Final Report:

Collaborative Research: Analysis and Interpretation of Multi-Scale Phenomena in Crustal Deformation Processes

Using Numerical Simulations of Complex Nonlinear Earth Systems

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Introduction

In both our past work and the work in progress we focused on understanding the physics and statistical patterns in earthquake faults and fault systems. Our approach had three key aspects. The first was to look for patterns of seismic activity in earthquake fault systems. The second was to understand the physics of a sequence of models for faults and fault systems that are increasingly more realistic. The third key element was to connect the two previous approaches by investigating specific properties found in models to see if they are indeed properties of real faults.

A specific example of how this approach works can be seen in the following: In the papers discussed below, we demonstrated that the cellular automaton (CA) versions of the slider block models with long range stress transfer are ergodic and could be described by a Boltzmann-Gibbs distribution in the meanfield limit. The ergodicity follows from the fact that the long range stress transfer makes the model meanfield. The meanfield nature of the CA models, generated by long range stress transfer, also allows a description of the CA models by a Langevin equation. The Langevin eqauation indicates that evolution of seismicity in the model over relatively short times is linear in time. This appears to be consistent with the success of a forcasting algorithm we have developed that is based on a linear evolution of seismicity patterns. This algorithm has had considerable success in that the regions of the Southern California fault system which have been predicted to have a higher probability of an event greater than magnitude 5 have consistently been the sites where such events occur.

These two results have led to the question as to whether the Southern California fault system is ergodic and can be described by a Langevin equation like the model. To answer this question we ran a series of tests for ergodicity very much like the ones run on the models. Our results, which have been accepted for publication in Physical Review Letters (*Tiampo et al.*, in press), demonstrate that the Southern California system is ergodic in the same way that is seen in the models. These results will be discussed in more detail below. However, the point that needs to be emphasized is that it was the combination of model investigation via theory and simulation coupled with assimilation and classification of real data and applying the methods of statistical mechanics to real fault systems that led to both a successful forecasting algorithm and a deeper understanding of the nature of earthquake fault systems.

Below we describe in some detail the results obtained in the previous funding period. We present these in three groups. A) Investigation of statistical physics models and applications. B) Earthquake fault systems and Greens functions for complex sources and C) Space time patterns, data analysis and forecasting.

Statistical Physics Models and Applications

Our papers completed during the funding cycle in this area, funded in whole or in part by US DoE include:

Rundle, J.B., Computational earth system science, *Computing in Science and Engineering (CiSE)*, **2** , 20-21, 2000.

Preston, E., JS de sa Martins, JB Rundle, M. Anghel and W. Klein, Models of earthquake faults with long-range stress transfer, *Computing in Science and Engineering*, **2**, 34-41, 2000.

Rundle, JB, DL Turcotte and W. Klein, Introduction, pp. 1-3, *GeoComplexity and the Physics of Earthquakes*, ed. JB Rundle, DL Turcotte and W. Klein, AGU Monograph 120, American Geophysical Union, Washington, DC, 2000.

Klein, W., CD Ferguson, M. Anghel, JB Rundle and JS Sa Martins, Cluster analysis in earthquake fault models with long-range interactions, *AGU Monograph " Geocomplexity and the Physics of Earthquakes"*, ed. JB Rundle, D. Turcotte and W. Klein, 2000.

JB Rundle and W. Klein, Coarse-grained models and simulations for nucleation, growth, and arrest of earthquakes, pp. 307-322 in Earthquake Thermodynamics and Phase Transformations in the Earth's Interior, ed. R. Teisseyre and E. Majewski, Academic Press, San Diego, 2000.

J.B. Rundle, W. Klein, D.L. Turcotte, B.D. Malamud, Precursory seismic activation and critical-point phenomena, *Pure Appl. Geophys*., **157**, 2165-2182, 2000.

(Book) Rundle, JB, DL Turcotte and W. Klein, *GeoComplexity and the Physics of Earthquakes*, AGU Monograph 120, American Geophysical Union, Washington, DC, 2000.

