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THE EFFECT OF MATERIAL HOMOGENIZATION IN CALCULATING THE
GAMMA-RAY DOSE FROM SPENT PWR FUEL PINS IN AN AIR MEDIUM

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ABSTRACT

The effect of material homogenization on the calculated dose rate was studied for several arrangements of typical PWR spent fuel pins in an air medium using the Monte Carlo code, MCNP.¹ The models analyzed increased in geometric complexity, beginning with a single fuel pin, progressing to “small” lattices, i.e., 3x3, 5x5, 7x7 fuel pins, and culminating with a full 17x17 pin PWR bundle analysis. The fuel pin dimensions and compositions were taken directly from a previous study² and efforts were made to parallel this study by specifying identical flux-to-dose functions and gamma-ray source spectra.

The analysis shows two competing components to the overall effect of material homogenization on calculated dose rate. Homogenization of pin lattices tends to lower the effect of radiation “channeling” but increase the effect of “source redistribution.” Depending on the size of the lattice and location of the detectors, the net effect of material homogenization on dose rate can be insignificant or range from a 6% decrease to a 35% increase relative to the detailed geometry model.

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I. INTRODUCTION

The dry-lattice problem is essentially a shielding problem composed of an array of two heterogeneous compositions: a highly attenuating medium (fuel pin) and low attenuating medium (air). In many practical analytical situations, it is beneficial to combine the two media into a single, homogeneous smear of fuel pin and air.

Although material homogenization saves time in eliminating detail from the calculational model and/or may be unavoidable given limitations of the analysis code being employed, this type of simplification can lead to errors in the calculated gamma-ray transmission through the lattice. Large differences in calculated dose have been reported² between the results of heterogeneous and homogeneous models of a PWR fuel pin lattice.

Consider the case where a “shield” of continuous material contains a random dispersion of a second material with different attenuation characteristics. Instances of the second material in the shield can be considered as discontinuities. Discontinuities within shields can be either random or ordered and can consist of highly attenuating material, “lumps,” or low attenuating material, “voids.” The magnitude of the discontinuity can be classified as large or small based on its optical thickness relative to the mean-free path of the continuous phase.

Various techniques have been employed to calculate the effect of discontinuities using ray theory³ on random and ordered discontinuities. However, to rigorously study the effect of discontinuities it is convenient to use Monte Carlo techniques. Application of the Monte Carlo method for particle transmission through random, binary stochastic media has also been studied.⁴

In this work, it is convenient to consider the fuel pin regions in the lattice as the discontinuities and the air surrounding the pins in the lattice as the continuous material. Clearly, the discontinuities are “large” and regularly spaced within the continuous air medium. Homogenization of materials within the lattice will serve to eliminate the channeling effect by removing the discontinuities. It is expected that this will tend to lower the outwardly directed gamma-ray transmission through the lattice. Direct measurements of fission-product gamma-ray transmission through low-enriched fuel pin lattices in air have been made characterizing the channeling effect for some lattice arrangements.⁵

A second effect of material homogenization is to redefine the source volume. Typically, the birth sites of all source photons are restricted to the cylindrical volume of the fuel pins. Once the fuel pin, cladding, and surrounding air are homogenized, the birth sites can exist at any location within the homogenized volume (source redistribution). This implies that outwardly directed source photons born close to the edge of the homogenized volume will undergo less attenuation. This effect is coupled to the traditional self-shielding effect that reduces the attenuation of gamma rays born within fuel regions. Therefore, the resulting gamma-ray transmission in a homogeneous model will depend on the competing effects of eliminating the channeling effect and source redistribution.

II. CALCULATIONAL MODELS

Intuitively, the removal of the channeling effect will tend to lower the dose and the redistribution of the source will tend to increase the dose. By comparing MCNP results

from heterogeneous material calculations and homogeneous material calculations, a determination can be made as to which effect dominates for PWR pin lattices.

The fuel pin and bundle used for this study are based on the Westinghouse Vantage-5 TM, 17x17 array with control rods and instrument tube. This is the same bundle previously studied² and the fuel bundle characteristics shown in Table I are taken directly from the reference. The gamma-ray source and material compositions used in this study are also taken from the reference and are shown in Table II and Table III, respectively.

