n)$$
$$t_{n+1} = t_n + \frac{2\pi}{\omega(E_{n+1})}$$

The system can be made Hamiltonian, and that was, in fact, done in Ref 1. (Essentially by making E/ω(E) a dynamical variable. Since ω(E) doesn't vary much over the range we are considering, we can stay with E as the independent variable here with no loss of generality and simplicity of presentation.) Thus Liouville's theorem is applicable and the consequences of that theorem were a source of many conversations at that time. However, the restrictions on stacking (see below), etc. are not relevant to our present discussion.

III. Buckets

For a given particle orbit, the successive points lie on one of the closed curves shown in Fig.1. We see that at resonance energies where the frequency of revolution is an integer sub-multiple of the rf frequency, there is a fixed point (in this case, at phase 0 or π) surrounded by a set of closed curves corresponding to particles trapped near the resonance energy. The region of closed curves was called a "bucket".

If we define the phase

$$\varphi_n = \omega_{rf} t_n - 2\pi h n$$

where the last term is inserted to make φ_n a slowly varying function of n for energies near the synchronous energy, then the equations of motion become

$$E_{n+1} = E_n + V_o \sin\varphi_n \quad ,$$
$$\varphi_{n+1} = \varphi_n + \frac{2\pi\omega_{rf}}{\omega(E_{n+1})} - 2\pi h \quad .$$

Near the synchronous energy, φ_n and E_n are slowly varying functions of n, and we can approximate these equations by the differential equations

$$\frac{dE}{dn} = V_o \sin\varphi \quad ,$$
$$\frac{d\varphi}{dn} = \frac{2\pi\omega_{rf}}{\omega(E)} - 2\pi h \quad ,$$

where n is now a continuous variable.

These equations are derivable from the Hamiltonian

$$\underline{H}(\varphi, E, n) = 2\pi \int_{E_s}^{E} \left[\frac{\omega_{rf}}{\omega(E)} - h \right] dE + V_o \cos\varphi \quad ,$$

which is independent of n and therefore a constant of the motion. A trapping region with curves of constant H is shown in Fig.2 (Fig. 3a of Ref 1).

IV. Motion Outside a Bucket

It is clear that the analytic result must break down in cases where two predicted buckets overlap. This will occur if for example in Fig.1 we increase the accelerating voltage so that the buckets become larger. Numerical results suggested that in such cases in the overlap region stochastic behavior results. A computed example is shown in Fig. 3 (Fig. 4a of Ref 1). The solid curves correspond to particle orbits whose successive points lie on closed curves. Two orbits are shown in the stochastic region. The successive points (circles and triangles respectively) appear randomly distributed and do not lie on closed curves.

Pursuing this idea, we calculated a case with two rf voltages applied to the accelerating gap, with nearby frequencies and with voltage adjusted so that the predicted buckets for each frequency applied singly would overlap. The results for three particles (closed circles, open circles, and triangles) are shown in Fig. 4 (Fig. 4b of Ref 1). It is clear that for each particle the phase points return randomly to points in the region occupied by the buckets. There are no closed curves. The two solid curves are the two predicted bucket boundaries (in the absence of the other bucket). Chirikov, Ref.2, has emphasized the chaotic behavior in the region of overlapping buckets

V. Stacking

The motivation for our study of longitudinal motion outside a bucket was to develop a method for building up an intense beam so as to obtain sufficient intensity as to make a colliding beam. In that connection it was very important to obtain a Hamiltonian formulation, so that general concepts could be invoked without going into the details of the process. Most importantly, it was Eugene Wigner who pointed out to us that we could greatly advance our understanding by using Liouville's theorem.

So the idea is to have particles circulating at some final energy and them bring up a new group in an rf bucket and drop them (turn down the rf voltage) where the others are circulating. The question was would the rf bucket bringing up the new group so disturb the circulating beam as to make the process not viable. Most particularly, how much phase space dilution would be caused by this process?

We studied the problem numerically – in fact this was our very first study – by putting 11 particles, at various phases, but all at the same energy, and then bringing a bucket up to them. To our surprise we found 8 were de-accelerated. We immediately realized, invoking Liouville's theorem, the idea of phase displacement. This concept has been used, subsequently, as an acceleration method.

