

Ultrahigh energy neutrinos, small x and unitarity

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The ultrahigh energy cross section for neutrino interactions with nucleons is reviewed, and unitarity constraints are discussed. We argue that existing QCD extrapolations are self-consistent, and do not imply a breakdown of the perturbative expansion in the weak coupling.

I. INTRODUCTION

Ultrahigh energy neutrinos are predicted from a number of sources. One source is from cosmic ray interactions with the microwave background radiation [1], producing charged pions which decay into neutrinos. Another possible source is decaying cosmic strings or extremely massive relics [2], which ultimately contribute to a cosmic neutrino flux. Detection of ultrahigh energy neutrinos may shed light on the observation of air shower events with energies in excess of 10^{11} GeV, reveal aspects of grand unification or yield some insight into the sources of the highest energy cosmic rays.

A number of detectors are able or will be able to detect neutrino induced showers. For example, the Auger experiment should be able to detect neutrino induced horizontal air showers initiated by neutrinos with energies above 10^9 GeV. The proposed OWL/EUSO satellite experiments should be able to detect upward air showers produced by $\nu_\tau \rightarrow \tau$ just below the Earth's surface. The event rates predictions depend on the ultrahigh energy neutrino cross section, extrapolated beyond the measured regime, as well as on the predicted neutrino fluxes.

Recent discussions by Dicus, Kretzer, Repko, and Schmidt [3] about the implications of perturbative unitarity have refocused attention on the ultrahigh energy extrapolation of the neutrino-nucleon cross section. In the next section we review the cross section evaluation including the extrapolation of the parton distribution function to small parton momentum fraction x . We examine to what extent the cross section may be sensitive to the presence of saturation effects in the evolution of the parton distributions. In the following section, we outline the unitarity argument and comment on what can and cannot be learned by relating the neutrino-nucleon forward scattering amplitude to the total neutrino-nucleon cross section.

II. NEUTRINO-NUCLEON CROSS SECTION

The expression for the neutrino-nucleon charged current cross section, for $\nu_l(k)N(p) \rightarrow l(k')X(p')$, is

$$\frac{d^2\sigma}{dx dQ^2} = \frac{G_F^2}{\pi} \left(\frac{M_W^2}{Q^2 + M_W^2} \right)^2 \cdot [q(x, Q) + (1-y)^2 \bar{q}(x, Q)] , \quad (1)$$

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in terms of $Q^2 = -(k - k')^2$, $x = Q^2/(2M\nu)$ and $y = \nu/E$, for neutrino energy E and lepton energy transfer $\nu = E - E'$ defined in the nucleon rest frame. The nucleon mass is M , and the center of mass energy squared is $S = 2ME$. In Eq. (1) we have followed Refs. [3, 4] to introduce effective quark and antiquark densities that contain the contributions from the various flavors as well as the appropriate electroweak mixing angles. The expression for the neutral current reaction $\nu_l(k)N(p) \rightarrow \nu_l(k')X(p')$ can be cast into an identical form, with the obvious replacement $M_W \rightarrow M_Z$ and with different effective quark densities. In what follows, we will only consider the neutrino-nucleon cross section. At high energies, the antineutrino cross section is expected to be very similar. We will neglect perturbative QCD corrections to the cross section which were found to be small in Refs. [3, 5]. Finally, we also neglect the contributions to the cross section arising from charm quarks in the initial state; these can be sizable at high energies, but are unimportant for our more qualitative purposes.

In Eq. (1), increasing Q^2 has two effects: as Q^2 rises, the cross section decreases due to the W propagator, but the contributions of the quark and antiquark distribution functions $q(x, Q)$ and $\bar{q}(x, Q)$ increase due to QCD evolution [6]. The propagator dominates and effectively cuts off the growth in Q^2 at $Q^2 \sim M_W^2$. As a consequence, the typical x value as a function of incident neutrino energy E is $x \sim M_W^2/(2ME)$, so ultrahigh neutrino energies translate to small parton x . For the highest neutrino energies considered, $E \sim 10^{12}$ GeV, $x \sim 10^{-8}$. HERA measurements of structure functions [7] extend to $x \sim 10^{-6}$; however, such low values of x are measured at $Q^2 < 0.1$ GeV². For $Q \sim M_W$, the structure functions are measured down to $x \sim 10^{-3}$ in the D0 experiment's analysis of inclusive jets [8]. Small x extrapolations of the parton distribution functions are therefore necessary to extend the predictions for the neutrino-nucleon cross section above $E \sim 10^7$ GeV.

