**PHASE I FINAL TECHNICAL REPORT
FOR**

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**MULTI-ATTRIBUTE SEISMIC/ROCK PHYSICS APPROACH
TO CHARACTERIZING
FRACTURED RESERVOIRS**

Prepared for

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by

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JUSTIFICATION

During Phase I, we performed our rock physics, geologic, and seismic feasibility analysis for fracture characterization. We addressed the problem of seismic fracture detection and characterization, in general, and specifically at our field site in West Texas. In this analysis, we were able to implement a number of fracture modeling procedures and to quantitatively assess the theoretical detectability of reservoir fractures. This allowed us to identify which of the several seismic attributes are most likely to be useful for fracture detection at the site. One of our results showed that Thomsen's parameters should be very useful for indicating gas-filled fractures at the site. We also outlined Monte Carlo approaches that can be applied for feasibility analysis at any site.

Prior to our study, most seismic fracture detection methods have focused on qualitatively detecting seismic anisotropy. While anisotropy is critical, when used alone it has left us with interpretation ambiguities and nonuniqueness. During Phase 1 of this project, we found that by statistically integrating geologic and seismic information we can develop nontraditional methods for fracture detection. For example, we found that quantifying the correlations between fracture occurrence and depositional environment, and between fracture spacing and layer thickness, we allow us to decrease the uncertainty of fracture interpretation using seismic anisotropy. One of the reasons for this is that different depositional environments (and rock facies) give rise to differences in their seismic signatures (impedances, velocities, and Poisson's ratio).

Phase II will allow us to validate and improve these methodologies by carrying out a small-scale field pilot study. Taking methods to the field, always allows us to make them more applicable. In the field, we will be able to more accurately assess the uncertainties introduced by measurement noise, source and receiver response, coupling, and near surface acoustic properties. We can then more realistically develop ways to reduce the uncertainty, as well as to identify which aspects of the geology and fracture distribution we can characterize most robustly. The small-scale field study will also help us to establish empirical fracture parameters that are needed as inputs to rock physics models. For example, ALL of the effective medium models for fracture-induced anisotropy require fracture stiffness as inputs (these are sometimes characterized as moduli or aspect ratios). There is no realistic way to assign these values

theoretically, so being able to calibrate them in a small pilot will give us the critical inputs we need for interpreting the full scale seismic survey in phase III. Finally, the data from the pilot will allow us to add an aspect of "data mining" to the study. In Phase I, we have exploited our best theoretical tools, but the real field data will give us the opportunity to discover other, unpredicted, signatures of fractures that might prove valuable.

Our accomplishments in Phase I suggest that Phase II will lead to fruitful and practical next generation fracture characterization methods.

The practical outcome from Phase I is a methodology for modeling the seismic velocity, impedance, and reflectivity signatures of fractured reservoirs, based on well logs and the geologic data for any site of interest. The first part of this methodology is characterizing the natural variability of porosity and acoustic properties of the unfractured (matrix rock). The second part of the methodology is the means to explore how the geophysical signatures will change when fractures are added to that matrix. The most important contribution of the Phase I work is an approach that any explorationist can use when assessing whether fractures will be detectable, and what the uncertainty or risk of that interpretation will be. The seismic signatures are almost always non-unique and often non-distinct. Our phase I methodology quantifies this uncertainty and allows estimates to be made about how good the seismic experiment, signal/noise etc. has to be before it would be of any use for fracture interpretation.

If an explorationist applied our methodology, he/she could use the results to make decisions about what sort of seismic experiment would be most likely to lead to successful fracture characterization. The results from such analyses could be used as inputs in other financial and economic analyses to decide how much money to spend on various seismic experiments. In many cases, the seismic measurement may not be able to 'image' the fracture directly, but the presence of fractures will give rise to various indirect seismic signatures that can be used in combination with geologic information to make interpretations about the fractures.

Tech-transfer and dissemination of our research results to the industry is an integral part of our efforts. Our industry partner on this project, Marathon Oil, will have access to the new methodologies coming out this project. We will publish our results in professional journals and present them at international conferences. This plays a role in informing other explorationists about new methods. From time to time, we also give industry courses, which incorporate our recent research results, thus facilitating tech-transfer.

We anticipate that the practical outcome of Phase II will be (1) validation and improvement of the modeling methodology from Phase I, and (2) development of methodologies that can be applied specifically to the processing and interpretation of field seismic data for characterizing fractured reservoirs. We expect that the outcome will be more reliable and realistic interpretation of seismic and geologic data for fracture detection and mapping.

INTRODUCTION

This project consists of three key interrelated Phases, each focusing on the central issue of imaging and quantifying fractured reservoirs, through improved integration of the principles of rock physics, geology, and seismic wave propagation. This report summarizes the results of Phase I of the project.

The key to successful development of low permeability reservoirs lies in reliably characterizing fractures. Fractures play a crucial role in controlling almost all of the fluid transport in tight reservoirs. Current seismic methods to characterize fractures depend on various anisotropic wave propagation signatures that can arise from aligned fractures.

We are pursuing an integrated study that relates to high-resolution seismic images of natural fractures to the rock parameters that control the storage and mobility of fluids. Our goal is to go beyond the current state-of-the art to develop and demonstrate next generation methodologies for detecting and quantitatively characterizing fracture zones using seismic measurements.

Our study incorporates 3 key elements:

(1) **Theoretical rock physics** studies of the anisotropic viscoelastic signatures of fractured rocks, including up scaling analysis and rock-fluid interactions to define the factors relating fractures in the lab and in the field;

(2) **Modeling of optimal seismic attributes**, including offset and azimuth dependence of travel time, amplitude, impedance and spectral signatures of anisotropic fractured rocks. We will quantify the information content of combinations of seismic attributes, and the impact of multi-attribute analyses in reducing uncertainty in fracture interpretations.

(3) **Integration and interpretation** of seismic, well log, and laboratory data, incorporating field geologic fracture characterization and the theoretical results of items 1 and 2 above.

The focal point for this project is the demonstration of these methodologies in the Marathon Oil Company Yates Field in West Texas.

SUMMARY OF PHASE I RESULTS

The focus of Phase I research was the basic rock physics of fractured rocks, in particular the rock physics appropriate to the selected field site.

Our Phase I objective was to establish the optimum seismic-to-rock properties transforms that relate observable seismic signatures – velocities, velocity dispersion, velocity anisotropy – to the rock, fracture, and pore fluid characteristics that control hydrocarbon storage and mobility.

During this Phase, we worked on several activities focused on establishing, refining, and understanding the basic relations linking realistic fracture and fluid parameters with their geophysical signatures. We give a very short overview here, and the details are given in the separate ATTACHMENTS.

State of the Art and “Best Practices.” . We prepared a comprehensive summary of the rock physics state of the art for seismic fracture detection and characterization. The ATTACHMENT A, “Fracture Rock Physics Handbook” includes discussions of fracture effective medium theory, fluid effects, laboratory studies, and current field practice.

The sensitivity of seismic wave propagation to cracks and fractures is one of the fundamental observations of Rock Physics. Cracks and fractures tend to lower seismic velocities; they are responsible for the almost universal stress dependence of velocities; they tend to increase wave attenuation; and they often create seismic anisotropy. The seismic sensitivity to pore fluids is often exaggerated in highly fractured rocks. Cracks and fractures tend to enhance most of the interesting rock physics connections between seismic attributes and the rock and fluid properties of interest. Hence, many different strategies for fracture detection are possible.

In ATTACHMENT A, we comment on our current understanding of the rock physics of seismic fracture detection. We begin with a short discussion of the relevant laboratory and theoretical rocks physics aspects and then mention some field techniques that have become fairly standard as well as some newer and more speculative fracture detection techniques. We have also added some reference materials, which are excerpts from previous publications.

In short, the state of the art is that seismic field techniques are sometimes successful at detecting anisotropy indicative of fractures. A number of convincing field successes have been reported. The usual seismic result is an estimate of the amount of elastic anisotropy and the approximate direction of the elastic symmetry planes. We usually infer the most likely direction of fracture alignment from the elastic symmetry directions, and assume that the regions of

highest fracture density correspond to those with the highest elastic anisotropy. However, our ability to *quantify* the underlying fracture density, shape, aperture, connectivity, and pore fluid in geologically reasonable terms is limited. The problem is not a shortage of theoretical models; instead, the difficulties arise from (1) overly-simplified descriptions of fracture geometries in the theoretical models, (2) poor knowledge of the fracture parameters to use in the fracture models, (3) fundamental non-uniquenesses in separating lithologic effects, fluid effects, fracture effects, and frequency effects, and (4) spatial heterogeneity.

As is often the case in other geophysical problems, the most reasonable and realistic fracture interpretation from the seismic requires a strong geologic input. Some interesting current research involves taking observed styles of fracture occurrence from outcrop studies, and then stochastically simulating geologically similar fracture distributions that are consistent with the seismic observations.

Geologic Model of Fracture Occurrence in the Yates Field. ATTACHMENT B summarizes the geologic setting of our field site in the Yates Field.

Yates Field is located in the Central Basin Platform of the Permian Basin in western Texas. It was discovered by surface geology in 1926. Oil in place is estimated in 4.2 MMB. After producing more than 1 MMB during 74 years, the field is still delivering 30,000 bopd. The geological model is based on well logs and production data. Seismic information is limited to four 2-D lines, very few for the 91 km² area. In spite of the high density of wells in the field, there remains some uncertainty regarding the existence of faults and the orientation of conductive fractures.

At the surface, the Yates Field is a very simple structure. It is a broad, subtle anticline involving the Lower Cretaceous beds. But the geology of the Yates Field is far from being simple. The trapping mechanism, for example, is a combination of both stratigraphic and structural features that lead to one of the largest oil accumulations in the basin. The Permian rocks at Yates Field are located at the highest structural position of the basin, at the southeastern edge of the Central Basin Platform. This platform is a major Permian tectonic feature bounded by normal faults. That structure apparently controlled the facies distribution of the main reservoir, the San Andres Formation. Limestone's were deposited within the platform, while sandstones and shale were deposited in the surrounding areas, Delaware and Midland basins, which were deeper basins adjacent to the platform.

In the same way, reservoir heterogeneities, that are controlling porosity and permeability, are relatively complicated. They can be summarized in three main groups: 1) Heterogeneities related to the stratigraphic architecture and the depositional environments, that generate vertical and

lateral variations of facies, 2) heterogeneities associated to small-scale faults and fractures, and 3) heterogeneities induced by diagenesis or geochemical alterations.

ATTACHMENT B is a summary of the geology of Yates field, focused on those aspects relevant for seismic modeling of fracture-induced velocity anisotropy. It is based on some studies and published papers related to Yates field or to the main reservoir, but is also supported by field observations made during a 10 day field trip with geoscientists A. Younes, R. Stever, J. Schlueter, M. Bitter and G. Leaf, from Marathon Oil Co. It is focused on the description of the structural heterogeneities.

Rock Physics And Fracture Modeling of The San Andres Reservoir. ATTACHMENT C gives an overview on the rock physics analysis of the San Andres reservoir. It also presents the Phase I results on the fracture modeling, including the interval properties (velocities and anisotropy parameters), as well as the interface properties (P-P reflectivity and Azimuth Variation with Offset and Azimuth attributes).

From the detailed core information available from one of the wells, combined with the sonic log data, we show that different depositional environments and their associated facies have different elastic signatures that can be seismically distinguished. We also show that there is a correlation between the fracture distribution and the depositional environments. The majority of the fractures are associated with the low energy, sub tidal depositional environment. This is an important result that can play a critical role in fracture delineation from seismic measurements.

Our fracture modeling suggests that, if there is no other source of anisotropy present in the reservoir except for the alignment of one set of vertical fractures, Thomsen's parameters can give us insights into the fracture network characteristics. The shear wave splitting parameter can be related to the fracture density, a very important characteristic of the fracture network. These anisotropy parameters might also help us discriminate between oil and gas filled fractures, under situations in which there is little fluid communication between the cracks and the matrix porosity. However, we also need to consider the case in which the fluid communication is more important.

For the AVOZ analysis, we consider the interface between Queen and Grayburg formations as cap rock, and the underlying San Andres reservoir rocks modeled with vertical fracture sets. The aligned fractures generate an azimuthally anisotropic medium from which we compute the P-P reflectivity as a function of angle of incidence and azimuth. We can conclude that the oil - saturated fracture models have a stronger azimuthal signature than the gas-filled fractures. However, as the structural information on the Yates Field suggests, we also need to account for the presence of a second set of fractures, orthogonal to the one considered. This will generate an anisotropic medium with orthorhombic symmetry.

ATTACHMENT C contains the details of the log analysis and fracture modeling summarized above.

Seismic Detection Of Pore Fluids: Pitfalls of Ignoring Anisotropy. ATTACHMENT D represents the SEG 2000 abstract, RPB 6.5.

Rocks are often anisotropic, but we may not have complete information about the anisotropic elastic stiffness tensor. We may have measurements of only one of the velocities along the principal directions. However, we need the full elastic tensor to do fluid substitution calculations in anisotropic rocks. How much of an error is introduced if we take the velocities on one direction only and do fluid substitution ignoring anisotropy? In the paper we present the results of a numerical experiment that aims to evaluate these errors. As presented in ATTACHMENT D, we model two types of anisotropy: anisotropy due to preferential alignment of cracks and anisotropy due to thin layering. We estimate and compare the fluid related changes in velocities using Gassmann (isotropic) and Brown & Korrington (anisotropic) low frequency models. We also present ultrasonic data from dry, anisotropic sandstone samples. We show that the errors in fluid substitution caused by ignoring anisotropy can be significant in some situations.

The work described in ATTACHMENT D was given as an oral presentation at SEG International Exposition and Seventieth Annual Meeting, Calgary, Alberta, August 6-11, 2000.

PLANS FOR PHASE II

The main objective of Phase II is to develop the prototype field data interpretation scheme.

Our purpose is to acquire and interpret a small-scale seismic data set to demonstrate the feasibility of our interpretation schemes for high-resolution detection and characterization of fracture systems, and in the same time to improve our models.

Options for this phase include: 2 or 3 intersection 2D surface seismic lines spanning a range of azimuths; a cross-well tomographic survey; or multioffset-multiazimuth VSP. The acquisition parameters will be designed for optimal recovery of the most informative fracture attributes that we identified in Phase I modeling.

For this purpose, we will continue and extend our analysis of log, core, and reservoir fluids data from the field site to refine the basic rock model and estimate natural variability in reservoir rock and fluid properties. Predictions will be made of the seismic fracture attributes, and the interpretation uncertainty, based on the well-calibrated rock model.

Once acquired, the field data will be processed and attributes extracted. Fractured rock parameters will be estimated using the interpretation models developed in Phase I. Training and

calibration probability density functions will be established for the field based on log data, fluid properties, and derived distributions to extend training data for fluid and pressure conditions not encountered in the wells.

The personnel at Marathon Oil will provide log data and expertise as needed. They will also take the lead in managing the practical aspects of the field pilot study, including design of the survey specifications, acquiring bids from seismic contractors, and performing basic processing of the data.

Stanford and Marathon will jointly participate in the analysis and discussions.

ATTACHMENT A

FRACTURE ROCK PHYSICS HANDBOOK

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INTRODUCTION

The sensitivity of seismic wave propagation to cracks and fractures is one of the fundamental observations of Rock Physics. Cracks and fractures tend to lower seismic velocities; they are responsible for the almost universal stress dependence of velocities; they tend to increase wave attenuation; and they often create seismic anisotropy. The seismic sensitivity to pore fluids is often exaggerated in highly fractured rocks. Cracks and fractures tend to enhance most of the interesting rock physics connections between seismic attributes and the rock and fluid properties of interest. Hence, many different strategies for fracture detection are possible.

In this report, I comment on our current understanding of the rock physics of seismic fracture detection. I begin with a short discussion of the relevant laboratory and theoretical rock physics aspects and then mention some field techniques that have become fairly standard as well as some newer and more speculative fracture detection techniques. Finally, I point out some possibly fruitful areas for research. I have also added some appendices of reference material which are excerpts from previous publications of myself and my colleagues.

In short, the state of the art is that seismic field techniques are sometimes successful at detecting anisotropy indicative of fractures. A number of convincing field successes have been reported. The usual seismic result is an estimate of the amount of elastic anisotropy and the approximate direction of the elastic symmetry planes. We usually infer the most likely direction of fracture alignment from the elastic symmetry directions, and assume that the regions of highest fracture density correspond to those with the highest elastic anisotropy. However, our ability to *quantify* the underlying fracture density, shape, aperture, connectivity, and pore fluid in geologically reasonable terms is very poor. The problem is not a shortage of theoretical models; instead, the difficulties arise from (1) overly-simplified descriptions of fracture geometries in the theoretical models, (2) poor knowledge of the fracture parameters to use in the fracture models, (3) fundamental non-uniquenesses in separating lithologic effects, fluid effects, fracture effects, and frequency effects, and (4) spatial heterogeneity.

As is often the case in other geophysical problems, the most reasonable and realistic fracture interpretation from the seismic requires a strong geologic input. Some interesting current research involves taking observed styles of fracture occurrence from outcrop studies, and then stochastically simulating geologically similar fracture distributions that are consistent with the seismic observations.

It appears that the near-term strategy that will most immediately benefit seismic fracture detection is to improve seismic modeling, field acquisition, and processing techniques aimed at detecting anisotropy. We know that fractures are often associated with seismic anisotropy, and anything that we can do to better image elastic anisotropy will be a benefit. For the foreseeable future, proof that the observed anisotropy is related to fractures will have to come from nonseismic methods.

As in much of the Earth sciences, the rock physics knowledge of fracture detection is imperfect. There are many articles, anecdotes, and experiments that bear on the problem, some of them conflicting. We attempt here to draw practical conclusions and generalizations where we can, recognizing that our current understanding of the problem is incomplete and does not allow a single recipe that will always work in every situation.

This report is not intended as a tutorial or an exhaustive review. More complete discussions of the various methods can be found in the Rock Physics Handbook (Mavko et al., 1998), and in the numerous references cited in this report.

THE LABORATORY BASIS FOR SEISMIC FRACTURE DETECTION

Evidence of Fractures in Velocity-Pressure Relations (Isotropic)

Seismic velocities in virtually all reservoir rocks almost always *increase* with effective pressure. This is a direct manifestation that seismic propagation is affected by cracks and fractures. Any bit of pore space tends to elastically soften a rock, by weakening the structure of the otherwise rigid mineral material. This decrease in elastic moduli usually results in a decrease of the rock P- and S-wave velocities. *Effective pressure* (confining pressure - pore pressure) acts to stiffen the rock frame by mechanically eliminating some of this pore space. Nur and Simmons (1969) were among the first to identify this pressure-dependence as resulting from the opening and closing of microcracks and crack-like features, such as grain boundaries. *It is now common to say that cracks are what create the pressure dependence.*

Nur (1971) first theoretically explained the pressure dependence using stress-dependent distributions of ellipsoidal (penny-shaped) cracks in an elastic matrix. Numerous later papers further explored theoretically the crack-induced pressure dependence (see, for example, Sayers, 1988a,b, 1995; Mavko et al, 1995, among others). This approach is described below.

Figure 1 shows examples of velocity vs. effective pressure measured on a limestone and a dolomite. In each case the pore pressure was kept fixed and the confining pressure was increased, resulting in frame stiffening which increased the velocities. This type of

Figure 1. Seismic P- and S-wave velocities vs. effective pressure in two carbonates.

behavior is relatively elastic and reversible in many sandstones, limestones, dolomites, and igneous and metamorphic rocks up to pressures of 30-60 MPa; i.e., reducing the effective stress *decreases* the velocities, just as increasing the effective stress *increases* the velocities (usually with only small hysteresis). The reversible elastic effect is often evident also in shales and chinks, although it is limited to lower pressures ~10 MPa, beyond which permanent damage is done to the rock.

- *The crack-induced stress effects that we show in Figure 1 are part of the basis for seismic monitoring of changes in pore pressure during reservoir production.*
- *Numerous authors have shown that these stress and pore pressure effects can be described using theoretical crack models. Nevertheless, our ability to predict the occurrence of cracks and the resulting sensitivity to pressure from first principles is poor.*

Stress-Induced Velocity Anisotropy

The closing of cracks or fractures under compressive stress tends to increase the effective elastic moduli of rocks. When the crack population is anisotropic, either in the original unstressed condition, or as a result of the stress field, then these can impact the overall elastic *anisotropy* of the rock. Laboratory demonstrations of stress-induced anisotropy have been reported by numerous authors (e.g., Nur and Simmons, 1969; Lockner et al., 1977; Zamora and Poirier, 1990; and Sayers et al., 1990; Yin, 1992; Cruts et al., 1995).

The simplest case to understand is a rock with a random (isotropic) distribution of cracks imbedded in an isotropic mineral matrix. In the initial unstressed state the rock is elastically isotropic. If a *hydrostatic* compressive stress is applied, as shown in Figure 1, then cracks in all directions respond similarly, and the rock remains isotropic, although it becomes stiffer. However, if a *uniaxial* compressive stress is applied, then cracks with normals parallel or nearly parallel to the applied stress axis will tend to close preferentially, and the rock will take on an axial or transversely isotropic symmetry.

The historical basis for virtually all modern seismic methods for fracture detection was the classic stress-induced velocity anisotropy experiment by Nur and Simmons (1969), illustrated in Figure 2. The experiment was performed on a sample of dry granite under various levels of uniaxial compressive stress. The plots show P-wave and two polarizations of S-wave velocity versus direction of wave propagation relative to the direction of applied pressure. We can see immediately that cracks are present, because the velocities are well below the velocities of a pure quartz-feldspar mix (i.e., there must be porosity), and because the velocities are highly stress-dependent (i.e., cracks are opening and closing with stress). We infer that the cracks are initially isotropically distributed, because all the velocities are independent of direction at zero stress.

As uniaxial stress is applied, crack anisotropy is induced. The velocities clearly vary with direction relative to the stress-induced crack alignment. It is exactly this kind of P- or S-wave anisotropy that we look for in the field as evidence of aligned fractures.

Figure 2. Ultrasonic P- and S-wave velocities versus the direction of wave propagation relative to the direction of applied uniaxial stress. The sensitivity of velocity to stress is a strong indicator of microcracks. The stress-induced anisotropy indicates that the fracture population evolves from an isotropic to aligned distribution.

It is useful to point out that seismic waves are sensitive to crack and fracture alignment, but cannot necessarily reveal the mechanism for fracture alignment. In principle, the wave propagation will be the same whether we begin with an unfractured rock and add aligned fractures, or begin with an isotropically fractured rock and preferentially close some of the fractures.

- *A rule of thumb is that a seismic wave is most sensitive to cracks and fractures when its direction of propagation or direction of polarization is perpendicular (or nearly so) to the crack faces.*
- *The opening and closing of cracks and fractures is usually considered to be most affected by the component of compression normal to the crack faces. This is implicit in most seismic fracture interpretations. Nevertheless, other stress relationships can exist, affected by fracture interaction, fracture and fault curvature, elastic heterogeneity, and grain scale stress-concentrations. The most comprehensive fracture direction models incorporate as much of the geometric complexity as possible.*

It is also important to remember that there are non-fracture sources of seismic anisotropy, such as layering and mineralogic fabric. The part of the anisotropy that is sensitive to stress is most likely related to cracks and fractures. In practice, seismic waves always see the superposition of the different anisotropies. Table 1 summarizes the elastic symmetries that result when various applied stress fields interact with various initial crack symmetries (Paterson and Weiss, 1961; Nur, 1971).

Table 1. Dependence of Symmetry of Induced Velocity Anisotropy on Initial Crack Distribution, Applied Stress, and Its Orientation

Symmetry of Initial Crack Distribution	Applied Stress	Orientation of Applied Stress	Symmetry of Induced Velocity Anisotropy	Number of Elastic Constants
Random	Hydrostatic		Isotropic	2
	Uniaxial		Axial	5
	Triaxial*		Orthorhombic	9
Axial	Hydrostatic		Axial	5
	Uniaxial	Parallel to axis of symmetry	Axial	5
	Uniaxial	Normal to axis of symmetry	Orthorhombic	9
	Uniaxial	Inclined	Monoclinic	13
	Triaxial*	Parallel to axis of symmetry	Orthorhombic	9
	Triaxial*	Inclined	Monoclinic	13
Orthorhombic	Hydrostatic		Orthorhombic	9
	Uniaxial	Parallel to axis of symmetry	Orthorhombic	9
	Uniaxial	Inclined in plane of symmetry	Monoclinic	13
	Uniaxial	Inclined	Triclinic	21
	Triaxial*	Parallel to axis of symmetry	Orthorhombic	9
	Triaxial*	Inclined in plane of symmetry	Monoclinic	13
	Triaxial*	Inclined	Triclinic	21

*Three generally unequal principal stresses.

The most common approach to modeling the stress-induced anisotropy is to assume angular distributions of idealized penny-shaped cracks (Nur, 1971; Sayers, 1988a, 1988b; Gibson and Toksöz, 1990). The stress-dependence is introduced by assuming or inferring distributions or spectra of crack aspect ratios with various orientations.

The assumption is that a crack will close when the component of applied compressive stress normal to the crack faces causes a normal displacement of the crack faces equal to the original half-width of the crack. This allows us to estimate the crack *closing stress*:

$$\sigma_{\text{close}} = \frac{3\pi(1-2\nu)}{4(1-\nu^2)} \alpha K_0 = \frac{\pi}{2(1-\nu)} \alpha \mu_0$$

where α is the aspect ratio of the crack, and ν , μ_0 and K_0 are the Poisson's ratio, shear modulus, and bulk modulus of the unfractured rock (see Appendix A on the Deformation of Inclusions and

Cavities in Elastic Solids). Hence, the thinnest cracks will close first, followed by thicker ones. This allows one to estimate, for a given aspect ratio distribution, how many cracks remain open in each direction for any applied stress field. These inferred crack distributions and their orientations can be put into one of the popular crack models (e.g., Hudson, 1981) to estimate the resulting effective elastic moduli of the rock. While these penny-shaped crack models have been relatively successful and provide a useful physical interpretation, they are limited to low crack concentrations, and may not represent well a broad range of crack geometries (see Appendix B on Hudson's Model for Cracked Media).

As an alternative, Mavko et al. (1995) presented a simple recipe for estimating stress-induced velocity anisotropy directly from measured values of isotropic V_p and V_s versus hydrostatic pressure. This method differs from the inclusion models, because it is relatively independent of any assumed crack geometry and has no limitation to small crack densities. To invert for a particular crack distribution, one needs to assume crack shapes and aspect ratio spectra. However, if we do not invert for a crack distribution, but instead directly transform hydrostatic velocity-versus-pressure data to stress-induced velocity anisotropy, we can avoid the need for parameterization in terms of ellipsoidal cracks and the resulting limitations to low crack densities. In this sense, the Mavko et al. method provides not only a simpler, but also more general solution to this problem, since ellipsoidal cracks are just one particular case of the general formulation.

Most theoretical approaches assume an isotropic, linear, elastic solid mineral material. Methods based on ellipsoidal cracks or spherical contacts are limited to these idealized geometries, and low crack densities.

