SOME PHILOSOPHIC BASES UNDERLYING MATHEMATICS

INTERPRETATIONS IN SECONDARY SCHOOLS

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CHAPTER I

INTRODUCTION

Purpose of the Study

As a usual thing, philosophy is regarded as something entirely apart from the average, everyday existence. It is an undefinable, non-understandable, abstract, metaphysical subject far removed from the ordinary affairs of life. This is the opinion of the average individual. But, on the contrary, philosophy is one of the strongest motivating forces in civilization, and it affects the lives of all the people in many ways every day of their life. For philosophy, basically speaking, is ideas, and ideas are what makes people do things.

The part philosophy plays in everyday life has been especially noticeable in education. All of us are familiar with the changes in aims in education, with the changes in the ways of teaching, and with the changes in the curriculum. These changes, fundamentally speaking, are due to changes in the philosophy of the people, which, in turn, is due to changing conditions of society. The purpose of this study is to make a survey of some changes in the aims, techniques, and methods of teaching mathematics in the secondary schools and to some philosophical bases underlying the changes.
Limitations of the Study

No attempt will be made to give a detailed study of any philosophy but tendencies and trends will be studied. The study of these trends will not be limited to merely those in America, but this phase will be universal. The practical application of philosophical aims to the teaching of secondary mathematics will be confined to a study of the secondary schools in the United States.

Source of Data

The data for this study have been taken from various sources and represent a gradual accumulation of ideas as well as facts developed in continued study of education as a graduate student. For the background study, several basic philosophical textbooks were read, and the history of the teaching of mathematics in secondary schools in America was traced in various educational reports made by educational associations. In addition to this, examination was made of textbooks used in various stages of the development of the teaching of mathematics and courses of study for Texas. The aims of education as a whole were surveyed in various educational literature of various kinds. An attempt has been made to organize this data and present it in sequence and in a manner easily understood by the average reader.
Method of Procedure

In a recent article, "Changing Philosophies in the Teaching of Mathematics," Leskow divided the teaching of mathematics into different periods according to the aims. These are meaning and understanding, disciplinary, utilitarian, and the social or cultural. For the sake of unity and coherence, this study has been divided into an examination of each of these types of teaching and the basic philosophical aims underlying it.

The purpose of the study, the limitations, source of material and the plan for developing the examination are given in the first chapter of the study. Chapter II seeks to outline the early teaching of secondary mathematics and the principles that dominated that type of teaching. The formal disciplinary teaching is given attention in Chapter III. The utilitarian aim of education is examined in a fourth part of the study, and the causes are sought in a study of changing social concepts and in the light of new inventions and discoveries. The swing back to the cultural aim in teaching is analyzed in the fifth chapter, and the significance of the lessons learned from World War II is given attention. The findings of the study are given in the concluding chapter.

CHAPTER II

EARLY AIMS AND CONCEPTS OF THE TEACHING
OF MATHEMATICS IN SECONDARY SCHOOLS
IN THE UNITED STATES

The development of the secondary school system in the United States is of comparatively recent origin. In the early days of America, the only free schools established were those of the elementary grades. Universities were early established for the training of men for the professions, but the bridge between the universities and the elementary schools was not bridged by high schools as we know them today. Instead, there were private academies established by the different colonies, and which were open only to the children of the very "well-to-do" people. Mathematics was one of the strongest subjects taught in these private schools, and the purpose in its teaching was to foster reflective thought and develop the powers of reasoning.

The roots of this type of teaching extend back to England, the mother country of most of the colonies. During the latter part of the seventeenth and most of the eighteenth century, a lifeless formalism had grown up in all of Europe in religion and in morals. The revolt that culminated in the establishment of Puritanism in England occurred in the latter part of
the eighteenth century. The Puritans, however, merely set up another kind of formalism in religion, and this was eventually rebelled against by a group of intellectuals who believed that human happiness and a true knowledge of life could only be obtained through the use of the reasoning faculties of the individual.

According to Monroe the aims of this group of intellectuals were:

. . . to liberate the mind from the dominance of supernatural terrorism; to establish the moral personality of the individual, independent of ecclesiastical and social forms; to demonstrate the intellectual freedom and sufficiency of man. . . . The Enlightenment asserted a supreme faith in the reason of the individual, in justice to the state, in toleration in religious beliefs, in liberty in political action, and in the rights of man. The entire period was controlled by a profound belief in the prerogative of the individual, his right to individual judgment, and to the determination of every question uninfluenced by the beliefs and superstitions of the Church and the traditions of society. Freedom of thought, liberty of conscience, sufficiency of reason for the conduct of life, were thus the watchwords and the keys of interpretation of this eighteenth century movement. 1

Those of us who are familiar with the individualistic philosophy of the early American settlers can readily recognize familiar thought patterns in Monroe's discussions of ways of English thinking. Such thoughts had become prevalent in England; it was natural that the people who had the courage to cross a wide ocean seeking freedom of thought and of religion were also imbued with them. They were the kind of people

who thought strongly on most subjects, and who believed in the value of individual reasoning.

