Seismic Attenuation for Reservoir Characterization
DE-FC26-01BC15356

Quarterly Report
Oct. 1 – Dec 31, 2001
Issued January, 2002

Contributors
Dr. Joel Walls*
Dr. M. T. Taner*
Dr. Gary Mavko**
Dr. Jack Dvorkin**

*Principal Contractor:
Rock Solid Images
2600 S. Gessner Suite 650
Houston, TX, 77036

**Subcontractor:
Petrophysical Consulting Inc.
730 Glenmere Way
Emerald Hills, CA, 94062

Disclaimer

This report was prepared as an account of work sponsored by the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Contents

Abstract.....	3
Section 1: Rock Physics	4
Introduction.....	4
Results and Discussion.....	4
Conclusions	8
Work Planned for Next Period.....	8
Problems Encountered This Period	9
References.....	9
Section 2: Q and Dispersion Computation Using Gabor-Morlet Transform.....	10
Introduction:	10
Method:	10
Theory:	11
Gabor -Morlet Transform Method:	12
Gabor-Morlet Wavelet Specifications	12
Computational Procedure:.....	14
Conclusions:.....	19
References:.....	20

Abstract

In Section 1 of this first report we will describe the work we are doing to collect and analyze rock physics data for the purpose of modeling seismic attenuation from other measurable quantities such as porosity, water saturation, clay content and net stress. This work and other empirical methods to be presented later, will form the basis for “Q pseudo-well modeling” that is a key part of this project. In Section 2 of this report, we will show the fundamentals of a new method to extract Q, dispersion, and attenuation from field seismic data. The method is called Gabor-Morlet time-frequency decomposition. This technique has a number of advantages including greater stability and better time resolution than spectral ratio methods.

Section 1: Rock Physics

Introduction

In this first phase of the project our focus is to assemble a knowledge-base of existing rock physics data and relations on seismic attenuation and how it relates to lithology and pore fluids.

The goal of this task is to develop the methods to predict Q pseudo-logs. In other words, we will take existing well log quantities, such as V_p , V_s , porosity, V_{shale} , and S_w , and use these to predict a corresponding estimate of Q vs. depth, as well as perturbations to these Q estimates, corresponding to gas and other hydrocarbons.

These pseudo-logs will be the key inputs to the systematic forward modeling of synthetic seismograms, from which Q -attributes will be extracted and calibrated to the actual interval values. The purpose of this modeling is to establish the sensitivity of the attributes to the properties of interest, and to help separate the contributions of both intrinsic Q , which relates to the rock and fluid properties, and scattering Q , which relates to the geometric arrangement of layers. We view this as a natural extension to the Q -domain, of the usual forward modeling studies that are used in any careful rock physics study of the signatures of rock and fluid properties for exploration and reservoir characterization and monitoring. It is analogous to the “what if” exercises that are now routinely used, for example, in AVO modeling.

Results and Discussion

In this section, we show examples of the attenuation data that we are compiling. The key points that we will look for are robust relations that will allow us to predict Q , or how Q changes with changes in lithology, fluid content, and pressure.

Figure 1 shows the variations V_p , V_s , Q_p , Q_s , and moduli (bulk and shear) with pressure in unconsolidated water-saturated sands. The data taken from Prasad and Meissner (1992) were measured at 100 kHz. The two symbols denote sands of different grain sizes: solid symbols mark a finer grained sand (220 μm median grain size) and open symbols mark a coarser grained sand (550 μm median grain size). Velocity and moduli increase with increasing pressure. However, there is a difference in the attenuation

change with pressure: S-wave attenuation is high at low pressures and decreases with pressure. P-wave attenuation on the other hand, does not change much with pressure.

We need to understand this difference in Qp vs Qs pressure behavior. Is it a general property? Is it only the case in water-saturated rocks? Do we see it in other data sets?

Figure 1. V_p , V_s , Q_p , Q_s , and moduli (bulk and shear) with pressure in unconsolidated water-saturated sands.