W. Klein, J Rundle, M. Anghel, J.S.S. Martins, and C. Ferguson, Scaling and phase transitions in models of earthquake faults, pp. 189-190, *Proceedings 2nd ACES Workshop*, ed. M. Matsu'ura, K. Nakajima and P. Mora, University of Queensland Press, Brisbane, AU (2001).

J.B. Rundle, K.F. Tiampo, W. Klein and J.S.S. Martins, Self-organization in leaky threshold systems: The influence of near mean field dynamics and its implications for earthquakes, neurobiology and forecasting, *Proc. Nat. Acad. Sci*. USA, **99**, Supplement 1, 2514-2521, (2002)

D.L. Turcotte and J.B. Rundle, Self-organized complexity in the physical, biological, and social sciences, *Proc. Nat. Acad. Sci*. USA, **99**, Supplement 1, 2463-2465, (2002).

JS de Sa Martins, JB Rundle, M. Anghel, W. Klein, Transition from simplicity to complexity in models of frictional sliding, *Phys. Rev. E*, **65**, 056117(1-4) (2002).

JB Rundle, DL Turcotte, C Sammis, W Klein and R. Shcherbakov, Statistical physics approach to understanding the multiscale dynamics of earthquake fault systems (invited), Rev. Geophys. Space Phys., in press (2002).

Klein, W, H. Gould, J. Tobochnik, FJ Alexander, M Anghel and G Johnson, Clusters and fluctuations in mean field critical points and spinodals, *Phys. Rev. Lett*., 85, 1270-1273, 2000.

Klein, W., T. Lookman, A Saxena and D Hatch, Nucleation in systems with elastic forces, *Phys. Rev. Lett*., 88, 085701(1-4), 2002.
M. Anghel, W. Klein, JB Rundle, and JS de Sa Martins, Scaling in a cellular automaton model of earthquake faults, *Phys. Rev. E*, submitted, 2002.

P Vincent, JB Rundle, R. Bilham and PA Rosen, Aseismic Mw=5.3 and 5.6 slip events along the Southern San Andreas fault system captured by radar interferometry, submitted to *Nature*, 2001.

E.F. Preston, J.S.S. Martins and J.B. Rundle, Abelian threshold models and forced weakening, Phys. Rev. Lett., in review, (2001).

N. Gulbahce, H. Gould and W. Klein, Phys. Rev. E in press, (2003)

In these papers, we have looked at a sequence of models with long range stress transfer. If the stress transfer is infinite in that a failed block transfers stress to blocks in an infinitely large region, then the model is ergodic. This means that time averages and and spatial averages of dynamical quantities give that same result. It also suggests that the system may be in equilibrium and could be described by Boltzmann-Gibbs(BG) statistical mechanics.

We tested this hypothesis and indeed found that the models can be described by equilibrium BG statistical mechanics. The test took the form of simulating the model and measuring the probability that the system has an energy E. These results led to a derivation of a Langevin equation for the stress that is only exactly valid for models with infinite range stress transfer.

When the stress transfer region is large but finite the system behavior is more complicated. In this case the system behaves as if it is in a metastable state. Hence for some time, as in thermal systems, the system acts as if it is in equilibrium with the associated BG statistics but large events (nucleation) occur that drive the system out of equilibrium. The occurance of the large events as well as their statistical distribution is predicted by the Langevin equation, however the ensuing processes are not. In the measurement of ergodicity instead of seeing a measure of ergodicity that last through the simulation run, one sees what we will refer to as punctuated equilibrium. Namely, there are periods where the system appears to be ergodic which are interrupted by a large event. This event takes the system out of equilibrium and some time is required for it to settle down to a new "equilibrium" state. After this relaxation time the system again appears to be in equilibrium until the next large event.

We also investigated the structure of critical phenomena fluctuations in long range and infinite range stress transfer models. The question we addressed is whether these models of earthquake faults and fault systems showed any difference in the physics when the events were of different sizes. For example; Is a "magnitude" 2 event in the model a different kind of event than a magnitude 7? Our first step was to understand the structure of fluctuations in this class of models. To do this we used the fact that the Langevin equation for the meanfield CA models is precisely the same as the Langevin equation for Ising models. Hence, any scaling found in the slider block models will be the same as the scaling in meanfield Ising models.