Another characteristic of the thought patterns of the early Americans was the tendency to stress the importance of a few. Theoretically, the Americans declared all men "free and equal," but in actual practice the leaders favored an aristocracy of intelligence and wealth. Education was not for the masses but for the favored few. It's main aim was to develop the reflective powers of reasoning of a selected class of individuals.

These feelings of the people extended over into their ways of living and to the methods and aims of education. No early attempt was made to establish schools for all the children of all the people; the academies were founded for the purpose of training students for the universities, and this idea for a long time dominated the secondary free schools that were established. In 1819, a textbook entitled The Scholar's Arithmetic was printed in New Hampshire, which featured meaning and understanding. One of the quotations in the book exemplified the educational philosophy as well as the aims of the mathematics teaching of the period. It was:

As much as possible, endeavor to do everything of yourself; one thing found out by your own thought and reflection, will be of more real use, than twenty things told you by an instructor. Understand everything as you go along.²

²Quoted in Ben A. Sueltz, "Adams Did It 125 Years Ago," The Mathematics Teacher, XXXVI (April, 1943), 183-186.
In other words, the aim of education was to aid the student to understand the world in which he lived, and to figure it out for himself through his own reasoning powers.

The philosophical basis for such types of thought go back to the time of Plato. This philosopher sought truth through the direct vision of reason as developed in the consciousness of man. The duty of each man was to know himself. Through the realization of his own nature, man must work out for himself the things that life is to be lived for. Education should aim to develop the pupil's powers of reflection and thought so that he should know himself, and know best how to plan his life.

The principles laid down by Plato were reborn in the liberal movement of the Renaissance, and education was defined in the same terms as those used by Plato, by Aristotle and Quintilian. The aim of education was to produce the perfect man fitted for taking part in all life's activities.

Paulus Vergerius, a professor in the University of Padus, wrote a treatise on education in 1374 which was widely influential and which was used as a text in schools. In this he formulated the conception of education as follows:

We call those studies liberal which are worthy of a free man; those studies by which we attain and practice virtue and wisdom; that education which calls forth, trains and develops those highest gifts of body
and mind, which ennoble men and which are rightly judged to rank next in dignity to virtue only.\textsuperscript{3}

Two other ideas of education, the practical and the scholastic, were also prevalent at this time, and Vergerius sought to differentiate his idea from these by saying:

For to the vulgar temper, gain and pleasure are the one aim of existence; to a lofty nature, moral worth and fame.\textsuperscript{4}

In the light of these expressions, the little arithmetic known as The Scholar's Arithmetic which was used in the New Hampshire schools in 1819 is highly illuminating and interesting. Adams, the author of the book, advises the users to "Remember that youth, like the morning, will soon be past, and that opportunities once neglected, can never be regained."\textsuperscript{5}

Adams stressed self-reliance and arranged the exercises in such a way as to develop this. Analysis was used to a large extent in solving problems. Adams wanted his scholars to develop independence of attack based upon understanding of the process and its usefulness.

This type of teaching is not generally accredited to this period. In both the Tenth and Sixteenth Yearbooks of the National Council of Teachers of Mathematics, we find such statements as "Children should understand and see the

\textsuperscript{3}Quoted in Monroe, \textit{op. cit.}, p. 166.

\textsuperscript{4}Ibid.

\textsuperscript{5}Quoted in Suelz, \textit{op. cit.}, p. 183.
sense of what they are doing. Adams had this in mind when he wrote this textbook. He advised, "These questions the pupil should be made to study and reflect upon, till he can of himself devise the proper answer."

There were a great many things in the textbook, too, besides simple arithmetic problems. Section III of the book dealt with the treatment of square and cube root, Fellowship, Barter, Loss and Gain, Duodecimals, Measuring Wood, Painter's and Joiner's Work, Gauging, Mechanical Powers, Alligation, Position, and various other things that have been strangers in the modern textbooks. Not many arithmetic teachers of today are familiar with the terms "Alligation" and "Position."

Sueltz states that there were other textbooks by Deboll, Pike, Colburn, and Root which followed the same philosophy as regards the aims of mathematics. He says they are worth re-studying.

The significant thing about this textbook is that it represented a type of teaching which was more or less pragmatic at a time when such a word was an unknown term in education. Generally speaking, the pioneer education of the United States is regarded as an offshoot of the rationalist school of philosophy.

It is true that the dominant philosophy of the early Americans was rationalistic in nature. The great majority

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6 Ibid. 7 Ibid. 8 Ibid.
CHAPTER III

THE DISCIPLINARY CONCEPT OF TEACHING MATHEMATICS AND ITS PHILOSOPHICAL BASE

The roots of the disciplinary philosophy extend back to the Middle Ages. In the various solutions of the moral problem of life offered by Plato, Aristotle, and other Greek philosophers, the key to the riddle was found in the intellectual nature of man. Opposed to this, Christianity offered the solution found in man's moral nature. It substituted new ideals of life and new motives of conduct.

A complete readjustment of social and educational life resulted from the Christians' concept of life. Religion with the Greeks had been chiefly a political concern. They worshipped pagan gods who were never supposed to have had much concern with personal morality and right conduct. Ethics had been associated with philosophy. Under Christianity, religion was dissociated from politics. Ethics and morality became a part of the Christian creed. Moral and religious elements replaced the intellectual, aesthetic, and physical elements in the dominant educational ideals and practices.

