Bianucci, Mannella, and Grigolini Reply: In a recent Letter [1] we have discussed the linear response theory (LRT) and shown that the breakdown of this theory occurring at intermediate times, observed in an earlier paper [2] as well as in [1], disappears upon an increase of the number of degrees of freedom. In a Comment to [1] Falcioni and Vulpiani [3] claim that this breakdown is rather a consequence of the lack of mixing: according to them, regardless of the number of degrees of freedom, mixing is the key ingredient behind the LRT.

We agree with them that, in the absence of mixing, it is difficult to define a unique system equilibrium, and that the two-dimensional system of [1] lacks mixing. However, the breakdown of the LRT, whatever its origin might be, should be discussed at a dynamic level, leaving therefore arbitrary the choice of the initial conditions. Thus, provided that the condition of equilibrium in the absence of perturbation is fulfilled, this choice must be made so as to get simple predictions. We note that the microcanonical distribution, as any function of the unperturbed Hamiltonian, is invariant in the absence of perturbation, and therefore it is a genuine equilibrium distribution even if not unique in a system lacking mixing. Furthermore, it also results in a simple prediction. A microcanonical initial condition, in conclusion, fits the main requirements of equilibrium and simplicity, and, in fact, is adopted in [1].

With this initial condition, we switch on the perturbation, monitor the ensuing nonequilibrium dynamics, average on the initial condition, and compare the numerical results to the theoretical prediction. To be more specific, we determine numerically the time evolution of 10^6 trajectories driven by the perturbed Hamiltonian, with random initial conditions extracted from the unperturbed microcanonical distribution. This “cloud” of trajectories is used to define, at any given time t , the corresponding “distribution,” and thus the mean value of the observables of interest. We stress that at large times the susceptibility obtained averaging over this distribution is numerically coincident with the theoretical susceptibility computed over the perturbed microcanonical distribution, even in the absence of mixing (see also [2]).

In principle, increasing the number of degrees of freedom might produce mixing. Thus, on the basis of the results of [1] alone, we could not rule out the theoretical perspective of Falcioni and Vulpiani [3] as the correct interpretation of both validity and failure of the LRT.

However, we are forced to so on the basis of an earlier paper [2]. Let us consider the system of Eq. (4.3) of [2] and that of Eq. (4.1) of the same paper [2]. The former

turns out to be mixing within the limits of the computational accuracy and the latter, although two dimensional, is proved theoretically [4] to be mixing: according to [3], both systems should exhibit LRT. Yet, both are shown [2] to violate the LRT prediction. Most of the trajectories evolving from the microcanonical initial condition turn out to be unstable, and at the statistical level this fact is mirrored by the breakdown of the LRT. This means that the well known van Kampen’s arguments apply only to the low-dimensional case and mixing is actually a source of temporary deviation from the LRT [1,2].

In both the chaotic and nonmixing case of [1] and the mixing cases of [2], the LRT prediction is recovered at large times, because, as time increases, the fragmentation of the Liouville density becomes so high as to be virtually indistinguishable from a smooth phase space. The same smoothing effect, and this is the central result of [1], is produced by increasing the number of degrees of freedom.

In conclusion, mixing ensures the condition for the linear response at large times, but it is not a sufficient condition at intermediate times. Neither mixing seems to be a necessary condition, since, as shown in [1], the LRT is recovered at large times also by the two-dimensional case, which is not mixing. Thus, even if the arguments used in [1] are not mathematically compelling, they seem to afford the only exhaustive answer to the LRT debate.

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