Figure 2 shows the sensitivity of V_p - V_s , Q_p - Q_s , and k - μ ratios to pressure in unconsolidated water-saturated sands. The measurements were made at 100 kHz (data from Prasad and Meissner, 1992). The two symbols denote sands of different grain sizes: solid symbols mark a finer grained sand (220 μm median grain size) and open symbols mark a coarser grained sand (550 μm median grain size). The upper row shows data for a larger pressure range, the lower row presents the same data at low pressures. The main change is observed in all plots at low pressures. Above about 5 MPa, the pressure sensitivity is not observed.

Figure 2. The sensitivity of V_p - V_s , Q_p - Q_s , and k - μ ratios to pressure in unconsolidated water-saturated sands.

Figure 3 shows how lithology, grain size, and pressure effects can be separated on a loss diagram. Blue symbols mark a water-saturated sandstone measured at 1 MHz frequency. All other symbols represent measurements in water-saturated sands made at 100 kHz. Sands are characterized by high V_p/V_s and Q_p/Q_s values, whereas the opposite is true for sandstones. This implies that in sandstones, shear losses are as high as bulk losses in dry sands whereas in saturated sands, shear losses are higher. The different colored symbols in sands denote sands of different grain sizes: purple symbols mark a finer grained sand (220 μm median grain size) and red symbols mark a coarser grained sand (550 μm median grain size). The arrow marks trends in the plots with decreasing differential pressures (equivalent to increasing pore pressures).

Figure 3. Evidence that lithology, grain size, and pressure effects can be separated on a loss diagram

Figure 4 shows how saturation effects can be separated on a loss diagram. Water-saturated sands (blue symbols) are characterized by high V_p/V_s and Q_p/Q_s values, whereas the dry sands (red symbols) have low V_p/V_s values but high Q_p/Q_s . This implies that the shear losses are as high as bulk losses in dry sands whereas in saturated sands, shear losses are higher. All measurements were made at 100 kHz.

Figure 4. Separation of saturation effects using velocity and Q ratios.

Conclusions

We conclude from these results that Q provides additional information about fluid saturation and net stress in sandstones over and above what is available from velocity alone. This means that analysis of Q , dispersion and attenuation can potentially indicate zones of gas (or perhaps oil) saturation in-situ, plus it may help in detecting zones of over-pressure. Of course the lab results given above apply to higher frequency (100 KHz) than are used in the field (10 Hz to 10KHz for seismic and well logs) so the relative magnitudes of the changes may be different in-situ. In our next report we will include information relating to lower frequency measurements also.

Work Planned for Next Period

We continue to search for existing data to add to the database. We have received some data from Burlington Resources but we do not yet know if it is suitable for the project. Our next step is to experiment with robust functional forms that will allow us to capture the key Q behavior vs. porosity, saturation, rock type, and pressure. This will include evaluating and improving known relations such as those of Klimentos and McCann. Even more fruitful, however, will be model-based formulas that exploit the robust ratios such as Q_p/Q_s .

We are also testing a few commercial software tools that will be needed for the later synthetic seismic modeling tasks. We are also conducting research on extraction of attenuation from reflection seismic data. Development of the background methodology is described in Section 2.

Problems Encountered This Period

No significant problems have been encountered in our work so far. We are 3-4 weeks behind schedule, primarily because of previous time commitments of the primary researchers. The project will be back on schedule no later than mid 2002. All indications are that the project will progress nicely over the next several months.

References

Prasad, M. and Meissner, R., 1992, Attenuation mechanisms in sands: Laboratory versus theoretical (Biot) data: *Geophysics*, 57, no. 5, 710-719.

Klimentos, T. and McCann, C., 1990, Relationships between compressional wave attenuation, porosity, clay content, and permeability of sandstones: *Geophysics*, 55, no. 8, 998-1014.

Section 2: Q and Dispersion Computation Using Gabor-Morlet Transform

M. Turhan Taner
Rock Solid Images

Introduction:

Absorption, dispersion and the related Q quality factor are some of the more important seismically measurable factors that relate to porosity and rock physics. Unfortunately, many of the previous attempts to measure these properties from seismic data contain high degrees of uncertainty due to the very subtle change of seismic data characteristics over the measured distance. However, robust computation of these parameters will greatly improve our ability to estimate reservoir characteristics. The purpose of this paper is to discuss one method to calculate attenuation, Q and dispersion from instantaneous spectra. Instantaneous spectra can either be obtained using the Wigner-Ville transform, or by the Gabor transform (Gabor, 1946), which we will discuss in this report.