Under this new concept of thought, education became a rigid regime in preparation for some future state. All natural interests were to be suppressed through discipline
of the colonists were people who had fled Europe because of religious persecution. Religion was authoritative in nature; it followed established laws, and accepted unquestioningly the thesis that man's destiny was pre-ordained. Since the colonists were deeply religious, it follows that the institutions they established were of an authoritarian nature.

The schools were no exception to this rule. Yet this mathematics text of Adams indicates that an undercurrent of idealism or pragmatism was present. The history of philosophy shows that a philosophy may be dominant at a certain time, but that underneath are always other currents of thought. Almost a century after the time of Adams, James, a great American philosopher, developed a theory which has been called "pragmatism." This theory has transformed the teaching of mathematics in the secondary schools of the United States. To the student of today's developments in mathematics, this type of mathematics teaching in the early days of the United States is revealing.
of self; this discipline was more effective if it was more harsh. In the view of Christianity everything connected with this world and its activities was evil; all consideration for the development of personality and the cultivation of the aesthetic taste or intellectual activity was a gross sin. From the sixth to the thirteenth century, the intellectual element was practically eliminated from education, and when it did reappear it was under the guise of a formal training which paid more attention to form than intellectual thought.

This Christian philosophy was the dominant one through the Middle Ages, and it materially influenced the development of education. Intellectual life was bound within narrow limits. The purpose of education was instruction in church rituals and church doctrines. Abasement of self was the major purpose. For these reasons education was conceived as a discipline. The harder it was, the more effective it was. It could not be based on interest, for that would defeat its purpose. Therefore, it was formal, rigorous, and repressive.

Even after the revolt of the Protestant Churches from the Roman Catholic Church, this doctrine or disciplinary philosophy still prevailed. The Puritan doctrine was a direct outgrowth of the religious differences that prevailed in England between the Established Church and people who had the courage to differ with its doctrines. The Puritans were
the first people to settle in the new world, America, and their religion was the main purpose in their lives. They wanted freedom to worship as they pleased, but this worship was according to rules the same as that of the Roman Catholic Church or the Established Church of England. The Puritans themselves had made their rules, and they were strict observers and permitted no deviation.

The first schools in the new land were established by the Puritan immigrants. These were for the instruction of the very young and for the training of ministers for the churches. Since the Puritan doctrine was one of stern discipline, it is to be expected that the school curriculum would be also stern and disciplinary.

Scholasticism, in many ways, was closely akin to this disciplinary concept of life. In the beginning, Scholasticism was a revival of the liberal learning of the Greeks, but it degenerated into a study of the Greek classics for the value of their form and style and not their content. In this respect, drill and analysis were considered more valuable than searching for thought content. This conception of education took root in the Renaissance and spread all over Europe in the centuries that followed. It fastened this type of teaching in the schools, and made education a dull, lifeless affair in which the word "interest" was an unknown quantity.¹

The Puritan fathers, then, when they came to the new world, were imbued with a Christian belief that discipline was a worthy aim of education and that this education should be formal and consist of exercises and drills without too much stress on the thought content of the subject matter. The schools that they established reflected these aims and the subject matter and methods of teaching were likewise affected.

Smith, in a discussion before the National Council of Teachers of Mathematics, in 1925, described the ways in which arithmetic, an integral part of mathematics, was taught.² He said that it was felt, by early educators of the disciplinary school, that the subject should be hard in order to be valuable, and that it did not make so much difference what the subject was; the object was to make the student hate it. The old idea that this was good for the mind and the soul still existed. There was also prevalent the idea that as many applications of arithmetic should be introduced as the time allowed. The fact that the pupil did not understand many of these problems and that they were far removed from his daily life was of no significance.

Algebra, Smith said, was taught as if it were a purely mathematical discipline "unrelated to life except as life

might enjoy the meaningless puzzle. The pupils regarded it as a fairly interesting way of getting nowhere.

The algebra textbook was arranged on the same principle.

On the theory that we must scientifically define all terms before they can safely be used, the algebra textbooks began with a long series of definitions which were to be memorized without much relation to the problem to be worked later. Next in order a number of strange terms—monomial, binomial, residual, polynomial, coefficient, and exponent—were introduced to be learned for future use. There were no attempts to develop the real applications of the science, but algebra was taught as a mental gymnastic exercise. A typical problem is given below:

Bought 3 horses, a number of cows, and 100 sheep for $2500.00. The number of cows was equal numerically to 4 times the price of a sheep, and a sheep and a horse cost $5.00 less than 1/5 the cost of all the cows. Find the cost of a horse, and a sheep, and the number of cows, if a cow costs $40.00.

Geometry, in the disciplinary type of school, consisted of at least one year of plane geometry, following the course in algebra, and at least a half-year in solid geometry. In most schools there was a great deal of memorizing of demonstrations and the original exercises were not stressed heavily. They were regarded as an intellectual grind, without either purpose or pleasure.