The source position was sampled uniformly in the radius of the fuel pellet and a cosine probability density function (at the beginning of exposure) was used to sample the axial position in the pin. Each pin in the bundle was assumed to have the same power density and therefore assumed to have equal probability of emitting a delayed fission-product gamma ray.

Ring detectors perpendicular to the vertical axis were used to tally the gamma-ray flux on the outside of the bundle at various radial distances from its center. A flux-to-dose function was added to convert the MCNP tally to dose. The geometry, source, compositions, and tallies in this study are all consistent with that in the previous work.²

II.A Single Pin and Small Lattice Calculations

To isolate the effect of source redistribution, a heterogeneous and homogeneous model of a single PWR fuel pin was constructed. In the absence of any other pins to provide a channeling effect, the difference in the calculated dose should be a direct result of the

source redistribution. Similarly, to isolate the channeling effect, 3x3, 5x5, and 7x7 PWR pin lattices were modeled with both homogeneous and heterogeneous materials.

Table IV shows the results of the single pin and small lattice calculations using a set of five ring detectors. The detector radii start at a minimum value, determined by adding 0.1 cm to the diagonal of the entire lattice, and increase to a value of 19.0 cm beyond the minimum radius. Two additional ring detectors are placed at 100.0 cm and 200.0 cm distance from the center of the pin or lattice. All the ring detectors are at the mid-point of the model relative to the vertical axis. The ratios versus detector distance from Table IV are plotted in Figure 1.

The data confirm that source redistribution results in a higher dose for the homogeneous model. As the distance to the detector increases, this effect lessens due to the source volumes in both cases approaching a line-source, parallel to the axis of the ring detector. The effect, however, remains significant even at 200.0 cm where the homogeneous dose is 17% higher than that calculated in the heterogeneous model.

The results of the 3x3 PWR fuel bundle pin lattice are shown in Table V. These results begin to show the effect of homogenization of materials in a lattice. The channeling in the heterogeneous model competes with the source redistribution of the homogeneous model, resulting in a lower ratio of the homogeneous to heterogeneous dose. Figure 2 shows the homogeneous dose rate to heterogeneous dose rate ratio as a function of distance from the center of the lattice. Again, the ratios decrease with distance as the two source volumes appear more as a line-source relative to the detectors. However, the dose rate calculated in the homogeneous case remains nearly 10% higher than the heterogeneous dose rate.

As the lattices grow in size, the magnitude of the channeling effect is reduced. However, the data shown in Table VI and VII and plotted in Figure 3 for the 5x5 and 7x7 PWR fuel pin lattices show less of a difference between the homogeneous and heterogeneous model calculated doses. This suggests that a boundary effect is involved in the competition between heterogeneous channeling and the homogeneous source redistribution.

As previously discussed, one of the primary benefits from source redistribution as a result of homogenization is the potential for a gamma-ray birth site to be near the boundary of the unit cell, thereby increasing the escape probability of the gamma ray (a surface-to-volume effect). The escape probability decreases the further the source site is from the lattice boundary. Therefore, for source redistribution to have a larger effect, a significant fraction of the gamma-ray birth sites need to be well within a mean-free path of the edge of the lattice.

The probability of selecting a birth site within a mean-free path of the boundary is proportional to the ratio of the desired birth-site volume to the total volume of the unit cell. Given constant material properties and lattice characteristics, the mean-free path of a photon will not change with the size of the lattice, causing the volume of desirable birth sites to increase less rapidly than the total volume of the unit cell.

Figure 4 shows the relationship between the escape probability and volume fraction of desired birth sites for a photon with $\mu=0.67 \text{ cm}^{-1}$. Considering a volume within one half of a mean-free path (61% escape probability) to a boundary, the corresponding volume ratios for a 3x3, 5x5, 7x7, and 17x17 PWR lattices are 0.63, 0.42,

0.31, and 0.13, respectively. The overall effect of increasing the lattice size is to reduce the magnitude of source redistribution in the homogeneous case.

II.B. Complete Fuel Bundle Calculations

MCNP models for the entire PWR fuel bundle, as described in Reference 1, were created using both heterogeneous and homogeneous materials. An isotopic mass balance of the homogenized materials was performed to ensure that the mass of each isotope was conserved.

The model consists of all the control rod tubes and the instrumentation tube assuming that the control rods themselves are absent and that the instrumentation tube is empty. An air medium surrounds the bundle, pins, control rod tubes and instrumentation tube. It is assumed that air occupies the inside of the control rod tubes and instrumentation tube, as well.