Perturbative QCD governs the small x extrapolations. The sea quark distributions dominate the cross section at high energies. Sea quarks are produced by gluon splitting $g \rightarrow q\bar{q}$, so the gluon distribution $g(x, Q)$ dictates the eventual quark and antiquark distributions at small x . The gluon distribution is parameterized as $xg(x, Q_0) \sim x^{-\lambda}$ for $x \ll 1$ at a reference scale Q_0 . Approximate small- x DGLAP evolution [9], for λ close to 0.5, yields a gluon distribution function of the same form, at a larger value of Q : $xg(x, Q) \sim x^{-\lambda}$ [10]. As a practical matter, λ was determined at $Q = M_W$ to extrapolate the parton distribution functions [4], for example, those by the CTEQ collaboration [11], below $x = 10^{-5}$. Glück, Kretzer, and Reya [5] have checked that the full DGLAP evolution of the Glück, Reya, and Vogt [12] distribution functions yields only some 20% difference at $x = 10^{-8}$ compared with the power law extrapolation of the CTEQ densities. Kwiecinski, Martin, and Stasto [13] have performed a BFKL-type [14] evolution, yielding results in substantial agreement at the highest energies considered (10^{12} GeV). The resulting total neutrino-nucleon cross sections can be parameterized by power laws for 10^7 GeV $< E < 10^{12}$ GeV. For example, the charged current neutrino-nucleon cross section, using the CTEQ4 parton distribution functions, scales as $\sigma = 5.5 \times 10^{-36} (E/\text{GeV})^{0.36}$ cm² [4].

Ultimately, the growth of the parton distribution functions – and hence that of the cross section – predicted by both DGLAP and BFKL evolution will have to slow down when the gluon densities become large enough that $gg \rightarrow g$ recombination processes become important [15]. The regime in x, Q^2 in which this happens is referred to as “saturation region” and can be estimated. It can be roughly characterized by the condition [16]

$$Q_s^2(x) = (1\text{GeV}^2) \cdot \left(\frac{x_0}{x}\right)^\lambda \quad (2)$$

with λ and x_0 obtained from a fit to HERA data: $\lambda = 0.288$, $x_0 = 3.04 \times 10^{-4}$. Q_s is the “saturation scale” – saturation should roughly occur when $Q < Q_s(x)$ at a given x . Clearly, as x decreases, saturation effects are expected to become relevant already at larger and larger Q^2 .

In Figs. 1 and 2 we examine whether the neutrino-nucleon cross section at ultrahigh energy, $E = 10^{12}$ GeV, might be sensitive to such saturation effects. Figure 1 shows $\log_{10}(d^2\sigma/d\log_{10}(x)d\log_{10}(Q^2))$, evaluated using the parton distribution functions of [12], as a function of $\log_{10} x$ and $\log_{10} Q^2$. It is evident that scales $Q \sim M_{W,Z}$ dominate the cross section. Note that the parton distributions of [12] can be used also down to rather small $Q^2 \sim 1$ GeV². This is convenient because, in order to obtain the total neutrino-nucleon cross section one needs to integrate the expression in Eq. (1) over the range $0 < Q^2 < xS$. For $Q < 1$ GeV, we have frozen the scale in the parton distribution functions to 1 GeV. Fortunately, as can be seen from Fig. 1, the region of $Q^2 < 1$ GeV² contributes only very little to the total cross section.

The line in the x, Q^2 plane corresponding to the saturation condition, Eq. (2), is also shown in Fig. 1. In Fig. 2 we follow Ref. [13] to look at a projection onto the x, Q^2 plane. The contours are the lines at which the cross section $d^2\sigma/dx dQ^2$ has fallen to $10^{-31-0.5n}$ cm², where $n = 1, \dots, 6$. From these figures, it becomes evident that, even at the highest neutrino energies, contributions to the cross section resulting from the regime sensitive to gluon recombination effects are marginal. For our example at $E = 10^{12}$ GeV, the “saturation region” contributes far less than 1% to the total cross section.

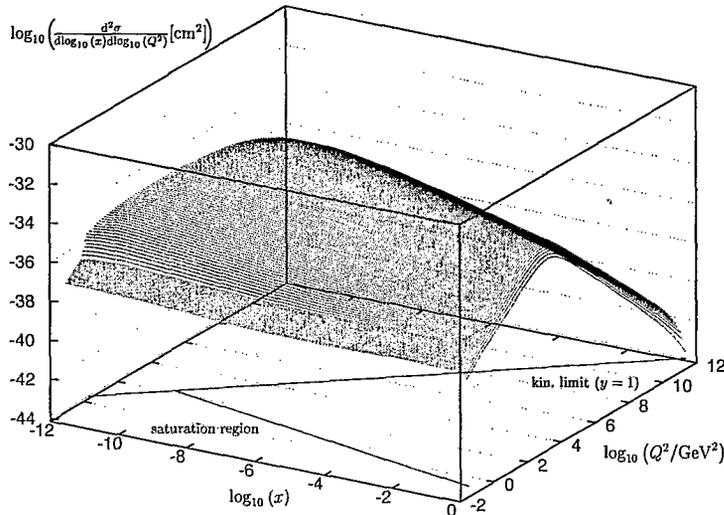


FIG. 1: The neutrino-nucleon cross section $d^2\sigma/dx dQ^2$ at $E = 10^{12}$ GeV as a function of x, Q^2 . The “saturation region” is derived from Eq. (2).