Yale (personal communication) has used laboratory measurements of stress-induced anisotropy to infer the directions of nonhydrostatic in situ stress. In this method, he measures shear wave splitting under hydrostatic atmospheric pressure, to identify the elastic symmetry planes. He assumes that the anisotropy is the result of a preferred alignment of microcracks which are the result of stress unloading as the sample was brought from in situ conditions to the surface. If the rock is isotropic under in situ nonhydrostatic stress, then bringing the sample to the surface effectively superimposes a nonhydrostatic unloading stress. In other words, he observes an *unloading* stress-induced anisotropy in the lab. He claims that the method gives estimates of the insitu stress directions consistent with other indicators.

The Seismic Wave Attenuation Signature of Fractures

Seismic waves tend to be more highly attenuated in cracked and fractured rocks. One cause is scattering; another is squirt dispersion related to wave-induced fluid motion in the fractures.

Figure 3 shows laboratory measurements of P- and S-wave attenuation on dry sandstone samples by Lucet (1989). Both ultrasonic and sonic (resonant bar) measurements are shown. The resonant bar measurements are at about 1 kHz; with this method E and S waves are measured from which the P-wave velocity and attenuation can be estimated. The drastic decrease of attenuation with increasing effective pressure is key evidence that the attenuation is microfracture related.

In fact, we make the claim that virtually any geophysical observation that is sensitive to pressure is a possible, and perhaps likely, indicator of fractures.

Figure 3. Wave attenuation vs. effective pressure in dry limestone samples (Lucet, 1989). Curves marked br are sonic frequency measurements using the resonant bar technique. Curves label us are ultrasonic data. The large decrease in attenuation with pressure is a strong indicator of microfractures.

Since the samples are dry, we know that the attenuation is not related to any wave-induced fluid motion, although it might be related in part to surface forces (see, for example, Tutuncu,

1992, and Tutuncu and Sharma, 1992). We suspect that scattering from the fractures is the dominant mechanism of the attenuation. Any heterogeneities in acoustic impedance tend to scatter wave energy, giving an apparent decrease in Q . Gas-filled fractures have drastically different acoustic impedance than the surrounding mineral, and are therefore excellent scatterers. Hudson gives theoretical estimates of the scattering attenuation expected for distributions of penny-shaped cavities. (see Appendix B on Hudson's Model for Cracked Media).

Figure 4 shows ultrasonic laboratory waveforms from Yin (1992), in an experiment similar to that by Nur and Simmons, shown in Figure 2. In this case Yin took cubes of dry rock, roughly 8 cm on a side. These were mounted in an apparatus in which separate levels of normal compression could be applied to each face. Stacks of transducers were placed on each face so that P-waves and two polarizations of S-waves could be propagated in each of the three principal directions. The resulting waveforms are shown for a sample of limey shale. The 9 traces on the left of Figure 4 were recorded when a very small level of hydrostatic stress was applied to the sample; the traces in the middle, when a small increment of compression was added in the Z-direction; and the traces on the right when an additional increment of compression was added in the Y-direction.

When the compressive stress was increased in the Z-direction, Yin observed an increase in velocities for all waves either propagating or polarized in the Z-direction, indicating a preferential closing of microcracks perpendicular to Z. These velocity increases are not easily detectable in Figure 4. More visible, however, is an increase in wave amplitude and wavelet frequency in those same components. A similar increase in amplitude and frequency content is seen in waves propagating or polarized in the Y-direction when the increase of compression in Y is added.

NEXT PAGE: Figure 4. Ultrasonic waveforms recorded by Yin (1992). First trace label refers to the direction of propagation; the second, the direction of polarization. The 9 traces on the left were recorded when a very small level of hydrostatic stress was applied to the sample; the traces in the middle, when a small increment of compression was added in the Z-direction; and the traces on the right when an additional increment of compression was added in the Y-direction.

This result is the anisotropic analog of Lucet's result – ultrasonic wave attenuation is larger when more microfractures are present. Manipulation of the fracture population with stress causes the attenuation to change. In this case, the attenuation *anisotropy* is changed. Since the rock is dry, we again suspect that scattering is the attenuation mechanism.

The squirt mechanism is now widely recognized as one of the dominant sources of velocity dispersion and wave attenuation in fluid-bearing rocks. At sonic to ultrasonic frequencies, the fluctuating stresses of passing waves induce grain-scale pressure gradients between crack-like features with different stiffnesses and different orientations. The resulting fluid motion dissipates energy. Mavko and Jizba (1991) showed how the crack content, revealed by the pressure dependence of dry rock velocities, can be used to quantify the squirt dispersion in fluid-bearing rocks. Figure 5 shows high frequency measurements of velocity vs. effective pressure in dry and water saturated granite samples. As discussed earlier, the pressure dependence is a key indicator of microcracks. The curve labeled Gassmann is the low frequency prediction of the water-saturated velocity using the dry data as inputs. The difference between the measured saturated velocities and the Gassmann prediction is the velocity dispersion. The observation that the dispersion (and necessarily the associated attenuation) decreases with effective pressure is a clue that microfractures cause the dispersion and attenuation. The excellent agreement of the crack-based Mavko-Jizba theory with the observed velocities is another indicator that fractures are involved.

Figure 5. Ultrasonic velocity vs. effective stress in dry and water-saturated granite samples (dots). The difference between the measured saturated values and the Gassmann prediction is the dispersion. The observation that this dispersion decreases with effective pressure is a strong indication that it is associated with microcracks.

ELASTIC THEORY OF FRACTURES

The role of elastic fracture models is to help provide a link between seismic observables – such as elastic constants, anisotropy, shear wave splitting, and directions of symmetry planes and the physical rock and fracture properties that we wish to know – fracture length, orientation, aperture, geometry, connectedness, permeability.

The physical mechanism of the elastic fracture effect is very simple: fractures, whether wet or dry, soften the rock in a way that depends on the fracture direction. Normal and shear stresses acting on the plane of the crack induce more overall strain than would be observed in the host rock without the fractures. There are two general approaches to quantifying the effect of cracks and fractures on elastic anisotropy. One approach considers cracks and fractures as isolated “inclusions” or cavities in an elastic matrix (Eshelby, 1957; Walsh, 1965; Wu, 1966; O’Connell and Budiansky, 1974; Hudson, 1980, 1981, 1990, 1994; Nishizawa, 1982; Crampin, 1984). Usually these are assumed to be ellipsoids or “penny-shaped” cavities of finite size, although a few other non-elliptical shapes have been considered (see, for example, Mavko and Nur, 1978). The penny shaped cracks are usually considered to have all dimensions much smaller than a seismic wavelength. The second modeling approach, popularized by Schoenberg and Muir (1989), represents fractures as planes of discontinuous slip or very thin planar zones that are more compliant than the unfractured rock. The planes extend infinitely far. Both approaches allow for different angular distributions of fractures.

Teng (1998) has shown that to first order the slip-interface model of Schoenberg and Muir (1989) is mathematically equivalent to that of Hudson (1981). In other words, for any distribution of Schoenberg-Muir fractures, we can find some distribution of Hudson penny shaped cracks that gives the same anisotropic effective elastic medium, and vice versa. The good news is that we can choose the approach that better describes the physical problem of interest. For example, we might use distributions of ellipsoidal cracks to describe microcracks and grain boundaries in the stress-induced anisotropy experiments of Nur and Simmons (Figure 2), while we might use slip interfaces to describe extensive regional fractures. The bad news is that if we seismically determine fracture-related elastic anisotropy, we can say very little about the fracture configuration without additional geologic and rock mechanics inputs.

Details of the Hudson penny-shaped crack model are given in Appendix B, and a comparison of Hudson’s model with the slip-interface models is given in Appendix D. We do not repeat the equations here, but instead attempt to draw some general conclusions about the state of elastic fracture modeling.

- If we are able to seismically measure the anisotropic elastic constants of a formation, then we can always find equivalent distributions of penny-shaped cracks or slip planes that can explain those constants, within the error of the measurements.
- From seismically determined elastic constants, we can make quantitative statements about crack density, but the densities only have meaning for the idealized model being considered. A large density of aligned, but randomly located penny shaped cracks, might give the same P- and S-wave anisotropy as fewer fractures clustered as coplanar sets. Similarly, intersecting fractures might give smaller elastic constants than an equal density of non-intersecting fractures.
- Even for a given fracture modeling approach, the inferred fracture density and orientations will be nonunique, in part from uncertainty in the moduli and anisotropy of the background unfractured medium. i.e. We often cannot tell whether the observed seismic anisotropy is from anisotropy in the matrix, anisotropic distributions of microcracks, or anisotropic distributions of macrofractures.
- Uncertainties in the fracture-filling fluid can also lead to uncertainties in the fracture density that we infer from the elastic constants. For example, a smaller number of dry cracks might give similar P-wave anisotropy as a larger number of liquid-saturated cracks.
- In analogy to isotropic rocks, we know how to model some of the *difference* between the anisotropic elastic properties of rocks with different pore fluids. These distinctions should be relatively independent of fracture model. In other words, we have techniques for doing fluid substitution in anisotropic rocks. These are discussed in the next section.
- We can say, with very poor accuracy, something about the fracture aperture, but it will only have meaning for idealized ellipsoidal fracture models.
- We can not practically distinguish, from elastic measurements alone, between
 - perfectly parallel fractures and fractures with modest angular distributions
 - fractures that do and do not intersect
 - short fractures and long fractures

FLUID SUBSTITUTION

Experience with unfractured isotropic rocks has taught us that seismic velocities are often sensitive to pore fluids. It should be no surprise that fractured rocks, and anisotropic rocks in general, show a similar sensitivity. As always, there are two fluid effects that must be considered: the change in rock bulk density, and the change in rock elastic moduli.

With anisotropic rocks we must also consider the difference between high frequency effects and low frequency effects, which are defined as whether or not pore fluids have time to flow and equilibrate within the various parts of the fractures and matrix pore space during a seismic period. In simple terms, we can define three fluid states:

1. *very high frequency*: pore fluids do not have time during a seismic period to flow from microcrack to microcrack (the same as isotropic high frequency squirt) and do not have time to flow from the matrix to the macro fractures.
2. *very low frequency*: pore fluids have time during a seismic period to completely equilibrate throughout all parts of the pore space.
3. *intermediate*: pore fluids have time to locally equilibrate among microcracks (low frequency squirt), but do not have time to flow from matrix to macrofractures.

In the next sections we comment on each of these states and give some simple recipes for estimating the corresponding fluid substitution. A number of authors (Frisillo and Thomsen, 1991; Thomsen, 1991; Pointer et al, 1996; Hudson et al. 1996) have all addressed aspects of this problem, although most of the time they end up making it more complicated than it needs to be.

Very low frequency

For *isotropic* rocks, fluid effects at low frequencies are well described with Gassmann's equations:

$$K_{sat} = K_{dry} + \frac{\left(1 - \frac{K_{dry}}{K_0}\right)^2}{\frac{\phi}{K_{fl}} + \frac{1 - \phi}{K_0} - \frac{K_{dry}}{K_0^2}}$$

$$\mu_{sat} = \mu_{dry}$$

where K_{sat} , K_{dry} , K_0 , and K_{fl} are the bulk moduli of the saturated rock, dry rock, mineral, and pore fluid, and μ_{sat} and μ_{dry} the shear moduli of the saturated and dry rock.

Implicit in the Gassmann relations is that the frequency is low enough that pore fluids can completely flow and equilibrate any small scale wave-induced pore pressure gradients during the seismic period.

Brown and Korrington (1975) derived the *anisotropic* version of Gassmann's relations:

$$S_{ijkl}^{(dry)} - S_{ijkl}^{(sat)} = \frac{(S_{ij\alpha\alpha}^{(dry)} - S_{ij\alpha\alpha}^0)(S_{kl\alpha\alpha}^{(dry)} - S_{kl\alpha\alpha}^0)}{(S_{\alpha\alpha\beta\beta}^{(dry)} - S_{\alpha\alpha\beta\beta}^0) + (\beta_{fl} - \beta_0)\phi}$$

where

- $S_{ijkl}^{(dry)}$ effective elastic compliance tensor of dry rock
- $S_{ijkl}^{(sat)}$ effective elastic compliance tensor of rock saturated with pore fluid
- S_{ijkl}^0 effective elastic compliance tensor of mineral material making up rock
- β_{fl} compressibility of pore fluid
- β_0 compressibility of mineral material = $S_{\alpha\alpha\beta\beta}^0$
- ϕ porosity

The Brown and Korrington relation also assumes that the pore fluids are equilibrated throughout the pore space – the very low frequency case.

For fractured rocks, very low frequency fluid substitution can be performed by taking the elastic compliances of the dry fractures sitting in a dry matrix and putting them into the Brown and Korrington formula. The porosity to use is the sum of the matrix and fracture porosities.

The dry compliances that we put into the Brown-Korrington relations can come from dry rock measurements, or from application of a dry rock theory, such as Hudson's. Similarly the difference between the moduli of a fractured rock saturated with one fluid can be related to the moduli with a different fluid can be modeled by using the relation twice.

Caution: “Dry rock” is not the same as gas-saturated rock. The dry rock modulus refers to the incremental bulk deformation resulting from an increment of applied confining pressure, with pore pressure held constant. This corresponds to a “drained” experiment in which pore fluids can flow freely in or out of the sample to insure constant pore pressure. Alternatively, it can correspond to an undrained experiment in which the pore fluid has zero bulk modulus, so that pore compressions do not induce changes in pore pressure – this is approximately the case for an air-filled sample at standard temperature and pressure. However, at reservoir conditions, gas takes on a non-negligible bulk modulus, and should be treated as a saturating fluid.

Very high frequencies

There are two conceptually identical approaches to modeling very high frequency fluid substitution. Mavko and Jizba (1991) in their model of isotropic squirt showed a method for determining the crack content from dry rock velocity vs. pressure data and transforming that into high frequency fluid-saturated behavior. Mukerji and Mavko (1994) extended this to general anisotropic behavior. The advantages of these methods are that they do not depend on idealized penny-shaped crack models.

A second, somewhat equivalent method is to model dry rock behavior with distributions of penny-shaped cracks using a model like that of Kuster and Toksöz (1974) or Hudson. Then, the crack models are recomputed using the same distributions of cracks, but filled with the pore fluid of interest. The reason that this works is that the crack models always implicitly consider the cracks to be isolated from each other with respect to fluid flow. This is in contrast to a Gassmann or Brown-Korrington approach which implicitly considers that the cracks are well connected. Remember to use one distribution of ellipsoidal cracks to model the matrix pore space, and another set to represent the macrofractures.

Intermediate frequencies

Intermediate frequencies can be handled very simply using a step-wise application of the very high and very low frequency methods. Step one is to model fluid substitution in the rock matrix without macrofractures using the Gassmann relations. Step two is to use the resulting matrix properties as the unfractured rock background and add isolated macrofractures using fluid-filled Hudson cracks.

Which are high frequencies and which are low?

The characteristic frequency that separates high frequency conditions and low frequency conditions for local squirt flow is estimated by O'Connell and Budiansky (1977) as:

$$f_1 = \frac{K_0}{\eta} \left(\frac{c}{a} \right)^3$$

where c/a is the aspect ratio of the cracks; K_0 is the solid matrix bulk modulus; and η is the fluid viscosity. This estimate of characteristic frequency applies to fluid flow between closely spaced micro cracks. For water-saturated rocks, many sandstones appear to have characteristic frequencies of 10^3 - 10^4 Hz. Higher viscosity fluids push this to lower frequencies.

The problem of flow in larger fractures with permeable walls was considered by Dvorkin et al. (1990). The characteristic frequency for flow between the fracture and matrix is given by

$$f_{II} = 2\pi\omega = 2\pi \frac{\kappa\phi K_{fl}}{c^2\eta}$$

where c is the fracture aperture, κ the matrix permeability, ϕ the porosity, η the fluid viscosity, and K_{fl} the fluid bulk modulus. This is the characteristic frequency corresponding to the second type of local fluid flow between fractures and permeable matrix. It shows that the larger the matrix permeability and porosity, the easier the fluid flow, hence the larger the characteristic frequency. On the other hand, the more viscous the fluid, the more difficult the flow, and the lower the characteristic frequency.

To explore this relation, Teng (1998) considered three sets of sandstone porosity and permeability data imbedded with fractures. These sandstones vary from tight gas sands to high porosity loosely consolidated sandstones (not likely to be fractured). Figure 6 shows estimates of the characteristic frequency versus porosity for the sandstones when the fluid is water (viscosity 1cP), normal oil (viscosity 10cP), and heavy oil (viscosity 500cP), respectively. We can see that the characteristic frequency ranges from smaller than 0.1Hz to larger than 10^5 Hz, depending on both the fluid viscosity and the rock matrix permeability and porosity.

Figure 6. Estimates of characteristic frequency vs. porosity for a wide range of pore fluids and sandstones.

This work suggests that for fractured, permeable rocks, we cannot simply treat laboratory ultrasonic measurements as high frequency conditions, and seismic surveys as low frequency conditions. It depends on the properties of fracture, pore fluid, and rock matrix.

VELOCITY DISPERSION

Fluid-related mechanisms dominate velocity dispersion in the crust. Velocity dispersion is relatively easy to model using the fluid substitution methods just discussed. The very high frequency model will yield the largest elastic constants, the very low frequency the lowest elastic constants. Measurement of velocity dispersion from field seismic data will be very uncertain, except for special survey techniques such as cross-well seismic.

MICROCRACKS VS. MACROFRACTURES

As we have already discussed, the mathematics that is used for elastic fracture modeling is essentially the same for microcracks (submillimeter) and large macrofractures (centimeters to kilometers). Hudson's model, for example, has been applied equally often to describe microcracks affecting ultrasonic waves in core plugs and to joints and macrofractures affecting VSP and 3-D seismic data in fractured reservoirs. This apparent mathematical similarity accounts, in part, for the strong laboratory roots of the current field techniques.

One of the biggest differences between microfractures and macrofractures, besides their size, is their geometry, most notably the spacing and orientation. Microcracks and crack-like features tend to be randomly distributed, controlled by grains and grain contacts. Even under the uniaxial stress experiments of Nur and Simmons (Figure 1), there is a wide angular distribution of the microcracks. In contrast, field scale joints and macrofractures often occur in more discrete aligned or subparallel sets (Figure 7), often reflecting the regional stress field or structural features such as folds. Multiple fracture sets at different orientations can effectively kill the seismic anisotropy, as least to our crude ability to seismically detect it.

Figure 7. Orthogonal regional fractures in Devonian Antrim shale, Michigan Basin

The different styles of occurrence in part influence how we look for fractures. In the laboratory, we might infer microcracks from low velocities or stress-dependent velocities, even if the velocities are isotropic. *In the field, we tend to think “fracture” only when we see velocity anisotropy, even though fairly isotropic fracture distributions are possible.* Modelers usually

represent macrofractures in the field with a single set of cracks aligned along the observed elastic symmetry plane.

In many tight reservoirs naturally occurring fractures provide the only economic means of production. Because one of the principal stresses is usually vertical, the least principal stress is often horizontal, resulting in a vertical open fracture set. Multiple fracture sets can exist, with some inherited from earlier stress fields; but often a single set perpendicular to the present-day least compressive stress is thought to dominate the elastic and permeability anisotropy. Work by Barton suggests that shear faults sometimes enhance permeability. Yet, in virtually all seismic field studies the target is open near-vertical joints.

A very brief discussion of fracture occurrence is given by Lorenze et al (1996). An extensive structural geology literature exists for fracture occurrence, including numerous papers by Aydin and Pollard (Pollard et al., 1992; Segall and Pollard, 1983; Aydin and Johnson, 1983, Nicholson and Pollard, 1985, Olson and Pollard, 1989; Helgesen and Aydin, 1991; Du and Aydin, 1991)

The ability to describe the very different scales of fractures with a single model, such as Hudson's, indicates how simple it is to create elastic anisotropy with cracks. At the same time, it highlights the fundamental difficulty of constraining geologically realistic fracture information from seismic. Being able to fit field anisotropy with idealized penny-shaped cracks has, perhaps, led seismologists into a false sense of achievement.

FIELD TECHNIQUES

In the next sections I review a few field techniques for seismic fracture detection. In almost all cases, the only thing that the techniques seek is *anisotropy* – in travel time, amplitude, AVO, or frequency. The result is usually a map of elastic anisotropy, including some measure of the degree of anisotropy and the approximate direction of the symmetry plane. The association of these with fractures is an interpretation and not really a seismic result.

FIELD TECHNIQUES - SHEAR WAVE SPLITTING

Fracture detection is one of the few areas of geophysics where shear wave methods can be called the older, “conventional” approach, and P-wave methods are newer and more experimental. Until recently, virtually all field shear wave techniques exploited *shear wave splitting* or *birefringence*.

The most common field target is sites with approximately vertical, subparallel fractures. If the unfractured rock mass were homogeneous and isotropic, then such a fracture set would create a transverse isotropy with horizontal axis of symmetry (normal to the fracture faces), which is sometimes call TIH. In fact, unfractured sedimentary rocks tend to already be anisotropic due to depositional fabric and layering. If the bedding is horizontal, then it might be approximately transversely isotropic with a vertical symmetry axis, sometimes called TIV. Vertical fractures cutting a TIV rockmass tends to create a material with orthorhombic symmetry. In either case, the vertical fractures create an azimuthal anisotropy of elastic properties.

In either the TIH or Orthorhombic cases, vertically propagating shear waves that are polarized parallel to the fracture planes will travel faster than shear waves polarized perpendicular to the fracture planes. Any vertically propagating shear wave incident on the medium at arbitrary polarization will immediately split into the two modes, which are generally called the *fast shear wave* and the *slow shear wave*. This is shear wave splitting. Any method that can detect the separate polarizations of the fast and slow shear waves can yield information on the fracture-induced anisotropy. Shear wave splitting in the field has been detected on cross-dipole logs, multicomponent VSPs, 2-D and 3-D shear surveys, and converted shear wave surveys.

Field methods based on shear wave splitting have several distinct advantages. First, since the velocities of the fast and slow waves are seldom more than a few percent different, both waves tend to follow the same ray path. Hence, there is little risk that we are sampling heterogeneity instead of anisotropy. Second, shear wave methods work well for vertical or near vertical ray

paths. Hence, there often is not a need for far offsets. *This latter point will have to be rethought for converted shear waves.*

Shear wave splitting – the Alford Rotation

A shear wave source polarized in the fast direction (parallel to the fractures) will generate only the fast wave. A shear wave source polarized in the slow direction (perpendicular to the fractures) will generate only a slow wave. A shear wave source polarized in an arbitrary direction will generate both modes.

In the field, we often don't know the elastic symmetry direction before collecting our shear wave data. Hence the inline shear wave source—not necessarily in a symmetry direction—will generate both fast and slow modes, and both modes will be recorded on both inline and crossline receivers. In principle, the field experiment, including source and receivers, can be rotated by trial and error until the symmetry directions are discovered – that is, until the inline source generates only a single mode recorded on the inline receiver, and the crossline source generates only a single mode recorded only on the crossline receiver.

It is much more efficient to simulate the rotation on the computer. Alford (1986) suggested the simple idea, which is now widely applied to cross-dipole shear logs, shear-wave VSPs, and surface shear reflection data. The rotation is simply a tensor transformation:

$$S'(t)_{ij} = \beta_{ik}\beta_{jl}S_{kl}(t)$$

where $S_{kl}(t)$ are the four shear wave traces (inline source-inline receiver, inline source-crossline receiver, crossline source-crossline receiver, and crossline source-inline receiver) before rotation, $S'(t)_{ij}$ are the four seismic shear wave traces after rotation, and β_{ij} are the direction cosines between the new rotated coordinates and the original coordinates. The standard procedure is to try various rotation angles until the cross term reflected energy (inline source-crossline receiver and crossline source-inline receiver) is minimized.

Figure 8 from Chen (1995) shows an example of the four shear wave components both before and after the rotation. The correct rotation angle is identified by the disappearance of coherent reflectors on the cross components.

Figure 8. Four-component shear wave stacks, (top) before Alford rotation, (bottom) after rotation. In each set the upper left is inline source-inline receiver; upper right, inline-crossline; lower left crossline-inline; lower right, crossline-crossline.

Shear wave splitting travel time differences

During the last 15 years, numerous field studies have demonstrated shear-wave splitting travel time anisotropy – the difference in arrival time of the fast and slow waves. For example, Alford (1986), Willis (1986), Mueller (1991), Winterstein, (1992), Chen (1995), Lynn et al. (1996), Corrigan et al (1996) and Teng (1998) [among others] measured shear wave splitting travel time differences in surface seismic shear-wave surveys. Beydoun et al., 1985, Crampin et al, 1986, Johnston, 1986; Queen and Rizer, 1990, and Teng (1998) [among others] used shear wave splitting in multicomponent VSPs as an indicator of fractures.

Some of the most comprehensive field studies of fractures have been carried out at the Conoco Borehole test facility (e.g., Queen and Riser, 1990; Enru et al., 1991; Majer et al., 1997). Their mapped set of fractures is also consistent with permeability anisotropy (Queen and Rizer, 1990), and with the region NE-SW trending maximum horizontal compression (Zoback and Zoback, 1981).

Shear wave reflectivity

Since the fast and slow shear waves see a different shear velocity, they likewise see a different shear wave impedance (velocity times density). Hence reflectivity of fast and slow shear waves at the same reflector point will tend to be different.

Mueller (1991) applied this simple idea to show differences in fast and slow shear wave reflection amplitudes in fractured formations. In his example, the fast polarization wave, which travels roughly at the velocity of the unfractured rock, showed a larger and more continuous reflectivity than the slower polarization wave. The reflectivity of the slower polarization wave also showed more lateral variability, which Mueller assumed was related to lateral variations of fracture intensity. Other causes for reflectivity changes might be roughness of the reflecting boundary, related to the fracturing.

Converted shear waves

Converted shear waves are being researched more and more for a variety of reservoir characterization uses. Since shear waves are some of the best fracture indicators, converted shear looks particularly promising. We can speculate that fracture-related anisotropy might be detected in converted shear wave splitting travel time differences, azimuthal variations in P-to-S reflectivity and AVO, azimuthal variations in P-to-S moveout velocity, and polarization-dependent reflectivity. Michelena (1995) and Ata et al., (1995) reported fracture-related converted shear wave anisotropy in surface seismic experiments.

FIELD TECHNIQUES - P WAVES

Conventional 3D P-wave surveys have a number of distinct advantages over shear wave surveys. P-wave acquisition and processing is routine and relatively cheap. Field crews are relatively plentiful, and P-wave surveys can be both land and marine. The data volume to collect and process is also substantially smaller in P-wave surveys. Hence, there are natural economic and operational incentives for learning to detect fractures with P-waves.

There is no question that P-waves “see” fractures. The laboratory and theoretical results discussed above illustrate that velocity and attenuation are both affected.

The P-wave methods summarized below all rely on good azimuthal coverage at far offsets. Many conventional 3D surveys have limited offsets in the cross-line direction, severely limiting the effectiveness of applying the methods. This could be a particular problem for marine surveys.