The identical gamma-ray source described in Table II is used, with the source-sites sampled uniformly from all fuel pins. The radial distribution within a fuel pin is flat and the axial distribution is cosine shaped, as with the small lattice models.

Ring detectors are placed at various radii perpendicular to the vertical axis of the bundle starting at 15.2 cm and extending to 500.0 cm. Three separate calculations were performed with the set of ring detectors at $z = 0.0$ cm (axial mid-point), $z = 100.0$ cm, and $z = 200.0$ cm. Given the axial symmetry of the model, the solution at any height, z , should identically equal the solution at $-z$. The ring detector tallies are modified by the same flux-to-dose function used in the small lattice calculations.

Two axial regions were modeled in the homogeneous case into which masses from the heterogeneous model were uniformly distributed: (1) the fuel region, (2) the top

and bottom plenum regions. The volume of the homogeneous fuel region was determined by calculating the dimensions of a parallelepiped having sides equal to 17 unit cells. Each unit cell had length and width dimensions equal to the lattice pitch. The axial height remained the same for both the heterogeneous and homogeneous cases. This resulted in a parallelepiped having dimensions of 21.42 cm x 21.42 cm x 365.76 cm. The mass of all materials located in the fuel region (fuel meat, fuel cladding, control rod cladding, instrumentation tube cladding, and interstitial air) were uniformly distributed into this volume. The same procedure was used for the top and bottom plenum regions and the materials located in these regions (cladding, springs, and interstitial air) were uniformly distributed into this volume. Gamma-ray source sites were sampled uniformly within the x - y plane and sampled according to a cosine-shape in the axial dimension. A plot on the x - y plane at the bundle mid-point is shown in Figure 6, as rendered by the MCNP plotter, with the center portion magnified to provide greater detail.

Calculated dose rates for the $z = 0$ cm case are shown in Table VIII. Figure 7 shows the dose rate results for the homogeneous and heterogeneous cases along with the ratio of the homogeneous to heterogeneous dose rates. There exists a very slight over prediction of approximately 1% in the homogeneous case but otherwise no significant differences in dose rates were visible as a result of homogenization. This result is in line with the expectations. A large lattice will see less of an effect from source redistribution as a result of homogenization. The channeling effect of the heterogeneous model nearly balances (within about 1%) the increase in dose rate from source redistribution.

Calculated dose rates for the $z = 100.0$ cm case are shown in Table IX. Figure 8 shows the dose rate results for the homogeneous and heterogeneous cases along with the

ratio of the homogeneous to heterogeneous dose rates. There exists a slight under prediction close to the lattice, but the trend is increasing with detector radius and is approximately unity at 500.0 cm from the center of the lattice.

The gamma-ray source at this axial height ($z = 100.0$ cm) is considerably less than at the axial mid-plane ($z = 0.0$), where the maxima of the cosine-shaped distribution is located. Gamma rays born at locations closer to the mid-plane have to travel nearly vertically in order to contribute to the detector tallies close to the bundle. This eliminates the benefit from radial source redistribution since the photons may travel a significant vertical distance through the lattice. The channeling effect present in the heterogeneous case, however, facilitates transport of photons both radially and vertically, causing the homogeneous case to under predict the dose rates at these locations.

As the detector radii increase, photons born towards the axial mid-plane have less of a vertical path to the detector and the radial effects of source redistribution again become a factor. At the 300.0 cm detector, the homogeneous and heterogeneous dose rates are very nearly equal with a slight increasing trend noted at the 400.0 cm and 500.0 cm detectors.

The final set of results has the ring detectors at a vertical position of ± 200.0 cm, which is slightly greater than that of the fueled region of the bundle. The calculated dose rates and dose rate ratios are shown in Table X. Figure 9 shows the dose rate results for the homogeneous and heterogeneous cases along with the ratio of the homogeneous to heterogeneous dose rates.