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3Ibid. 4Ibid., p. 24.
Algebra and geometry and arithmetic as mathematics were all taught separately as if there was no connection at all between them. This method of teaching was called the "water-tight compartment" method. The student seldom realized the connection or relationship existing between different subjects. It was believed that mathematics in the secondary schools should be studied for the habits of logical thinking, precise and accurate work, and that it would aid pupils in other subjects and establish certain life habits.

The foregoing, in general, is the picture of secondary mathematics as taught in the secondary schools of the United States prior to 1900. In some sections of the country, especially in the early Puritan schools, the disciplinary ideas prevailed from the beginning. In others, in the colonies established by the wealthy and more cultured people, the liberal type of teaching prevailed for awhile. Gradually the disciplinary concept spread until it dominated the curriculum of all the schools.

This disciplinary concept it is seen, was an integral part of the philosophy of rationalism, and stressed authority, conformity, and formalism. Little attempt was made to link the educations of the youth to his daily interests or to his practical needs. Skills and not attitudes were sought, and style instead of content was the goal. Philosophy, it is apparent, was the motivating force in determining the
educational aims, and as such influenced the purposes, means, and techniques of teaching secondary mathematics in the United States for more than half a century.
CHAPTER IV

THE UTILITARIAN CONCEPT OF TEACHING MATHEMATICS AND ITS PHILOSOPHICAL BASE

The beginnings of the utilitarian aims in education go back to the days of old Roger Bacon, the great Franciscan monk philosopher of England. Bacon lived in the thirteenth century and he even then caught a glimpse of the coming changes in education and in the character of the intellectual life of the nation. He held that truth could be reached by experiments a thousand times better than by reading in books. He said if he could have his way he would burn all the books of Aristotle for the study of them could only be a waste of time.\textsuperscript{1} He would teach the things that could be of direct use to the pupil and things he could learn for himself from nature.

Another Bacon in the seventeenth century in England was to give new weight to this utilitarian view. Francis Bacon, English philosopher, developed the theory that knowledge of nature was the source of all power. Observation, investigation, experimentation, were the chief means of acquiring knowledge. He said that knowledge could not be gained by the old

\textsuperscript{1}James Harvey Robinson, \textit{A History of Western Europe}, p. 273.
scholastic method of definition and drill and that truth is not reached by the mere accumulation of a number of similar instances. 

This doctrine of Bacon's gave a new tendency to education and the consequences were that there was a veering away from the formalism of the old learning toward the realism of the new based on experimentation and science. From dealing with words and abstractions, education came to deal with objects and ideas.

Bacon advanced still another idea that was to have a profound effect upon education. This was his desire to reorganize the entire realm of human knowledge for the improvement of the human race. This thought for human welfare was a new note in world society. It is true that Christianity had been founded upon the principles of human welfare, but this had been a matter of religion and was not a social or educational aim. From this time on more and more will be heard of educating the individual for citizenship, of aims for bettering the general welfare of all the people. Education takes on a hitherto unknown social value.

Out of these philosophical opinions and reasonings came a new concept of knowledge, a new concept of how it should be gained. Comenius, one of the great early European teachers, formulated the following rules for teaching:

\[\text{Monroe, op. cit., p. 231.}\]
1. Whatever is to be known must be taught (that is, by presenting the object or the idea directly to the child, not merely through its form or symbol).

2. Whatever is taught should be taught as being of practical application in everyday life and of some definite use.

3. Whatever is taught should be taught straightforwardly, and not in a complicated manner.

4. Whatever is taught must be taught with reference to its true nature and its origin; that is to say, through its causes.

5. If anything is to be learned, its general principles must first be explained. Its details may then be considered, and not until then.

6. All parts of an object (or subject), even the smallest, without a single exception, must be learned with reference to their order, their position and their connection with each other.

7. All things must be taught in due succession, and not more than one thing should be taught at a time.

8. We should never leave any subject until it is thoroughly understood.

9. Stress should be laid on the differences between things, in order that what knowledge is acquired may be clear and distinct. 

Under the impact of such writings as these, the entire concept of education began to change gradually. Like all new ideas its progress was slow, but it smoldered under the formal scholastic teachings, and gradually emerged in the nineteenth century as a full-blown educational theory which was to change almost overnight the accepted formal theory of teaching.

Practically all of the subjects taught in school came under the searching glare of the new aims and concepts. Mathematics, because it had been taught in such a formal abstract fashion and because it so closely touched the lives of all

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3Quoted in Monroe, op. cit., p. 242.
the children, was given a severe drubbing at the hands of educators. Swenson said:

Mathematics . . . is too mechanical and formal. The reasoning is deductive while practical living demands induction. It is artificial. The time at which it is studied, the time devoted to it, the manner in which it is taught, the amount to be covered, are determined by tradition, not by any consideration of the need of the child. The subject is difficult.\(^4\)

In England Perry started a movement in 1901 to discard the pure mathematical syllabus that had been used in the formal drill teaching and to find a new method, with the idea that usefulness must determine what subjects should be taught and in what ways. He did not discard the teaching of pure mathematics, but he wanted to reserve this type of teaching for those who were mathematically inclined. Formerly the aim, it appeared, had been to make all boys mathematicians. The consequences of such teaching were that many of the boys, because they could not comprehend the subject, learned to hate it. Mathematics was the "schoolboy's horror."