We would like to point out that Morlet and his associates introduced Gabor's work to the geophysical industry. He modified Gabor's sub-division of the frequency domain that retained the wavelet shape over equal octave intervals. This is now called the Gabor-Morlet transform. This is also recognized as the first expression of the generalized wavelet transform. We have included references to a number of papers by Morlet. Further development of the idea was conducted by Koehler which resulted in a number of theoretical and practical papers (1983, 1984). The application presented here represents one of the results.

Method:

We apply conventional spectral division on the Gabor-Morlet decomposed data rather than in the Fourier domain. By definition, absorption relates to the energy loss per cycle and dispersion relates to propagation velocity varying as function of frequency. The energy loss affects the amplitude spectra of the wavelets. Waves propagating through any medium lose some of their energy by conversion to heat or by plastic deformation,

hence the spectrum of a transmitted wavelet will contain less energy than the incident wavelet. If the propagation velocity is constant for all frequencies, this loss relates to a percentage of the energy loss per cycle. Since the same distance will be traversed by more cycles of higher frequencies than lower frequencies (longer wavelength), the higher frequencies will naturally suffer more losses than the lower frequencies, but the phase spectrum will remain the same. In a medium where there is dispersion and energy loss, both the amplitude and the phase spectra will change according to the characteristics of that medium. It is interesting to note that pressure and shear waves will have different dispersion and attenuation characteristics which can be put to effective use in seismic reservoir characterization.

Theory:

We assume a constant Q condition, i.e. energy loss relates to the number of cycles over a travel distance spanned. In this case, the original amplitude spectrum of the seismic wavelet $A_0(f)$ will be changed to:

$$|A_t(f)| = |A_0(f)| \cdot \exp(-\pi f t / Q) \quad (1)$$

where t is the travel time from origin to the target.

Q is estimated from the ratio of the amplitude spectra of the wavelets obtained above and below the area of interest.

$$\ln \left| \frac{A_2(f)}{A_1(f)} \right| = -\pi f (t_2 - t_1) / Q \quad (2)$$

The main problem stems from the zero or near zero values of $A_1(f)$ which give rise to unusable values and large estimation errors. However, over coherent zones the ratio gives estimates that are more accurate. To take advantage of this we use the amplitude of the spectra as weights in a least square line fitting for Q estimation. The spectral ratio's problem with zeros on the unit circle are due to computational inaccuracies of autocorrelation functions or to the effects of various forms of noise, reflectivity series and the man-made effects of using notch filters. Since spectral division is the same as z-polynomial division, we can obtain the desired results by dividing two stable polynomials. These stable polynomials are conventionally computed by unit-step prediction error, better known as spiking operators. These operators are the minimum phase inverse of the minimum phase equivalent of the seismic wavelet from a particular computation zone. Since these operators are minimum phase, they can be inverted or

used in polynomial division without any instability. These inverse wavelets can be computed from auto-correlation functions using the Wiener-Levinson algorithms.

Gabor -Morlet Transform Method:

The Gabor-Morlet transform is performed by filtering the seismic data with a series of Gabor-Morlet wavelets. The results are narrow-band analytic traces. The amplitude and phase of each narrow band filtered output represent the average amplitude and phase of the narrow-band part of the input trace.

The proposed method includes the computation of the analytic traces from the original input. A user-selected number (N) of Gabor-Morlet wavelets are convolved with the data to produce N sub-band analytic traces. These sub-band traces are normalized by dividing them by the original trace envelope. This will remove the amplitude variation of individual reflected events, leaving only the variations between the individual sub-band traces. These trace amplitudes can be displayed as instantaneous amplitude spectra of the input trace. Similarly, joint time-frequency phase spectra are generated as the arc-tangent of the imaginary-to-real parts of each sub-band. These are displayed as the instantaneous time-frequency phase spectra.