We found that the phase space dilution was minimal and thus the idea of stacking was valid. It was subsequently employed, for example, on the CERN ISR.

VI. Stochastic Accelerators

Proposals to utilize stochastic behavior to accelerate particles were made as early as 1948 by Kolomenskij.[3] Experiments are reported by Keller.[4]

The authors considered, during the time period of this report and without knowledge of Kolomenskij's work, stochastic acceleration. However, nothing was ever published. The idea was suggested by Donald Kerst, who seeing our results about bucket overlap, conceived of a DC accelerator with many rf systems, closely spaced in frequency so that their buckets overlapped. Particles would be injected continuously, some would be accelerated a bit, but then get de-accelerated and come back out the injector, but others – a steady stream of them – would come out at top energy.

In order to study this idea, we remember making a large graph, extending all the way from the floor to the ceiling of energy vs. phase. Each injected particle was given a color and its jumping around, from turn to turn, was duly recorded. Soon we had a very colorful chart. The idea works, but the process of acceleration took so long that gas scattering destroyed most of the particles in the process and, therefore, it was not interesting as a practical device. Because it didn't prove useful as an accelerator, we never even wrote up or published the work.

VII. Conclusion

It is unfortunate that chaotic behavior which is now of considerable interest, and which was observed and familiar to accelerator physicists fifty years ago, was of so little practical interest in accelerator design that there was little mention of it in the literature of that time. We can be thankful that others, certainly Boris Chirikov, at first, realized the importance of stochasticity and have built a great, and important, edifice.

That he did this "on his own", without the benefit of the humble observations of accelerator physicists, is most impressive and certainly worthy of the deep appreciation we are all showing him at this Conference.

By now, chaos theory has even penetrated the accelerator community. If one turns to a recent accelerator physics conference, say the one of 1994, then one finds a number of papers on chaos [5-10], including references to Chirikov.

Acknowledgment

All the figures are directly taken, without alteration, from Ref 1 with the permission of CERN.

References

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Figure Captions

Fig. 1. A phase plot, energy E versus radio-frequency (rf) phase, ϕ. Plotted are points at the corresponding energy and phase, for each arrival at the accelerating gap. Sub-harmonics of one oscillator can be seen.

Fig. 2. Stationary (analytic) bucket obtained using the equations of Section III.

Fig. 3. Scattered points and smooth trajectories of a high-voltage bucket. The solid curves correspond to particle orbits whose successive points lie on closed curves. Two orbits are shown in the stochastic region. The successive points (circles and triangles respectively) appear randomly distributed and do not lie on closed curves.

Fig. 4. Scattered points when two buckets overlap. The results are shown for three particles (closed circles, open circles, and triangles). For each particle the phase points return randomly to points in the region occupied by the buckets. There are no closed curves. The two solid curves are the two predicted bucket boundaries (in the absence of the other bucket).