At this height, gamma rays born at any location in the fuel need to travel a vertical distance through the lattice before contributing to the detectors close to the bundle. As

the detector radii increase, the fraction of the vertical distance within the fuel bundle decreases. The peculiar maximum in the calculated dose rates at 25 cm to 30 cm is a result of the shielding from fuel and plenum regions reducing the dose rate. As the detector radii increase, the shielding decreases faster than the geometric attenuation leading to a maximum between 25 cm and 30 cm. As in the previous case, the channeling effect facilitates the transport of gamma rays through the lattice vertically leading to a slight under prediction of the homogeneous dose rate. As the detector radii increase further, the vertical distance traveled by gamma rays contributing to the detector tallies decreases. The effect of source redistribution begins to compensate for the lack of channeling in the homogeneous model, explaining the increasing trend in the homogeneous to heterogeneous ratios.

The results of the full PWR fuel bundle analyses demonstrate that the effect of homogenizing the fuel region, consistent with the method of this study, does impact the calculated dose rates on the outside of the bundle. The largest effects are noted at locations away from the axial mid-point and close to the bundle. Within three to five meters distance of the bundle, the dose rates of the homogeneous and heterogeneous models are nearly identical.

III. CONCLUSIONS

The competing effects of source redistribution and removing the channeling on the calculated dose rate outside a typical PWR fuel pin lattice was investigated using MCNP. Several calculations were performed using a single pin, small lattices of pins, and finally a 17x17 pin standard PWR fuel bundle. Comparisons of the heterogeneous and homogeneous dose rates showed that for the single pin and small lattices, the

homogenization of materials resulted in an over prediction of the dose rate on the outside of the lattice. This effect decreased as the lattice became larger and the heterogeneous and homogeneous dose rates for the 17x17 case are nearly identical, at the bundle axial mid-point (see Table XI and Table XII).

This is significant in that it differs substantially from the previously published study² that suggests the peak homogeneous to heterogeneous ratios can be as high as 13 in the $z = 0$ and $z = +100$ cases, and 69 in the $z = +200$ cm case. The results of this work suggest that if the analyst is interested only in the total dose outside a fuel bundle, it may not be necessary to include all the detail of the heterogeneous model, specifically if the calculation is at the bundle axial mid-point.

Additionally, this work supports the current approach used in calculating gamma-ray dose to the pressure vessel and the response of gamma-ray detectors external to the core where, in both cases, the fuel bundles are generally modeled as homogeneous regions. However, if the dose rate on the outside of a lattice of fuel pins as a function of angle is required, or dose rate in close proximity to the bundle or at heights other than the bundle midpoint is desired then full heterogeneous modeling is required.

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Table I. Characteristics of PWR fuel bundle used in this study.

Fuel element array	17 x 17
Number of fuel elements	264
Number of control rod channels	24
Number of instrument tubes	1
Assembly dimension (cm)	21.42 x 21.42
Pellet-pellet gap (cm)	0.3456
Fuel element outside diameter (cm)	0.9144
Fuel element height (cm)	386.80
Cladding thickness (cm)	0.0572
Pellet-clad gap (cm)	0.0082
Top and Bottom plenum length (cm)	19.37
Fuel pellet outside diameter (cm)	0.7836
Fuel rod length (cm)	365.76
Control rod channel outside diameter (cm)	1.0922
Cladding thickness (cm)	0.04064
Instrumentation tube outside diameter (cm)	1.204
Cladding thickness (cm)	0.04064

Table II. Energy group structure and gamma-ray emission rates for PWR bundle.

Group	Energy Range (MeV)	Average Energy (MeV)	Energy Release Rate ^a (MeV/fission)	Photon Release Rate (γ /s)
1	0.0 to 1.0	0.5	7.54E-3	1.511E-2
2	1.0 to 2.0	1.5	1.52E-4	1.013E-4
3	2.0 to 3.0	2.5	3.78E-4	1.511E-4
4	3.0 to 4.0	3.5	1.38E-7	3.935E-8
5	4.0 to 5.0	4.5	4.34E-22	9.640E-23
6	5.0 to 7.5	6.25	1.83E-22	2.937E-23
7	7.5 to 20.0	13.75	6.44E-24	4.689E-25

^aEnergy release rates are given after 1 yr cooling time following 40000 h of service in the core.

Table III. PWR Fuel bundle material compositions.