The envelope peaks of the input trace correspond to the time where all the sub-band components are in-phase. If we pick the envelope and phase values of each sub-band, we will have the specific amplitude and phase spectra content of the input wavelet. Absorption and dispersion estimates are then obtained from the differences of log amplitude and phase between adjacent wavelets in the time direction. We will cover the details of the computation below. Gabor wavelet theory is reviewed in an excellent paper by Koehler (1983). Here, I will describe the practical application of the Gabor-Morlet wavelet theory.

Gabor-Morlet Wavelet Specifications

The time domain response of the wavelet:

$$g_j(t) = \exp(-a_j t^2) \cdot \exp(i \omega_j t) \quad (3)$$

The corresponding frequency domain response is:

$$G_j(\omega) = \int_{-\infty}^{\infty} g_j(t) \exp(i \omega t) dt = \sqrt{\frac{P}{a_j}} \cdot \exp\{-(\omega - \omega_j)^2 / 4a_j\} \quad (4)$$

$$\Delta t_j = k_1 / \omega_j \quad \text{width of j'th wavelet in time domain,} \quad (5)$$

$$\mathbf{D}\mathbf{w}_j = k_2 \mathbf{w}_j \quad \text{width of } j\text{'th wavelet in frequency domain,}$$

(6)

where:

$$\mathbf{D}_j \mathbf{D}\mathbf{w}_j = k_1 k_{2j} \quad \text{constant}$$

(7)

The "width" of a function is defined as the interval between which the function is equal to or more than one-half its maximum value: i.e.,

$$\exp\{-a_j (\mathbf{D}_j / 2)^2\} = 1/2 \quad \text{and,} \quad (8)$$

$$\exp\{-(\mathbf{D}\mathbf{w}_j / 2)^2 / 4a_j\} = 1/2. \quad (9)$$

From equations 6 and 7 we get,

$$a_j (\mathbf{D}_j)^2 / 4 = \ln(2) \quad \text{and}$$

$$(\mathbf{D}\mathbf{w}_j)^2 / 16a_j = \ln(2).$$

We compute:

$$(\mathbf{D}_j)^2 (\mathbf{D}\mathbf{w}_j)^2 = 64.(\ln 2)^2 \quad \text{this will result in:}$$

$$\mathbf{D}_j \mathbf{D}\mathbf{w}_j = 8 \ln 2 \quad (10)$$

If we choose $k_1 = 4\mathbf{p}$ and $\mathbf{D}_j = 4\mathbf{p} / \mathbf{w}_j$, this makes the wavelet amplitude equal to one-half of its maximum at an interval of one period on each side of the maximum point. From equations 7 and 10 we get:

$$k_2 = \frac{2}{\mathbf{p}} \ln(2) \quad \text{and} \quad \mathbf{D}\mathbf{w}_j = \frac{2 \ln 2}{\mathbf{p}} \mathbf{w}_j. \quad (11)$$

The value of a_j is determined from equation 8 or 9;

$$a_j = \frac{\ln 2}{4\mathbf{p}^2} \mathbf{w}_j^2. \quad (12)$$

The user specifies the usable frequency band for spectra computation which is sub-divided into equal intervals in an octave representation of the frequency axis.

Figure 1 shows the results of the decomposition. We have designed 17 Gabor-Morlet sub-band complex filters. The real and imaginary parts of the sub-band data was

generated by convolving the input data with corresponding filters. The amplitude spectra is generated in the conventional way as the square root of the sum of the squares of the real and imaginary parts of the sub-band traces. The phase spectrum is the arc-tangent of the ratio of the imaginary to the real part of the sub-band trace. Figure 1A shows the input data taken from a 3-D stacked and migrated data set. Figure 1B is the Joint Time/Frequency Analysis amplitude spectra. Figure 1C is the phase spectra. On both of the displays, the horizontal axis is frequency and the vertical axis is time. The seismic data shows that there is a zone of thin-bedded sequences between 500 milliseconds and 1400 milliseconds. The amplitude spectra in this zone are wider band. The limits of this zone are marked by low amplitude areas. Events below 1500 milliseconds exhibit a different character, more widely spaced and separated by low reflectivity events. This also has shown on the amplitude spectra. The phase spectra is not influenced by trace amplitude, thus it appears with uniform scale. The colors represent the phase angle.