P-wave stacking velocity anisotropy

Vertically propagating P-waves will generally not be very sensitive to vertical fractures. To first order, we expect the fractures to be seismically invisible on near offsets. However, as offsets increase and propagation paths take on larger angles of incidence a natural fracture-induced velocity anisotropy can exist (Figure 9). Source-receiver azimuths parallel to the fracture strike will tend not to see the fractures, while source-receiver azimuths perpendicular to the fractures will be slowed. The greater the angle of incidence while the waves are in the fractured layer, the more they are slowed by the fractures. Hence the moveout or stacking velocity will be azimuthally anisotropic and possibly non-hyperbolic.

Figure 9. left: P-wave survey line perpendicular to fracture trend will have lower interval and moveout velocities. right: survey line along fracture strike will not be much affected by the fractures.

Several studies in the Powder River Basin of Wyoming found evidence of such a stacking velocity anisotropy. The target was vertically fractured tight gas formations at depths exceeding 11000 ft. Chen (1995) examined 2D P-wave lines with azimuths at approximately right angles to each other. The line more parallel to the fracture strike showed faster moveout velocities than the line more perpendicular to the fractures strike. Figure 10 shows CDP gathers from the two lines at their point of intersection. Both were moved out with the same velocity function. The events on the left are nicely flattened, while the events on the right are overcorrected. Figure 10 also shows limited range stacks from the two lines, again using the same stacking velocities for both lines. The near offset stack shows good continuity from one line to the next at their intersection. The far offset stack shows a jump of 15 milliseconds, again highlighting the overcorrection of moveout in the faster (fracture-parallel direction).

Figure 10. CDP gathers at the same midpoint but on orthogonally trending lines. Both are moved out with the same stacking velocity. (a) CDP gather flattened on line 1. (b) Overcorrected CDP gather on line 2. (c) Tie of events on near-offset stacks at the intersection of lines 1 and 2. (d) Mistie of events on the far-offset stacks at the intersection of lines 1 and 2.

Corrigan et al. (1996) studied azimuthal variations in moveout velocity on 3D P-wave data at the same site. They found small, but definite stacking velocity anisotropy, which was consistent with Chen's results and the known fracture trend. Similar results have been reported elsewhere by Blackhawk Geosciences (1995), Lynn et al. (1996), Craft et al. (1997), and others.

One of the difficulties with detection of azimuthal stacking velocity variations is that azimuthally different far-offset propagation paths sample different parts of the earth. Hence, there is the possibility of mistaking velocity *heterogeneity* for velocity *anisotropy*. *Nevertheless, azimuthal stacking velocity anisotropy is probably the simplest and most robust P-wave method for detecting fractures.*

Western's Fractogram

Western Geophysical is marketing fracture detection via its *FractogramTM*. Western advertises seismic modeling, survey design, acquisition and processing of 3-D azimuthal P-wave data for fracture detection. The approach is a simple combination of detection of azimuthal variations of P-wave stacking velocities and azimuthal variations of reflectivity or AVO.

The azimuthal variation of velocity is found by azimuthally binning prestack traces and performing velocity analysis within each azimuth range. Then, the Dix equation is applied to find azimuthal variations of interval velocity, which are expected to have a cosine(2q) dependence. The azimuth of vertical fractures is assumed to lie in the direction of maximum stacking velocity. Details are discussed by Craft, et al. (1997).

The azimuthal variations of far offset amplitudes are also expected to have a cosine(2q) dependence. (See our discussion of AVO-AVOZ below). The Fractogram also searches for this. Details can be found in Mallick et al. (1996).

The Fractogram appears to be reasonable application of current technology.

Attenuation and frequency loss

Yin's data, shown in Figure 4, illustrate the potential of using attenuation as a fracture indicator. Chen (1995), using simple spectral analysis of stacked traces, found a rough correlation of lower average frequency and fracture occurrence on 2D P-wave lines. Figure 11 below illustrates his result.

Stacking-chart display of the center frequencies of the pre-stack P-wave data on line 2.

Figure 11. Surface-consistent corrected center frequencies. Region with low center frequency corresponds to higher fracture density.

Measurement of intrinsic Q is extremely difficult, even in the lab. Nevertheless, a number of convincing field studies of attenuation have appeared, though not necessarily related to fracture detection. For example, Dilay and Eastwood (1995) showed that steam injection correlated with a pronounced loss of high frequencies in P-wave data. The key was careful relative comparison of spectra on shallow and deep windows in and out of the producing zone. Yamamoto et al. (1994, 1995) mapped spatial variations in P-wave velocity dispersion using broad-band cross-

well imaging. Dispersion is directly relatable to attenuation via the Kramers-Kronig relations (Bourbié et al., 1987; Lucet, 1989).

Much less quantitative is the general observation that P-waves tend to be attenuated when gas is present. Since many fractured targets are tight-gas formations, the gas as well as the fractures could be associated with attenuation.

Discovery Bay, a small start-up company, is offering software for extracting Q as an attribute from stacked data. The methodology is based on a spectral ratio technique applied to continuously sliding small data windows on the seismic traces.

AVO-AVOZ

Appendix F summarizes some of the theory of fracture-related azimuthal variations in AVO. The mechanism is based on the simple theorem stated earlier: waves will be slowed by fractures when the direction of propagation or direction of polarization have a component normal to fractures.

The thing I point out here is that some of the fracture and fluid AVOZ signatures can be counterintuitive. Figure 12 compares the theoretical AVOZ at the boundary between an isotropic cap rock and a vertically fractured reservoir rock. The colored circles show P-P reflection amplitude vs. azimuth and angle of incidence out to 35 degrees. The sinusoidal plots below show the amplitude vs. azimuth at a fixed offset of 30 degrees. Note the greatest azimuthal fracture signature occurs with the water saturated fractures.

The reason for this is that dry fractures cause both P- and S-wave anisotropy, while water saturated fractures cause mostly S-wave anisotropy with less P-wave anisotropy. Since AVO involves the ratio of the two, the saturated fractures cause the biggest signature.

Figure 12. Comparison of AVOZ in water-saturated vs. gas saturated fractures.

Figure 13 shows a comparison of AVOZ for dry fractures in a low Poisson ratio rock vs. a high Poisson ratio rock. The biggest azimuthal variation is seen in the high Poisson ratio rock.

The conclusion we draw is very simple. The best way to predict and interpret fracture related AVOZ signatures is to combine careful rock physics modeling with careful anisotropic seismic modeling.

Figure 13. AVOZ for high and low Poisson ratio fractured rock.

THE QUANTITATIVE INTERPRETATION CHALLENGE

As I have mentioned several times in this report, there are severe limitations on what seismic data alone can tell us about fractures. Certainly, there are a number of P- and S-wave attributes in which we can detect anisotropy. And certainly, aligned fractures can be a cause of this anisotropy. Beyond that, we can say very little from seismic alone. We pointed out earlier that virtually any observed elastic anisotropy can be explained with distributions of finite, isolated penny shaped cracks, or equally well with distributions of slip-discontinuity planes of infinite extent. Similarly, we cannot seismically distinguish between perfectly parallel cracks or small angular distributions of cracks; we cannot distinguish between connected cracks and

isolated cracks; we cannot distinguish between microcracks and macrofractures; and we cannot distinguish between long cracks and short cracks.

It is my opinion that seismic models are best used to locate the locations and orientations of fractures that we already suspect exist based on other indicators.

CONCLUSIONS

The state of the art is that seismic field techniques are sometimes successful at detecting anisotropy indicative of fractures. A number of convincing field successes have been reported. The usual seismic result is an estimate of the amount of elastic anisotropy and the approximate direction of the elastic symmetry planes. We usually infer the most likely direction of fracture alignment from the elastic symmetry directions, and assume that the regions of highest fracture density correspond to those with the highest elastic anisotropy. However, our ability to *quantify* the underlying fracture density, shape, aperture, connectivity, and pore fluid in geologically reasonable terms is very poor. The problem is not a shortage of theoretical models; instead, the difficulties arise from (1) overly-simplified descriptions of fracture geometries in the theoretical models, (2) poor knowledge of the fracture parameters to use in the fracture models, (3) fundamental non-uniquenesses in separating lithologic effects, fluid effects, fracture effects, and frequency effects, and (4) spatial heterogeneity. As is often the case in other geophysical problems, the most reasonable and realistic fracture interpretation from the seismic requires a strong geologic input.

A number of companies, including ARCO and AMOCO, have used shear wave splitting techniques almost routinely for locating “sweet spots” to drill in certain tight gas formations. In many of these cases, the formations were known to contain fractures, based on other nonseismic information, such as geologic studies and wells. The seismic anisotropy was used to locate the most likely fracture direction (direction of elastic symmetry planes) and locations of highest fracture densities (highest degree of anisotropy).

Without independent information on the fracture occurrence, there will be ambiguity in interpreting the seismic anisotropy. Anisotropy can indicate fractures, but it can also reflect stress-induced anisotropy caused by the opening and closing of microcracks in response to the in situ stresses. Anisotropy can also be caused by rock fabric.

It appears that the near-term strategy that will most immediately benefit seismic fracture detection is to improve seismic modeling, field acquisition, and processing techniques aimed at detecting anisotropy. We know that fractures are often associated with seismic anisotropy, and anything that we can do to better image elastic anisotropy will be a benefit. A second promising

area is to apply existing rock physics fracture models to explore fracture signature as a function of lithology, stress, and fluid type. For the foreseeable future, proof that the observed anisotropy is related to fractures will have to come from nonseismic methods.

Shear wave splitting yields the most unambiguous measures of anisotropy; P-waves also yield measures of anisotropy, but more care must be taken to separate heterogeneity from anisotropy.

A Modeling Recipe

When designing a field acquisition program, or interpreting field data for fractures, we suggest the following modeling approach.

1. Define the velocity and density model for the background unfractured rock. This is the same as with any seismic modeling. The velocities can be estimated from logs or from the fast P- and S-wave directions of VSP or surface seismic data.
2. Define the fracture sets to be considered. Usually these are assumed to be near vertical. An arbitrary number of fracture sets, each with its own fracture azimuth, can be considered, depending on the geologic setting. Most of the published examples for successful seismic fracture detection in the field have assumed only a single fracture set. (An important uncertainty in analyzing fracture anisotropy is whether the anisotropy represents a single fracture set, or two perpendicular fracture sets with one more dominant than the other. A highly fractured rock mass with a wide distribution of fracture directions can look approximately isotropic. Hence, methods based on anisotropy alone will miss the fractures. Comparison of the velocities with log estimates of the unfractured rock velocity helps to resolve this.)
3. Choose Hudson's model to estimate the anisotropic elastic constants corresponding to the fracture sets. Assigning fracture densities for the various fracture sets is somewhat arbitrary, since all of the idealized fracture models are difficult to relate to real fractures. The approach I recommend is to target a desired degree of anisotropy. For example, fracture induced P- and S-wave anisotropy of 5-7% is reasonable. We might want to model several levels of anisotropy ranging from 0% (isotropic) up to 7%. So, we find, by trial and error, the Hudson fracture density that will give these desired levels of anisotropy. Hence, the role of the Hudson model is simply to generate fracture-consistent sets of anisotropic elastic constants, rather than to represent precise geologic fracture types.
4. Pore fluids are modeled using fluid substitution. The various sources of fluid-related velocity dispersion are difficult to define. I recommend including the fluid effects in two steps. First, use the usual isotropic Gassmann's relations to saturate the unfractured rock mass. Then use Hudson's fluid-saturated formulation to add the macrofractures with the desired pore fluids.

5. Use the anisotropic elastic model computed in steps 1-4 as an input to an anisotropic seismic modeling code, such as Nucleus. Simulate the relevant field geometry.

SUGGESTED RESEARCH

Seismic modeling methods

Most of the P- and S-wave fracture detection techniques are really methods to detect elastic anisotropy. As with *isotropic* seismic methods, interpretation of seismic data for interval properties is most successful when model-based. It appears that improvement of numerical wave propagation techniques in anisotropic media will yield the most immediate benefits. These would include travel time prediction, amplitudes, AVO, converted waves. Such modeling methods should be used when designing surveys, as well as interpreting or inverting the results.

Since we are often looking for azimuthal anisotropy, it is important to model carefully the azimuthally-dependent source radiation pattern and receiver response. Remember too, that P-wave and converted S-wave methods might require small offsets, particularly in the cross-line direction.

Rock physics models

It is my sense that there are enough elastic fracture models. I say this, not because we know how to effectively model every detail of realistic fracture geometries, but because seismically determined elastic constants tell us very little about fracture details. There is a fundamental non-uniqueness.

I believe that rock physics can best be used to apply currently available fracture models to explore how fracture signatures depend on lithology, unfractured porosity, and pore fluids. We also need to explore how to separate, if possible, the effects of pore fluids, pressure, and crack aspect ratio – perhaps in a way that is analogous to the use of V_p and V_s in unfractured rocks to separate fluid effects from lithology and stress. We also need to refine better the frequency dependence that results from wave-induced fluid motion.

Fracture physical constants

One of the difficulties in quantitatively relating elastic constants to real physical fractures is ignorance of fracture parameters. For example, when applying the Schoenberg-Muir model, what fracture stiffnesses are reasonable? Or when using Hudson's model, what aspect ratio will mechanically represent real fracture geometries? There has been some laboratory work in the Civil Engineering field that has attempted to measure fracture stiffness, but there is a suggestion that the results depend too much on scale. Perhaps a more reliable approach will be to find geologically well-characterized fractures in the field, measure their seismic signatures, and invert for the equivalent fracture properties. I suspect that this will be very difficult.

Attenuation

Rock physics laboratory and theoretical work suggests that attenuation can be a powerful attribute for determining rock and fluid properties. An intriguing research area will be to improve seismic acquisition and processing techniques to better estimate Q or relative Q .

Integrated geology/geostat tools

Determination of rock fracture parameters from seismic data is inherently very nonunique. One approach that I am excited about is the use of geostatistical techniques to combine geologically realistic descriptions of fracture styles with the seismic constraints on elastic anisotropy. One approach that we have started to explore is to generate stochastic fracture maps that are consistent with the seismic data and that incorporate geologically realistic fracture lengths, connectivity, apertures, and geometries. These give us models that can be flow simulated. They also allow us to explore which details are well determined by the seismic and which are not.

APPENDICES

The following 6 appendices are excerpts from previous publications of the author and his colleagues. These are particularly relevant reference materials for fracture analysis, and are provided for the convenience of the reader.

Appendix A – Deformation of Inclusions and Cavities in Elastic Solids

Appendix B - Hudson's Model

Appendix C – Anisotropic Form of Hooke's Law

Appendix D – Thomsen's Notation for Weak Elastic Anisotropy

Appendix E – Comparison of Some Elastic Fracture Models

Appendix F – AVOZ in Anisotropic Environments

Appendix A – Deformation of Inclusions and Cavities in Elastic Solids (adapted from “The Rock Physics Handbook,” Mavko et al, 1998)

Many problems in effective fracture modeling can be solved or estimated in terms of the elastic behavior of cavities and inclusions. In this modeling context, fractures represent simply a portion of the pore space.

Some static and quasistatic results for cavities are presented here. It should be remembered that these are often also valid for certain limiting cases of dynamic problems. Excellent treatments of cavity deformation and pore compressibility are given by Jaeger and Cook (1969) and Zimmerman (1991).

It is our intention here to outline elasticity results for the deformation of fractures and to show both quantitatively and conceptually how they impact elastic moduli. The more specific application to *anisotropic* fracture distributions are described in Appendixes B (Hudson’s model) and Appendix E (comparison of various fracture models).

General crack and pore deformation: Effective dry compressibility. Consider a homogeneous linear elastic solid that has an arbitrarily shaped pore space – either a single cavity, or a collection of pores. The effective dry compressibility (reciprocal of the dry bulk modulus) of the porous solid can be written as

$$\frac{1}{K_{\text{dry}}} = \frac{1}{K_0} + \frac{\phi}{v_p} \left. \frac{\partial v_p}{\partial \sigma} \right|_{\text{dry}}$$

where

K_{dry}	effective bulk modulus of dry porous solid
K_0	bulk modulus of the solid mineral material
ϕ	porosity
v_p	pore volume
$\left. \frac{\partial v_p}{\partial \sigma} \right _{\text{dry}}$	derivative of pore volume with respect to externally applied hydrostatic stress

and where we assume no inelastic effects such as friction or viscosity are present. This is strictly true, regardless of pore geometry and pore concentration. This can be slightly rewritten as

$$\frac{1}{K_{\text{dry}}} = \frac{1}{K_0} + \frac{\phi}{K_\phi}$$

where

$$K_\phi = v_p / \left. \frac{\partial v_p}{\partial \sigma} \right|_{drv}$$

is defined as the dry pore space stiffness.

These equations state simply that the porous rock compressibility is equal to the intrinsic mineral compressibility, plus an additional compressibility due to the pore space.

Caution: “Dry rock” is not the same as gas-saturated rock. The dry frame modulus refers to the incremental bulk deformation resulting from an increment of applied confining pressure, with pore pressure held constant. This corresponds to a “drained” experiment in which pore fluids can flow freely in or out of the sample to insure constant pore pressure. Alternatively, it can correspond to an undrained experiment in which the pore fluid has zero bulk modulus, so that pore compressions do not induce changes in pore pressure – this is approximately the case for an air-filled sample at standard temperature and pressure. However, at reservoir conditions, gas takes on a non-negligible bulk modulus, and should be treated as a saturating fluid.

Stress-induced pore pressure. If this arbitrary pore space is filled with a pore fluid with bulk modulus, K_{fl} , the saturated solid is stiffer under compression than the dry solid, because an increment of pore fluid pressure is induced which resists the volumetric strain. The ratio of the induced pore pressure, dP , to the applied compressive stress, $d\sigma$, is sometimes called **Skempton’s coefficient** and can be written as

$$B \equiv \frac{dP}{d\sigma} = \frac{1}{1 + K_\phi \left(\frac{1}{K_{fl}} - \frac{1}{K_0} \right)} = \frac{1}{1 + \phi \left(\frac{1}{K_{fl}} - \frac{1}{K_0} \right) \left(\frac{1}{K_{dry}} - \frac{1}{K_0} \right)^{-1}}$$

where K_ϕ is the dry pore space stiffness defined above. For this to be true, the pore pressure must be uniform throughout the pore space as would be the case if (i) there is only one pore, (ii) all pores are well connected and the frequency and viscosity are low enough for any pressure differences to equilibrate, or (iii) all pores have the same dry pore stiffness. Given these, there is no additional limitation on pore geometry or concentration. All of the necessary information about pore stiffness and geometry is contained in the parameter K_ϕ .

When fractures are an important part of the pore space, K_ϕ tends to be small, and dP/ds is large.

Saturated stress-induced pore volume change. The corresponding change in fluid-saturated pore volume, v_p , caused by the remote stress is

$$\left. \frac{1}{v_p} \frac{dv_p}{d\sigma} \right|_{\text{sat}} = \frac{1}{K_{\text{fl}}} \frac{dP}{d\sigma} = \frac{1/K_{\text{fl}}}{1 + K_{\phi} \left(\frac{1}{K_{\text{fl}}} - \frac{1}{K_0} \right)}$$

Low-frequency saturated compressibility. The low frequency saturated bulk modulus, K_{sat} , can be derived from Gassmann's equation. One equivalent form is

$$\frac{1}{K_{\text{sat}}} = \frac{1}{K_0} + \frac{\phi}{K_{\phi} + \frac{K_0 K_{\text{fl}}}{K_0 - K_{\text{fl}}}} \approx \frac{1}{K_0} + \frac{\phi}{K_{\phi} + K_{\text{fl}}}$$

where again all of the necessary information about pore stiffness and geometry is contained in the dry pore stiffness K_{ϕ} , and we must insure that the stress-induced pore pressure is uniform throughout the pore space.

3-D ellipsoidal cavities: Many fractured media models are based on ellipsoidal inclusions or cavities. These are mathematically convenient shapes and allow quantitative estimates of, for example, K_{ϕ} , which is defined above. Eshelby (1957) discovered that the strain, ϵ_{ii} , inside an ellipsoidal inclusion is homogeneous, when a homogenous strain, ϵ_{ii}^0 , (or stress) is applied at infinity. Since the inclusion strain is homogeneous, operations such as determining the inclusion stress or integrating to get the displacement field are simple.

Penny-shaped crack – oblate spheroid. Consider a dry penny-shaped ellipsoidal cavity, with semi-axes $a \ll b = c$. When a remote uniform tensional stress, $d\sigma$, is applied normal to the plane of the crack, each crack face undergoes an outward displacement, U , normal to the plane of the crack, given by the radially symmetric distribution:

$$U(r) = \frac{4(1-\nu^2)c\sqrt{1-(\frac{r}{c})^2}}{3\pi K_0(1-2\nu)} d\sigma$$

where r is the radial distance from the axis of the crack. For any arbitrary homogeneous remote stress, $d\sigma$ is the component of stress *normal* to the plane of the crack. Thus, $d\sigma$ can also be thought of as a remote hydrostatic stress field.

This displacement function is also an ellipsoid, with semiminor axis $U(r=0)$ and semimajor axis, c . Therefore the volume change, dv , which is simply the integral of $U(r)$ over the faces of the crack, is just the volume of the displacement ellipsoid:

$$dv_p = \frac{4}{3}\pi U(0)c^2 = \frac{16c^3}{9K_0} \frac{(1-\nu^2)}{(1-2\nu)} d\sigma$$

Then the volumetric strain of the cavity is

$$\epsilon_{ii} = \frac{dv_p}{v_p} = \frac{4(c/a)}{3\pi K_0} \frac{(1-\nu^2)}{(1-2\nu)} d\sigma$$

and the pore stiffness is

$$\frac{1}{K_\phi} = \frac{1}{v_p} \frac{dv_p}{d\sigma} = \frac{4(c/a)}{3\pi K_0} \frac{(1-\nu^2)}{(1-2\nu)}$$

An interesting case is when an applied compressive stress causes a displacement $U(0)$ equal to the original half-width of the crack, causing the crack to close. Setting $U(0) = a$ allows one to compute the *closing stress*:

$$\sigma_{\text{close}} = \frac{3\pi(1-2\nu)}{4(1-\nu^2)} \alpha K_0 = \frac{\pi}{2(1-\nu)} \alpha \mu_0$$

where $\alpha = (a/c)$ is the aspect ratio.

Now, under a remotely applied homogeneous *shear stress*, τ_o , corresponding to remote shear strain $\epsilon_o = \tau_o/2\mu_0$, the effective *shear strain* in the cavity is

$$\epsilon = \tau_o \frac{2(c/a)}{\pi \mu_0} \frac{(1-\nu)}{(2-\nu)}$$

Appendix B - Hudson's Model Hudson's Model for Cracked Media (adapted from “The Rock Physics Handbook,” Mavko et al, 1998)

This commonly used model is based on a scattering theory analysis of the mean wave field in an elastic solid with thin penny-shaped ellipsoidal cracks or inclusions, (Hudson, 1980, 1981). The effective moduli c_{ij}^{eff} are given as

$$c_{ij}^{\text{eff}} = c_{ij}^0 + c_{ij}^1 + c_{ij}^2$$

where c_{ij}^0 are the isotropic background moduli and c_{ij}^1 , c_{ij}^2 are the first and second order corrections respectively. (Note that Hudson uses a slightly different definition for the shortened 2-subscript elastic notation, and there is an extra factor of 2 in his c_{44} , c_{55} and c_{66} . This makes the equations given in his paper for c_{44}^1 and c_{44}^2 slightly different from those given below which are consistent with the more standard notation)

For a single crack set with crack normals aligned along the 3-axis, the cracked media shows transverse isotropic symmetry and the corrections are

$$c_{11}^1 = -\frac{\lambda^2}{\mu} \epsilon U_3$$

$$c_{13}^1 = -\frac{\lambda(\lambda + 2\mu)}{\mu} \epsilon U_3$$

$$c_{33}^1 = -\frac{(\lambda + 2\mu)^2}{\mu} \epsilon U_3$$

$$c_{44}^1 = -\mu \epsilon U_1$$

$$c_{66}^1 = 0$$

and (superscripts on the c_{ij} denote second order – not quantities squared)

$$c_{11}^2 = \frac{q}{15} \frac{\lambda^2}{(\lambda + 2\mu)} (\epsilon U_3)^2$$

$$c_{13}^2 = \frac{q}{15} \lambda (\epsilon U_3)^2$$

$$c_{33}^2 = \frac{q}{15} (\lambda + 2\mu) (\epsilon U_3)^2$$

$$c_{44}^2 = \frac{2}{15} \frac{\mu(3\lambda + 8\mu)}{\lambda + 2\mu} (\epsilon U_1)^2$$

$$c_{66}^2 = 0$$

where

$$q = 15 \frac{\lambda^2}{\mu^2} + 28 \frac{\lambda}{\mu} + 28$$

$$\epsilon = \frac{N}{V} a^3 = \frac{3\phi}{4\pi\alpha} = \text{crack density}$$

The isotropic background elastic moduli are l and m while a and α are the crack radius and aspect ratio respectively. The corrections c_{ij}^1 and c_{ij}^2 obey the usual symmetry properties for transverse isotropy or hexagonal symmetry. U_1 and U_3 depend on the crack conditions. For dry cracks

$$U_1 = \frac{16(\lambda + 2\mu)}{3(3\lambda + 4\mu)}$$

$$U_3 = \frac{4(\lambda + 2\mu)}{3(\lambda + \mu)}$$

For "weak" inclusions (i.e. when $ma/[K' + (4/3)m']$ is of the order 1 and is not small enough to be neglected),

$$U_1 = \frac{16(\lambda + 2\mu)}{3(3\lambda + 4\mu)} \frac{1}{(1 + M)}$$

$$U_3 = \frac{4(\lambda + 2\mu)}{3(\lambda + \mu)} \frac{1}{(1 + \kappa)}$$

where

$$M = \frac{4\mu'(\lambda + 2\mu)}{\pi\alpha\mu(3\lambda + 4\mu)}$$

$$\kappa = \frac{[K' + (4/3)\mu'](\lambda + 2\mu)}{\pi\alpha\mu(\lambda + \mu)}$$

with K' and m' the bulk and shear modulus of the inclusion material. The criterion for an inclusion to be "weak" depends both on its shape or aspect ratio a as well as the relative moduli of the inclusion and matrix material. Dry cavities can be modeled by setting the inclusion moduli to zero. Fluid-saturated cavities are simulated by setting the inclusion shear modulus to zero.

Caution: Since the cavities are isolated with respect to flow, this approach simulates very high frequency behavior, appropriate to ultrasonic lab conditions. At low frequencies, when there is time for wave-induced pore pressure increments to flow and equilibrate, it is better to find the effective moduli for dry cavities, and then saturate them with the Brown and Korrington low frequency relations. This should not be confused with the fact that this is often termed a low frequency theory as crack dimensions are assumed to be much smaller than a wavelength.

Hudson also gives expressions for infinitely thin fluid filled cracks:

$$U_1 = \frac{16(\lambda + 2\mu)}{3(3\lambda + 4\mu)}$$

$$U_3 = 0$$

These assume no discontinuity in the normal component of crack displacements and therefore predict no change in the compressional modulus with saturation. There is, however, a shear displacement discontinuity, and a resulting effect on shear stiffness. This case should be used with care.