Material	Fuel	Air	Zircaloy	Type 304 Stainless Steel
Density (g/cm ³)	10.5	1.296E-3	6.5	0.972
Element	Weight Fractions			
Uranium	0.8815			
Oxygen	0.1185	0.2315		
Nitrogen		0.7552		
Argon		0.0133		
Zirconium			0.9850	
Tin			0.0120	
Iron			0.0030	0.6950
Chromium				0.1900
Manganese				0.0200
Nickel				0.0950

Table IV. Dose rate comparison for single PWR fuel bundle pin modeled heterogeneously and homogeneously.

Ring Detector Position	Distance from Center of Pin (cm)	Heterogeneous Dose Rate* (Sv/s)	Homogeneous Dose Rate* (Sv/s)	Ratio (Hom. / Het.) $\pm$ 95% CI
Minimum	1.0	1.1515E-15	1.5525E-15	1.35 $\pm$ 0.05
+ 1.0	2.0	5.7146E-16	7.2275E-16	1.26 $\pm$ 0.03
+ 4.0	5.0	2.2470E-16	2.8118E-16	1.25 $\pm$ 0.02
+ 9.0	10.0	1.1227E-16	1.3872E-16	1.24 $\pm$ 0.01
+ 19.0	20.0	5.4690E-17	6.7154E-17	1.23 $\pm$ 0.01
100.0	100.0	8.2131E-18	9.7579E-18	1.188 $\pm$ 0.004
200.0	200.0	2.8383E-18	3.3214E-18	1.170 $\pm$ 0.004

*MCNP Calculated dose rates are normalized to a single source gamma ray.

Table V: Dose rate comparison for a 3x3 lattice of PWR fuel bundle pins modeled heterogeneously and homogeneously.

Ring Detector Position	Distance from Center of Lattice (cm)	Heterogeneous Dose Rate* (Sv/s)	Homogeneous Dose Rate* (Sv/s)	Ratio (Hom. / Het.) $\pm$ 95% CI
Minimum	2.78	3.2553E-16	3.6942E-16	1.13 $\pm$ 0.03
+ 1.0	3.78	2.2634E-16	2.5513E-16	1.13 $\pm$ 0.03
+ 4.0	6.78	1.2125E-16	1.3534E-16	1.12 $\pm$ 0.02
+ 9.0	11.78	6.8764E-17	7.6477E-17	1.11 $\pm$ 0.01
+ 19.0	21.78	3.6251E-17	4.0026E-17	1.10 $\pm$ 0.01
100.0	100.0	6.1116E-18	6.7094E-18	1.098 $\pm$ 0.005
200.0	200.0	2.1473E-18	2.3453E-18	1.092 $\pm$ 0.004

*MCNP Calculated dose rates are normalized to a single source gamma ray.

Table VI: Dose rate comparison for a 5x5 lattice of PWR fuel bundle pins modeled heterogeneously and homogeneously.

Ring Detector Position	Distance from Center of Lattice (cm)	Heterogeneous Dose Rate* (Sv/s)	Homogeneous Dose Rate* (Sv/s)	Ratio (Hom. / Het.) $\pm$ 95% CI
Minimum	4.56	1.5296E-16	1.6888E-16	1.10 $\pm$ 0.03
+ 1.0	5.56	1.2024E-16	1.2987E-16	1.08 $\pm$ 0.03
+ 4.0	8.56	7.3392E-17	7.9721E-17	1.09 $\pm$ 0.02
+ 9.0	13.56	4.5476E-17	4.8818E-17	1.07 $\pm$ 0.01
+ 19.0	23.56	2.5476E-17	2.7255E-17	1.07 $\pm$ 0.01
100.0	100.0	4.7349E-18	5.0543E-18	1.07 $\pm$ 0.01
200.0	200.0	1.6775E-18	1.7850E-18	1.064 $\pm$ 0.005

*MCNP Calculated dose rates are normalized to a single source gamma ray.

Table VII: Dose rate comparison for a 7x7 lattice of PWR fuel bundle pins modeled heterogeneously and homogeneously.