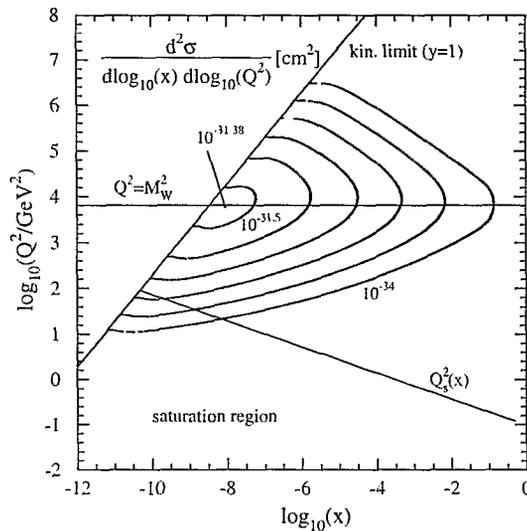


FIG. 2: Contour plot of the neutrino-nucleon cross section in Fig. 1 in the x, Q^2 plane.

III. UNITARITY CONSIDERATIONS

Dicus, Kretzer, Repko, and Schmidt in Ref. [3] have brought unitarity considerations to the fore. A restatement of the optical theorem relates the total neutrino-nucleon cross section to the neutrino-nucleon forward elastic scattering amplitude. The latter can be written in terms of the differential elastic cross section, evaluated at Mandelstam variable $t = 0$, which, with some approximations, yields:

$$\left. \frac{d\sigma_{\text{el}}}{dt} \right|_{t=0} \geq \frac{1}{16\pi} \sigma_{\text{tot}}^2. \quad (3)$$

One can view this as a lower bound on the forward scattering elastic cross section, or as an upper bound on the total cross section. In Ref. [3], the authors observe that the inequality is saturated at a relatively low energy by using the lowest order, G_F^2 , contribution for the elastic cross section on the left, and the G_F^4 contribution that comes from the inclusive cross section (1) on the right. Specifically, using the leading term in G_F for the elastic differential cross section, they conclude that

$$\sigma_{\text{tot}} \lesssim 9.3 \times 10^{-33} \text{ cm}^2, \quad (4)$$

which already is violated for $E \gtrsim 2 \times 10^8$ GeV. From this they deduce that at yet higher energies, where the right-hand side of Eq. (3) increases, while the left is constant (at $\mathcal{O}(G_F^2)$), previously neglected terms that are higher order in the weak coupling g , in particular, g^6 or g^8 terms, *must* become important. They go on to suggest that this signals a breakdown in perturbation theory in the weak coupling, g . This is a striking implication indeed, especially given the small size of the cross section in Eq. (4).

There is another, and we believe more natural, interpretation of the equality when $E \gtrsim 2 \times 10^8$ GeV. First, we observe that the forward elastic cross section receives two qualitatively different and quantum mechanically incoherent contributions. The first of these describes the coherent elastic scattering of the entire nucleon through weak vector boson exchange, which begins at tree level, that is, at G_F^2 in the cross section. The second is the contribution of high- Q^2 virtual states that results from the *incoherent* scattering of partons. The latter, not the former, is related independently by the optical theorem to the inelastic cross section on the right-hand side of Eq. (3), and will saturate that inequality identically at order G_F^4 , regardless of its size, just as at order G_F^2 the forward cross section is identically equal to the corresponding contribution from the square of the real part, which has been neglected on the right of Eq. (3).

That being said, we may still ask whether the dominance of the partonic part of the cross section, higher-order by g^2 compared to the elastic part, might not be a sign of large contributions from yet higher orders in the weak coupling. Integrating the factorized form Eq. (1), over x and Q^2 , however, shows that at very high energy the square of the total cross section behaves as G_F^2 times $[g^2(S/M_W^2)^\lambda]^2$. This is to be compared to G_F^2 on the left-hand side of Eq. (3). The factor g^2 is the default size of a higher-order electroweak correction. The factor $(S/M_W^2)^\lambda$ is due to the large number of partons of size $1/M_W$ at $x \sim M_W^2/S$. For higher orders in g^2 to contribute at a similar level, they would have to come accompanied by a similar large counting factor. At the leading power in $1/M_W$, which is given by Eq. (1), this cannot happen, simply because $q(x)$ and $\bar{q}(x)$ already count the partons. It would still be possible if more partons are involved in the hard scattering, but this involves going to higher twist, that is, to explicit suppression by additional powers of $1/M_W$, which would have to be compensated for by higher-twist multi-parton matrix elements. While such contributions are not very well-known even at low momentum transfers, there is no experimental indication of such large scales implicit within the nucleon.

The forgoing arguments, of course, assume that the unaided QCD extrapolations described above are equal to the task of so many orders of magnitude. We have shown above the self-consistency of these extrapolations, and that they do not, by themselves, lead to problems with unitarity, or give evidence of a breakdown in perturbation theory in the weak coupling [17]. The very fact of the self-consistency of the QCD extrapolations shows that ultra high energy neutrinos offer an exploration of the strong interactions, as well as of cosmic dynamics, into unprecedented length scales.

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