Figures

Fig. 1

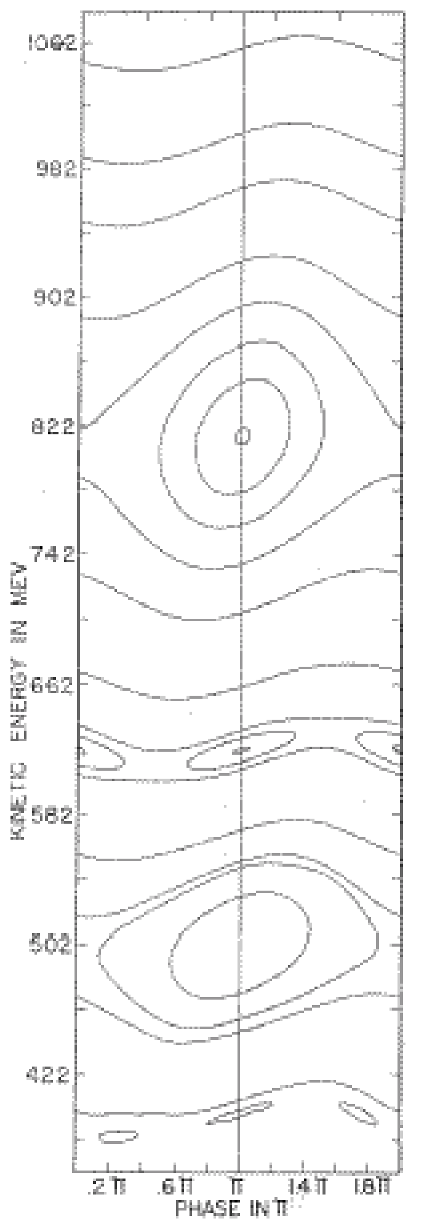

Fig. 2

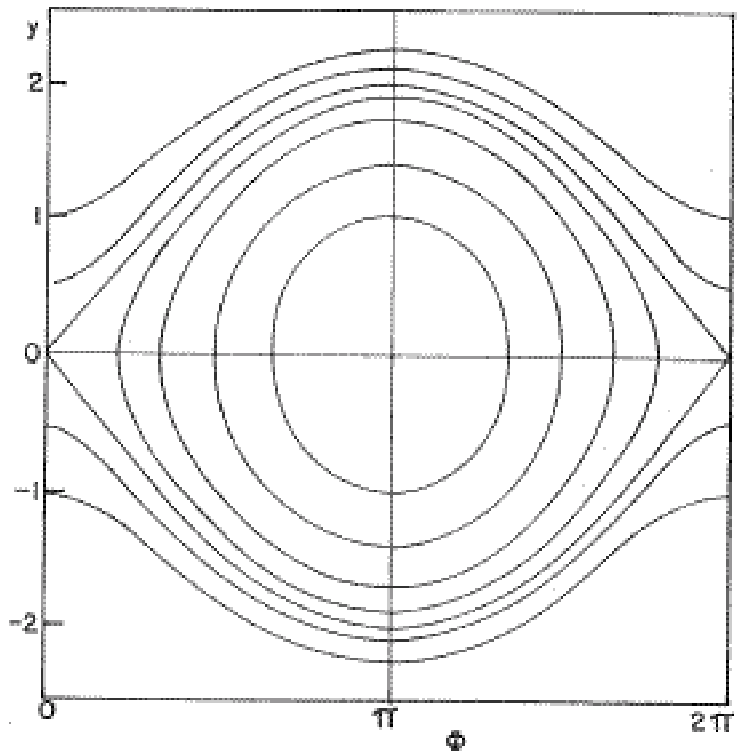

Fig. 3

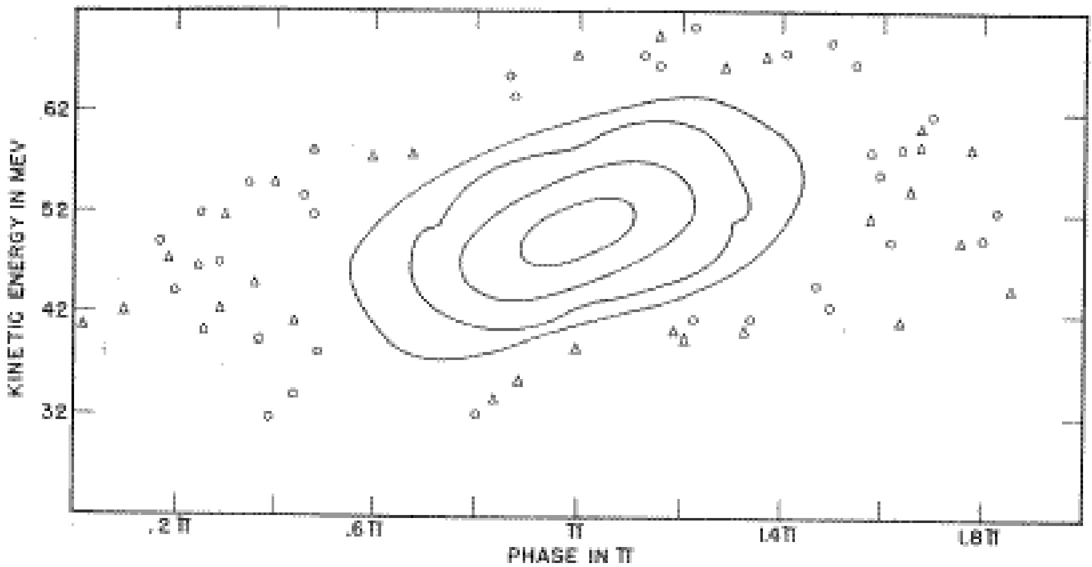

Fig. 4