We found that in the meanfield models with infinite range stress transfer there are clusters, which we will refer to as fundamental clusters, that are the building blocks of all of the structures such as fluctuations and nucleation droplets. These fundamental clusters have an extremely low density compared to fluctuations and nucleating droplets. However, they do scale; that is $n_s \sim s^{-1.5}$ which translates into a b value of 0.5. We also were able to identify fluctuations that are made up of fundamental clusters but are much denser. They also scale with the same exponent. Finally, we were able to identify nucleation droplets which are the densest of the objects which scale with and exponent of 2. This leads to a b value of 1.

We then looked for these objects in simulations of the CA models and identified them. This differentiation leads to a Gutenburg-Richter(GR) scaling plot with two different slopes. One, at the smaller end, with a b value of 1 and the other, at the larger end with a b value of 2. It is important to note here that this result implies that these models produce “earthquake” events of different types that lie on a GR scaling plot. These different types of events have very different physical characteristics with very different origins. Moreover, when studying the long range, rather than infinite range, versions of these models via simulations the same structures appear. However, there also appears larger events that are not properties of an equilibrium system, do not fall on the GR scaling plot and are not described by the Langevin equation. These events appear to involve a significant fraction of the entire system and may be triggered by one of the smaller GR events.

Since nucleation events appear to be important in these CA models we asked the question; If nucleation is important in earthquake faults then is the nucleation droplet of the same type as found in long range Ising models? Will the fact that the stress transfer Green’s function, which comes from linear elasticity, has the form r^{-3} rather than the flat interaction with a cutoff used in theoretical and simulation studies of Ising models change the properties of the nucleation droplet. The answer was no. The flat interaction with the cutoff and the r^{-3} “interaction” from elasticity have the same form of nucleation droplet.

We have begun to investigate the properties of the Burridge-Knopoff (BK) model when the springs are not just nearest neighbor, as in the original, but they reflect the long range nature of the stress transfer on real faults. We have begun our investigation with the d=1 form of the model with a velocity weakened friction force and springs that attach a given block to 2R other blocks. The spring constants are all taken to be the same but they are scaled by 2R. The data we have taken so far indicates that there are two scaling regions in the GR plots, one with slope 1.5(b value .5) and one with slope 2.(b value 1). This is a preliminary indication that the physics of the long range BK model is the same as that of the long range CA model described above.

As we have pointed out systems with long but finite range interactions or stress transfer are not meanfield. We will refer to these systems as near meanfield. In meanfield systems there is a spinodal line. That is, there is a line of critical points that is responsible for the GR scaling. The question remains as to the nature of the spinodal, or as we will refer to it, the pseudo-spinodal when the stress transfer range or interaction is not infinite. To explore this question we have adopted an approach due to *Yang and Lee* (1952) which was used to answer the question of how phases appeared as the system size became infinite when it was clear that finite size systems had no phase transitions.

Lee and Yang noted that phase transitions arose from zeroes of the partition function. They showed that all of the zeroes of the partition function of a fluid in a finite volume were in the complex activity plane where the activity $z = e^{\beta\mu}$. Here β is the inverse temperature and μ is the chemical potential. There is a gap in the distribution of the zeroes in the neighborhood of the real-positive z axis. This is the axis of the physical values of z. This gap goes to zero as the system size goes to infinity. Namely the distance of the closest zero to the real axis decreases as the size goes to infinity.

The same approach works for Ising models with a Hamiltonian given by

$$-\beta H = J\Sigma_{ij}S_iS_j - h\Sigma_iS_i$$

where J is a coupling constant, h is an external magnetic field and $S_i = \pm 1$ The sums are over the lattice sites on which the spins represented by the S_i are located. For this model Lee and Yang showed that the zeroes of the partition function lie on the imaginary h axis with a gap centered around $h = 0$ for finite systems. This gap closes as the system size goes to infinity.