Computational Procedure:

In Q computation, we need to compute the amplitude spectra ratio of two adjacent events. The joint Time/Frequency analysis provides us the spectra of all events in the seismic trace and we can compute the log of the amplitude ratio between any two events. Since we are interested in the slope of this ratio, the mean amplitude differences of the two events will not adversely affect our computation. It may be necessary to run the analysis with several wide-band decompositions to establish the frequency band over which more reliable results may be obtained. If high frequency noise is present it will result in erroneous results. Tuning thickness may result in peaks at various frequency bands. Once the usable bandwidth is established, a section showing the interval Q estimates can be generated. These values can also be used for lithological classification.

The phase spectra will provide information for dispersion estimation. Attributes picked at the peak of the envelope represent the average of the wavelet attribute which is why we pick the amplitude spectrum at the time of envelope peak for Q computation. The phase spectra is picked the same way. If we look at Figure 1C we observe that most of the spectra of the events are horizontal which means that these wavelets exhibit a constant phase rotation, and their rotation angle is the phase corresponding to the phase at the envelope peak. Therefore, computation of dispersion consists of determining the phase differences at each sub-band trace and computing an average phase delay per cycle. Since dispersion is related to absorption, higher levels of dispersion may be diagnostic of

higher levels of absorption, which may indicate fracturing in carbonates or unconsolidated sands in clastic environment.

By definition absorption is a measure of loss of energy per cycle or per unit travel distance. We will use the time-related definition and compute absorption as loss of energy in dB/cycle. Here we are assuming that absorption, dispersion and Q factor are frequency constant. This is principally true for low lossy media with $Q > 10$. (Johnston and Toksoz, 1981). We will compute dispersion as phase delay per cycle and Q will be a dimensionless quantity representing the ratio of stored energy to dissipated energy.

Let $\mathbf{D}$ be the time difference between two consecutive pick times given in seconds, f center frequency of the Gabor sub-band given in cycles per second, then $\mathbf{D} \cdot f$ gives the number of cycles traversed between the two picks. Therefore the phase differences, or amplitude ratios we measure will correspond to delays and losses we measure on Gabor sub-bands. Let $\mathbf{a}$ represent the absorption coefficient, $\mathbf{b}$ represent the dispersion coefficient and $\mathbf{Q}$ represent the quality factor. Also let $A_1(f), A_2(f)$ be the measured amplitudes, and $\mathbf{f}_1(f), \mathbf{f}_2(f)$ be measured phases at the first and the second locations on the corresponding Gabor sub-band, respectively. We can compute the desired parameters by minimizing the following expressions:

Absorption coefficient (dB/Cycle):

$$\mathbf{a} \int_{f=f_1}^{f_2} \left[20 \log_{10} |A_2(f) / A_1(f)| - \mathbf{a} \mathbf{D} \cdot f - \mathbf{b} \right]^2 df = \epsilon \quad (13)$$

Q quality factor (dimensionless ratio):

$$\mathbf{a} \int_{f=f_1}^{f_2} \left[\ln |A_2(f) / A_1(f)| - \mathbf{p} \mathbf{D} \cdot f / \mathbf{Q} - \mathbf{b} \right]^2 df = \epsilon \quad (14)$$

Dispersion coefficient (Degrees/Cycle):

$$\mathbf{a} \int_{f=f_1}^{f_2} \left[\mathbf{f}_2(f) - \mathbf{f}_1(f) - \mathbf{h} \mathbf{D} \cdot f - \mathbf{b} \right]^2 df = \epsilon \quad (15)$$

where b is a DC component of the differences, that depends on the amplitude and the phase characteristics of the wavelets, not the frequency-dependent coefficients we seek. The equations above will results in a 2-by-2 linear system of equations, which are easily solved. This computation will produce three additional attributes.