The first order changes l_1 and m_1 in the isotropic elastic moduli l and m of a material containing randomly oriented inclusions are given by

$$\mu_1 = -\frac{2\mu}{15} \varepsilon (3U_1 + 2U_3)$$

$$3\lambda_1 + 2\mu_1 = -\frac{(3\lambda + 2\mu)^2}{3\mu} \varepsilon U_3$$

These results agree with the self-consistent results of Budiansky and O'Connell (1974).

For two or more crack sets aligned in different directions, corrections for each crack set are calculated separately in a crack-local coordinate system with the 3-axis normal to the crack plane, then rotated or transformed back into the coordinates of c_{ij}^{eff} and finally the results are added to get the overall correction. Thus for three crack sets with crack densities $\varepsilon_1, \varepsilon_2$ and ε_3 with crack normals aligned along the 1-, 2- and 3-axis respectively, the overall first order corrections to $c_{ij}^0, c_{ij}^{(3 \text{ sets})}$, may be given in terms of linear combinations of the corrections for a single set with the appropriate crack densities as follows (where we have taken into account the symmetry properties of c_{ij}^1):

$$c_{11}^{(3 \text{ sets})} = c_{33}^1(\varepsilon_1) + c_{11}^1(\varepsilon_2) + c_{11}^1(\varepsilon_3)$$

$$c_{13}^{(3 \text{ sets})} = c_{13}^1(\varepsilon_1) + c_{13}^1(\varepsilon_2) + c_{13}^1(\varepsilon_3)$$

$$c_{12}^{(3 \text{ sets})} = c_{12}^1(\varepsilon_1) + c_{12}^1(\varepsilon_2) + c_{12}^1(\varepsilon_3)$$

$$c_{22}^{(3 \text{ sets})} = c_{11}^1(\varepsilon_1) + c_{33}^1(\varepsilon_2) + c_{11}^1(\varepsilon_3)$$

$$c_{23}^{(3 \text{ sets})} = c_{12}^1(\varepsilon_1) + c_{13}^1(\varepsilon_2) + c_{13}^1(\varepsilon_3)$$

$$c_{33}^{(3 \text{ sets})} = c_{11}^1(\varepsilon_1) + c_{11}^1(\varepsilon_2) + c_{33}^1(\varepsilon_3)$$

$$c_{44}^{(3 \text{ sets})} = c_{44}^1(\varepsilon_2) + c_{44}^1(\varepsilon_3)$$

$$c_{55}^{(3 \text{ sets})} = c_{44}^1(\varepsilon_1) + c_{44}^1(\varepsilon_3)$$

$$c_{66}^{(3 \text{ sets})} = c_{44}^1(\varepsilon_1) + c_{44}^1(\varepsilon_2)$$

Note that $c_{66}^1 = 0$ and $c_{12}^1 = c_{11}^1 - 2c_{66}^1 = c_{11}^1$.

Hudson (1981) also gives the attenuation coefficient ($\gamma = \omega Q^{-1}/2V$) for elastic waves in cracked media. For aligned cracks with normals along 3-axis, the attenuation coefficients for P-, SV- and SH-waves are

$$\gamma_P = \frac{\omega}{V_S} \varepsilon \left(\frac{\omega a}{V_P} \right)^3 \frac{1}{30\pi} \left[A \cdot U_1^2 \sin^2 2\theta + B \cdot U_3^2 \left(\frac{V_P^2}{V_S^2} - 2 \sin^2 \theta \right)^2 \right]$$

$$\gamma_{SV} = \frac{\omega}{V_S} \varepsilon \left(\frac{\omega a}{V_S} \right)^3 \frac{1}{30\pi} [A \cdot U_1^2 \cos^2 2\theta + B \cdot U_3^2 \sin^2 2\theta]$$

$$\gamma_{SH} = \frac{\omega}{V_S} \varepsilon \left(\frac{\omega a}{V_S} \right)^3 \frac{1}{30\pi} [A \cdot U_1^2 \cos^2 \theta]$$

$$A = \frac{3}{2} + \frac{V_S^5}{V_P^5}$$

$$B = 2 + \frac{15}{4} \frac{V_S}{V_P} - 10 \frac{V_S^3}{V_P^3} + 8 \frac{V_S^5}{V_P^5}$$

In the above expressions V_P and V_S are the P and S velocities in the uncracked isotropic background matrix, ω the angular frequency and θ is the angle between the direction of propagation and the 3-axis (axis of symmetry).

For randomly oriented cracks (isotropic distribution) the P and S attenuation coefficients are given as

$$\gamma_P = \frac{\omega}{V_S} \varepsilon \left(\frac{\omega a}{V_P} \right)^3 \frac{4}{(15)^2 \pi} \left[A \cdot U_1^2 + \frac{1}{2} \frac{V_P^5}{V_S^5} \cdot B(B-2) U_3^2 \right]$$

$$\gamma_S = \frac{\omega}{V_S} \varepsilon \left(\frac{\omega a}{V_S} \right)^3 \frac{1}{75\pi} [A \cdot U_1^2 + \frac{1}{3} B \cdot U_3^2]$$

The fourth power dependence on ω is characteristic of Rayleigh scattering.

Hudson (1990) gives results for overall elastic moduli of material with various distributions of penny-shaped cracks. If conditions at the cracks are taken to be uniform so that U_1 and U_3 do not depend on the polar and azimuthal angles q and f , the first order correction is given as

$$c_{ijpq}^1 = -\frac{A}{\mu} U_3 [\lambda^2 \delta_{ij} \delta_{pq} + 2\lambda\mu(\delta_{ij} \varepsilon_{pq} + \delta_{pq} \varepsilon_{ij}) + 4\mu^2 \varepsilon_{ijpq}] \\ - A\mu U_1 (\delta_{jq} \varepsilon_{ip} + \delta_{jp} \varepsilon_{iq} + \delta_{iq} \varepsilon_{jp} + \delta_{ip} \varepsilon_{jq} - 4\varepsilon_{ijpq})$$

where

$$A = \int_0^{2\pi} \int_0^{\pi/2} \varepsilon(\theta, \phi) \sin\theta \, d\theta \, d\phi$$

$$\varepsilon_{ij} = \frac{1}{A} \int_0^{2\pi} \int_0^{\pi/2} \varepsilon(\theta, \phi) n_i n_j \sin\theta \, d\theta \, d\phi$$

$$\varepsilon_{ijpq} = \frac{1}{A} \int_0^{2\pi} \int_0^{\pi/2} \varepsilon(\theta, \phi) n_i n_j n_p n_q \sin\theta \, d\theta \, d\phi$$

and n_i are the components of the unit vector along the crack normal, $\mathbf{n} = (\sin q \cos f, \sin q \sin f, \cos q)$ while $\varepsilon(q, f)$ is the crack density distribution function such that $\varepsilon(q, f) \sin q \, dq \, df$ is the density of cracks with normals lying in the solid angle between $(q, q + dq)$ and $(f, f + df)$.

Special cases of crack distributions:

a) When cracks with total crack density e_t have all their normals aligned along $q = q_0, f = f_0$:

$$\varepsilon(\theta, \phi) = \varepsilon_t \frac{\delta(\theta - \theta_0)}{\sin\theta} \delta(\phi - \phi_0)$$

and then

$$A = \varepsilon_t$$

$$\xi_{ii} = n_i^0 n_i^0$$

$$\xi_{ijpq} = n_i^0 n_j^0 n_p^0 n_q^0$$

where

$$n_1^0 = \sin\theta_0 \cos\phi_0, n_2^0 = \sin\theta_0 \sin\phi_0, n_3^0 = \cos\theta_0.$$

b) Rotationally symmetric crack distributions with normals symmetrically distributed about $q = 0$, that is, e is a function of q only:

$$A = 2\pi \int_0^{\pi/2} \varepsilon(\theta) \sin\theta d\theta$$

$$\xi_{12} = \xi_{23} = \xi_{31} = 0$$

$$\xi_{11} = \xi_{22} = \frac{\pi}{A} \int_0^{\pi/2} \varepsilon(\theta) \sin^3\theta d\theta = \frac{1}{2} (1 - \xi_{33})$$

$$\xi_{1111} = \xi_{2222} = \frac{3\pi}{4A} \int_0^{\pi/2} \varepsilon(\theta) \sin^5\theta d\theta = 3\xi_{1122} = 3\xi_{1212}, \text{ etc.}$$

$$\xi_{3333} = \frac{8}{3} \xi_{1111} - 4\xi_{11} + 1$$

$$\xi_{1133} = \xi_{11} - \frac{4}{3} \xi_{1111} = \xi_{2233} = \xi_{1313} = \xi_{2323}, \text{ etc.}$$

Elements other than those related to the above by symmetry are zero. A particular rotationally symmetric distribution is the Fisher distribution for which $e(q)$ is

$$\varepsilon(\theta) = \frac{\varepsilon_t}{2\pi} \frac{e^{(\cos\theta)/\sigma^2}}{\sigma^2(e^{1/\sigma^2} - 1)}$$

For small σ^2 this is approximately a model for a Gaussian distribution on the sphere:

$$\varepsilon(\theta) \approx \varepsilon_t \frac{e^{-\theta^2/2\sigma^2}}{2\pi\sigma^2}$$

The proportion of crack normals outside the range $0 \leq \theta \leq 2\sigma$ is approximately $1/e^2$. For this distribution,

$$A = \varepsilon_t$$

$$\xi_{11} = \frac{-1 + 2\sigma^2 e^{1/\sigma^2} - 2\sigma^4(e^{1/\sigma^2} - 1)}{2(e^{1/\sigma^2} - 1)} \approx \sigma^2$$

$$\xi_{1111} = \frac{3}{8} \left[\frac{-1 + 4\sigma^4(2e^{1/\sigma^2} + 1) - 24\sigma^6 e^{1/\sigma^2} + 24\sigma^8(e^{1/\sigma^2} - 1)}{(e^{1/\sigma^2} - 1)} \right] \approx 3\sigma^4$$

This distribution is suitable when crack normals are oriented randomly with a small variance about a mean direction along the 3-axis.

c) Cracks with normals randomly distributed at a fixed angle from the 3-axis forming a cone. In this case ϵ is independent of f and is zero unless $q = q_0$, $0 \leq \phi \leq 2\pi$.

$$\epsilon(\theta) = \epsilon_t \frac{\delta(\theta - \theta_0)}{2\pi \sin \theta}$$

which gives

$$A = \epsilon_t$$

$$\epsilon_{11} = \frac{1}{2} \sin^2 \theta_0$$

$$\epsilon_{1111} = \frac{3}{8} \sin^4 \theta_0$$

and the first order corrections are

$$\begin{aligned} c_{1111}^1 &= \frac{-\epsilon_t}{2\mu} [U_3(2\lambda^2 + 4\lambda\mu \sin^2 \theta_0 + 3\mu^2 \sin^4 \theta_0) + U_1\mu^2 \sin^2 \theta_0(4 - 3\sin^2 \theta_0)] \\ &= c_{1111}^1 \end{aligned}$$

$$c_{3333}^1 = \frac{-\epsilon_t}{\mu} [U_3(\lambda + 2\mu \cos^2 \theta_0)^2 + U_1\mu^2 4 \cos^2 \theta_0 \sin^2 \theta_0]$$

$$c_{1122}^1 = \frac{-\epsilon_t}{2\mu} [U_3(2\lambda^2 + 4\lambda\mu \sin^2 \theta_0 + \mu^2 \sin^4 \theta_0) - U_1\mu^2 \sin^4 \theta_0]$$

$$\begin{aligned} c_{1133}^1 &= \frac{-\epsilon_t}{\mu} [U_3(\lambda + \mu \sin^2 \theta_0)(\lambda + 2\mu \cos^2 \theta_0) - U_1\mu^2 2\sin^2 \theta_0 \cos^2 \theta_0] \\ &= c_{1133}^1 \end{aligned}$$

$$\begin{aligned} c_{2323}^1 &= \frac{-\epsilon_t}{2} \mu [U_3 4\sin^2 \theta_0 \cos^2 \theta_0 + U_1(\sin^2 \theta_0 + 2\cos^2 \theta_0 - 4\sin^2 \theta_0 \cos^2 \theta_0)] \\ &= c_{2323}^1 \end{aligned}$$

$$c_{1212}^1 = \frac{-\epsilon_t}{2} \mu [U_3 \sin^4 \theta_0 + U_1 \sin^2 \theta_0 (2 - \sin^2 \theta_0)]$$

Assumptions and limitations

- Idealized crack shape (penny-shaped) with small aspect ratios and crack density. Crack radius and distance between cracks much smaller than a wavelength. The formal limit quoted by Hudson for both first and second order terms is ϵ less than 0.1.
- The second order expansion is not a uniformly converging series and predicts increasing moduli with crack density beyond the formal limit, (Cheng, 1993). Better results will be obtained by using just the first order correction than inappropriately using the second order correction. Cheng gives a new expansion based on Padé approximation which avoids this problem.
- Cracks are isolated with respect to fluid flow. Pore pressures are unequilibrated and adiabatic. Appropriate for high frequency lab conditions. For low frequency field situations use Hudson's dry equations and then saturate using Brown and Korrington relations. Should not be confused with the fact that this is often termed a low frequency theory as crack dimensions are assumed to be much smaller than a wavelength.

- Sometimes a single crack set may not be an adequate representation of crack induced anisotropy. Need to superpose large number of crack sets with angular distributions.

Appendix C – Anisotropic Form of Hooke's Law

(adapted from “The Rock Physics Handbook,” by Mavko et al., 1998)

Hooke's law for a general anisotropic linear elastic solid states that the stress s_{ij} is linearly proportional to the strain e_{ij} :

$$\sigma_{ij} = c_{ijkl} \epsilon_{kl}$$

with summation implied over the repeated subscripts k and l . The *elastic stiffness tensor*, c_{ijkl} , is a fourth rank tensor obeying the laws of tensor transformation and has a total of 81 components. However not all 81 components are independent. The symmetry of stresses and strains implies that

$$c_{ijkl} = c_{jikl} = c_{ijlk} = c_{jilk}$$

reducing the number of independent constants to 36. Also the existence of a unique strain energy potential requires that

$$c_{ijkl} = c_{klij}$$

further reducing the number of independent constants to 21. This is the maximum number of elastic constants that any medium can have. Additional restrictions imposed by symmetry considerations reduce the number much further. Isotropic linear elastic materials, which have the maximum symmetry, are completely characterized by 2 independent constants while materials with Triclinic symmetry (lowest symmetry) have all 21 constants.

Alternatively, the strains may be expressed as a linear combination of the stresses:

$$\epsilon_{ij} = s_{ijkl} \sigma_{kl}$$

In this case s_{ijkl} is called the *elastic compliance tensor* and has the same symmetry as the corresponding stiffness tensor.

It is a standard practice in elasticity to use an abbreviated notation for the stresses, strains, and the stiffness and compliance tensors, as it simplifies some of the key equations (Auld, 1990). In this abbreviated notation the stresses and strains are written as six-element column vectors rather than as nine-element square matrices:

$$T = \begin{bmatrix} \sigma_1 = \sigma_{11} \\ \sigma_2 = \sigma_{22} \\ \sigma_3 = \sigma_{33} \\ \sigma_4 = \sigma_{23} \\ \sigma_5 = \sigma_{13} \\ \sigma_6 = \sigma_{12} \end{bmatrix} \quad E = \begin{bmatrix} e_1 = \epsilon_{11} \\ e_2 = \epsilon_{22} \\ e_3 = \epsilon_{33} \\ e_4 = 2\epsilon_{23} \\ e_5 = 2\epsilon_{13} \\ e_6 = 2\epsilon_{12} \end{bmatrix}$$

Note the factor of 2 in the definitions of strains.

The four subscripts of the stiffness and compliance tensors are reduced to two [each pair of indices $ij(kl)$ being replaced by one index $I(J)$] using the following convention:

$ij(kl)$	$I(J)$
11	1
22	2
33	3
23,32	4
13,31	5
12,21	6

The relation, therefore, is $c_{IJ} = c_{ijkl}$ and $s_{IJ} = s_{ijkl} \cdot N$ where

$$N = \begin{cases} 1 & \text{for } I \text{ and } J = 1, 2, 3 \\ 2 & \text{for } I \text{ or } J = 4, 5, 6 \\ 4 & \text{for } I \text{ and } J = 4, 5, 6 \end{cases}$$

Note the difference in the definition of s_{IJ} from that of c_{IJ} . This results from the factors of two which are introduced in the definition of strains in the abbreviated notation. **Caution:** Different definitions of strains are sometimes adopted which move the factors of 2 and 4 from the compliances to the stiffnesses. However, the form given above is the more common convention. In the two-index notation c_{IJ} and s_{IJ} can be conveniently represented as 6x6 matrices. However, they no longer follow the laws of tensor transformation. Care has to be taken when transforming from one coordinate system to another. One way is to go back to the 4-index notation and then use the ordinary laws of coordinate transformation.

The non-zero components of the more symmetric anisotropy classes commonly used in modelling rock properties are given below:

Isotropic – 2 independent constants:

$$\begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{bmatrix} \quad c_{12} = c_{11} - 2c_{44}$$

The relations between the c's and the Lamé's parameters λ and μ of isotropic linear elasticity are

$$c_{11} = \lambda + 2\mu \quad c_{12} = \lambda \quad c_{44} = \mu$$

Cubic – 3 independent constants:

$$\begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{bmatrix}$$

**Hexagonal or Transversely Isotropic –
5 independent constants:**

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \quad c_{66} = \frac{1}{2}(c_{11} - c_{12})$$

Orthorhombic – 9 independent constants:

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}$$

For *isotropic* symmetry the phase velocity of wave propagation is given by

$$v_P = \sqrt{\frac{c_{11}}{\rho}} ; v_S = \sqrt{\frac{c_{44}}{\rho}}$$

where ρ is the density.

In *anisotropic* media there are, in general, 3 modes of propagation – quasilongitudinal, quasishear and pure shear – with mutually orthogonal polarizations. For a medium with *transversely isotropic* (hexagonal) symmetry, the wave slowness surface is always rotationally symmetric about the axis of symmetry. The velocities of the three modes in any plane containing the symmetry axis are given as

quasilongitudinal mode (transversely isotropic)

$$V_P = \{c_{11}\sin^2\theta + c_{33}\cos^2\theta + c_{44} + \sqrt{M}\}^{1/2}(2\rho)^{-1/2}$$

quasishear mode (transversely isotropic)

$$V_{SV} = \{c_{11}\sin^2\theta + c_{33}\cos^2\theta + c_{44} - \sqrt{M}\}^{1/2}(2\rho)^{-1/2}$$

pure shear mode (transversely isotropic)

$$V_{SH} = \left(\frac{c_{66}\sin^2\theta + c_{44}\cos^2\theta}{\rho} \right)^{1/2}$$

where

$$M = [(c_{11} - c_{44})\sin^2\theta - (c_{33} - c_{44})\cos^2\theta]^2 + (c_{13} + c_{44})^2\sin^2 2\theta$$

and θ is the angle between the wave vector and the axis of symmetry. The 5 components of the stiffness tensor for a transversely isotropic material are obtained from 5 velocity measurements: $V_P(0^\circ)$, $V_P(90^\circ)$, $V_P(45^\circ)$, $V_{SH}(90^\circ)$ and $V_{SH}(0^\circ) = V_{SV}(0^\circ)$.

$$c_{11} = \rho V_{P(90^\circ)}^2$$

$$c_{12} = c_{11} - 2\rho V_{SH(90^\circ)}^2$$

$$c_{33} = \rho V_{P(0^\circ)}^2$$

$$c_{44} = \rho V_{SH(0^\circ)}^2$$

and

$$c_{13} = -c_{44} + \sqrt{4\rho^2 V_{P(45^\circ)}^4 - 2\rho V_{P(45^\circ)}^2 (c_{11} + c_{33} + 2c_{44}) + (c_{11} + c_{44})(c_{33} + c_{44})}$$

For the more general **orthorhombic** symmetry the velocities of the three modes propagating in the three symmetry planes – XZ, YZ and XY are given as follows:

quasilongitudinal mode (orthorhombic – XZ plane)

$$V_P = \{c_{55} + c_{11}\sin^2\theta + c_{33}\cos^2\theta + \sqrt{(c_{55} + c_{11}\sin^2\theta + c_{33}\cos^2\theta)^2 - 4A}\}^{1/2} (2\rho)^{-1/2}$$

quasishear mode (orthorhombic – XZ plane)

$$V_{SV} = \{c_{55} + c_{11}\sin^2\theta + c_{33}\cos^2\theta - \sqrt{(c_{55} + c_{11}\sin^2\theta + c_{33}\cos^2\theta)^2 - 4A}\}^{1/2} (2\rho)^{-1/2}$$

pure shear mode (orthorhombic – XZ plane)

$$V_{SH} = \left(\frac{c_{66}\sin^2\theta + c_{44}\cos^2\theta}{\rho} \right)^{1/2}$$

where

$$A = (c_{11}\sin^2\theta + c_{55}\cos^2\theta)(c_{55}\sin^2\theta + c_{33}\cos^2\theta) - (c_{13} + c_{55})^2 \sin^2\theta \cos^2\theta$$

quasilongitudinal mode (orthorhombic – YZ plane)

$$V_P = \{c_{44} + c_{22}\sin^2\theta + c_{33}\cos^2\theta + \sqrt{(c_{44} + c_{22}\sin^2\theta + c_{33}\cos^2\theta)^2 - 4B}\}^{1/2} (2\rho)^{-1/2}$$

quasishear mode (orthorhombic – YZ plane)

$$V_{SV} = \{c_{44} + c_{22}\sin^2\theta + c_{33}\cos^2\theta - \sqrt{(c_{44} + c_{22}\sin^2\theta + c_{33}\cos^2\theta)^2 - 4B}\}^{1/2} (2\rho)^{-1/2}$$

pure shear mode (orthorhombic – YZ plane)

$$V_{SH} = \left(\frac{c_{66}\sin^2\theta + c_{55}\cos^2\theta}{\rho} \right)^{1/2}$$

where

$$B = (c_{22}\sin^2\theta + c_{44}\cos^2\theta)(c_{44}\sin^2\theta + c_{33}\cos^2\theta) - (c_{23} + c_{44})^2 \sin^2\theta \cos^2\theta$$

quasilongitudinal mode (orthorhombic – XY plane)

$$V_P = \{c_{66} + c_{22}\sin^2\phi + c_{11}\cos^2\phi + \sqrt{(c_{66} + c_{22}\sin^2\phi + c_{11}\cos^2\phi)^2 - 4C}\}^{1/2} (2\rho)^{-1/2}$$

quasishear mode (orthorhombic – XY plane)

$$V_{SH} = \{c_{66} + c_{22}\sin^2\phi + c_{11}\cos^2\phi - \sqrt{(c_{66} + c_{22}\sin^2\phi + c_{11}\cos^2\phi)^2 - 4C}\}^{1/2} (2\rho)^{-1/2}$$

pure shear mode (orthorhombic – XY plane)

$$V_{SV} = \left(\frac{c_{55}\cos^2\phi + c_{44}\sin^2\phi}{\rho} \right)^{1/2}$$

where

$$C = (c_{66}\sin^2\varphi + c_{11}\cos^2\varphi)(c_{33}\sin^2\varphi + c_{66}\cos^2\varphi) - (c_{13} + c_{66})^2\sin^2\varphi\cos^2\varphi$$

and the angles θ and φ are measured from the Z- and X-axis respectively.

Appendix D – Thomsen's Notation for Weak Elastic Anisotropy (adapted from "The Rock Physics Handbook," by Mavko et al., 1998)

A transversely isotropic elastic material is completely specified by 5 independent constants. In terms of the shortened notation the elastic constants can be represented as

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}, \text{ where } c_{66} = \frac{1}{2}(c_{11} - c_{12})$$

where the 3-axis (z-axis) lies along the axis of symmetry.

Thomsen (1986) suggested a convenient notation for this type of material, when only weakly anisotropic, in terms of the P-wave and S-wave velocities, α and β , propagating along the symmetry axis plus three additional constants:

$$\begin{aligned} \alpha &= \sqrt{c_{33}/\rho} \\ \beta &= \sqrt{c_{44}/\rho} \\ \epsilon &= \frac{c_{11} - c_{33}}{2c_{33}} \\ \gamma &= \frac{c_{66} - c_{44}}{2c_{44}} \\ \delta &= \frac{(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2}{4c_{33}c_{44}} \end{aligned}$$

In terms of these constants, the three phase velocities can then be written conveniently as

$$\begin{aligned} V_P(\theta) &\approx \alpha (1 + \delta \sin^2\theta \cos^2\theta + \epsilon \sin^4\theta) \\ V_{SV}(\theta) &\approx \beta \left(1 + \frac{\alpha^2}{\beta^2} (\epsilon - \delta) \sin^2\theta \cos^2\theta \right) \\ V_{SH}(\theta) &\approx \beta (1 + \gamma \sin^2\theta) \end{aligned}$$

where V_{SH} is the wavefront velocity of the pure shear wave which has no component of polarization in the vertical (z) direction, V_{SV} is the pseudo-shear wave polarized normal to the pure shear wave, and V_P is the pseudo-longitudinal wave. θ is the phase angle between the wavefront normal and the symmetry axis (z-axis).

The constant ϵ can be seen to describe the fractional difference of the P-wave velocities in the vertical and horizontal directions:

$$\epsilon = \frac{V_P(90^\circ) - V_P(0^\circ)}{V_P(0^\circ)}$$

and therefore best describes what is usually called the “P-wave anisotropy”.

Similarly, the constant g can be seen to describe the fractional difference of the SH-wave velocities between vertical and horizontal directions, which is equivalent to the difference between the vertical and horizontal polarizations of the horizontally propagating S-waves:

$$\gamma = \frac{V_{SH}(90^\circ) - V_{SV}(90^\circ)}{V_{SV}(90^\circ)} = \frac{V_{SH}(90^\circ) - V_{SH}(0^\circ)}{V_{SH}(0^\circ)}$$

Appendix E – Comparison of Some Elastic Fracture Models (Adapted from Teng, 1998)

Introduction

Fracture detection and characterization is of great importance to hydrocarbon recovery and waste isolation because fractures greatly impact the overall permeability and fluid flow (Nelson 1985). In order to characterize the physical properties of fractures from the measurements of rock mechanical properties including seismic velocity, we need to use the elasticity theories of fractured rocks that link fracture physical descriptions to rock elasticity.

There are three widely used fracture models: Hudson's (1980, 1981, 1990, 1994) penny-shaped-crack model, Schoenberg-Muir's (1983, 1988, 1989) thin-layer model, and Pyrak-Nolte's (1990a, 1990b) slip-interface model. They have different assumptions, and are suitable for different fracture configurations. Hudson's and Schoenberg-Muir's models are the long-wavelength-limit effective medium theories. In contrast, Pyrak-Nolte's model is not for long wavelength limit. It takes fracture inertial effect into account and gives a frequency-dependent seismic wave velocities. We show how these models are mathematically equivalent to each other in the first order. Their equivalence reveals that penny-shaped cracks and equivalent amount of parallel-wall long fractures can generate the same seismic anisotropic response. The only way to distinguish them is to rely on the in situ fracture observations in the cores and outcrops.