We used the same approach, looking for zeroes of the partition function, for finite range interactions rather than finite system sizes, to see if we can characterize the pseudo-spinodal. We found that the zeroes of the partition function in the Ising model lie in the complex temperature-magnetic field space for finite interaction range R.(Gulbahce et al 2003) There is a gap in the distribution of zeroes around the real temperature-magnetic field plane that shrinks as $R \to \infty$. Since the meanfield CA model and the meanfield Ising model are described by the same Langevin equation this means that the pseudo-spinodal in both models have the same structure. This demonstrates how spinodals, or more accurately, pseudo spinodals give rise to scaling in CA models. Given the results outlined above it also explains the origin of GR scaling in BK models with long range springs.

Earthquake Fault Systems and Greens Functions for Complex Sources

Papers in this area, supported in whole or in part by US DoE, include:

Folch, A., J. Fernandez, JB Rundle and J. Marti, Ground deformation in a viscoelastic medium composed of a layer overlying a half-space: A comparison between point and extended sources, *Geophys. J. Int.*, **40**, 37-50, 2000.

Tiampo, K.F., John B. Rundle, J. Fernandez, and J.O. Langbein, Spherical and ellipsoidal volcanic sources at Long Valley caldera, California, using a genetic algorithm technique, *J. Vol. Geotherm. Res.*, **102**, 189-206, 2000.

PB Rundle, JB Rundle, KF Tiampo, J. Martins, S. McGinnis, W. Klein, Triggering dynamics on earthquake fault networks, pp. 305-317, *Proc. 3rd Conf. San Andreas Fault System*, ed. G. Bokelmann and RL Kovach, Stanford Univ. (2000).

J. Fernández, M Charco, M, K.F. Tiampo, G. Jentzsch, JB Rundle, Joint interpretation of displacement and gravity data in volcanic areas. A test example: Long Valley Caldera, California**,** *Geophys. Res. Lett.* **28**, 1063-1066, 2001.

J. Fernandez, KF Tiampo, G. Jentzsch, M. Charco, JB Rundle, Inflation or deflation? New results for Mayon volcano applying elastic-gravitational modeling, *Geophys. Res. Lett*., **28**, 2349-2352, (2001).

PB Rundle, JB Rundle, KF Tiampo, J.S. S. Martins, S. McGinnis, W. Klein, Nonlinear network dynamics on earthquake fault systems, *Phys. Rev. Lett*., 87, 148501 (2001).

JB Rundle, A. Donnellan, G. Fox, W. Klein, P Rundle and K Tiampo, General Earthquake Models: Progress and prospects, pp. 147-150, *Proceedings 2nd ACES Workshop*, ed. M. Matsu'ura, K. Nakajima and P. Mora, University of Queensland Press, Brisbane, AU (2001).

J. Fernandez, KF. Tiampo, and J.B. Rundle, Viscoelastic displacement and gravity changes due to point magmatic intrusions in a gravitational layered solid earth, *Geophys. J. Int*, **146**, 155-170 (2001).

KF Tiampo, JB Rundle, P. Hopper, J.Sa Martins, S. Gross and S. McGinnis, Parallelization of a large-scale computational earthquake simulation program, *Concurrency & Computation, Practice and Experience*, 14, 531-550 (2002). ACES Special Issue.

JB Rundle, PB Rundle, W Klein, J Martins, KF Tiampo, A Donnellan and LH Kellogg, GEM plate boundary simulations for the Plate Boundary Observatory: Understanding the physics of earthquakes on complex fault systems, *PAGEOPH*, 159, 2357-2381 (2002).

J. Parker, A.Donnellan, G. Lyzenga, JB Rundle and T.Tullis, Performance modeling codes for the QuakeSim problem solving environment, *Computational Science – ICCS 2003*, proc. Int. Conf. Comp. Sci., Part III, 845-862 (Springer-Verlag, Berlin 2003).