Computation of Absorption:

To solve for the absorption, we take the partial derivatives of equation (13) with respect to the unknowns and set them equal to zero;

$$\begin{aligned} \frac{\partial \epsilon}{\partial a} &= -2 \left\{ \frac{\partial}{\partial a} \left[20 \log_{10} |A_2(f) / A_1(f)| \right] \right|_{f=f_1} \mathbf{D} \cdot \mathbf{f} - a \frac{\partial}{\partial a} (\mathbf{D} \cdot \mathbf{f})^2 - b \frac{\partial}{\partial a} \mathbf{D} \cdot \mathbf{f} \} = 0 \\ \frac{\partial \epsilon}{\partial b} &= -2 \left\{ \frac{\partial}{\partial b} \left[20 \log_{10} |A_2(f) / A_1(f)| \right] \right|_{f=f_1} - a \frac{\partial}{\partial b} (\mathbf{D} \cdot \mathbf{f}) - b \frac{\partial}{\partial b} 1 \} = 0 \end{aligned} \quad (16)$$

This will give us the desired normal equations;

$$\begin{aligned} \frac{\partial}{\partial a} \left[20 \log_{10} |A_2(f) / A_1(f)| \right] \right|_{f=f_1} \mathbf{D} \cdot \mathbf{f} &= a \frac{\partial}{\partial a} (\mathbf{D} \cdot \mathbf{f})^2 + b \frac{\partial}{\partial a} (\mathbf{D} \cdot \mathbf{f}) \\ \frac{\partial}{\partial b} \left[20 \log_{10} |A_2(f) / A_1(f)| \right] \right|_{f=f_1} &= a \frac{\partial}{\partial b} (\mathbf{D} \cdot \mathbf{f}) + b \frac{\partial}{\partial b} 1 \end{aligned} \quad (17)$$

This could be simplified, if we represent these equations by simple symbols as;

$$\begin{aligned} d_1 &= \frac{\partial}{\partial a} \left[20 \log_{10} |A_2(f) / A_1(f)| \right] \right|_{f=f_1} \mathbf{D} \cdot \mathbf{f} , \\ d_2 &= \frac{\partial}{\partial b} \left[20 \log_{10} |A_2(f) / A_1(f)| \right] \right|_{f=f_1} , \\ c_{11} &= \frac{\partial}{\partial a} (\mathbf{D} \cdot \mathbf{f})^2 , \\ c_{12} &= \frac{\partial}{\partial b} (\mathbf{D} \cdot \mathbf{f}) , \\ c_{22} &= \frac{\partial}{\partial b} (1) . \end{aligned} \quad (18)$$

The, the equation (17) can be written as;

$$\begin{aligned}
c_{11}\mathbf{a} + c_{12}\mathbf{b} &= d_1 \\
c_{12}\mathbf{a} + c_{22}\mathbf{b} &= d_2
\end{aligned} \tag{19}$$

From which , we can obtain the solution for absorption as;

$$\mathbf{a} = (d_1 \cdot c_{22} - d_2 \cdot c_{12}) / (c_{11} \cdot c_{22} - c_{12} \cdot c_{12}) \tag{20}$$

Computation of Q:

Q computation will be similar to absorption computation, we take the partial derivatives with respect to the unknowns. However $\mathbf{Q}$ appears as inverse on equation (14). In order to make the formulation similar to absorption, I will introduce $\mathbf{P}$ as the inverse of $\mathbf{Q}$ and replace it into the equation (14)

$$\mathbf{\dot{a}} \left[\ln \left| A_2(f) / A_1(f) \right| - \mathbf{P} \mathbf{D} \cdot f \cdot \mathbf{P} - b \right]^2 = \epsilon^2 \tag{21}$$