Perry advocated letting the boys find out things for themselves because that discovery is good for them and is of more real value. He maintained that people use mathematics in their everyday life, and that the school children should be given the mathematics that they were going to need in

their practical activities. In other words, the subject should be made utilitarian.

Perry also favored education for citizenship. "He believed that everybody should study mathematics not just for knowledge but for scientific habits of thought. He said:

I believe that men who teach demonstrative geometry and orthodox mathematics generally are not only destroying what power to think already exists, but are producing a dislike, a hatred, for all kinds of computation, and therefore for all scientific study of nature, and are doing incalculable harm."

In America in the beginning there was no separating of arithmetic, algebra, and geometry into separate "water-tight" compartments, but by 1787 a tendency began to be evident to draw a sharp line of separation between the branches of mathematics. Mathematics was taught at first mainly in the colleges, and the subject was first separated into parts here. Arithmetic, algebra, and geometry were taught as separate subjects, and when the colleges began to require these subjects for college entrance the practice of separation was handed down to secondary schools. The colleges in setting up their entrance requirements, designated exactly the subject matter to be covered and very often the textbooks to be used. The subject matter was that which was taught in

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the colleges, and very often the textbooks recommended for secondary teaching were those used in the colleges. In this way the courses of study which were written and developed by college professors for college students were introduced into the secondary schools. Under such conditions it was natural that the subject matter should be arranged logically rather than psychologically. For these reasons our high schools inherited courses of study wherein mathematics was broken up into "water-tight" compartments.

The results of such a program were that high school curricula were formal and the teaching was of the same pattern. The separation of mathematics into separate studies and the stress placed on discipline and drill did not tend to popularize the subject.

There were changes in the social and economic life of the world, too, that were conducive to educational change in a great many ways. In the beginning, life was comparatively simple in the United States. When the outlines of our school system were formed there was not a great separation into rich and poor; there were no great railroads, no large cities, no great industrial enterprises. The invention of the steam engine, the telegraph, the telephone, the railway, electric lights, and numerous others changed the country from a rural one to a highly complex, industrial civilization.
These changes had a great effect on all manners of life, and not the least of these were upon the schools. For one thing, they altered the number and types of children that attended school. In the colonial and pioneer days of America, there were no great systems of public schools like we have today. The secondary schools, in particular, were private schools and only the children of the very well-to-do attended. A great many of these children became professional people of some kind, and the type of training offered them was intellectual. In most instances such training was not amiss. But with the development of the country, and the use of machines there were a great many more children free to attend school. When every small home was a factory and when farming all had to be done by hand, the children were too busy to attend secondary school in particular. In the average family the main purpose of education was achieved when the children learned the rudiments of education - the three R's.

The industrial age, however, changed this picture. Manufacturing was taken over by great factories, and power machinery of all kinds eliminated much of the labor hitherto required of children on the farms. Then, too, the tendency in the new world had been towards universal education and to this end schools had been established and fostered. The free public school system was well established by 1875 in the United States. As the population increased, and the number
of children who attended school also increased, the growth of the high schools was phenomenal. In formulating the curricula for these different schools, teachers had to take into consideration that they were no longer dealing with a select group of children, but children in the mass.

This universal education has differentiated the problems of education that we have had to face in the United States from those of European countries where a more select group of children have been in the schools.

Philosophy, too, had a great deal to do with the changing viewpoint on methods and aims of teaching. John Dewey, a great American philosopher, advanced the premise that "education is life"; it is not a preparation for life, but it is life itself. In the traditional school the teacher was the autocratic dispenser of the school program, and it was a "cut-and-dried" formal process. In the new education recommended by Dewey, the teacher must present the subject matter in such a way as to satisfy the needs of the learner. In order to know these needs, the teacher must have a knowledge of the laws of learning, of the individual students, and of their environmental history and needs.

The teachers of the country became more aware themselves that some changes would have to be made in education

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7John Dewey, Experience and Education, p. 85.
to meet the changing conditions in society. On August 26, 1857, an association of teachers was first formed at Philadelphia, Pennsylvania, under the name of The National Teachers Association. This Association continued to grow, and in 1907 it was incorporated as the National Education Association of the United States. Its purposes were stated as follows:

Purposes--To elevate the character and advance the interests of the profession of teaching, and to promote the cause of popular education in the United States. 8

This Association has been one of the most influential factors in investigating the need for changes in educational procedure and in recommending changes. It early gave its attention to the improvement of the teaching of mathematics in the United States. As early as 1893 the Association appointed a Committee of Ten to investigate and recommend a course of study for the teaching of mathematics in secondary schools. The same Committee also recommended parallel courses in algebra and demonstrative geometry in the tenth and eleventh grades, but there was nothing said about uniting the different subjects or making them practical.