K.F. Tiampo, J.B. Rundle, W. Klein and J.S.S. Martins, Ergodicity in natural fault systems, *PAGEOPH*, special issue, in press (2003).

M. Charco, J. Fernández, K. Tiampo, M. Battaglia, L. Kellogg, J. McClain, and J. B. Rundle, Study of volcanic sources at Long Valley Caldera, California, using gravity data and a genetic algorithm inversion technique, *PAGEOPH*, (2002).

JB Rundle, LH Kellogg, KF Tiampo, W. Klein and PB Rundle, Integrating NASA Space Geodetic Observations with Numerical Simulations of a Changing Earth, *Proc. 3rd ACES Workshop*, edited P. Mora and A. Donnellan, University of Queensland Press, submitted, (2002).

K.F. Tiampo, J.B. Rundle, W. Klein, J.S.S. Martins, and C.D. Ferguson, Ergodic dynamics in a natural threshold system, *Phys. Rev. Lett*., in press (2003).

JB Rundle, LH Kellogg, JSS Martins, KF Tiampo, W Klein, PB Rundle and A Donnellan, Illuminating the relationship between observable data and unobservable dynamics using numerical simulations of a changing earth, *PAGOPH*, submitted, (2002).

K.F. Tiampo, J.B. Rundle, J.S.S. Martins, W. Klein and S. McGinnis, Methods for the evaluation of geodetic data and seismicity developed with numerical simulations: Review and applications, PAGEOPH, submitted (2003).

K.F. Tiampo, J. Fernandez, M. Charco, G. Jentzsch, and J.B. Rundle, New results at Mayon, Philippines, from a joint inversion of gravity and deformation measurements, *PAGEOPH*, submitted, (2003).

J.Fernández, K. F. Tiampo, J. B. Rundle, and Gerhard Jentzsch, On gravity gradients produced by magmatic intrusions, *J. Geodynamics*, submitted (2003).

K.F. Tiampo, J. Fernandez, G. Jentzsch, M. Charco, and J.B. Rundle, Inverting for the parameters of a volcanic source using a genetic algorithm and a model for magmatic intrusion in elastic-gravitational layered earth models, *Computers and Geosciences*, submitted (2003).

A. Donnellan, JB Rundle, J. Ries, GC Fox, M. Pierce, J. Parker, R. Crippen, E. DeJong, B. Chao, W. Kuang, D. McLeod, M. Matsu'ura, J. Bloxham, Illuminating the earth's interior through advanced computing, *Comp. in Sci. Eng*., in press, (2003).

In these papers, we developed models in which the topological structure of the major strike-slip faults in the Southern California fault system are included explicitly. We refer to this model as Virtual California.(VC) We investigated the effect on the seismicity of the VC system of changing the stress dissipation properties of the friction laws on individual faults. Specifically we investigated the influence of an additional precursory aseismic slip which occurs on a much longer time scale than associated with the sudden seismic slip. The inclusion of this term was motivated by the laboratory experiments of Karner and Marone.

The effect of the aseismic term can be parameterized by a constant α. For $\alpha < 0$ we obtain stress roughening due to the aseismic slip. For $\alpha > 0$ we obtain stress smoothing. The events for $\alpha > 0$ tend to be larger and less complex than those in which $\alpha < 0$. Investigation of the effect of α is that it tends to smooth the stress distribution on the individual faults leading to larger events. By contrast $\alpha = 0$ tends to concentrate the stress release in smaller events. Since the parameter α is a determined by the network topology and the nature of the stress Greens function our results indicate that the network topology as well as the properties of individual faults will determine the nature of earthquakes on individual faults. Moreover, since the nature of the stress Greens function, and the fault system topology, can change with time the nature of the events on the fault can as well.

The fact that the models we study are in a semi or punctuated equilibrium state is an important aspect of their physics. This determines to a great extent that the systems will exhibit scaling and allows us to predict the scaling exponents. In addition it provides us with a framework which we can use to understand the nature of the earthquake events and, perhaps, use this understanding to develop forecasting techniques. It is clearly important then to understand whether real earthquake fault systems are in this punctuated equilibrium state, which motivated our work on the fluctuation ergodicity metric, discussed above.