Then , the partial derivatives are given as;

$$\begin{aligned}
\frac{\mathbf{1}\epsilon^2}{\mathbf{1P}} &= -2 \left\{ \mathbf{\dot{a}} \left[\ln \left| A_2(f) / A_1(f) \right| \right] \mathbf{P} \mathbf{D} \cdot f - P \mathbf{\dot{a}} (\mathbf{P} \mathbf{D} \cdot f)^2 - b \mathbf{\dot{a}} \mathbf{P} \mathbf{D} \cdot f \right\} = 0 \\
\frac{\mathbf{1}\epsilon}{\mathbf{1b}} &= -2 \left\{ \mathbf{\dot{a}} \left[\ln \left| A_2(f) / A_1(f) \right| \right] - P \mathbf{\dot{a}} (\mathbf{P} \mathbf{D} \cdot f) - b \mathbf{\dot{a}} \mathbf{1} \right\} = 0
\end{aligned} \tag{22}$$

We can simplify these expressions the same way as before, by substitution;

$$\begin{aligned}
d_1 &= \mathbf{\dot{a}} \left[\ln \left| A_2(f) / A_1(f) \right| \right] \mathbf{P} \mathbf{D} \cdot f , \\
d_2 &= \mathbf{\dot{a}} \left[\ln \left| A_2(f) / A_1(f) \right| \right] , \\
c_{11} &= \mathbf{\dot{a}} (\mathbf{P} \mathbf{D} \cdot f)^2 , \\
c_{12} &= \mathbf{\dot{a}} (\mathbf{P} \mathbf{D} \cdot f) , \\
c_{22} &= \mathbf{\dot{a}} (1) .
\end{aligned} \tag{23}$$

Then, the normal equation (22) can be written as;

$$\begin{aligned} c_{11}P + c_{12}b &= d_1 \\ c_{12}P + c_{22}b &= d_2 \end{aligned} \quad (24)$$

From which , we can obtain the solution for inverse of Q as;

$$1/Q = P = (d_1 \cdot c_{22} - d_2 \cdot c_{12}) / (c_{11} \cdot c_{22} - c_{12} \cdot c_{12}) \quad (25)$$

Computation of Dispersion:

The partial derivatives of equation (15) with respect to the unknowns will give us the desired normal equations;

$$\begin{aligned} \frac{\partial \mathbf{e}}{\partial \mathbf{a}} &= -2 \left\{ \frac{\partial}{\partial \mathbf{a}} [\mathbf{f}_2(f) - \mathbf{f}_1(f)] \mathbf{D} \cdot f - \frac{\partial}{\partial \mathbf{a}} (\mathbf{D} \cdot f)^2 - b \frac{\partial}{\partial \mathbf{a}} \mathbf{D} \cdot f \right\} = 0 \\ \frac{\partial \mathbf{e}}{\partial \mathbf{b}} &= -2 \left\{ \frac{\partial}{\partial \mathbf{b}} [\mathbf{f}_2(f) - \mathbf{f}_1(f)] - \frac{\partial}{\partial \mathbf{b}} (\mathbf{D} \cdot f) - b \frac{\partial}{\partial \mathbf{b}} 1 \right\} = 0 \end{aligned} \quad (26)$$

By the following substitutions, we can simplify the normal equations;

$$\begin{aligned} d_1 &= \frac{\partial}{\partial \mathbf{a}} [\mathbf{f}_2(f) - \mathbf{f}_1(f)] \mathbf{D} \cdot f, \\ d_2 &= \frac{\partial}{\partial \mathbf{a}} [\mathbf{f}_2(f) - \mathbf{f}_1(f)], \\ c_{11} &= \frac{\partial}{\partial \mathbf{a}} (\mathbf{D} \cdot f)^2, \\ c_{12} &= \frac{\partial}{\partial \mathbf{a}} (\mathbf{D} \cdot f), \\ c_{22} &= \frac{\partial}{\partial \mathbf{a}} (1). \end{aligned} \quad (27)$$

Then, the normal equation (26) can be written as;

$$\begin{aligned} c_{11}\mathbf{b} + c_{12}b &= d_1 \\ c_{12}\mathbf{b} + c_{22}b &= d_2 \end{aligned} \tag{28}$$

From which , we can obtain the solution for dispersion as;

$$\mathbf{b} = (d_1 \cdot c_{22} - d_2 \cdot c_{12}) / (c_{11} \cdot c_{22} - c_{12} \cdot c_{12}) \tag{29}$$

Note that the denominator of equation (29) is same as the denominator of the equation (20).