Relationship Between Hudson's and Schoenberg-Muir's Models

We first review the two effective medium theories given by Hudson (1980, 1981, 1990, 1994), and Schoenberg and Muir (1983, 1988, 1989), and then show their equivalence.

Hudson's model is an intuitive effective medium theory that assumes an elastic solid with an internal distribution of thin penny-shaped ellipsoidal cracks. Hudson used crack density (ϵ) and aspect ratio (α) to describe the structure of fracture systems. The crack density is defined as:

$$\epsilon = \frac{N}{V} a^3 = \frac{3\phi}{4\pi\alpha} \quad (\text{E.1})$$

where a is crack radius, N/V is the number of cracks per unit volume, and ϕ is crack-induced porosity. The effective moduli are given as:

$$C_{ij}^{eff} = C_{ij}^0 + C_{ij}^1 + C_{ij}^2 \quad (\text{E.2})$$

where C_{ij}^0 are the isotropic background moduli and C_{ij}^1 , C_{ij}^2 are the first and second order corrections respectively that depend on the crack orientation, density, and aspect ratio. We can calculate crack-induced porosity based on Equation E.1, and apply fluid substitution relationships in this model. Moreover, this theory is well developed for diverse crack distributions, including one or more sets of parallel fractures, conical distributions, and random distributions. The major limitation is that it only works for small crack density ($e < 0.1$) and small aspect ratio. For a single crack set with crack normals aligned along the 3-axis, the cracked media shows transversely isotropic symmetry and the first order corrections are

$$C_{11}^1 = -\frac{\lambda^2}{\mu} \varepsilon U_3 \quad (\text{E.3a})$$

$$C_{13}^1 = -\frac{\lambda(\lambda + 2\mu)}{\mu} \varepsilon U_3 \quad (\text{E.3b})$$

$$C_{33}^1 = -\frac{(\lambda + 2\mu)^2}{\mu} \varepsilon U_3 \quad (\text{E.3c})$$

$$C_{44}^1 = -\mu \varepsilon U_1 \quad (\text{E.3d})$$

$$C_{66}^1 = 0 \quad (\text{E.3e})$$

where U_1 , U_3 depend on crack conditions. For dry cracks:

$$U_1 = \frac{16(\lambda + 2\mu)}{3(3\lambda + 4\mu)} \quad (\text{E.4a})$$

$$U_3 = \frac{4(\lambda + 2\mu)}{3(\lambda + \mu)} \quad (\text{E.4b})$$

For weak inclusions:

$$U_1 = \frac{16(\lambda + 2\mu)}{3(3\lambda + 4\mu)} \frac{1}{1 + M} \quad (\text{E.5a})$$

$$U_3 = \frac{4(\lambda + 2\mu)}{3(\lambda + \mu)} \frac{1}{1 + \kappa} \quad (\text{E.5b})$$

where

$$M = \frac{4\mu'(\lambda + 2\mu)}{\pi\alpha\mu(3\lambda + 4\mu)} \quad (\text{E.5c})$$

$$\kappa = \frac{[K' + (4/3)\mu'](\lambda + 2\mu)}{\pi\alpha\mu(\lambda + \mu)} \quad (\text{E.5d})$$

with K' and μ' the bulk and shear modulus of the inclusion material.

Schoenberg-Muir's slip-interface model is based on the Backus average (1962) and represents fractures by infinitely thin layers. This model works for one or more sets of parallel fractures, and it does not have the small density limitation. However, since it uses an infinitely thin layer description that does not contain porosity information, we cannot apply fluid substitution here. For one set of parallel fractures with normals aligned along the 3-axis, the effective compliance matrix is:

$$S_{ij}^{eff} = S_{ij}^0 + \Delta S_{ij} \quad (E.6)$$

where S_{ij}^0 is the isotropic background compliance, and

$$\Delta S_{ij} = h_f * \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_N & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_T & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (E.7)$$

with Z_N and Z_T the normal and shear compliance, and h_f the volume fraction of the fractures. Schoenberg (1983) also gave the effective stiffness expressions:

$$C_{11} = [1 - 4(1 - \gamma)E_N]C_{33} \quad (E.8a)$$

$$C_{13} = (1 - 2\gamma)C_{33} \quad (E.8b)$$

$$C_{33} = \frac{\mu}{\gamma + E_N} \quad (E.8c)$$

$$C_{44} = \frac{\mu}{1 + E_T} \quad (E.8d)$$

$$C_{66} = \mu \quad (E.8e)$$

where

$$E_N = \frac{\mu}{H\kappa_N} = \frac{N}{L} \frac{\mu}{\kappa_N} \quad (E.9a)$$

$$E_T = \frac{\mu}{H\kappa_T} = \frac{N}{L} \frac{\mu}{\kappa_T} \quad (E.9b)$$

$$\gamma = \frac{V_s^2}{V_p^2} = \frac{\mu}{\lambda + 2\mu} \quad (E.9c)$$

Here, κ_N and κ_T are the normal and shear specific stiffness of the fractures, H is the fracture spacing, L is the length of a rock sample, and N is the number of fractures inside the rock sample.

Using the relationship between stiffness matrix and compliance matrix:

$$\tilde{C} = (\tilde{S})^{-1} \quad (E.10)$$

we transform Equation E.7 to stiffness domain by:

$$\begin{aligned} \Delta\tilde{C} &= \tilde{C}^{eff} - \tilde{C}^0 \\ &= (\tilde{S}^{eff})^{-1} - \tilde{C}^0 \\ &= (\tilde{S}^0 + \Delta\tilde{S})^{-1} - \tilde{C}^0 \\ &= \left((\tilde{C}^0)^{-1} + \Delta\tilde{S} \right)^{-1} - \tilde{C}^0 \end{aligned} \quad (E.11)$$

The matrix computation results of Equation E.11 are:

$$\Delta C_{11} = -\frac{\lambda^2 h_f Z_N}{1 + (\lambda + 2\mu) h_f Z_N} \quad (E.12a)$$

$$\Delta C_{13} = -\frac{\lambda(\lambda + 2\mu) h_f Z_N}{1 + (\lambda + 2\mu) h_f Z_N} \quad (E.12b)$$

$$\Delta C_{33} = -\frac{(\lambda + 2\mu)^2 h_f Z_N}{1 + (\lambda + 2\mu) h_f Z_N} \quad (E.12c)$$

$$\Delta C_{44} = -\frac{\mu^2 h_f Z_T}{1 + \mu h_f Z_T} \quad (E.12d)$$

$$\Delta C_{66} = 0 \quad (E.12e)$$

Comparing Equation E.12 with equations E.3 and E.8, we get:

$$\frac{1}{H\kappa_N} = h_f Z_N = \frac{U_3 \varepsilon}{\mu - (\lambda + 2\mu) U_3 \varepsilon} \quad (E.13a)$$

$$\frac{1}{H\kappa_T} = h_f Z_T = \frac{U_1 \varepsilon}{\mu - \mu U_1 \varepsilon} \quad (E.13b)$$

The above formulae demonstrate that the first-order Hudson model is equivalent to the Schoenberg-Muir slip-interface model. With this relationship, we can derive the porosity information from Schoenberg-Muir's parameters by transforming the spacing and specific stiffness of fractures to crack density and aspect ratio, and then use fluid substitution. Equations E.13a and E.13b also explain Hudson's small crack density limitation: large crack density will yield unrealistic negative fracture stiffness.

Relationship between Pyrak-Nolte's Model and Two Effective Medium Theories

Pyrak-Nolte characterized fractures by fracture spacing and specific stiffness as in Schoenberg-Muir's model. But she took the fracture inertial effect into account by considering each fracture as a scattering boundary. This model gives a frequency-dependent group velocity. The P-wave group velocity propagating perpendicular to the fracture plane is:

$$U_{eff} = \frac{U \left\{ + [\omega / (2\kappa_N / Z)]^2 \right\}}{1 + [\omega / (2\kappa_N / Z)]^2 + (NUZ / 2L\kappa_N)} \quad (E.14)$$

with κ_N the normal specific stiffness of fractures, ω the wave frequency, and Z the seismic impedance of the isotropic background rock:

$$Z = \rho U \quad (E.15)$$

In order to find the connection between the Pyrak-Nolte and Schoenberg-Muir models, we compare the group velocity derived from the two theories. At Pyrak-Nolte's low frequency long wavelength limit ($\omega \rightarrow 0$), Equation E.14 reduces to:

$$U_{eff} = \frac{U}{1 + (NUZ / 2L\kappa_N)} \quad (E.16)$$

The first order approximation is:

$$U_{eff} = U \left[1 - (NUZ / 2L\kappa_N) \right] = U \left[1 - \frac{\lambda + 2\mu}{2H\kappa_N} \right] \quad (E.17)$$

On the other hand, since for wave propagating along the symmetry axis of a transversely isotropic medium, the phase velocity and the group velocity are the same, we can derive the group velocity from Schoenberg-Muir's Equation E.8 as follows,

$$C_{33} = \frac{\mu}{\gamma + E_N} \approx (\lambda + 2\mu) \left[1 - \frac{\lambda + 2\mu}{H\kappa_N} \right] \quad (\text{E.18})$$

$$U_{eff} = V_{eff} = \sqrt{\frac{C_{33}}{\rho}} = \sqrt{\frac{(\lambda + 2\mu) \left[1 - \frac{\lambda + 2\mu}{H\kappa_N} \right]}{\rho}} \approx U \left[1 - \frac{\lambda + 2\mu}{2H\kappa_N} \right] \quad (\text{E.19})$$

Equation E.19 gives the same group velocity expression as Equation E.17. This shows that Schoenberg-Muir's effective medium theory is equivalent to Pyrak-Nolte's low frequency limit. Moreover, because Hudson's first order theory is equivalent to Schoenberg-Muir's model, it should also be consistent with Pyrak-Nolte's low frequency limit. This suggests that the inertial effect is indeed a geometrical dispersion effect, and that we should be aware of it when the wavelength is not long compared to the fracture geometry.

Discussion

Fracture model construction is important for fracture characterization from rock mechanical properties such as elastic wave velocity. We proved that the first order Hudson's model, Schoenberg-Muir's model, and the low frequency limit of Pyrak-Nolte's model are mathematically equivalent in the first order. This tells us that penny-shaped cracks and long parallel-wall fractures are seismically indistinguishable. We need to rely on the geological observations to determine which model is more suitable for seismic fracture characterization in the field.

The relationships among the three models enable us to take advantage of each of them. We can transform the Schoenberg-Muir's description of fractures to Hudson's description, and conduct fluid substitution. We can also use the Pyrak-Nolte's description of fractures to estimate the magnitude of the frequency dispersion effects when seismic waves pass through fractured rocks.

Appendix F - AVOZ in Anisotropic Environments

(adapted from “The Rock Physics Handbook,” Mavko et al, 1998)

An incident wave at a boundary between two anisotropic media can generate reflected quasi P-waves and quasi S-waves, and transmitted quasi P-waves and quasi S-waves (Auld 1990). In general, the reflection and transmission coefficients vary with offset and azimuth. **AVOZ** (amplitude variation with offset and azimuth) can be detected by 3D seismic surveys and is a useful seismic attribute for reservoir characterization.

Brute force modeling of AVOZ by solving the Zoeppritz (1919) equations can be complicated and unintuitive for several reasons: for anisotropic media in general, the two shear waves are separate (shear wave birefringence), the slowness surfaces are non-spherical and not necessarily convex, and the polarization vectors are neither parallel nor perpendicular to the propagation vectors.

Schoenberg and Protázio (1992) give explicit solutions for the plane wave reflection and transmission problem in terms of submatrices of the coefficient matrix of the Zoeppritz equations. The most general case of the explicit solutions is applicable to monoclinic media with a mirror plane of symmetry parallel to the reflecting plane. Let **R** and **T** represent the **reflection and transmission matrices**:

$$\mathbf{R} = \begin{bmatrix} R_{PP} & R_{SP} & R_{TP} \\ R_{PS} & R_{SS} & R_{TS} \\ R_{PT} & R_{ST} & R_{TT} \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} T_{PP} & T_{SP} & T_{TP} \\ T_{PS} & T_{SS} & T_{TS} \\ T_{PT} & T_{ST} & T_{TT} \end{bmatrix}$$

where the first subscript denotes the type of incident wave and the second subscript denotes the type of reflected or transmitted wave. For “weakly” anisotropic media, the subscript P denotes the P-wave, S denotes one quasi-S wave, and T denotes the other quasi-S wave, i.e. the tertiary or third wave. As a convention for real s_{3P}^2 , s_{3S}^2 , and s_{3T}^2 ,

$$s_{3P}^2 < s_{3S}^2 < s_{3T}^2$$

where s_{3i} is the vertical component of the phase slowness of the i th wave type when the reflecting plane is horizontal. An imaginary value for any of the vertical slownesses implies that the corresponding wave is inhomogeneous, or evanescent. The impedance matrices are defined as

$$\mathbf{X} = \begin{bmatrix} e_{P1} & e_{S1} & e_{T1} \\ e_{P2} & e_{S2} & e_{T2} \\ -(C_{13}e_{P1}+C_{36}e_{P2})s_1 & -(C_{13}e_{S1}+C_{36}e_{S2})s_1 & -(C_{13}e_{T1}+C_{36}e_{T2})s_1 \\ -(C_{23}e_{P2}+C_{36}e_{P1})s_2 & -(C_{23}e_{S2}+C_{36}e_{S1})s_2 & -(C_{23}e_{T2}+C_{36}e_{T1})s_2 \\ -C_{33}e_{P3}s_{3P} & -C_{33}e_{S3}s_{3S} & -C_{33}e_{T3}s_{3T} \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} -(C_{55}s_1+C_{45}s_2)e_{P3} & -(C_{55}s_1+C_{45}s_2)e_{S3} & -(C_{55}s_1+C_{45}s_2)e_{T3} \\ -(C_{55}e_{P1}+C_{45}e_{P2})s_{3P} & -(C_{55}e_{S1}+C_{45}e_{S2})s_{3S} & -(C_{55}e_{T1}+C_{45}e_{T2})s_{3T} \\ -(C_{45}s_1+C_{44}s_2)e_{P3} & -(C_{45}s_1+C_{44}s_2)e_{S3} & -(C_{45}s_1+C_{44}s_2)e_{T3} \\ -(C_{45}e_{P1}+C_{44}e_{P2})s_{3P} & -(C_{45}e_{S1}+C_{44}e_{S2})s_{3S} & -(C_{45}e_{T1}+C_{44}e_{T2})s_{3T} \\ e_{P3} & e_{S3} & e_{T3} \end{bmatrix}$$

where s_1 and s_2 are the horizontal components of the phase slowness vector, e_P , e_S , and e_T are the associated eigenvectors evaluated from the Christoffel equations (see Section 3.2), and C_{IJ} denotes elements of the stiffness matrix of the incident medium. $\mathbf{X}'$ and $\mathbf{Y}'$ are the same as above, except with primed parameters (transmission medium) replacing unprimed parameters (incidence medium). When neither $\mathbf{X}$ nor $\mathbf{Y}$ is singular and $(\mathbf{X}^{-1}\mathbf{X}'+\mathbf{Y}^{-1}\mathbf{Y}')$ is invertible, the reflection and transmission coefficients can be written as

$$\mathbf{T} = 2(\mathbf{X}^{-1}\mathbf{X}'+\mathbf{Y}^{-1}\mathbf{Y}')^{-1}$$

$$\mathbf{R} = (\mathbf{X}^{-1}\mathbf{X}'-\mathbf{Y}^{-1}\mathbf{Y}')(\mathbf{X}^{-1}\mathbf{X}'+\mathbf{Y}^{-1}\mathbf{Y}')^{-1}$$

Schoenberg and Protázio (1992) point out that a singularity occurs at a horizontal slowness for which an interface wave, e.g., a Stoneley wave, exists. When $\mathbf{Y}$ is singular, straightforward matrix manipulations yield

$$\mathbf{T} = 2\mathbf{Y}'^{-1}\mathbf{Y}(\mathbf{X}^{-1}\mathbf{X}'\mathbf{Y}'^{-1}\mathbf{Y}+\mathbf{I})^{-1}$$

$$\mathbf{R} = (\mathbf{X}^{-1}\mathbf{X}'\mathbf{Y}'^{-1}\mathbf{Y}+\mathbf{I})^{-1}(\mathbf{X}^{-1}\mathbf{X}'\mathbf{Y}'^{-1}\mathbf{Y}-\mathbf{I})$$

Similarly, $\mathbf{T}$ and $\mathbf{R}$ can also be written without $\mathbf{X}^{-1}$ when $\mathbf{X}$ is singular:

$$\begin{aligned}\mathbf{T} &= 2\mathbf{X}'^{-1}\mathbf{X}(\mathbf{I}+\mathbf{Y}^{-1}\mathbf{Y}'\mathbf{X}'^{-1}\mathbf{X})^{-1} \\ \mathbf{R} &= (\mathbf{I}+\mathbf{Y}^{-1}\mathbf{Y}'\mathbf{X}'^{-1}\mathbf{X})^{-1}(\mathbf{I}-\mathbf{Y}^{-1}\mathbf{Y}'\mathbf{X}'^{-1}\mathbf{X})\end{aligned}$$

Alternate solutions can be found by assuming that $\mathbf{X}'$ and $\mathbf{Y}'$ are invertible:

$$\begin{aligned}\mathbf{R} &= (\mathbf{Y}'^{-1}\mathbf{Y}+\mathbf{X}'^{-1}\mathbf{X})^{-1}(\mathbf{Y}'^{-1}\mathbf{Y}-\mathbf{X}'^{-1}\mathbf{X}) \\ \mathbf{T} &= 2\mathbf{X}'^{-1}\mathbf{X}(\mathbf{Y}'^{-1}\mathbf{Y}+\mathbf{X}'^{-1}\mathbf{X})^{-1}\mathbf{Y}'^{-1}\mathbf{Y} \\ &= 2\mathbf{Y}'^{-1}\mathbf{Y}(\mathbf{Y}'^{-1}\mathbf{Y}+\mathbf{X}'^{-1}\mathbf{X})^{-1}\mathbf{X}'^{-1}\mathbf{X}\end{aligned}$$

These formulas allow more straightforward reflectivity and transmissivity calculations when the media have at least monoclinic symmetry with a horizontal symmetry plane.

For a wave traveling in anisotropic media, there will generally be out-of-plane motion unless the wave path is in a symmetry plane. These symmetry planes include all vertical planes in **TIV** (transversely isotropic with vertical symmetry axis) media, the symmetry planes in **TIH** (transversely isotropic with horizontal symmetry axis), and orthorhombic media. In this case, the quasi P- and the quasi S-waves in the symmetry plane uncouple from the quasi S-wave polarized transversely to the symmetry plane. For weakly anisotropic media, we can use simple analytical formulas (Banik, 1987; Thomsen, 1993; Rüger, 1995,1996; Chen, 1995) to compute AVOZ responses at the interface of anisotropic media that can be either TIV, TIH, or orthorhombic. The analytical formulas give more insight into the dependence of AVOZ on anisotropy.

Thomsen (1986) introduced the following notation for weak transversely isotropic media with density ρ :

$$\begin{aligned}\alpha &= \sqrt{\frac{C_{33}}{\rho}} & \epsilon &= \frac{C_{11} - C_{33}}{2C_{33}} \\ \beta &= \sqrt{\frac{C_{44}}{\rho}} & \gamma &= \frac{C_{66} - C_{44}}{2C_{44}} \\ \delta &= \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})}\end{aligned}$$

The P-wave reflection coefficient for weakly anisotropic TIV media in the limit of small impedance contrast is given by (Thomsen, 1993):

$$R_{PP}(\theta) = R_{PP-isol}(\theta) + R_{PP-aniso}(\theta)$$

$$R_{PP\text{-iso}}(\theta) \approx \frac{1}{2} \left(\frac{\Delta Z}{\bar{Z}} \right) + \frac{1}{2} \left[\frac{\Delta \alpha}{\bar{\alpha}} - \left(\frac{2\bar{\beta}}{\bar{\alpha}} \right)^2 \frac{\Delta G}{\bar{G}} \right] \sin^2(\theta) \\ + \frac{1}{2} \left(\frac{\Delta \alpha}{\bar{\alpha}} \right) \sin^2(\theta) \tan^2(\theta)$$

$$R_{PP\text{-aniso}}(\theta) \approx \frac{\Delta \delta}{2} \sin^2(\theta) + \frac{\Delta \epsilon}{2} \sin^2(\theta) \tan^2(\theta)$$

where

$$\begin{aligned} \bar{\theta} &= (\theta_2 + \theta_1)/2 & \Delta \epsilon &= \epsilon_2 - \epsilon_1 & \Delta \gamma &= \gamma_2 - \gamma_1 \\ \bar{\rho} &= (\rho_1 + \rho_2)/2 & \Delta \rho &= \rho_2 - \rho_1 & \Delta \delta &= \delta_2 - \delta_1 \\ \bar{\alpha} &= (\alpha_1 + \alpha_2)/2 & \Delta \alpha &= \alpha_2 - \alpha_1 & & \\ \bar{\beta} &= (\beta_1 + \beta_2)/2 & \Delta \beta &= \beta_2 - \beta_1 & & \\ \bar{G} &= (G_1 + G_2)/2 & \Delta G &= G_2 - G_1 & G &= \rho \bar{\beta}^2 \\ \bar{Z} &= (Z_1 + Z_2)/2 & \Delta Z &= Z_2 - Z_1 & Z &= \rho \alpha \end{aligned}$$

In the above and following equations, Δ indicates a difference and an overbar indicates an average of the corresponding quantity. In TIH media, reflectivity will vary with azimuth, ϕ , as well as offset or incident angle θ . Rüger (1995, 1996) and Chen (1995) derived the P-wave reflection coefficient in the symmetry planes for reflections at the boundary of two TIH media sharing the same symmetry axis. At a horizontal interface between two TIH media with horizontal symmetry axis x_1 and vertical axis x_3 , the P-wave reflectivity in the vertical symmetry plane parallel to the x_1 symmetry axis can be written as

$$R_{PP}(\phi=0^\circ, \theta) \approx R_{PP\text{-iso}}(\theta) + \left[\frac{\Delta \delta^{(V)}}{2} + \left(\frac{2\bar{\beta}}{\bar{\alpha}} \right)^2 \Delta \gamma \right] \sin^2(\theta) \\ + \frac{\Delta \epsilon^{(V)}}{2} \sin^2(\theta) \tan^2(\theta)$$

where azimuth ϕ is measured from the x_1 -axis and incident angle θ is defined with respect to x_3 . The isotropic part $R_{PP\text{-iso}}(\theta)$ is the same as before. In the above expression

$$\alpha = \sqrt{\frac{C_{33}}{\rho}} \quad \epsilon^{(V)} = \frac{C_{11} - C_{33}}{2C_{33}}$$

$$\beta = \sqrt{\frac{C_{44}}{\rho}} \quad \delta^{(V)} = \frac{(C_{13} + C_{55})^2 - (C_{33} - C_{55})^2}{2C_{33}(C_{33} - C_{55})}$$

$$\beta^\perp = \sqrt{\frac{C_{55}}{\rho}} \quad \gamma^{(V)} = \frac{C_{66} - C_{44}}{2C_{44}} = -\frac{\gamma}{1+2\gamma}$$

In the vertical symmetry plane perpendicular to the symmetry axis, the P-wave reflectivity is the same as the isotropic solution:

$$R_{PP}(\phi=90^\circ, \theta) = R_{PP-iso}(\theta)$$

In non-symmetry planes, Rüger (1996) derived the P-wave reflectivity $R_{PP}(\phi, \theta)$ using a perturbation technique:

$$R_{PP}(\phi, \theta) \approx R_{PP-iso}(\theta) + \left\{ \left[\frac{\Delta\delta^{(V)}}{2} + \left(\frac{2\beta}{\alpha} \right)^2 \Delta\gamma \right] \cos^2(\phi) \right\} \sin^2(\theta)$$

$$+ \left\{ \left[\frac{\Delta\epsilon^{(V)}}{2} \right] \cos^4(\phi) + \left[\frac{\Delta\delta^{(V)}}{2} \right] \sin^2(\phi) \cos^2(\phi) \right\} \sin^2(\theta) \tan^2(\theta)$$

Similarly, the anisotropic parameters in orthorhombic media are given by Chen (1995) and Tsvankin (in press):

$$\epsilon^{(1)} = \frac{C_{22} - C_{33}}{2C_{33}}$$

$$\epsilon^{(2)} = \frac{C_{11} - C_{33}}{2C_{33}}$$

$$\delta^{(1)} = \frac{(C_{23} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})}$$

$$\delta^{(2)} = \frac{(C_{13} + C_{55})^2 - (C_{33} - C_{55})^2}{2C_{33}(C_{33} - C_{55})}$$

$$\gamma_2 = \frac{C_{44} - C_{55}}{2C_{55}}$$

The parameters $\epsilon^{(1)}$ and $\delta^{(1)}$ are Thomsen's parameters for the equivalent TIV media in the 2-3 plane. Similarly, $\epsilon^{(2)}$ and $\delta^{(2)}$ are Thomsen's parameters for the equivalent TIV media in the 1-3 plane; γ_2 represents the velocity anisotropy between two shear-wave modes traveling along the z-axis. The difference in the approximate P-wave reflection coefficient in the two vertical symmetry planes (with x_3 as the vertical axis) of orthorhombic media is given by (Rüger (1995, 1996))

$$\begin{aligned}
R_{\text{PP}}^{[x_1, x_3]} - R_{\text{PP}}^{[x_2, x_3]} \approx & \left[\frac{\Delta\delta^{(2)} - \Delta\delta^{(1)}}{2} + \left(\frac{2\beta}{\alpha} \right)^2 \Delta\gamma_2 \right] \sin^2(\theta) \\
& + \left[\frac{\Delta\epsilon^{(2)} - \Delta\epsilon^{(1)}}{2} \right] \sin^2(\theta) \tan^2(\theta)
\end{aligned}$$

The equations are good approximations up to 30-40° angle of incidence.

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ATTACHMENT B

**GEOLOGIC MODEL OF FRACTURE OCCURRENCE IN
YATES FIELD**

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INTRODUCTION

Yates field is located in the Central Basin Platform of the Permian Basin in western Texas (fig.1). It was discovered by surface geology in 1926 (Craig, 1988). Oil in place is estimated in 4.2 MMB. After producing more than 1 MMB during 74 years, the field is still delivering 30.000 bopd. The geological model is based on well logs and production data. Seismic information is limited to four 2-D lines, very few for the 91 km² area. In spite of the high density of wells in the field, there still remain some uncertainty regarding the existence of faults and the orientation of conductive fractures.