We used the Thirumalai-Mountain ergodicity metric to test the central and southern California fault system for effective ergodicity. Our approach was to bin the system over the region 32^o to 40^o latitude and 115^o to 125^0 longitude into boxes 0.1^o to a side. Within these boxes we count the number of events in time periods of one year which we call $R_j(t)$. Hence, in this study $R_j(t) = E_j(t)$ We take these events out of two catalogs, the Southern California Earthquake Center (SCEC) data base available at www.scecdc.scec.org and the Northern california Seismic Network(NCSN) database available at quake.geo.berkeley.edu. The SCEC catalog is used for the region between 32^o and 36^o latitude and the NCSN catalog between 36^o and 40^o. We should note here that we have done the same calculation with both larger and smaller box sizes with the same results.

We examined the inverse metric for the number of events in the real seismicity data set and for a slider block simulation. We found that the curves were very similar, consisting of straight lines (where the system is presumably ergodic) interupted by kinks or jogs, where the system is undergoing transition from one potential energy minimum to another. These kinks correlate with large(M>5) events on the fault system. Our conclusion is that the central and southern California fault system is in punctuated equilibrium similar to the CA models.

We also continued our program of computation of Greens functions for layered elastic media, in association with the development of topologically realistic models of tectonic and volcanic sources. We are using and systematically testing these Greens functions not only in earthquake codes such as Virtual California, but also in codes to model volcanic regions such as Mayon in the Philippines, and the volcanoes of the Canary Islands.

Space-Time Patterns, Data Analysis and Forecasting

Papers in this area include:

Rundle, J.B., W. Klein and K. Tiampo, Linear pattern dynamics in nonlinear threshold systems, *Phys. Rev. E*, **61**, 2418-2431, 2000.

Rundle, JB, W. Klein, K. Tiampo and S. Gross, Dynamics of seismicity patterns in systems of earthquake faults, *AGU Monograph " Geocomplexity and the Physics of Earthquakes"*, ed. JB Rundle, D. Turcotte and W. Klein, 2000.

KF Tiampo, JB Rundle, S. McGinnis, SJ Gross and W. Klein, Observation of systematic variations in non-local seismicity patterns from southern California, *AGU Monograph " Geocomplexity and the Physics of Earthquakes"*, ed. JB Rundle, D. Turcotte and W. Klein, 2000.

K. Tiampo, J Rundle, S. McGinnis and W. Klein, Pattern dynamics and forecast methods in seismically active regions, pp. 471-476, *Proceedings 2nd ACES Workshop*, ed. M. Matsu'ura, K. Nakajima and P. Mora, University of Queensland Press, Brisbane, AU (2001).

KF Tiampo, JB Rundle, S. McGinnis, SJ Gross and W. Klein, Systematic variations in non-local seismicity patterns in southern California, pp. 283-292 in *Seismotectonics in Convergent Plate Boundaries*, ed. Y. Fujinawa and A. Yoshida, Terra Scientific Publishing (TerraPub), Tokyo, Japan (2002) [ISBN 4-88704-129-2].

KF Tiampo, JB Rundle, S. McGinnis and W. Klein, Pattern dynamics and forecast methods in seismically active regions, *PAGEOPH*, **159**, 2429-2467 (2002).

KF Tiampo, JB Rundle, S. McGinnis, S. Gross and W. Klein, Mean field threshold systems and phase dynamics: An application to earthquake fault systems, *Europhys. Lett.*, **60**, 481-487, (2002).

KF Tiampo, JB Rundle, S.J. Gross, S. McGinnis and W. Klein, Eigenpatterns in southern California seismicity, *J. Geophys. Res.*, 107, B12, 2354, doi:10.1029/2001JB000562 (2002).