Since we are measuring the differences of log amplitudes and phase, we must observe these differences over coherent frequency bands rather than noise zones. Therefore, before attenuation parameters are computed, we should compute a Time-Frequency analysis and determine the coherent band-width. These bands are observed as laterally coherent events on the seismic section, and higher amplitude spectral bands and laterally continuous phase zones on the Time-Frequency displays.

Since we are observing the energy decay with respect to travel distance or time, we have to make sure that the log amplitude difference between top and the bottom of the beds are consistently positive, indication some loss. Since we can not gain any energy by traveling longer distance, in the negative cases, we can identify either to top or the base of the bed identified as not correctly picked..

Conclusions:

In this report, we have presented the Joint Time-Frequency analysis using Gabor-Morlet decomposition. This analysis makes it possible to measure absorption, Q quality parameter and dispersion directly between two events. Time-Frequency display is a valuable tool in itself, since it shows the major boundaries where considerable change of Q and/or dispersion may take place

References:

Arens, G., Fourgeau, E., Giard, D. and Morlet, J., 1980, Signal filtering and velocity dispersion through multilayered media, 50th Annual Internat. Mtg., Soc. Expl. Geophys., Reprints: , Session:G.70.

Gabor, D., 1946, Theory of Communication; Jour. IEEE, v 93, p. 429-441.

Goupillaud, P., Grossmann, A. and Morlet, J., 1983, Cycle-octave representation for instantaneous frequency spectra: 53rd Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, , Session:S24.5.

Goupillaud, P. L., Grossmann, A. and Morlet, J., 1984, A simplified view of the cycle-octave and voice representations of seismic signals: 54th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, Session:S1.7.

Johnston, D.H. and Toksoz, M.N., 1981, Seismic Wave Attenuation, Toksoz, M.N. and Johnston, D.H.,Eds., Seismic Wave Attenuation: Society of Exploration Geophysics, Geophysics Reprint Series no. 2, 1-5.

Millouet, J. and Morlet, J., 1965, Utilisation d'un central de digitalisation d'enregistrements sismiques: Geophys. Prosp., 13, no. 03, 329-361.

Morlet, J., 1981, Sampling theory and wave propagation, 51st Annual Internat. Mtg., Soc. Expl. Geophys., Reprints: , Session:S15.1.

Morlet, J., Arens E., Fourgeau, E. and Giard D., 1982, Wave propagation and sampling theory-Part 1; Complex signal and scattering in multilayer media. Part I; Geophysics, v.47 no. 2, p. 203-221.

Morlet, J., Arens E., Fourgeau, E. and Giard D., 1982, Wave propagation and sampling theory-Part 1; Sampling theory and Complex waves. Part II; Geophysics.v.47 no. 2, p. 222-236. (* Discussion in GEO-49-09-1562-1563; Reply in GEO-49-09-1564-1564)

Morlet, J., 1984, Reply to discussion of 'Wave propagation and sampling theory - Part I: Complex signal and scattering in multi-layered media', by Morlet, J., et al (GEO-47-02-0203-0221): Geophysics, 49, no. 09, 1564.

Morlet, J. and Schwaetzer, T., 1962, Mesures d'amplitude dans Les sondages le log d'attenuation: Geophys. Prosp., 10, no. 04, 539-547.

Koehler, F., 1983, Gabor Wavelet Theory; SRC Internal report.

Koehler, F., 1984, Gabor Wavelets, Transforms, and Filters. 1, 2, 3 Dimensions. Continuous and Discrete Theory and application to Migration; SRC Internal Report.

Qian, S. and Chen D., 1999, Joint Time-Frequency Analysis; IEEE Signal Processing Magazine, Vol. 16, No. 2, 52-67.

Qian, S. and Chen D., 1996, Joint Time-Frequency Analysis; Englewood Cliffs NJ, Prentice Hall.

Figure 1. Gabor-Morlet decomposition

**A) Seismic
Data**

**B) Time/Frequency
Amplitude Spectra**

**C) Time/Frequency
Phase Spectra**