Another strong association which has influenced the development of mathematics is that of the American Mathematical Society. In its ninth meeting in Chicago in 1902 a university

professor, E. H. Moore, made a speech which greatly changed the aims and methods of teaching mathematics. As a pure mathematician, he told the society that he favored correlation of the teaching of mathematics with the problems of physics and chemistry and engineering. He thought that such teaching would make it impossible to give very young students a great body of the essential notions of trigonometry, analytical geometry, and the calculus. He thought that the fundamental problem in the pedagogy of elementary mathematics was the unification of pure and applied mathematics. He said:

Engineers tell us that in the school algebra is taught in one water-tight compartment, geometry in another, and physics in another, and that the student learns to appreciate (if ever) only very late the absolutely close connection between these different subjects and then if he credits the fraternity of teachers with knowing the closeness of this relation, he blames them most heartily for their unaccountably stupid way of teaching him. . . .

Would it not be possible to organize the algebra, geometry, and physics of the secondary school into a thoroughly coherent four year course, comparable with the strength and closeness of the course in Latin. 9

This was a clear-cut expression for the formulation of definite courses of study in the teaching of mathematics. There had been some exponents of the correlation theory prior to Moore, and his espousal of the cause created a favorable impression. The interest that was generated by Moore's talk culminated in a number of organized associations of mathematics teachers for study.

One of the most active of these organizations was the
Central Association of Science and Mathematics Teachers.
Not long after its organization in Chicago in 1905 the math-
ematics section became interested in the reorganization of
mathematics in the secondary schools. In 1906 a Committee
of Five was appointed to prepare a course of study in alge-
bra. This Committee prepared a report the next year and
published it in the official organ of the Association. The
Committee suggested that the connection between arithmetic
and algebra be made as close as possible and that mensura-
tion problems of areas and volumes be used in the algebra
course. The Committee made the following statement in re-
gard to the unification of the different mathematical sub-
jects:

To bring the different branches of mathematics
into closer relationship and to preserve the unity
of mathematical work is so much to be desired in
secondary schools that the simultaneous teaching of
arithmetic, algebra, and geometry would seem indis-
pensable. By this is not meant that so many hours
per week be devoted to one subject and the remain-
ing hours to another - not parallel teaching - but
the actual blending of the subjects wherever the
relations are so close that such a blending will be
forced.\textsuperscript{10}

The direct outgrowth of this report was the appointment
of another committee to study the unification of mathematics.

\textsuperscript{10}Charles Ammerman and Others, "Preliminary Report of
the Committee of the Central Association on Algebra in the
Secondary Schools," \textit{School Science and Mathematics}, \textit{VII} (1908),
674-685.
The preliminary report of this committee was published in *School Science and Mathematics* before the annual meeting of the Association in 1908.\textsuperscript{11} This report stressed the need for breaking away from tradition and from a cut-and-dried method of teaching. Arithmetic, algebra, and geometry should be taught in such a way as to emphasize their relations to each other. Unification of mathematics was also stressed.

These reports, discussions and articles started a wave of correlated teaching of mathematics in the secondary schools. So much emphasis was placed on correlation by some enthusiasts that many teachers began to react unfavorably to correlation. A greater demand rose for a reform in the teaching of mathematics.

The Mathematical Association of America appointed a committee in 1916 of the leading high school and college mathematics teachers to study the problem of the teaching of mathematics. One of the factors that influenced the course of the discussions was the great number of students attending the secondary schools. The committee was asked to study the situation of meeting the needs of all these students and to determine some program of teaching mathematics to all the children of all the people.

A preliminary report was made in 1920 by the Committee, and the final report was published in 1923. This report recommended some fundamental changes in the teaching of mathematics in the secondary schools. It recommended that arithmetic, intuitive geometry, algebra, numerical trigonometry, and a small amount of demonstrative geometry be taught in the junior high schools. The aim of the Committee was stated. It follows:

To the end that all pupils in the field of secondary education shall early gain a broad view of the whole field of elementary mathematics, and, in particular, in order to insure contact with this important element in secondary education on the part of very large numbers of pupils who, for one reason or another, drop out of school by the end of the ninth year, the National Committee recommends emphatically that the course of study in mathematics during the seventh, eighth, and ninth years contain the fundamental notions of arithmetic, of algebra, of intuitive geometry, of numerical trigonometry, and at least an introduction to demonstrative geometry, and that this body of material be required of all secondary school pupils.\textsuperscript{12}

Mathematics, it will be seen, was no longer to be considered in "water-tight" compartments but was to be presented in such a way that the pupil would gain a broad view of the whole field of elementary mathematics early in his high school course. Schorling defines the new method in this way:

General mathematics is an introductory, basic exploratory course in which the simple and significant principles of arithmetic, algebra, geometry,

\textsuperscript{12}The Mathematical Association of America, The Reorganization of Mathematics in Secondary Education, A Report by the National Committee on Mathematical Requirements, p. 3.
intuitive geometry, statistics, and numerical trigonometry are taught so as to emphasize their natural and numerous interrelations. 13

The aims of the new type of teaching mathematics were listed as: (1) Practical or utilitarian, (2) disciplinary, and (3) cultural. The one general aim was:

The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and of space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and of action which will make these understandable. 14

It is evident from this literature that the old formal aim of teaching mathematics was completely outmoded, in theory at least. Mathematics was to be practical, it was to give the student an understanding of the importance of mathematics, and it was to be a discipline in that it developed mental habits through the necessary operations which were valuable through transfer to other situations.