JB Rundle, W Klein, KF Tiampo, Andrea Donnellan, and GC Fox, Strategies for the detection and analysis of space-time patterns in complex driven threshold systems, *Computational Science – ICCS 2003*, *Proc. Int. Conf. Comp. Sci.*, Part III, 827-844 (Springer-Verlag, Berlin 2003).

K.F. Tiampo, J.B. Rundle, W. Klein, Y. Ben-Zion, and S. McGinnis, Using eigenpattern analysis to constrain seasonal signals in southern California, *PAGEOPH*, special issue, (2003).

W. Klein, J. B. Rundle and K. Tiampo, Theoretical investigation of the principal component analysis of earthquake fault systems, *Phys. Rev. Lett.*, submitted (2003).

José Fernández, Juan. A. Infante, Ting-To Yu, John B. Rundle, Displacement produced by thrust faulting in a viscoelastic-gravitational plane earth model composed of two layers overlying a half-space, *Earth Planet. Sci Lett.*, submitted, 2002.

We have developed methods in which space-time patterns of earthquakes can be identified by means of the method of Principal Components (eigenfunction or eigenpattern analysis method). This method uses catalogs of seismic activity to define matrix operators on a coarse-grained lattice of sites within a spatial region. The equal time matrix operators are diagonalized to obtain the eigenvectors, which physically represent eigenpatterns. If the correlation operator is used, the eigenvalues represent probabilities, whereas if the covariance operator is used, the eigenvalues represent the variability of a pattern. Any given episode of seismic activity can be represented as a linear sum over eigenpatterns. Systematic study of these linear sums in southern California has shown that activity precursory to large earthquakes can be identified as a series of growing peaks in the power spectrum. These peaks can then be used to reconstruct the possible spatial locations of the impending large event.

We have also used these methods as a basis for the development of earthquake forecast algorithms. Using both simulations and observed earthquake data, we showed that the space-time patterns of earthquakes can be represented by a time-dependent system state vector in a Hilbert space. The length of the state vector represents the temporal frequency of events throughout the region, and is closely related to the rate at which stress is dissipated by the fault system. We further deduced that the information about the system state is represented solely by the phase angle of the state vector, hence the term "phase dynamics". Changes in the norm of the state vector therefore represent only random, Boltzmann-type fluctuations, and can be essentially removed by requiring the system state vector to have a constant norm.

Using these ideas, we analyzed data from southern California since 1932 between 32^o and 37^o north latitude, and 238^o to 245^o east longitude. Visual inspection of our results clearly shows that the method has forecast skill, but rigorous statistical testing is needed. We used two types of null hypotheses to test the forecasts. 1) We constructed thousands of random earthquake catalogs from the observed catalog by using the same total number of events, but assigning occurrence times from a uniform probability distribution over the years 1932-1991, and distributing them uniformly over the original locations. This procedure produces a Poisson distribution of events in space with an exponential distribution of inter-event times. Randomizing the catalog in this way destroys whatever coherent space-time structure may have existed in the data. These random catalogs are used to construct a set of null hypotheses, since any forecast method using such a catalog cannot, by definition, produce useful information. 2) For the second null hypothesis, we used the seismic intensity data directly as a probability density at $\mathbf{x}_i$,, as has been proposed in the literature for the "standard null hypothesis".

We carried out Maximum Likelihood tests to evaluate the accuracy with which our probability measure can forecast "future" "large" ($m \geq 5.0$) events, relative to forecasts from the null hypotheses. We compute the Likelihood function $L \equiv \prod_j \{ \mathcal{P}[\mathbf{x}(e_j)] \big/ \sum_i \mathcal{P}[\mathbf{x}_i] \}$ for: 1) $\mathrm{Log}_{10}[L]$ for 500 random catalogs of the first type (histogram); 2) $\mathrm{Log}_{10}[L]$ for the standard seismic intensity map; and 3) $\mathrm{Log}_{10}[L]$ for our forecast of the 10 years 2000-2010. We found that the Likelihood ratio test confirms what visual inspections indicated, that the method indeed has considerable forecast skill. In view of the considerable and fundamental importance of this work, we are actively pursuing refinement of the technique under the current proposed work.