Ring Detector Position	Distance from Center of Lattice (cm)	Heterogeneous Dose Rate* (Sv/s)	Homogeneous Dose Rate* (Sv/s)	Ratio (Hom. / Het.) $\pm$ 95% CI
Minimum	6.35	8.9405E-17	9.5946E-17	1.07 $\pm$ 0.03
+ 1.0	7.35	7.3635E-17	7.8461E-17	1.07 $\pm$ 0.03
+ 4.0	10.35	4.9458E-17	5.2141E-17	1.05 $\pm$ 0.02
+ 9.0	15.35	3.2226E-17	3.4116E-17	1.06 $\pm$ 0.01
+ 19.0	25.35	1.8993E-17	1.9888E-17	1.05 $\pm$ 0.01
100.0	100.0	3.8132E-18	4.0149E-18	1.05 $\pm$ 0.01
200.0	200.0	1.3664E-18	1.4286E-18	1.046 $\pm$ 0.005

*MCNP Calculated dose rates are normalized to a single source gamma ray.

Table VIII: MCNP calculated dose rates for heterogeneous and homogeneous models of 17x17 fuel bundle at the bundle mid-plane ($z = 0$).

Detector Radius	Het. Dose Rate*		Hom. Dose Rate*		Dose Rate Ratio	
	(Sv/s)	Rel. Error	(Sv/s)	Rel. Error	(Hom. / Het.)	95% CI
15.2	2.0608E-17	0.0103	2.0641E-17	0.0106	1.0016	0.0296
20	1.3985E-17	0.0071	1.4069E-17	0.0074	1.0060	0.0205
25	1.0641E-17	0.0059	1.0757E-17	0.0059	1.0108	0.0167
30	8.6163E-18	0.0052	8.6730E-18	0.005	1.0066	0.0144
40	6.2411E-18	0.0043	6.3030E-18	0.0042	1.0099	0.0120
50	4.8239E-18	0.0037	4.8726E-18	0.0037	1.0101	0.0105
75	2.9775E-18	0.003	3.0079E-18	0.003	1.0102	0.0085
100	2.0619E-18	0.0027	2.0860E-18	0.0027	1.0117	0.0076
200	7.4184E-19	0.0024	7.4760E-19	0.0023	1.0078	0.0066
300	3.7027E-19	0.0024	3.7235E-19	0.0022	1.0056	0.0065
400	2.1711E-19	0.0023	2.1949E-19	0.0023	1.0109	0.0065
500	1.4203E-19	0.0024	1.4339E-19	0.0023	1.0096	0.0066

*MCNP Calculated dose rates are normalized to a single source gamma ray.

Table IX: MCNP calculated dose rates for heterogeneous and homogeneous models of 17x17 fuel bundle at the bundle mid-plane ($z = \pm 100.0$).

Detector Radius	Het. Dose Rate*		Hom. Dose Rate*		Dose Rate Ratio	
	(Sv/s)	Rel. Error	(Sv/s)	Rel. Error	(Hom. / Het.)	95% CI
15.2	1.6720E-17	0.0124	1.6128E-17	0.0118	0.9646	0.0343
20	1.1365E-17	0.008	1.0984E-17	0.0081	0.9665	0.0228
25	8.6460E-18	0.0066	8.3723E-18	0.0065	0.9683	0.0186
30	6.9764E-18	0.0056	6.8591E-18	0.006	0.9832	0.0164
40	5.1151E-18	0.0049	4.9718E-18	0.0049	0.9720	0.0139
50	3.9630E-18	0.0043	3.8621E-18	0.0047	0.9745	0.0127
75	2.4403E-18	0.0034	2.3874E-18	0.0033	0.9783	0.0095
100	1.7062E-18	0.0031	1.6721E-18	0.003	0.9800	0.0086
200	6.4422E-19	0.0025	6.3843E-19	0.0025	0.9910	0.0071
300	3.3552E-19	0.0025	3.3552E-19	0.0024	1.0000	0.0069
400	2.0378E-19	0.0023	2.0415E-19	0.0023	1.0019	0.0065
500	1.3547E-19	0.0023	1.3610E-19	0.0023	1.0046	0.0065

*MCNP Calculated dose rates are normalized to a single source gamma ray.

Table X: MCNP calculated dose rates for heterogeneous and homogeneous models of 17x17 fuel bundle at the bundle mid-plane ($z = \pm 200.0$).