In many respects these aims were the direct outgrowth of the changing philosophies of the times. John Dewey's outspoken comment that "Education was life" put a completely new aspect on all kinds of teaching. Dewey held that education is growth, not a process of learning a mass of facts

13 Raleigh Schorling, "Suggestions for the Solution of an Important Problem that has Arisen in the Last Quarter of a Century," The First Yearbook, National Council of Teachers of Mathematics, p. 100.

14 A Report of the National Committee on Mathematical Requirements, p. 4.
and figures. Life, he said, is development, and development and growth are life. The education process was an end in itself, and the educational process was one of continued reorganizing, reconstructing, and transforming.

These ideas of Dewey's had a profound influence on the development of education. They transformed methods and aims. For example, he thought that values were instrumental. They help a person to gain ends. Progress occurs if these ends are gained. He asserted that the aim of education, and that the object or reward of learning is continued capacity for growth. Accordingly, the aims of education were reformulated in many instances by educational associations and these aims held up as standards for formulating curriculums.

The report of the Committee on the Reorientation of Secondary education recommended that general mathematics be taught in the junior high school grades instead of by separate subjects. It also recommended that much of the emphasis which formerly had been placed upon formal exercise be shifted to the concrete or "verbal" problem. The practical problems should be closely correlated with the child's real life and fit into life situations. The problems should be selected also to correlate the work in mathematics with the other courses of the curriculum, especially in connection with courses in science.
The Committee recommended a definite course of study for this general mathematics, and stated that all the children attending school in grades seven, eight, and nine be required to take it. There was no certain formula prescribed for teaching, but it was suggested that all topics, processes and drills which did not directly contribute to the development of the desired powers be eliminated. The Committee stated that it felt that logical organization of subject matter was of less importance than the acquisition, on the part of the pupils, of experience as to facts and methods of attack on significant problems, of the power to see relations, and of training in accurate thinking in terms of such relations.

In its recommendations for the teaching of mathematics in the tenth, eleventh, and twelfth grades, the Committee based its material on the principles outlined for the teaching of general mathematics, but qualified its recommendations as follows:

1. At this period, some attention should be given to the student's vocational or later educational needs.

2. The material for these years should include those mathematical ideas and processes that have the most important applications in the modern world.

3. An increasing amount of time should be directed to the logical organization of the material.
It was also recommended that electives be given to those students who had satisfactorily completed the junior high school work as outlined.

This was the new program as set up by the Committee on the Reorganization of Secondary Education and based on the new pragmatic philosophy that all education is life and growth combined. The next chapter will attempt to evaluate some aspects of this concept of teaching and the effects it has had upon the mathematical training given by the schools.
CHAPTER V

SOME RESULTS OF THE NEW PROGRAM OF TEACHING
MATHMATICS IN THE SECONDARY SCHOOLS

There will be no effort to evaluate the results of
the new program of teaching general mathematics in the
junior high schools and of permitting senior high schools
to let the students "elect" whether to take any more math-
ematics than for the necessary credits for graduation.
Rather, a number of recent reports by different investigat-
ing bodies will be presented as well as a number of views
from teachers in the field of mathematics. We are too
close to the movement to evaluate it and can only point
out future possible developments.

The Report of the Committee for the Reorganization of
Secondary Education was made in 1923. Betz, writing in
1940, stressed the national picture of mathematics in the
United States at that time.¹ He quotes Bell:

In the coming tempest only those things will
be left standing that have something of demonstrable
social importance to stand on. . . .

¹William Betz, "The Present Situation in Secondary
Mathematics with Particular Reference to the New National
Reports on the Place of Mathematics in Education." The
Mathematics Teacher, XXXIII (August, 1940), 339-360.
The harsh attrition has already begun. Are not mathematicians and teachers of mathematics in liberal America facing the bitterest struggle for their continued existence in the history of our Republic? American mathematics is exactly where, by common social justice, it should be - in harassed retreat, fighting a desperate rear-guard action to ward off annihilation. Until something more substantial than has yet been exhibited, both practical and spiritual, is shown the non-mathematical public as a justification for its continued support of mathematics and mathematicians, both the subject and its cultivators will have only themselves to thank if our immediate successors exterminate both.

Taking a realistic view of the facts, anyone but an indurated bigot must admit that mathematics has not yet made out a compelling case for democratic support. . . . This must be done, and immediately, if mathematics is to survive in America.2

In other words, mathematics had not justified its existence in a democracy. It would have to make out a better case than it had if it survived.

A few years before this, Betz stated, a Professor Seidlin published the findings based on a questionnaire sent to all the State Superintendents of Public Instruction in the United States.3 He reported that nearly twenty states, including New York State, were no longer requiring a single hour of mathematics beyond the eighth school year. The survey showed that courses in algebra and geometry, as state required courses for graduation from high school were "losing ground." Among the reasons listed for this development, the following were given most frequently by the superintendents:

2Ibid.