At surface, Yates field looks like a very simple structure. It is a broad, subtle anticline depicted by the Lower Cretaceous beds. But indeed the geology of Yates is far from being simple. The type of trap, for example, is a combination of both stratigraphic and structural factors that lead to one of the largest oil accumulations in the basin. The Permian rocks at Yates are located at the highest structural position of the basin, at the southeastern edge of the Central Basin Platform (Galloway *et al*, 1983). This platform is a major Permian tectonic feature limited by normal faults. That structure apparently controlled the facies distribution of the main reservoir, the San Andres Formation. Dolomitized limestones were deposited within the platform, while sandstones and shale were deposited in the surrounding areas, Delaware and Midland basins, which were deeper basins adjacent to the platform (figure 1).

In the same way, reservoir heterogeneities, that are controlling porosity and permeability, are relatively complicated. They can be summarized in three main groups: 1) Heterogeneities related to the stratigraphic architecture and the depositional environments, that generate vertical and lateral variations of facies, 2) heterogeneities associated to small-scale faults and fractures, and 3) heterogeneities induced by diagenesis or geochemical alterations.

This is a summary of the geology of Yates field, focused on those aspects relevant for seismic modeling of fracture-induced velocity anisotropy. It is based on some studies and published papers related to Yates field or to the main reservoir, but is also supported by field observations made during a 10 days field trip, with the geoscientists A. Younes, R. Stever, J. Schlueter, M. Bitter and G. Leaf, from Marathon Oil Co. It is focused on the

description of the structural heterogeneities, since it was the original purpose of this work.

Figure 1. Location of Yates field. The main paleogeographic elements of the Permian basin and oil fields producing from the San Andres Fm. are also shown. Craig (1990) in Tinker (1995).

GENERAL GEOLOGIC SETTING

Yates field is located about 100 miles south of Midland Texas, in a region dominated by very flat topography that reflects the smooth continuous layering of the cretaceous beds in the area. At Yates, and nearby regions, normal faults are affecting the flat-lying

cretaceous strata. The age of that tectonic activity can not be precisely defined, it could be as old as early Tertiary (Paleocene).

General stratigraphy

The stratigraphy at Yates is summarized in figure 2. You can divide it in four main sequences separated by three main unconformities: The pre-Permian, Permian-Triassic, and the Triassic-Cretaceous unconformities. The four sequences are pre-Permian, Permian, Triassic and Cretaceous. The top of the Cretaceous can be considered as another unconformity, since recent deposits are deposited directly on lower cretaceous rocks.

I am considering as the pre-Permian sequence, all the Paleozoic and Precambrian rocks underlying the Pennsylvanian-Permian unconformity, which are not relevant for the purpose of this report. The Permian sequence is composed of the Wolfcamp, Leonardian, Guadalupian and Ochoan series. Relevant for Yates are the Guadalupian and Ochoan series, which at this area are composed by sequences of dolomitized limestones, mixed siliciclastic-carbonates rocks and evaporites (anhydrite-gypsum and salt). The Guadalupian series, at Yates, is composed by the San Andres, Grayburg, Queen, Seven Rivers, Yates and Tansill formations (Donogue and Gupton, 1956). The Ochoan series is reduced to a thin section (25-200 feet) of a mixture of shale anhydrite and dolomite.

The San Andres Formation is mainly composed of dolomitic limestone deposited in a shallow marine platform, close to a ramp (Kerans *et al*, 1994; Sonnenfeld, 1991). The location of the ramp was controlled by some normal faults. There is an eustatic unconformity between the Grayburg and the upper San Andres. Above the unconformity a relatively thin package of shale, dolomitic mudstones and sandstones is found, corresponding to the Grayburg and Queen Formations (Donoghue and Gupton, 1956).

Overlying the Queen Formation there is a thick section (about 400 feet) of anhydrite, named as Seven Rivers Formation. On top of the Seven Rivers, there are 55-90 feet of sandstone. The Guadalupian Series ends with a thin section of shale, sandstone and anhydrite, Tansill Formation, that laterally correlates with a thick section of salt, prominent in the surrounding areas of the field, but virtually absent in this part of the platform (Donogue and Gupton, 1956).

The Triassic system is reduced, in this area, to a thin section (0-200 feet) of sandstones and claystones. It is limited by both underlying and overlying unconformities: The Triassic-Permian unconformity (below), and the Cretaceous-Triassic unconformity (above). The big time gap represented by the Cretaceous-Triassic unconformity indicates that this area was above the stratigraphic base level during a long period of time (about 100 m.y).

The Cretaceous (Middle to Upper Albian) form a depositional cycle that starts with sandstones (Trinity Formation) and continues with shale and limestones (Fredericksburg-Washita). The remarkable lateral continuity of the limestone suggests deposition in a shallow-marine uniform platform. These cretaceous rocks are almost undeformed, exception made of some normal faults and very subtle folds, one of them lead to the discovery of Yates field.

General tectonic setting

Figure 3 is a W-E regional cross section across the Central Basin Platform, from the southern extension of the Guadalupe Mountains to the eastern border of Midland Basin. This cross section summarizes the tectonic setting of Yates. Basically, it is located on a pre-Permian horst that controlled the facies distribution of the Permian deposits. Carbonates were deposited in the higher areas, like the Central Basin Platform, whereas shale and sand was deposited in the surrounding, deeper basins. After the Permian, this structure has been affected by tectonic stresses at different geologic periods.

The main unconformities observed in the stratigraphy of Yates are related to major tectonic events that occurred in the past. In order to understand the geology of this area it is better to try to infer the possible tectonic scenarios at different times, when each one of the events that produced such unconformities happened. These possible tectonic settings are described from the Permian to the present, in order to show the possible sequence of events that give rise to the structure that exists today.

The Central Basin Platform, on which is Yates located, is considered to be a major Permian horst or positive structure that controlled the facies distribution during the accumulation of the San Andres Formation and the overlying Permian formations. The

structure is trending NNW, and according to the cross section (Renfo *et al*, 1984), it was formed during the Late Pennsylvanian.

Figure 2. Composite section of the stratigraphy penetrated at Yates. Modified from Donogue and Gupton (1956).

Figure 3. Regional cross section (from W to E) showing the tectonic setting of the Central Basin Platform (CBP). The gray colors correspond to the Permian sequence, the green colors to the Cretaceous sequence. After Renfro *et al* (1984).

The tectonic setting for the time lapse between the Permian and the Cretaceous can not be deduced from the geology observed at Yates. As explained above, the time gap between the Triassic and the Cretaceous suggests that this area was above the base level, and therefore subjected to erosion, or at least no deposition, during a long period of time.

Currently Yates field is located in an inactive tectonic area, and considering the low deformation observed in the Cretaceous rocks, it is inferred that Yates area has not suffered major tectonic deformation since the Upper Cretaceous. It is inferred that this region was part of the foreland basin during the Laramide Orogeny. Probably it has been exposed to erosion since the early Tertiary.

About 200 km west of Midland and Yates is the Rio Grande rift basin, which is part of the Basin and Range extensional province. This is an extensional tectonic environment that has been active during the Cenozoic (Russel and Snelson, 1994). Whether this extensional tectonic regime has or has not affected Yates is difficult to determine, but the field observations indicate a post-cretaceous extensional stress regime. The orientation of drilling-induced tensile fractures in some wells suggests that the maximum compressive stress (or the minimum tensile stress) is oriented in direction WNW (290). Even though the normal faults observed at surface could be as old as early Tertiary, their orientation is in concord with the stress field inferred from the FMI interpretation.

The Central Basin Platform structure does not seem to have had a major impact in the facies distribution of the Cretaceous sediments, at least locally. Nevertheless, the fact that the subtle anticline structure resembles the structure of the Permian rocks at subsurface (Hennen and Metcalf, 1929) indicates that this subsurface structure is still impacting the deformation in the area. Earlier studies (Adams, 1940) suggest that Yates anticline was developed by differential compaction, probably related to the underlying structure.

RESERVOIR HETEROGENEITIES OF SAN ANDRES FORMATION

The geological heterogeneities can be classified into three main groups: stratigraphic or sedimentologic, structural or geomechanical, and geochemical.

Stratigraphic and sedimentologic heterogeneities refer to the changes in rock composition derived from the sequence of deposition of the different layers of sediment. Following the terminology established by Dunham (1962), Kerans *et al* (1994) and Hovorka *et al* (1993) grouped the carbonates of San Andres in four main rock types: mudstones, wackestones, packstones and grainstones. Each one of these types of carbonates has its own petrophysical properties. The distribution of these rock types is controlled by their genesis, and is intimately related to the sequence stratigraphy (Tinker *et al*, 1995). This facies distribution can also be controlling the occurrence of caves, which are associated with high productivity areas (Craig, 1988).

Structural heterogeneities refer to folds, faults, fractures, or changes induced in rock properties derived from mechanical deformation. The possible presence of normal faults at Yates is inferred from the geology observed at surface. Additionally, the consistent similarity between the orientation of fractures at Yates, both at surface and subsurface, and the orientation of fractures at the Guadalupe Mountains, where the San Andres Formation crops out, suggests a consistent pattern of regional fractures. This similarity also allows to estimate the joint density at subsurface.

Geochemically induced heterogeneities are those generated by chemical process, usually known as diagenetic processes. Hovorka *et al* (1993) analyzes the diagenetic features observed in the Permian rocks at the Guadalupe Mountains. They have found that in the San Andres Formation the dissolution of carbonate grains and resulting

precipitation of inter-granular cement produced a well-defined zone of high moldic porosity but relatively low permeability. The origin of this zone is related to exposure to meteoric waters during the early stages of cementation. By this type of diagenetic process, it is possible to generate a completely irregular zone of high or low porosity that may modify the seismic response of rocks.

Stratigraphic heterogeneities

Kerans *et al* (1994) and Sonnenfeld (1991) studied the sequence stratigraphy of the San Andres Formation in the areas where its depositional setting was quite similar to Yates. Tickler *et al* (1995) present a summary of the stratigraphy of the upper San Andres at Yates. Basically it can be divided in three genetic cycles within a major stratigraphic sequence identified as US4. Dolomitic wackestone and carbonate mudstone to the west, and dolomitic fusulinid packstone and grainstone to the east compose each cycle (fig. 4). The development of caves to the top of every cycle is interpreted to be the consequence of syndepositional subaerial exposure of the packstone to grainstone facies. (Ticker *et al*, 1995; Craig, 1988).

Since caves are having a major impact in rock porosity and density, it can be expected that their presence will generate a conspicuous anomaly in the seismic attributes. Figure 4 illustrates the spatial distribution of caves and its relationship to the area of the Stanford Seismic Project.

At the western sector of Yates the upper San Andres can be divided in three major stratigraphic units based on the log response. Higher velocities are associated to low-porosity dolomitic mudstone and wackestone, whereas lower velocities are associated to higher-porosity packstone. The distribution of the velocities for P and S waves for each one of these units shows a wide range of scattering, which could be related to the different distribution of pore shapes and sizes within the rocks. Even for rocks with the same porosity, the elastic behavior may be very different. For example, vuggy porosity may generate a different seismic response than inter-crystalline porosity. The dissolution of fossil molds can generate a patchy distribution of large pores, whereas a dolomite with sucrosic texture may have a very uniform distribution of pores.

Figure 4. Map of Yates (above), and stratigraphic cross section of San Andres Formation at Yates. The map shows the location of the pilot seismic project, with the western boundaries of the occurrence of caves. Taken from Tinker *et al* (1995).

Structural heterogeneities

From observations made in the field, it can be deduced that faults and fractures are the main structural heterogeneities at Yates. It can be argued that there is not direct evidence that the observed faults at surface can be extended to the reservoir, in fact previous studies interpreted these normal faults as a consequence of salt mobilization (Adams, 1940). Nevertheless, there is a consistent pattern between the orientation of faults and fractures observed at different localities (tables 1 and 2) and the orientation of fractures within the San Andres Formation at Yates (fig. 12). There is also a systematic pattern of normal faults that is coincident with the orientation of major lineaments, at surface, productivity trends at subsurface (Tinker and Mruk, 1995), and the current orientation of the stress in the field.

Faults and Fractures at Yates

Two main sets of normal faults were observed within and around Yates field: WNW (290°) and NE (040°). The strike of the younger set is WNW (290°). These faults are 2 to 5 km long, and spacing between them varies from 0.2 to 1.0 km (figure 14). Observed vertical slip ranges from 2 to 25 meters. In the only place where a fault zone was observed, it is composed of very well cemented gouge and breccia (fig.12). The surrounding damage zone and the tips of the faults are areas of high-density of fractures.

The azimuth of the older set of fractures is 040° (N40°E). These faults have less continuity since are cut by the younger ones. The amount of slip seems to be lower than the WNW faults. This set is more systematic and continuous further east of Yates, where the other WNW set is not well developed.

Four main sets of joints were found at outcrops in Yates (table 1), the orientations of these sets are 290° (WNW), 040° (NE), 340° (NNW) and 70° (ENE). These are the same dominant orientations of fractures reported from the FMI logs (fig. 10). The abutting relationship shows that the WNW set is younger than the NE set. Age relationships were not sorted out for the other sets. Both WNW and NE sets are consistent and present not only at Yates but also further west, at Guadalupe Mountains. The ENE and NNW sets are just locally dominant.

Fracture systems in the San Andres Formation at surface

Outcrops of the San Andres Formation were observed at three different localities in the Guadalupe Mountains, west of Carlsbad (New Mexico). The area is located west of the Permian Basin, and to the east of the Rio Grande graben, Large NW-trending normal faults, parallel to the graben trend, are found to the east of the Guadalupe Mountains.

Four localities were visited: The Last Chance Canyon, the Algerita escarpment at Lawyer Canyon, exposures along the highway 82, and exposures along the highway 380 near Picacho. The Algerita escarpment is just west of a zone of large normal faults, whereas

Table 1. Rose diagrams of fracture orientations at different localities within Yates oil field, in the Cretaceous Limestone

the outcrops of Last Chance and Wilson canyons are about 10 miles away from that fault zone. The exposures along the highway 82 and 380 are far from this normal fault zone, in an area where the beds are flat to slightly folded. In that region, major NE-trending lineaments have been identified (Russell *et al*, 1982). These lineaments seem to be related to large dextral strike-slip faults.

At Last Chance and Wilson canyons, the western border of the Guadalupe Mountains is part of the Huapache monocline, where the beds are gently dipping to the east due to the presence of a fault (the Huapache fault) in the subsurface. The structural position and the facies of the San Andres Formation in these area somewhat resemble that of the reservoir in the Yates field.

At each locality, but Last Chance Canyon, scanlines along beds of different lithologies were done in order to evaluate fracture density. Fracture orientation, spacing and height were recorded along every scanline. Bed thickness and lithology was recorded as well. A fracture set is defined as a group of fractures with approximately the same orientation. It is assumed that all the fractures of a given set are genetically related, even though this is not necessarily true since the presence of subordinate fractures like cross-joints or splay cracks may be misleading. The spacing for every fracture set has been calculated from the scanline data using simple trigonometric relations.

Three main fracture sets were found in the area: N 40° E, N 40° W and N 70° W (Table 2). The age relationships of these sets are similar to those observed at Yates; the NE set is older than the NW sets (figure 5). The N 40° W set is younger than the N 70° W, which is not well developed in this area, in comparison to Yates. A distinction between the two NW sets is complicated by the fact that the orientation of a given set may vary slightly due to differences in lithology. The N 40° E set has a consistent dextral strike-slip shear component, that is also observed in some large scale features like two lineaments identified in the area (Russell, 1982). Locally there are other sets of fractures and small faults, like those associated to a second order thrust faults that were found close to a larger feature with incipient strike-slip displacement.

Lack of extensive exposures did not allow the study of aspect ratios. Additionally, apertures near large escarpments are not reliable because of the local tensile stresses generated by these free faces. Nevertheless, there is a conspicuous difference in aperture between the NE and the NW sets. Joints of the NW sets have apertures of about 1 cm and are filled with calcite, while apertures of the N 40 E set are hairline to 1 mm. When calcite cement was found in the N 40 E set, it was confined within two N 40 W fractures, suggesting that the cement is filling a pre-existing feature (figure 5). Fracture density of the N 40° W set is also higher than the N 40°E set, as will be shown later.

Figure 5. Filling and age relationship of the main two joint sets. Calcite is filling the NW set joints, parallel to the red pen. Joints of the NE set are closed and just locally filled with calcite at the intersections with the NW set. It indicates that the NE set is older.

Table 2. Rose diagrams of fracture orientations in the Permian outcrops

PHYSICAL PARAMETERS FOR FRACTURE-INDUCED ANISOTROPY

In order to incorporate field observations to rock physics approach, and specifically to evaluate the applicability of Hudson's model or other methods to estimate the fracture-induced anisotropy, it is necessary to evaluate the relationship between the physical quantities measured in the field and the physical parameters needed for that models. Since we are dealing with layered media and theoretical studies suggest that there may be a relationship between spacing and bed thickness (i.e. Bai and Pollard, 2000), in the following discussion it is assumed that this relationship exists. Some assumptions have to be made. First, the following discussion is limited to joints within a bed of specific thickness. Even though in reality some of the joints are larger than the bed thickness, we are evaluating the density within a given bed, and therefore the maximum height is the bed thickness. Second, we are assuming that these joints are ellipsoidal.

Crack density for joint saturated layered media

Opening-mode fractures (joints and veins) in layered sedimentary rocks are often confined by the layer boundaries with their height equal to the layer thickness (Helgeson and Aydin, 1991; Gross and Engelder, 1995). Fractures of the same set form under the same tectonic stress field and are parallel or sub parallel to each other (Pollard and Aydin, 1988). Although there are some exceptions, field observations reveal that joint spacing in layered rocks is roughly proportional to the bed thickness. Bai and Pollard (1999) separate four different ranges of fracture spacing to layer thickness ratio. Basically this ratio varies from 0.1 to 10:

Range I: $S/T > 1.2$

Range II: $0.8 < S/T < 1.2$

Range III: $0.3 < S/T < 0.8$

Range IV: $S/T < 0.3$.

According to Bai and Pollard (1999) there is a critical spacing at which the jointing process should stop because there is a change in the sign of the stress field. This critical spacing is explained as the minimum spacing, and the beds have reached their maximum saturation level. Spacing in range I are related to beds that have not reached the fracture saturation level. Spacing in range II corresponds to the critical spacing, where the beds are near the saturation level. For the spacing in Ranges III and IV, there are different explanations, one could be that fractures densities higher than the proposed by the critical spacing are indeed related to shearing and stress concentration at one of these

discontinuities. High densities of fractures related to sheared joints have been reported as well (Jamison, 1997; Florez, 2000).

This relationship between spacing and layer thickness allows us to establish a link between the crack density parameter (ϵ) and the ratio S/T . The crack density parameter is given by:

$$\epsilon = \frac{N}{V} a^3 ; \quad (1)$$

where N is the number of fractures, V is the volume and a is the half-fracture height. It is also related to fracture-induced porosity (ϕ) and the aspect ratio (α) by the expression:

$$\epsilon \approx \frac{3\phi}{4\pi\alpha} \quad (2)$$

considering a bed of thickness T , it can be deduced that for a unit of length T the number of fractures per volume will be $N = T/S$. Since the fracture height is constrained by the bed thickness, it follows that $a^3 = (\frac{1}{2}T)^3$. Assuming a ‘penny shape’ of half-height a , and that the piece of rock we are studying is T^3 , the crack density parameter may be written as:

$$\epsilon = \frac{T/S}{8} \quad (3)$$

This expression indicates that the crack density parameter is basically controlled by the ratio of fracture spacing to bed thickness, or fracture index (Narr, 1991). Nevertheless, it is intrinsically assuming that we are dealing with a well-developed set of joints, since we are considering that the mean fracture height equals the bed thickness. That is not necessarily true for an immature fracture set. In that case, it can be inferred that fracture heights will be lower than bed thickness along a randomly selected scanline. This situation is analyzed later.

Aspect ratio

The aspect ratio is known in the geological literature as the relationship between the fracture length and the displacement, which for the case of veins and joints is the relationship between aperture and length. From linear elastic fracture mechanics theory, it is predicted that there is a linear relationship between displacement and fracture length, and it has been studied for different authors. Vermilye and Scholz (1993) report aspect ratios varying from 0.001 to 0.01 according to the lithology. They also show that for different sets of data, aspect ratio varies from 1×10^{-2} to 1×10^{-4} . Jamison (1997) reports

aspect ratios varying from 1×10^{-4} to 1×10^{-1} , but with an exponential distribution that show a range of aspect ratios from 1×10^{-3} to 1×10^{-2} . He also reports a linear relationship between fracture aperture and volumetric strain, which is equivalent to the fracture-induced porosity.

The aspect ratio (α) for the equation (2), refers to the relationship between fracture aperture and fracture height. The ratio height/length is 1 for penny shape cracks, but Petit *et al* (1994) report height/length ratios from 0.1 to 0.5 for natural fractures. They consider the ratio of 0.5 as the more appropriate for a single fracture. Although is important to bear in mind, this observation does not change very much the range of variation of aspect ratio already chosen.

Another factor influencing the possible range of aspect ratios is the stress field. Mavko *et al* (1998) indicate that the closing stress is given by the expression:

$$\sigma_{close} = \frac{\pi}{2(1-\nu)} \alpha \mu_o \quad (4)$$

where α is the aspect ratio, ν is the poisson's ratio and μ_o is the shear modulus of the material. Considering that a reasonable value of μ is in the order of 1×10^4 MPa, it can be deduced that the closing stress for an aspect ratio of 1×10^{-4} is about 2 MPa. A normal gradient of overburden stress is 23 MPa/ km, therefore a fracture with this aspect ratio will be closed even at shallow depths, unless tensional stresses are present.

Considering the different ranges of S/T , it is possible to establish some boundaries for the parameters used in Hudson's Model. The relationships between spacing, aspect ratio and porosity may also allow us to estimate the range of values that can be used for modeling the fracture-induced anisotropy. The tables 3 and 4 show some of the ranges inferred for crack density and fracture-induced porosity.

Table 3. Estimated fracture porosity for $\alpha = 0.01$

Range of S/T	Crack density (ϵ)	Aspect ratio (α)	Fracture porosity (ϕ)
0.01-0.3	12.5-0.375	0.01	0.5-0.016
0.31-0.8	0.375-0.156	0.01	0.016-0.006
0.8-1.25	0.156-0.1	0.01	0.006-0.001
1.25-10	0.1-0.0125	0.01	0.001-0.00012

Table 4. Estimated fracture porosity for $\alpha = 0.001$

Range of S/T	Crack density (ϵ)	Aspect ratio (α)	Fracture porosity (ϕ)
0.01-0.3	12.5-0.375	0.001	0.05-0.016
0.31-0.8	0.375-0.156	0.001	0.016-0.006
0.8-1.25	0.156-0.1	0.001	0.006-0.001
1.25-10	0.1-0.0125	0.001	0.001-0.00012

Fracture-induced porosity and joint density

Fracture porosity (Φ_f) is defined as the ratio of volume filled with fractures (V_f) to the total volume (V) of rock (Jamison, 1997; Narr, 1991):

$$\Phi_f = \frac{V_f}{V} \quad (5)$$

Since we are assuming ellipsoidal fractures, the total volume occupied by the fractures is:

$$V_f = \frac{4}{3} \pi \frac{a}{2} \frac{h}{2} \frac{l}{2} N = \frac{\pi}{6} \frac{T}{S} a h l \quad (6)$$

where a is the mean fracture aperture, h is the mean fracture height and l is the mean fracture length. As explained by Wu and Pollard (1995) and Younes (1996), the length and the height to length ratio of a given fracture set are related to the degree of development of that set. Therefore, it can be assumed that the length to height ratio can be taken as a constant k indicative of the degree of maturity of a given set. Assuming $k = 1$, and combining this with equation (6) and the definition of aspect ratio (α), the equation (5) can be written as:

$$\Phi_f = \frac{\pi}{6} \alpha \frac{h^2}{T^2} \frac{h}{S} \quad (7)$$

This expression shows that fracture porosity is a function of the mean aperture, the mean spacing, and the mean-height to thickness ratio. When the height equals T , then fracture porosity will be a function of just the aperture and the fracture index (T/S). In any case, the fracture porosity will change according to the aspect ratio (α) and the height to length ratio (k). Taking this expression (7) into equation (2), it is possible to derive an expression to calculate the crack density parameter (ϵ) from the measured data:

$$\epsilon = \frac{\pi \alpha h^3}{8 \pi \alpha T^2 S} = \frac{1}{8} \frac{h^2}{T^2} \frac{h}{S} \quad (8)$$

The calculated crack density parameters for the different scanlines measured are shown in Table 6 and figure 7. Since expression (7) shows that the height to thickness ratio must be included to evaluate fracture density, it is proposed to consider another

parameter, which can be called the specific joint density. This parameter is derived from (7), assuming α and k equal to 1:

$$SJD = \frac{\pi}{6} \frac{h^2}{T^2} \frac{h}{S} \quad (9)$$

Fracture density from outcrop data

In geological and engineering literature different techniques and terms are used to describe the distribution of fracture intensity within a rock body. Measurements of fracture intensity are made either in one or two dimensions (Dershowitz and Herda, 1992), and they cannot be directly extrapolated as three-dimensional quantities. In some instances geologist refers to deformation intensity in a very qualitatively way (mildly, moderately, highly) but no attempts have been made to specify the fracture density associated with these terms. In sedimentary rocks, the distribution of fracture intensity is frequently described in terms of the fracture-spacing index (Narr, 1991). Fracture index is defined as the ratio of jointed-layer thickness (T) to median joint spacing (S). The relationship between fracture spacing and thickness has been studied by Wu and Pollard (1995) and Bai and Pollard (2000).

Spacing can be calculated either from the scanline method or the area method (Wu and Pollard, 1995; Younes, 1996). According to them, the results from the scanline method will under-estimate the fracture density unless the fracture set is well developed, but area method is limited to pavement exposures.

The measured variables used to calculate the fracture density are shown in table 5. Fracture height has been limited to the bed thickness, therefore for those fractures higher than the thickness of stratigraphic interval under study, a height equal to the bed thickness has been assigned. The resulting physical parameters calculated are shown in table 6. The fracture index is included in order to show the difference in respect to the volumetric joint density calculated using expression (7). The estimated fracture porosity (Φ_f) has been determined using both the volumetric joint density ($\Phi_f 1$) and the fracture index ($\Phi_f 2$), for constant aspect ratio ($\alpha=0.001$) and length/height ratio ($k=5$). Depending on the height/thickness ratio, these values can either be similar or may differ in one order of magnitude.

The combination of height/spacing ratio and the fracture index (figure 6) illustrates the variation in fracture density. It can be seen that the N 40°W set has got a higher density of fractures than the N 40°E and the N 70°W, regardless of the structural position.

Therefore the set (or orientation) seems to be the first controlling parameter on fracture density.

Figure 6. Height to spacing ratio vs. Fracture index. The colors stand for the different fracture sets: blue = NW, red = NE and yellow = W-NW. The shapes stand for different lithologies.

Separating the crack density parameter or the specific joint density by rock type, and taking into account the locality and orientation (figures 7 and 8), it can be shown that both structural position and lithology are the other two factors impacting fracture density. The same plot for the fracture index does not show any apparent trend (figure 9). Since the obtained results from figures 7 and 8 are consistent with the field observations, it is considered that both the crack density parameter and the specific joint density are more appropriate to quantify fracture density than the fracture index.