Detector Radius	Het. Dose Rate*		Hom. Dose Rate*		Dose Rate Ratio	
	(Sv/s)	Rel. Error	(Sv/s)	Rel. Error	(Hom. / Het.)	95% CI
15.2	1.3411E-18	0.0186	1.2657E-18	0.0212	0.9438	0.0562
20	1.4533E-18	0.0156	1.3890E-18	0.0161	0.9558	0.0449
25	1.4521E-18	0.0114	1.3933E-18	0.0125	0.9595	0.0338
30	1.4380E-18	0.0106	1.3564E-18	0.011	0.9432	0.0306
40	1.3134E-18	0.0078	1.2406E-18	0.0082	0.9445	0.0226
50	1.1844E-18	0.0067	1.1130E-18	0.0067	0.9397	0.0190
75	9.2841E-19	0.0048	8.9153E-19	0.005	0.9603	0.0139
100	7.6094E-19	0.0047	7.2931E-19	0.004	0.9584	0.0124
200	4.1090E-19	0.0029	4.0116E-19	0.0029	0.9763	0.0082
300	2.5300E-19	0.0026	2.5036E-19	0.0026	0.9896	0.0074
400	1.6857E-19	0.0028	1.6810E-19	0.0024	0.9972	0.0074
500	1.1805E-19	0.0024	1.1814E-19	0.0024	1.0008	0.0068

*MCNP Calculated dose rates are normalized to a single source gamma ray.

Table XI: MCNP calculated dose rate ratios for single PWR pin and small PWR pin lattices at the axial midpoint of the assembly.

Detector Radius from Center of Assembly (cm)	Dose Rate Ratio (Hom. / Het.)			
	Single Pin ($r_{min} = 1.0$ cm)	3x3 Lattice ($r_{min} = 1.78$ cm)	5x5 Lattice ($r_{min} = 3.56$ cm)	7x7 Lattice ($r_{min} = 5.35$ cm)
r_{min}	1.35	1.13	1.10	1.07
$r_{min} + 1$	1.26	1.13	1.08	1.07
$r_{min} + 4$	1.25	1.12	1.09	1.05
$r_{min} + 9$	1.24	1.11	1.07	1.06
$r_{min} + 19$	1.23	1.10	1.07	1.05
$r = 100$	1.19	1.10	1.07	1.05
$r = 200$	1.17	1.09	1.06	1.05

Table XII: MCNP calculated dose rate ratios for a 17x17 PWR fuel bundle at $z = 0.0$ cm (the axial midpoint), $z = +100$ cm, and $z = +200$ cm.

Detector Radius from Center of Assembly (cm)	Dose Rate Ratio (Hom. / Het.)		
	$z = 0.0$ cm (Axial midpoint)	$z = +100$ cm	$z = +200$ cm
15.2	1.0016	0.9646	0.9438
20	1.0060	0.9665	0.9558
25	1.0108	0.9683	0.9595
30	1.0066	0.9832	0.9432
40	1.0099	0.9720	0.9445
50	1.0101	0.9745	0.9397
75	1.0102	0.9783	0.9603
100	1.0117	0.9800	0.9584
200	1.0078	0.9910	0.9763
300	1.0056	1.0000	0.9896
400	1.0109	1.0019	0.9972
500	1.0096	1.0046	1.0008

Figure 1. Ratio of MCNP calculated dose rates for homogeneous and heterogeneous models of a single PWR bundle fuel pin as a function of detector distance.

Figure 2. Ratio of MCNP calculated dose rates for homogeneous and heterogeneous models of a 3x3 lattice of PWR bundle fuel pins as a function of detector distance.

Figure 3. Ratio of MCNP calculated dose rates for homogeneous and heterogeneous models of a 5x5 and 7x7 lattice of PWR fuel pins as a function of detector distance.

Figure 4: Escape probabilities and volume fractions of desired birth sites for a photon with $\mu=0.67 \text{ cm}^{-1}$ in a homogenized model of 3x3, 5x5, 7x7, and 17x17 PWR fuel pin lattices.

Figure 6: MCNP model of the 17x17 PWR fuel bundle with control rod channels and instrument tube at the assembly mid-point, with center section magnified.

Figure 7: MCNP calculated dose rates for heterogeneous and homogeneous models of 17x17 fuel bundle at the bundle mid-plane ($z = 0$) as a function of detector radius.

Figure 8: MCNP calculated dose rates for heterogeneous and homogeneous models of 17x17 fuel bundle at $z = \pm 100.0$ as a function of detector radius.

Figure 9: MCNP calculated dose rates for heterogeneous and homogeneous models of 17x17 fuel bundle at $z = \pm 200.0$ as a function of detector radius.