3Ibid.
(1) lack of clearly defined and convincing objectives;
(2) poor teaching; and (3) inadequate textbooks:

Betz lists other factors as:
1. A one-sided emphasis on "social utility".
3. The absence of a clearly formulated and generally
accepted philosophy of education.
4. The doctrines of "progressive education", with
their emphasis on immediate experience, individual interest
and "felt" needs, and the disregard of race experience and
sequential learning.
5. A policy of incoherent curriculum revision based
largely on momentary interests, "social reconstruction,"
superficial "orientation", to the exclusion of continuity
and foundational training in essential lines of work.
6. The problem of mass education, with the resultant
attempts at "adaptation" to individual needs and interests,
all of which attempts have been unsuccessful because they
have ignored basic causes and problems, have rejected stan-
dards, and have preferred an inconsequential tinkering with
opportunist expedients or surface adjustments.
7. A wrong psychology based on a mechanistic concep-
tion of the mind and avoiding real understanding, thus making
the ruination of mathematical instruction almost inevitable.
8. A narrowly specific and hence inadequate training of secondary teachers.

9. The uncertain economic outlook, with a resulting aimlessness and lack of enthusiasm among millions of our young people.

Betz was writing in 1940 a year before Pearl Harbor. Tragically, this was the event that precipitated a test of the opinions he expressed. Was he right or wrong? Did a democracy have any use of mathematical skills, or was it something that could be discarded at will and thrown in the ash heap?

The answer to this question is such recent history that it is obvious. World War II, with its powerful mechanized instruments of warfare, its chemicals, its great air forces, its atom bombs, called for mathematical ability of high order from the soldiers who were called to fight. They did not have it. Calls from the Army and the Navy for the schools to set up special training programs were broadcast, and appeals were made to the schools to teach the needed mathematics.

In 1943, a Committee of the United States Office of Education conferred with Army officers in charge of training enlisted men and made a report on the needed mathematics in which the boys were found lacking. This Committee then reported to the mathematics teachers of the nation the
minimum mathematical needs and made general recommendations on the subject. The subject was considered of such importance that the whole issue of *The Mathematics Teacher* of October, 1943, was devoted to it and recommendations for teachers. The Army Service Forces set up schools all over the nation to instruct soldiers in the needed mathematical skills. There is no doubt that the war effort was considerably slowed down by the lack of knowledge of mathematics which was needed in so many theatres of war. It was found that a democracy had a very real and special need for mathematics.

Within the last month, July, 1946, a planning commission of the National Council of Teachers of Mathematics has served notice that it intends to recommend stiffer courses in mathematics for the nation's high school students. The commission stated that in the recent war many boys from good high schools lacked familiarity with such elementals as scale drawing, maps, ratio, decimal fractions and geometric relatives. It is going to recommend that all high school students be required to have this knowledge upon graduation.

It appears as if the struggle between two schools of philosophic thought is still under way. We might call the first school that of rationalism. It is represented by the scholastic philosophers who stressed formalism, drill, and

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4The Fort Worth Star Telegram, July 6, 1946.
and style, and who stressed the presence of authoritative governing laws. The second school of thought has been represented by the modern pragmatic movement which holds that a thing is good or bad according to the use made of it and that there is no fundamental law preordaining it. Rationalists perhaps would be the word for describing those who would hold fast to fundamental principles; idealists for those who would experiment with developments from within unregulated by essential fundamentals.
CHAPTER VI

CONCLUSIONS

The study of the influence of some philosophies upon the teaching of mathematics in secondary schools has been very revealing. The following conclusions have been drawn from the study:

1. The social conditions of any country condition the philosophy of this country.

2. The philosophy of any given country materially influences the aims and practices of education.

3. The aims in teaching mathematics in the secondary schools have varied with the philosophies prevalent at different times.

4. In certain parts of the colonial America, a cultural aim dominated the teaching of mathematics in the pioneer era.

5. In certain other areas, the disciplinary concept dominated, and this gradually smothered out the cultural aim of some of the early schools.

6. The disciplinary aim of teaching mathematics stemmed directly from the philosophy of scholasticism or rationalism.

7. The utilitarian or social aim of teaching mathematics in the secondary schools stemmed from the pragmatic teachings
of John Dewey and from the concepts of the modern age.

3. If educational reports made by mathematics teachers are to be accepted, both systems of teaching mathematics in the secondary schools have broken or fallen down. At the present time there is a demand for the reorientation of the mathematic program for the secondary schools.

If a mathematics teacher who has experienced both types of teaching were to be allowed to prophecy as to what will be done in the future it would probably be something like this: There will be a middle ground found between the two, and the type of teaching that eventually emerges will be far superior to either yet developed.

This statement is based on the belief that the two theories of educational philosophy are necessary in the successful teaching of mathematics. The very nature of mathematics itself presupposes the existence of some fundamental laws governing its operation. The nature of the student, on the other hand, is such that he will do better work if he is interested in the subject matter, and if it has some practical relation to his everyday activities. One severe critic of the present system lays much of the blame at the door of inefficient, indifferent teachers. The improvement of the teachers' efficiency and the arousal of a genuine desire to teach mathematics as a necessary science may be the answers to many of the questions troubling educators in the field of mathematics today.
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