Figure 7. Crack density parameter split by rock type, orientation and locality. Colors indicate locality: yellow = Algerita, red and blue = Alamogordo. Shapes indicate fracture sets: triangles = NNW (340°), circles= NE (040°) and squares = WNW (290°).

Figure 8. Specific joint density split by facies. Again colors stand for locality and shapes for different fracture sets.

Figure 9. Fracture index (T/S) split by facies. Colors stand for location and shapes for different fracture sets. Note that in some cases the fracture index does not properly reflect the fracture density.

Table 5. Summary of data collected on San Andres Formation*

Locality & Set	Scanline Orientation (deg)	Rock Type	Mean Spacing (m)	Mean Height (m)	Bed thickness (m)
Algerita Lawyer Cn.					
NE (60)	30	MDST	0.36	0.29	0.74
WNW (289)	30	MDST	1.63	0.74	0.74
NW (324)	30	MDST	0.30	0.68	0.74
NE (59)	100	MDST	0.40	0.58	0.74
NW (321)	100	MDST	0.23	0.65	0.74
WNW (280)	26	WCKST	1.03	0.90	1.30
NW (324)	26	WCKST	0.55	0.98	1.30
Highway 82 Outcrops					
NE (40)	127	WCKST	1.86	1.03	2.36
NW (330)	127	WCKST	1.13	1.53	2.36
NE (42)	116	WCKST	0.93	1.06	1.86
NW (330)	116	WCKST	0.64	1.07	1.86
NE (43)	116	PCKST	2.12	1.43	3.49
NE (326)	116	PCKST	0.80	1.92	3.49

* Does not include the smaller fractures counted as “small fractures in between”, the threshold was ~0.2 m

Table 6. Ratios and parameters derived from outcrop data

Locality & Set	S/T	H/T	Rock Type	Crack density param. (ϵ)	Joint density	$\Phi f(1)$ $\alpha=0.001$ $l=5h$	Fracture Index	$\Phi f(2)$ $\alpha=0.001$ $l=5h$
Algerita Lawyer Cn.								
NE (60)	0.49	0.40	MDST	0.02	0.068	0.002	1.069	0.027
WNW (289)	1.00	1.00	MDST	0.06	0.239	0.006	0.236	0.006
NW (324)	0.41	0.92	MDST	0.24	1.011	0.025	1.286	0.032
NE (59)	0.54	0.79	MDST	0.11	0.474	0.012	0.976	0.024
NW (321)	0.31	0.89	MDST	0.28	1.184	0.030	1.708	0.043
WNW (280)	0.79	0.69	WCKS	0.05	0.222	0.006	0.662	0.017
NW (324)	0.42	0.76	WCKS	0.13	0.538	0.013	1.242	0.031
Highway 82 Outcrops								
NE (40)	0.79	0.44	WCKS	0.01	0.056	0.001	0.664	0.017
NW (330)	0.48	0.65	WCKS	0.07	0.298	0.007	1.097	0.027
NE (42)	0.50	0.57	WCKS	0.05	0.192	0.005	1.049	0.026
NW (329)	0.35	0.57	WCKS	0.07	0.288	0.007	1.516	0.038
NE (43)	0.61	0.41	PCKST	0.01	0.059	0.001	0.860	0.022
NW (326)	0.23	0.55	PCKST	0.09	0.378	0.009	2.283	0.057

CONCEPTUAL MODEL OF FAULTS AND FRACTURES AT THE STANFORD SEISMIC PROJECT AREA

Based on the data available and the field observations, a preliminary conceptual model is proposed for the area of the Stanford Seismic Project at Yates field. First of all, field observations show that Yates area was under a tensile stress regime at least during the Tertiary. Even though it could still be under such stress regime. That tensile-stress regime have produced a series of normal faults such those illustrated in figures 13 and 14. Although the normal faults observed at surface do not necessarily extend to the Permian rocks at subsurface, enough evidence has been seen to postulate that similar faults can be present in the reservoir. These faults may strongly control the movement of fluids within the reservoir.

The observed faults are relatively small. Fault length varies from 200 m to about 5 km and slip ranges from 2 to 25 meters. This kind of faults can hardly be identified even with a dense network of wells like that of Yates. They cannot even be observed at subsurface without good seismic resolution or using some seismic attribute analysis. A comparison of the data obtained from the Algerita escarpment and the exposures along the highway 82 shows that fracture density is higher nearby fault zones (figure 12). Fracture clustering and high-productivity trends identified at Yates (Tinker and Mruk, 1995) may be related to the fractured damage zone near to one of these small faults.

The orientation of effective and solution enhanced fractures in the area of the Stanford Seismic Project is presented in figure 11. Regardless of the scattering, in some wells is clear that the main fracture sets are the same observed at surface. It can also be seen that tensile fractures are oriented in direction WNW (290°), which seems to be the direction of the maximum compressive (or minimum tensile) stress. Orientation of breakouts at well YU-2935, suggest a rotation of almost 90° in the stress field, which can be related to slip in a NE-trending fault. The orientation of effective fractures in that well is significantly different that in the other wells, suggesting rotation of the stress field, which is also considered as an evidence of the presence of faults in the reservoir.

From outcrop data we have found that the NW set presents higher density of fractures in a specific area. Nevertheless, fracture density may vary laterally or vertically depending on different variables. Indeed, three main factors have been found to control fracture density:

- 1) The orientation itself, which means that in a given locality the density is different for each fracture set;
- 2) The structural position, since the fracture density increase towards the faults, and

- 3) The lithology, fracture density seems to be higher in dolomitized mudstones than in the packstones.

Figure 13 corresponds to a radar image of an area just north of Yates field. It illustrates the main structural heterogeneities that could be found at the area of the Stanford Seismic Project. The faults shown have throws of about 25 m, and extend from 1 to 5 km. Spacing between the faults varies from 200 to 500 meters. Following the observations made at the outcropping fault zone (figure 12), it is probable that the core zones of those faults are filled with cement and therefore may constitute barriers to the fluid flow. Similarly, it is inferred that the adjacent damage zones of those faults may correspond to sectors with high density of fractures and therefore areas of high productivity. The orientation of the maximum horizontal stress (WNW) may also help to increase the fracture permeability along the WNW-trending set. The result of this would be zones of high permeability separated by a barrier; the connectivity between these high-permeability zones will depend on the continuity of the fault zone.

The NE-trending is not only less well developed and less continuous, but also is oriented parallel to the minimum compressive (or maximum tensile) stress. As a consequence, it is probable that this set present lower permeability. The orientation of this set in respect to the stress field and the general lower density observed at outcrops lead to the conclusion that this set may have a lower impact on the induced velocity anisotropy of the seismic waves.

Seismic information may help not only to identify rock anisotropy and infer fracture orientation and density, but also to determine the presence of small scale faults such those observed at surface. Even if the seismic resolution would not be enough to see the displacement of these faults, seismic attributes or coherence analysis may help to identify those fault zones.

The reservoir model may also be improved by a more detailed analysis of the relationship between fracture density and the other factors controlling its distribution like faulting and lithology. A more detailed study of fault architecture may also be necessary, since the permeability along and across a fault may vary according to the slip distribution and fault segmentation.

Figure 10. Rose diagrams of the orientation of effective (blue) and solution enhanced (red) fractures, interpreted from FMI logs. Tensile fractures (magenta) and breakouts (brown) are shown in rose diagrams adjacent to the respective well. Data provided by Marathon.

Figure 11. San Andres Formation. Joints and sheared joints at the Algerita Escarpment (a and b) and Highway 82 (c and d) outcrops. Notice the higher fracture intensity in the Algerita exposures, which are located just west of a major normal fault.

Figure 12. Normal fault: corresponds to one of the lineaments observed in the radar image (fig.13). a) The main fault, throw 58 ft, and a subsidiary fault (red lines). b) Fault zone and damage zone of the subsidiary fault. c) Detail view of the fault zone.

Figure 13. Conceptual model: this structural map of the area located just north of Yates field illustrates the proposed model. Red lines correspond to normal faults; the large ones are WNW trending, similar to that described in figure 12. The thinner lines are smaller, NE-trending faults. Spacing between the faults range from 200 to 500 m.

CONCLUSIONS

A general overview of the geology of Yates field has been presented that, in junction with the field observations made both at Yates area and at the Guadalupe Mountains allows to provide a better insight of the reservoir heterogeneities.

There are three main types of rock heterogeneities: stratigraphic, structural and geochemical. Stratigraphic heterogeneities of the San Andres Formation have been well studied by Kerans (1994) and Sonnenfeld (1991) and Ticker et al (1995). At Yates they are related to three main stratigraphic cycles, with strong lateral facies variation within each cycle. Cave-prone facies, which correspond to packstones and grainstones, are concentrated to the eastern side of Yates.

Structural heterogeneities have not been well studied at Yates. Field observations indicate that Yates was affected by a post-cretaceous tensile stress regime that generate a series of systematic, relative small, normal faults. The presence, at subsurface, of this kind of faults is difficult to identify with just well data, given the complex stratigraphy. It is proposed that these faults may be impacting the fluid flow in the reservoir. The dominant orientations of faults and fractures at Yates are WNW (290°) and NE (040°).

Scanlines measures on San Andres outcrops at different localities around the Guadalupe Mountains provide a quantitative estimation of fracture density within this unit. Three main factors have been found to control fracture density: orientation, structural position and lithology.

Three main sets of fractures were found in the Guadalupe Mountains area: NW (320°), WNW (290°) and NE (040°). Enough data to compare fracture density between different localities was collected for the NW and NE sets. This data shows that fracture density of the NW set is higher than the NE set. A plot of height/spacing vs. fracture index (T/S) helped to illustrate this variation.

A link between fracture index (T/S) and the crack density parameter (ϵ) have been defined. Since implicit within the fracture index is the assumption that we are dealing with well-developed fracture sets, a new parameter, the specific joint density, has been defined in order to estimate the fracture density and the crack density parameter. This new parameter involves the height to thickness ratio (h/T), which in the fracture index is assumed to be 1.

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ATTACHMENT C

**ROCK PHYSICS AND FRACTURE MODELING OF SAN
ANDRES RESEVOIR**

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WELL LOG DATA

In this section we present the well log data that we use in our rock physics analysis of the San Andres reservoir of the Yates Field.

Figure 1 presents the well log data from well 1711, which is one of the key wells.

Figure 1: WELL 1711.

The first column gives the depth in feet and also the formation names. The main formations encountered by the wells are:

-“**Yates SS**”, a mainly sandstone formation.

-“**7 Rivers J**” is the Seven Rivers formation, an evaporate-dominated formation that serves as a principal fluid seal to the underlying reservoir sequence.

-“**Queen**” formation, whose thickness varies from 20 to 50 feet in the area, is a mixture of sandstones, silstones and carbonates.

-“**Grayburg**” formation, characterized by interbedded silstone and dolomite, with thickness varying from 10 to almost 100 feet.

-“**San Andres**”, the main producing formation. Its lithology is represented by a dolomitized limestone, with interbedded packstones, grainstones, wackstones, and mudstones.

Figure 1a: WELL 0504.

Figure 1b: WELL 15C1.

For all the figures from 1a to 1d, the second column represents the Gamma Ray log, the third column the P waves (in blue) and the S waves (in red) velocities (in km/s). The blue curve is the compressional velocity (V_p), while the red curve is the shear velocity (V_s). The 4th column is the bulk density (in g/cm³), while the 5th column presents the porosity. Finally, the last column presents the fluid saturations: brine (blue), oil (red) and gas (green).

Some well logs do not exhibit full coverage for all the formations, and some are completely missing. Figures 1a – 1d and Figure 2 show all the wells available for the analysis. The shear wave data is available just for wells 0504, 1711, and 1755. Well 1755 does not have porosity and density data (Figure 2). For this case we derived the density and porosity information using the data from Well 1711. Since both Gamma Ray and P wave velocity are available for Well 1711 as well as for Well 1755, we determined the following linear relation between both density and porosity and these two logs (Gamma Ray and V_p):

$$\rho = 2049.8 + 0.085V_p + 251.9v_{sh}$$

$$V_p[m/s]$$

$$\phi = 0.4192 - 0.44289V_p - 0.144v_{sh}$$

$$V_p[km/s]$$

$$v_{sh} = \frac{GR - GR_{min}}{GR_{max} - GR_{min}}$$

ρ is density (rhob on logs)

ϕ is porosity (phi on logs)

V_p is the P wave velocity

GR is the gamma ray (in API)

v_{sh} is the volume of shale

Figure 1c: WELL 16B8.

Figure 1d: WELL 2509.

Figure 2: WELL 1755 -missing density (rhob) and porosity (phi).

Figure 3: WELL 1755 -derived density (rhob) and porosity (phi).

Figure 3 shows the Well 1755 logs with the derived density (rhob) and porosity (phi) from the relations presented above.

WELL LOG DATA ANALYSIS

In this section we present the results of the rock physics analysis of the well log data for the reservoir rock, the San Andres formation.

Velocity – Porosity relation

One of the most common crossplots in rock physics analysis is *velocity – porosity* crossplot because the velocity-porosity trends can give insights in the pore geometry, as well as cementation process of the rocks.

Figure 4 shows the V_p – *porosity* scatter plot for Well 1711. The velocities correspond to the brine saturated state. On the same figure we superimposed data from other carbonates, whose pore types are known. We also superimposed theoretical predictions for different pore types

calculated using the self-consistent effective medium approach (Berryman, 1980). Different curves correspond to different pore shapes: from crack like pores (with small aspect ratio) to spherical pores (with aspect ratio equal to 1).

Figure 4: WELL 1711- Vp- Porosity scatter plot.

In any rock, rounded, nearly spherical pores are elastically very rigid and lead to high velocities, while flattened, crack-like pores are elastically deformable and lead to low velocities (Walsh, 1965; Nur and Simmons, 1969; Nur, 1971; Wang, 1997). In carbonates, pore shapes are highly variable, inherited in part, from carbonaceous shells and skeletons at deposition, and then modified by mechanical compaction, cementation, and dissolution. Wang (1997) summarizes six distinct pore types that most commonly impact our seismic work in carbonates:

- **Intercrystalline and Interparticle porosity** refers to the pore spaces between particles, grains and crystals. This type of porosity tends to be angular and irregular in shape, with a large pore surface area to volume ratio. This type of pore is elastically deformable. The

compliant pore shape tends to make seismic V_p and V_s relatively low at a given porosity, and it tends to make velocities sensitive to effective pressure.

- **Moldic and Intraparticle porosity** tends to be rounded, sometimes tube-like. Intraparticle porosity is often inherited from chambers or cavities in deposited shells and skeletons. Moldic porosity can result from borings or dissolution of particles after deposition. In both cases the pores are fairly rigid, leading to high seismic velocities and a small sensitivity of velocities to effective pressure.
- **Vuggy porosity** also tends to be rounded and equant, sometimes resembling spheres. Vuggy porosity results mainly from dissolution after deposition. Vugs are rigid, leading to some of the highest velocities at a given porosity and the least sensitivity of velocity to effective pressure.
- **Channel porosity** tends to be elongated and irregular in shape, often resulting from enlargement of fractures or other pore shapes. Channel porosity, particularly that resembling fractures, tends to be deformable, giving low velocities.
- **Fenestral porosity** generally refers to irregular openings larger than grain-supported interstices. They result from shrinkage of sediments, gas pockets, and decay of algae. Because their shapes tend to be highly irregular, they result in low velocities.
- **Fracture porosity** can be of many scales. Microfractures are grain-size, while macrofractures can be several kilometers long. Fractures tend to be highly deformable and cause seismic velocities to be low and very stress-dependent. While microfractures might be randomly oriented, macro-fractures are often aligned and cause velocity anisotropy.

The porosity –velocity crossplot for the Well 1711 log data shows a significant scatter that span a large range of theoretical pore shapes. We can also notice that the well 1711 data plot more or less on the top of the superimposed data from the other carbonates whose pore shapes are known. However, we do not encounter porosity values greater than 35-40% as the other carbonates exhibit.

In summary, we can conclude that there is a significant pore shape variability in the San Andres formation. Some of the data correspond to densely cemented carbonates, other to carbonates with interparticle or intraframe porosity, or with vuggy porosity, while other may correspond to carbonates with micro and moldic porosity, and some even with crack-like porosity. However, the significant scatter showed by well 1711 data in the V_p - porosity domain can be partially due to different degrees of dolomitization and calcite cementation, and not only to pore shape variability.

P-wave velocity – S-wave velocity relation

Another very important relation is the one between P wave velocity and S wave velocity, since this is the key to lithology and also to fluid discrimination from seismic or sonic log data. This relation is also very important for predicting V_s in cases when we have only P wave information, while the S wave velocity is missing. There is a relatively wide variety of published V_p - V_s relations and V_s prediction techniques, although one of the most common and successful empirical relations are the Greenberg - Castagna relations.

For the San Andres reservoir, shear wave data is available in wells 0504, 1711, 1755 in our study area, therefore we can directly use these measurements for the rock physics analysis. However, we are interested to see how the different published V_p - V_s relations fit our well log data in order to gain insights about the lithology of the San Andres formation and to calibrate the most appropriate V_p - V_s relations for our reservoir. Establishing a robust V_p - V_s relation is very important for predicting V_s in other wells where we do not have shear wave information.

Figure 5 and Figure 6a show the V_p – V_s crossplot for Well 0504 and Well 1711 respectively, corresponding to the brine saturated state. Since the geological information suggests that the rocks are mainly dolomitic, we superimposed, for comparison:

Pickett's (1963) **dolomite** relation (laboratory)

$$V_s = V_p / 1.8 \text{ [km/s]}$$

and a least squares linear fit for **dolomites** (Castagna et al., 1993)

$$V_s = .5832 V_p - .07776$$

Figure 5: WELL 0504 $V_p - V_s$ scatter plot with the superimposed empirical relations for dolomite - Castagna et al. (1993) and Pickett (1963).

Figure 6a: WELL 1711 $V_p - V_s$ scatter plot with the superimposed empirical relations for dolomite - Castagna et al. (1993) and Pickett (1963).

We can see that for this data set neither Greenberg & Castagna, nor Pickett's empirical relations for dolomites fit the data. For both Well 0504 and Well 1711 the data are shifted above the predicted lines, in the increasing direction of V_p .

In contrast, Figure 6b shows again the V_p - V_s relation for Well 1711, where we superimposed Pickett and Castagna's relations for **limestones**:

Pickett (1963)

$$V_S = V_P / 1.9$$

(km/s)

and the least squares polynomial fit derived by Castagna et al. (1993) for **limestones**

$$V_S = -.05508 V_P^2 + 1.0168 V_P - 1.0305$$

(km/s)

Figure 6b: WELL 1711 V_p - V_s scatter plot with superimposed empirical relations for limestone - Castagna et al (1993) and Pickett (1963).

We can see that both Pickett and Castagna's empirical relations for **limestones** fit the data better than their equivalents for dolomites. This might be explained by the fact that the dolomitization process of the limestone is not complete and also that the calcite reprecipitation is significant.

Figure 7: WELL 1711 and WELL 0504 V_p - V_s scatter plots with their linear least square fits.

In figure Figure 7 we show the V_p - V_s scatter plot for Wells 0504 and 1711 with a linear least square fit for each of the well data. We can see that the V_p - V_s relation differs a little from one well to another, with respect to the linear least square fit.

In summary, the most reliable and most often used V_p - V_s relations are empirical fits to the well log data. In addition, we can make use of the effective medium models that predict both P and S velocities assuming idealized pore geometries. These models are mainly useful for extending the empirical relations to different pore fluids or measurement frequencies. When a local calibration is not possible, the Greenberg & Castagna relations are among the best published empirical relations available. For this particular data set we can see that Greenberg & Castagna relation for limestones does a fairly good job (Figure 6b).

Gassmann fluid substitution

Fluid substitution is a very important task in seismic fluid detection. In our analysis we use Gassmann equations for fluid substitution. This equation assumes a low frequency model and isotropic rocks. The pore geometry and stiffness are incorporated automatically through V_p and V_s measurements.

Figure 8 shows a crossplot for the calculated P and S wave velocities for different fluid types, using the data in Well 1711. As expected, the fluid sensitivity is larger for softer rocks, characterized by small values of V_p and V_s . As the rocks become stiffer, closer to the mineral point, the fluid sensitivity decreases significantly.

Figure 8: WELL 1711 V_p - V_s crossplot for brine, oil, and gas saturated rock.

We know the average elastic fluid properties in our area of interest:

Bulk modulus:

K brine =2.39GPa

K oil =1.6GPa;

K gas =0.0037GPa;

Density:

ρ brine =1.018 g/cm³;

ρ oil =0.863 g/cm³;

ρ gas =0.029 g/cm³;

Figure 8 enables us to conclude that, because the elastic properties of brine and oil are very similar, the change in P velocity when going from brine to oil saturated state is not very large. This is also corroborated with the fact that the solid matrix is a relatively stiff carbonate, therefore fluid effects are smaller. However, gas properties are net different from those of the brine and oil, and the fluid related changes in P wave velocity are larger when going from brine to gas saturated state.

Depositional environments and facies of San Andres Formation reflected on well logs

The “San Andres” formation is the producing reservoir in Yates Field, deposited in a shallow marine environment in different cycles. The Upper San Andres formation comprises of at least 3 different cycles. The reservoir rock is mostly a dolomitized limestone, exhibiting different facies, represented by interbedded grainstones, packstones, wackstones, and mudstones.

Figure 9 shows in different colors three main distinct intervals on the logs of Well 0504. These 3 units differentiate both by their porosity and density, and also by the P and S wave velocities. In Figure 10 are represented six different crossplots on which we can distinguish the three regions coded in green, red and blue selected on the well logs. The green interval in Figure 9 corresponds to rocks with high density and small porosity. For this area the sonic log shows

relatively high average velocities, but with high-frequency variations. The red unit on the same figure displays significantly smaller densities and higher porosities. The velocities are smaller compared with those on the green unit, and they are more uniform. The last interval, colored in blue (Figure 9), corresponds to higher densities than those of the red coded region, but a little smaller than those from the interval colored in green. The velocities are also a little higher.

Figure 11 shows the same three different intervals, but on the logs of Well 1711. The uppermost part of the San Andres formation is not covered by the sonic log, hence the first region (coded in green on Well 0504 in Figure 9) is missing on the sonic log. However, on the density and porosity logs we can identify the same three different intervals as in Well 0504. Figure 12 gives again the same six different crossplots as for the Well 0504 in Figure 10, in which again we can see the corresponding units depicted from the well logs.

Figure 9: WELL 0504 – 3 different well log intervals.

Figure 10: WELL 0504.

Figure 11: WELL 1711 – 3 different well log intervals.

Figure 12: WELL 1711.

Next we use the detailed information available from core data from Well 2509 about the depositional environments, rock texture, and fracture distribution. We combine this information with the sonic log from the same well, in order to determine possible relations between elastic properties and environmental, textural and fracture distribution of the rocks.

Figure 13 is V_p – porosity scatter plot, coded with the information about the depositional environment and texture of the rocks available from the cores. We can again identify three main depositional environments in the San Andres formation present in this well: shoal, shoal-margin and low energy subtidal. We can also identify three main different textures: dolomitic mudstone, packstone and wackestone. From Figure 13 we can see that the dolomitic mudstones were deposited into a low energy- subtidal regime. Most of the packstones were deposited into a shoal regime, while wackestones were deposited in all environments, but especially in the shoal and shoal-margin environments.

From Figure 13a we can notice that the different depositional environments have different elastic signature, therefore they might be seismically distinguishable.

Figure 13a: WELL 2509 Vp-porosity crossplot that emphasizes different depositional environments and textures.

From the core data in Well 2509 we also have information about the matrix permeability. Figure 13b shows a crossplot of V_p – permeability. Permeability is in logarithmic scale and is given in millidarcy. We can notice a decreasing trend of the matrix permeability with increasing velocity, but there is a significant scatter in the data. However, we can see that the packstones have the largest matrix permeability and exhibit smaller P wave velocities, while mudstones have the smallest matrix permeability and higher P wave velocities.

Figure 13b: WELL 2509 Vp – Permeability crossplot that emphasizes different depositional environments and textures.

In terms of **fracture distribution** with respect to the depositional environments we see from the core data in Well 2509, that the majority of the fractures belong to the subtidal, low energy depositional environment (Figure 14a). However, this environment is not very well represented in this well, so for a more reliable fracture distribution in relation with different environments we need additional information from other core data.

The shoal environment presents the smallest number of fractures per unit length. Figure 14b shows the fracture distribution with respect to rock texture. We can see that the fractures are more uniformly distributed in all rock types, but they are slightly more present in mudstones, which were basically deposited in a low energy, subtidal environment.

Figure 14a: Fracture distribution and depositional environments from core data –WELL 2509.

Figure 14b: Fracture distribution with respect to rock texture from core data –WELL 2509.

FRACTURE MODELING

The role of elastic fracture models is to help provide a link between seismic observables – such as elastic constants, anisotropy, shear wave splitting, and directions of symmetry planes and the physical rock and fracture properties that we wish to know fracture length, orientation, aperture, geometry, connectedness, permeability.

The physical mechanism of the elastic fracture effect is very simple: fractures, whether wet or dry, soften the rock in a way that depends on the fracture direction. Normal and shear stresses acting on the plane of the crack induce more overall strain than would be observed in the host rock without the fractures. There are two general approaches to quantifying the effect of cracks and fractures on elastic anisotropy. One approach considers cracks and fractures as isolated “inclusions” or cavities in an elastic matrix (Eshelby, 1957; Walsh, 1965; Wu, 1966; O’Connell and Budiansky, 1974; Hudson, 1980, 1981, 1990, 1994; Nishizawa, 1982; Crampin, 1984). Usually these are assumed to be ellipsoids or “penny-shaped” cavities of finite size, although a few other non-elliptical shapes have been considered (see, for example, Mavko and Nur, 1978). The penny shaped cracks are usually considered to have all dimensions much smaller than a seismic wavelength. The second modeling approach, popularized by Schoenberg and Muir (1989), represents fractures as planes of discontinuous slip or very thin planar zones that are more compliant than the unfractured rock. The planes extend infinitely far. Both approaches allow for different angular distributions of fractures.

Figure 15: Vp distributions for different formations.

Figure 16: Vs distributions for different formations.

Teng (1998) has shown that to first order the slip-interface model of Schoenberg and Muir (1989) is mathematically equivalent to that of Hudson (1981). In other words, for any distribution of Schoenberg-Muir fractures, we can find some distribution of Hudson penny shaped cracks that gives the same anisotropic effective elastic medium, and vice versa. The good news is that we can choose the approach that better describes the physical problem of interest. For example, we might use distributions of ellipsoidal cracks to describe microcracks and grain boundaries in the stress-induced anisotropy experiments of Nur and Simmons (1969), while we might use slip interfaces to describe extensive regional fractures. The bad news is that if we seismically determine fracture-related elastic anisotropy, we can say very little about the fracture configuration without additional geologic and rock mechanics inputs. The important fact is that the fracture alignment can induce anisotropy in rock properties that can be detected from velocity measurements of waves traveling through reservoir.

For our fracture modeling of San Andres formation we use Hudson's formulation (1981) to predict the P and S wave velocities and anisotropy parameters. The details of the Hudson's model are given in Appendix B of the Fracture Rock Physics Handbook (Attachment A).

In this section we present the results of our fracture modeling both for the interval properties (interval velocities and anisotropy parameters) as well as for the interface properties (reflectivity and AVO attributes). The goal of this modeling is to assess the uncertainty in fracture detection from seismic data.

In order to quantify these uncertainties in the seismic anisotropy we do Monte Carlo simulation on the input parameters in Hudson's model. In doing this we use the P wave and S wave velocity distributions, as well as the porosity and density distributions obtained from all the logs available. Figures 15, 16 and 17 show the V_p , V_s and porosity histograms together with their corresponding cumulative distribution functions not only for the San Andres formation, but also for the Queen, Grayburg and the compact interval of anhydrite immediately above the Queen formation.

Figure 17: Porosity distribution for different formations.

We also take into account that the seismic resolution is smaller than that of the sonic well log. The seismic wavelength can be as large as 100 m and more, so the waves ‘see’ an average velocity over a thicker interval compared to the sonic log. Figure 18 shows the upscaling effect on the velocity distribution for the Queen and Grayburg formation together. This figure represents a mirror histogram for the P wave velocity to facilitate the comparison between the V_p distribution before the upscaling (on the left) and after upscaling (on the right). We can see, as expected, that the upscaling process narrows the V_p distribution, because the seismic waves smoothens the high frequency variations present on the sonic well logs, averaging the values over much thicker intervals.

Figure 18: V_p distribution for Queen + Grayburg formation before upscaling (left), and after upscaling (right).

Interval properties

In order to determine the interval velocities and anisotropy parameters of the fractured reservoir, we use Hudson’s model to simulate the vertical fractures observed in the San Andres formation. In this way we generate a transversely isotropic medium with horizontal axis of symmetry (HTI). We assume that the only cause of the anisotropy is the presence of the fractures. The cracks are 100% saturated with the same fluid (brine, oil or gas), but not necessary with the same fluid as in the matrix porosity. We substitute the fluids in the cracks by changing

the elastic moduli of the crack inclusions in Hudson's equations. This situation corresponds to the case when there is no communication between the fluids in the pores and fluids in the cracks.

Figure 19: Probability distribution functions in V_p - V_s domain for unfractured San Andres, and fractured – fast and slow directions; *gas filled cracks*.

Figure 19 gives the probability distribution functions in the V_p - V_s domain for the unfractured San Andres formation (blue), and also for the fractured velocities in the fast direction, parallel to the vertical fractures (red), and in the slow direction, perpendicular to the fractures (green). In this case the matrix is brine saturated, while the fractures are filled with gas. The crack density in this model is 0.7, while the crack aspect ratio is 0.001, which are reasonable values, in accordance with the geological information available. Besides that these values are in the range of the Hudson's model applicability. The scatter in the data is due to the variability in the matrix properties, obtained via Monte Carlo simulation, based on the well log distributions.

We can see that fractures filled with gas soften the rock, as expected. Both P and S wave velocities drop, and as expected, the decrease is larger in the direction perpendicular to the cracks. However, there is a significant decrease in V_p even in the fast direction, parallel to the vertical cracks.

Figure 20 gives the same representation as Figure 19, but this time the cracks are filled with

Figure 20: Probability distribution functions in Vp-Vs domain for unfractured San Andres, and fractured – fast and slow directions; *oil filled cracks*.

oil. We can see that in this case the P wave velocity does not change much, not even for the direction perpendicular to the cracks. This can be explained by the fact that oil is a much less compressible fluid than gas, and that is why the cracks will be stiffer in this case. However, for the slow direction, there is a more significant drop in the S wave velocity. In the case when there is no fluid communication between the fractures and matrix porosity, it will be difficult to distinguish between the fractured and unfractured zones based on the vertical velocities alone, if the cracks are 100% oil saturated.

Next we analyze Thomsen's (1986) anisotropy parameters: epsilon (ϵ), gamma (γ) and delta (δ), since they are related to fracture's characteristics. More details about these parameters are given in the Appendix D of the Fracture Rock Physics Handbook (Attachment A).

The parameter epsilon (ϵ) can be seen to describe the fractional difference of the P-wave velocities in the slow and fast directions. The parameter gamma (γ) can be seen to describe the fractional difference between the vertical and horizontal polarizations of the horizontally propagating S-waves and is often called the shear-wave splitting parameter.

Figure 21: Thomsen's γ versus ϵ for all the 9 possible combinations between pore fluids and crack fluids.

Figure 22: Thomsen's δ versus vertical V_p for all the 9 possible combinations between pore fluids and crack fluids.

In our modeling we consider all the theoretical combinations between the pore fluids and crack fluids. In other words the matrix porosity can be saturated with brine, oil or gas, and the same for the cracks. Theoretically we can have a total of nine different possibilities like: brine saturated matrix porosity with brine, oil, or gas saturated cracks, oil saturated matrix porosity with either brine, oil or gas filled cracks etc.

Figure 21 represents all of these nine possible combinations for Thomsen's γ - ϵ domain. We set the axes limits to be the same for all the crossplots to facilitate the comparison. We can see that for gas filled cracks the parameter ϵ has higher values than for oil or water filled cracks, independently of the fluid type that saturates the matrix porosity (brine, oil, or gas). That means that the presence of the gas in the fractures makes the P-wave anisotropy stronger compared to

the situations when we have oil or brine filling the fractures. However, the shear wave anisotropy represented by γ is not so much influenced by the difference in the fluid properties present in the cracks. The shear wave splitting parameter is influenced more by the presence of cracks, and less by the fluids present in the fractures.

Figure 22 shows again all of the nine theoretically possible situations, but this time in the δ - V_p domain. The P-wave velocities represent the vertical velocities, along the cracks, in other words in the fast direction. We can again notice the difference between gas filled cracks on one hand, and brine or oil saturated cracks on the other. The δ values for the case when the fractures are filled with gas are positive, in contrast with the cases when the fractures are either filled with oil or brine.

In conclusion, Thomsen's anisotropy parameters might help discriminate between gas filled cracks over the oil or brine filled cracks, but they are more difficult to use for distinguishing between oil and brine saturated fractures. One reason for this is that the elastic properties of oil and brine are very close, but very different from the gas properties. We can use parameters ϵ and δ to get insights on fluid type. The parameter γ is more sensitive to the presence of the cracks, rather than to different crack fluids. If there is no other source of anisotropy in the reservoir except for the alignment of fractures, Thomsen's parameters can give us insights in the fracture network parameters.

Interface properties and AVOZ analysis

In this subsection we present preliminary results on Amplitude Variation with Offset and Azimuth (AVOZ) analysis at the interface between San Andres reservoir and the above Grayburg and Queen formations.

We consider again the same model of the vertical fractures embedded into an isotropic background, represented by the unfractured San Andres dolomitic reservoir. This generates a transversely isotropic medium with horizontal axis of symmetry. This model corresponds to an azimuthally anisotropic medium. It is known that the azimuthal anisotropy impacts the shear waves that split into two components, traveling with different velocities. Several methods can be used to obtain the shear wave splitting parameter γ , in order to relate with the crack density, one of the most important parameters which gives insights into the permeability of the fracture network. Although shear wave methods have developed extensively lately, there are still very

costly and furthermore it is difficult to record good quality shear data. That is why it is still very important to extract more information about fractured reservoir using P-wave. A good example is the AVO analysis. Amplitude variation with offset analysis has proven its usefulness in fractured reservoir characterization, since it can provide local information about the anisotropy at the target horizon, and therefore it can give insights on the fracture network parameters.

In our modeling we use Ruger's (1998) approach to model the P-P reflectivity as a function of angle of incidence as well as a function of azimuth. We consider the interface between the Queen and Grayburg formations combined and the modeled San Andres formation. We consider Queen and Grayburg together because their individual thickness is small, and they also exhibit similar lithology: shale, dolomitic mudstones, silstones and sandstones. Our fracture model for the San Andres formation is the same as described in the preceding subsection on interval properties.

Figure 23: P-P reflectivity at 30 degrees angle of incidence as a function of azimuth for Queen+Grayburg/San Andres interface; San Andres is modeled as having a brine saturated matrix porosity with *vertical GAS filled fractures*.

Figure 23 shows the P-P reflectivity at 30 degrees angle of incidence as a function of azimuth. In this case the San Andres formation is modeled as having brine saturated matrix porosity and **gas** filled fractures. Different curves correspond to different Monte Carlo realizations of the background properties. We can not observe very significant azimuthal

variation of the reflectivity. However, if we have the same system of fractures, but 100% saturated with *oil*, then the azimuthal variation is stronger, as we can see in Figure 24.

Figure 24: P-P reflectivity at 30 degrees angle of incidence as a function of azimuth for Queen+Grayburg/San Andres interface; San Andres is modeled as having a brine saturated matrix porosity with *vertical OIL filled fractures*.

Figure 25: P-P reflectivity vs. angle of incidence for average values of the adjacent layers (Queen +Grayburg/San Andres) for different azimuths; San Andres model with *GAS* filled cracks.

Figure 26: P-P reflectivity vs. angle of incidence for average values of the adjacent layers (Queen +Grayburg/San Andres) for different azimuths; San Andres model with *OIL* filled cracks.

Next we consider also the average values of the background elastic properties (velocities and densities), besides the Monte Carlo realizations, both for Queen and Grayburg combined and for the modeled San Andres reservoir. Then we do as AVO analysis for this averaged, unique values. Figure 25 and 26 represent the P-P reflectivity as a function of angle of incidence for different azimuths: 0 azimuth (perpendicular to the fractures' strike), 45 degrees azimuth, and 90 degrees azimuth (parallel to the fractures' strike). We again consider for San Andres formation the case of brine saturated matrix porosity with **gas** filled cracks (Figure 25) and **oil** filled cracks (Figure 26).

We can see that for gas filled cracks, the azimuthal variation is not very significant at relatively small angle of incidence, but it becomes more evident at higher angles of incidence (>25 degrees). However, for oil saturated cracks, the azimuthal variation is more important and it can be observed for smaller angle of incidence, starting around 10 degrees.

Figure 27: AVO gradient vs. P-P reflectivity at normal incidence; upper plot: azimuth parallel to fracture strike; lower plot: azimuth perpendicular to fracture strike; *GAS* filled cracks.

Next we also analyze the **AVO gradient**. In the case of azimuthally varying reflectivity, the AVO gradient is composed of the isotropic part, which does not vary with azimuth plus the anisotropic contribution, which is a function of the azimuth (Ruger 1998).

We model again two cases: San Andres reservoir with gas filled fractures and with oil

Figure 28: AVO gradient vs. P-P reflectivity at normal incidence; upper plot: azimuth parallel to fracture strike; lower plot: azimuth perpendicular to fracture strike; *OIL filled cracks*.

saturated fractures, respectively. Figure 27 and Figure 28 show in the *AVO gradient - normal incidence reflectivity* domain the fractured and unfractured Monte Carlo realizations. The upper plots in both figures show the AVO gradient at 90 degrees azimuth (parallel to the fractures), while the lower plots show the AVO gradient at 0 degrees azimuth (perpendicular to the fractures).

From Figure 27, we can see that, if the fractures are saturated with gas, it is possible to separate the unfractured zones from the fractured ones both using the AVO gradient along the fractures' strike and perpendicular to the fracture strike. Also, it is easier to discriminate between fractured and unfractured regions if the P-P reflectivity at normal incidence is greater than about - 0.05. However, if the fractures are filled with oil (Figure 28) it is possible to separate the fractured zones from the unfractured ones using the AVO gradient at 0 azimuth, perpendicular to the fractures, for a wider range of normal incidence reflectivity.

However, it is difficult to discriminate between oil filled fractured and unfractured zones using the AVO gradient in the direction parallel to the fracture's strike, when we make the assumption that there is no fluid communication between cracks and matrix porosity.

In summary, our model suggests that the AVOZ analysis can help us differentiate between fractured and unfractured zones, and also it might allow us to distinguish between gas and oil saturated fractures.

For the future work, we also need to consider the case in which there is fluid communication between the cracks and the matrix porosity.

In addition, we also need to account for the presence of the horizontal layering which might induce an important transversely isotropic medium with vertical axis of symmetry. Structural information suggests that we should also include another set of fractures, orthogonal to the one considered. In summary, on this anisotropic background generated by the thin-layering, we superimpose the two orthogonal sets of fractures that generate an anisotropic medium with orthorombic symmetry.

SUMMARY AND CONCLUSIONS

ATTACHMENT C gives an overview on the rock physics analysis of the San Andres reservoir. It also presents the Phase I results on the fracture modeling, including the interval properties (velocities and anisotropy parameters), as well as the interface properties (P-P reflectivity and Azimuth Variation with Offset and Azimuth attributes).

From the detailed core information available from one of the wells, combined with the sonic log data, we show that different depositional environments and their associated facies have different elastic signatures that can be seismically distinguishable. We also show that there is a correlation between the fracture distribution and the depositional environments. The majority of the fractures are associated with the low energy, subtidal depositional environment. This is an important result that can play a critical role in fracture delineation from seismic measurements.

Our fracture modeling suggests that, if there is no other source of anisotropy present in the reservoir except for the alignment of one set of vertical fractures, Thomsen's parameters can give us insights on the fracture network characteristics. The shear wave splitting parameter can be related to the fracture density, a very important parameter of the fracture network. These anisotropy parameters might also help us discriminate between oil and gas filled fractures, under situations when there is a little fluid communication between the cracks and the matrix porosity. However, we also need to consider the case in which the fluid communication is more important.

For the AVOZ analysis we consider the interface between Queen and Grayburg formations as cap rock, and modeled the underlying San Andres reservoir rocks with vertical fracture sets. The aligned fractures generate an azimuthally anisotropic medium from which we compute the P-P reflectivity as a function of angle of incidence and azimuth. We can conclude that the oil saturated fractures models have a stronger azimuthal signature than the gas filled fractures. However, as the structural information on the Yates field suggests, we also need to account for the presence of a second set of fractures, orthogonal to the one considered. This will generate a anisotropic medium with orthorhombic symmetry.

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ATTACHMENT D

**SEISMIC DETECTION OF PORE FLUIDS: PITFALLS OF
IGNORING ANISOTROPY**

Seismic detection of pore fluids: Pitfalls of ignoring anisotropy

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Summary

Rocks are often anisotropic, but we may not have full information about the anisotropic elastic stiffness tensor. We may have measurements of only one of the velocities along the principal directions. However, we need the full compliance tensor to do fluid substitution calculations for anisotropic rocks. How much of an error is introduced if we take the velocity along one direction only, and do fluid substitution ignoring anisotropy? In this paper we present the results of a numerical experiment which aims to evaluate these errors. We model two cases: anisotropy due to preferential alignment of cracks, and anisotropy due to layering. We estimate and compare the fluid related changes in the velocities using Gassmann, and Brown & Korringa low frequency models. Finally, we present examples using dry ultrasonic data from anisotropic sandstone samples.

Introduction

Isotropy is a very common assumption in geophysics, since the data are seldom sufficient to completely characterize elastic anisotropy. A good example is well sonic data, which in most cases gives only 1-D P-wave information of the subsurface, and less often both P and S-wave. In spite of great advances in anisotropic imaging and data processing, most of the time we do not have the complete elastic tensor, and we assume isotropy in order to predict the changes due to fluids in the seismic response. How far off are the velocity changes predicted from this isotropic approach from those that would have been predicted if we had had the complete anisotropic tensor? How valuable is the information in the full elastic tensor for estimating pore fluid signatures? This paper addresses these questions.

We start with numerical modeling to generate anisotropic data from ultrasonic measurements on dry, isotropic sandstones. We model two different types of anisotropy: crack anisotropy, and thin-layer anisotropy. We then predict the changes in velocity due to fluids using the low frequency Gassmann (1951) and Brown & Korringa (1975) approaches. For isotropic rocks, these two models are equivalent. For anisotropic materials, the Brown & Korringa equation allows us to predict the low frequency fluid related changes in velocity, provided we have the complete elastic tensor. It should be remembered that at high frequency, such as lab ultrasonic, the saturated velocity can be different from the Brown & Korringa prediction due to fluid related dispersion (Mukerji and Mavko, 1994). Gassmann's equation gives a low frequency estimate of fluid effects, while ignoring anisotropy. Finally, we com-

pare the predicted changes in velocity for the two different types of anisotropy. We also present results of theoretical fluid substitution using lab data from anisotropic rocks.

Crack anisotropy

We create crack-anisotropic rocks using Hudson's model (1980) by embedding aligned ellipsoidal cracks with aspect ratio equal to 0.0001, in a porous, isotropic background represented by Han's (1986) dry ultrasonic data. We consider the case of a single set of horizontal cracks, which generates a transversely isotropic medium, with the axis of symmetry parallel to the vertical direction (TIV). The stiffness tensor of such a material is completely defined by 5 independent elastic constants (C_{ij}). First, we derive the P and S dry velocities orthogonal to the cracks from C_{33} and C_{44} , and parallel to the cracks from C_{11} and C_{66} . We then use these velocities in Gassmann's equation to estimate the saturated velocities along the vertical and horizontal directions, assuming the rock to be isotropic. Second, we use the Brown & Korringa method to derive the complete saturated stiffness tensor ($C_{ij,sat}$) from the dry tensor. From this stiffness tensor we extract the $C_{33,sat}$ and $C_{11,sat}$ components in order to compute the corresponding P-wave saturated velocities, perpendicular and parallel to the cracks. Finally, we compare the Gassmann predictions with the Brown & Korringa ones, in order to evaluate the errors caused by disregarding the anisotropy.

Fig. 1: V_p perpendicular and parallel to the cracks, for dry and water saturated rocks using isotropic (Gassmann - Gper, Gpar) and anisotropic (Brown & Korringa - BKper, BKpar) fluid substitution models, as a function of the crack density.

Figure 1 shows the P-wave velocities of dry and water saturated rocks, as a function of the crack density parameter. The P-wave velocities in the vertical direction,

Fluid effects in anisotropic rocks

perpendicular to the cracks, decrease significantly, while the velocities in the horizontal direction are almost not at all affected by the presence of the cracks. As expected, the fluid sensitivity in the direction orthogonal to the cracks increases with crack density. The change in velocity due to fluids predicted by Gassmann's equation, ignoring anisotropy, is larger than the one predicted by the Brown & Korrington approach, which uses the information contained in the full elastic tensor. The difference between the two predictions increases with crack density, such that at a value equal to 0.1, Gassmann's equation overestimates the change in velocity by a factor of 1.4.

Figure 2 shows the relative change in P-wave velocity from dry to water saturated for different synthetic crack-anisotropic rocks, with a fixed crack density parameter. For all of these samples, Gassmann's equation overpre-

Fig. 2: The relative change in V_p parallel and perpendicular to the cracks, predicted by isotropic (Gper, Gpar) and anisotropic (BKper, BKpar) fluid substitution, when going from dry to water saturated synthetic crack-anisotropic rocks. The crack-density is 0.07.

dicts the change in velocity by a factor up to 1.8 in the direction parallel to the symmetry axis of the TIV medium (perpendicular to the cracks). However, in the direction parallel to the cracks, the errors made by ignoring anisotropy are negligible.

Backus average anisotropy

Another type of anisotropy occurs due to layering, which can also correspond to a TI medium. Backus (1962) showed that in the long wavelength limit, a stratified medium composed of layers of transversely isotropic materials, each with its symmetry axis perpendicular to the bedding, is also effectively anisotropic. If the individual layers are isotropic, the effective medium is still transversely isotropic, completely characterized by five independent constants.

In the following experiment, we model the elastic properties of a medium composed of two types of isotropic, horizontal layers for different values of the volumetric fractions of the two components. We again look for the

changes in V_p due to fluids using Gassmann and Brown & Korrington approaches, both in the vertical, and horizontal directions.

Figure 3 shows the P-wave velocities for dry and water saturated rocks, as the volumetric fraction of the softer rock increases. In the limit, when the volumetric fraction of the slower velocity component is 0, the medium is isotropic. Obviously, in this case P-wave velocity is the same regardless of direction, and both Gassmann and Brown & Korrington predict the same velocity change. However, as the fraction of the softer rock increases, the horizontal velocities ($V_{p,par}$) decrease slower than the vertical velocities ($V_{p,per}$). Gassmann and Brown & Korrington give different estimates in the perpendicular direction to the bedding. These differences increase with the volumetric fraction of the slower velocity component.

Fig. 3: V_p perpendicular and parallel to the layering, for dry and water saturated rocks, using isotropic (Gper, Gpar) and anisotropic (BKper, BKpar) fluid substitution equations, as a function of the volumetric fraction of the softer rock.

For thin-layered anisotropy, we can identify two distinct situations. In the first case (Figure 3), the change in the P-wave velocity orthogonal to the layering given by Gassmann's equation is smaller than the correct Brown & Korrington prediction. However, in the second case (Figure 4) the Gassmann approach gives a higher estimate than the Brown & Korrington one in the same perpendicular direction with respect to the layering. This situation appears when the stiffer constituent of the layered medium has a saturated velocity smaller than its dry one, due to the density effect. Nevertheless, in both cases, the errors made by ignoring anisotropy are negligible in the direction parallel to the bedding.

Next we determine the errors in the vertical direction, as a function of the ratio between the dry P-wave velocities of the two isotropic constituents of the layered medium. We keep the dry velocity (V_{p1}) of the first component to be equal to 5000 m/s, and vary the V_{p1}/V_{p2} ratio. We assign the corresponding S-wave velocities, porosities, and densities for each constituent layer using Han's empirical relations for dry shaly sandstones, and then we evaluate the errors in fluid substitution caused by ignoring anisotropy. The relative error in the contour plot (Figure 5) represents

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Fig. 4: V_p perpendicular and parallel to the layering, for dry and water saturated rocks, using isotropic (Gper, Gpar) and anisotropic (BKper, BKpar) fluid substitution equations, as a function of P-wave anisotropy.

the normalized difference between fluid sensitivity estimated by Gassmann, as compared to the correct Brown & Korrington prediction $(\Delta V_{PGass} - \Delta V_{PBK}) / \Delta V_{PBK}$.

Fig. 5: Top: Contour plot of the relative error in % when ignoring anisotropy for a thinly layered medium. Bottom: P-wave anisotropy as a function of the volumetric fraction of the stiffer constituent, for different V_{p1}/V_{p2} ratios.

The differences between the predicted velocities obtained through isotropic and anisotropic fluid substitution models are larger for higher contrast between the dry velocities of the two constituents of the layered medium (V_{p1} and V_{p2}). The P-wave anisotropy, as defined for the dry effective medium by $(V_{ppar} - V_{pper}) / V_{pper}$, is maximum when the volumetric fractions of the two components are equal (Figure 5). However, as seen from the contour plot, the maximum relative errors caused by ignoring anisotropy, *do not* correspond to the maximum anisotropy. These peak values migrate to higher volumetric fractions of the stiffer component, as V_{p1}/V_{p2} increases. This means that, for a 2-constituent layered media, the relative error is higher in the case in which we have small volumetric fraction of the slower velocity component in the stiffer background, than in the case in which we have the same small

volumetric fraction, but of the higher velocity material in the slower velocity background.

Laboratory data example

The laboratory dataset we use consists of ultrasonic P and S wave velocities of dry TIV sandstone samples, with pore pressure at room condition, and increasing confining pressure from 1000 to 9000 psi. The samples belong to a fluvial-deltaic depositional environment, with porosities ranging from 17% to 23%. The clay content was not determined, but very fine and dilute shale layering is present in some of the samples. Each sample has a complete set of P and S velocity measurements that suffice to completely define the elastic TIV behavior of the rock: P-wave and S-wave velocities in the horizontal and vertical planes, and P wave velocity in an oblique direction of 45 degrees. From these measurements we derive the elements of the stiffness tensor corresponding to the dry state. Then we do theoretical fluid substitution, using both Gassmann and Brown & Korrington low frequency models, in the same way as described for the synthetic case.

Figures 6 and 7 show the dry measured velocities and the theoretical saturated ones, for two different samples, as a function of pressure. The theoretical saturated velocities are computed both by ignoring anisotropy (Gassmann), and by taking it into account (Brown & Korrington).

Fig. 6: P-wave velocities for the dry (measured) and the water saturated (theoretical) anisotropic rock, perpendicular and parallel to the bedding, as a function of pressure.

Figure 6 presents the case in which V_p perpendicular to the bedding predicted by Gassmann is larger than the Brown & Korrington estimate. This difference becomes smaller as the confining pressure increases. In the direction parallel to the layering, however, the two low frequency substitution models give very close results.

For the other example in Figure 7, Gassmann's equation predicts a smaller change in the vertical velocity than the Brown & Korrington model. In this case, the difference increases slightly with pressure.

Finally, we calculate the relative errors caused by ignoring anisotropy when doing fluid substitution for 4

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Fig. 7: P-wave velocities for the dry (measured) and the water saturated (theoretical) rock, perpendicular and parallel to the bedding, as a function of pressure.

different samples. Figure 8 shows these relative errors $[(\Delta V_{PGss} - \Delta V_{PBK}) / \Delta V_{PBK}]$ as a function of fluid sensitivity for all pressures. The relative errors are high on the left side of the schematic gray band (its limits should not be taken strictly). However, this area might not be very interesting, since the fluid sensitivity is small, and in practical situations may not be seismically resolvable from other effects. On the right side of the gray band, the fluid changes are easily detectable, and the relative errors may be too small to cause any significant difference. The area of greatest interest and caution corresponds approximately to the gray band, where the sensitivity to fluids is large enough to be of practical importance and, at the same time, the relative errors are also significant. This is the region in which we cannot accurately evaluate the changes due to fluids without taking anisotropy into account.

Fig. 8: Relative errors in the perpendicular direction to the bedding, as a function of fluid sensitivity of the samples for all pressures.

Conclusions

In this paper we show that the errors in fluid substitution caused by disregarding anisotropy can be significant in

some situations. For the transversally isotropic case, the errors may be large in the direction parallel to the symmetry axis of the TI medium, but they are negligible in the orthogonal direction.

For the crack-anisotropic rocks, ignoring anisotropy seems always to overpredict the change in the velocity perpendicular to a single set of aligned cracks. In contrast, for the thin-layer anisotropy, Gassmann's equation usually underestimates the fluid-related changes, but in particular situations it can also overpredict them, depending on the density effect. Our theoretical model suggests that, for a layered medium, large relative errors in ignoring anisotropy for fluid substitution could be encountered in materials composed of a stiff isotropic background with thin intercalations of a softer component, rather than in materials composed of a soft background with thin layers of a stiffer component.

The results of the theoretical fluid substitution on lab ultrasonic dry data show that ignoring anisotropy can lead to either underestimating or overestimating the changes in velocity due to fluid substitution. The errors can be significant especially in the direction parallel to the symmetry axis of TI medium. We need to pay particular attention to those cases when fluid sensitivity of the effective medium is not very large, but it is big enough to be seismically detectable. For those situations we cannot accurately evaluate the changes due to fluids if anisotropy is ignored.

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