**Full-Volume, Three-Dimensional, Transient Measurements of Bubbly
Flows Using Particle Tracking Velocimetry and Shadow Image
Velocimetry Coupled with Pattern Recognition Techniques**

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Final Report

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This is the third phase report of the present NEER project. The overall goal, as described in the initial report (report for phase 1), was to develop a state-of-the-art; non-intrusive diagnostic tool to perform simultaneous measurements of both the temporal and three-dimensional spatial velocity of the two phases of a bubbly flow. These measurements are required to provide a foundation for studying the constitutive closure relations needed in computational fluid dynamics and best-estimate thermal hydraulic codes employed in nuclear reactor safety analysis and severe accident simulation. Such kinds of full-field measurements are not achievable through the commonly used point-measurement techniques, such as hot wire, conductance probe, laser Doppler anemometry, etc. The results can also be used in several other applications, such as the dynamic transport of pollutants in water or studies of the dispersion of hazardous waste.

A hybrid of the stereoscopic Particle Tracking Velocimetry (PTV) and the Shadow Image Velocimetry (SIV) flow measurement techniques was developed to provide three-dimensional full-volume and transient velocity fields of the phases of a bubbly flow. This hybrid flow measurement technique was applied to provide us with the shape and trajectory of a single air bubble rising in stagnant water, in a restricted medium.

Pattern recognition algorithms were used to track the seed tracers embedded in the flow. These algorithms were proven to accurately describe the flow field in regions of low and high velocity gradients. Two tracking were used: an Architecture Resonance Theory 2 Neural Network (ART2 NN) and the Spring Model (SM). The ART2 NN tracks tracer particles during four consecutive frames, so it provides not only the velocity vector, but also the acceleration in a Lagrangian reference frame. For regions of high velocity gradient, the SM was able to accurately track the tracer particles within two consecutive frames. These two pattern recognition-based algorithms were applied independently to the flow images. By combining the information from both tracking techniques, the full-volume, three-dimensional velocity field was determined. The ART2 NN provided information of regions surrounding the rising bubble, while the SM described the flow in the bubble wake. The developed imaging and pattern recognition techniques used for this study can also be applied to medical imaging.

In the Phase I report, a description of the constructed test facility was presented. The camera calibration technique and a method to reduce refraction problems were developed and presented. In addition, the PTV and SIV flow measurement techniques and tracking routines the ART2 NN and the SM were described and validated. During Phase II, the bubble shape and trajectory of the dispersed phase were obtained utilizing the developed state-of-the-art SIV technique. The wall influence on the shape and motion of the bubble was investigated. The magnitude and coefficients of the drag and lift forces acting on the bubble were also inferred from the information of the SIV and PTV techniques. In addition, the transient behavior of the total and turbulent kinetic energy in the test volume was presented. Results about the pseudo-turbulence generated by a rising bubble were introduced and discussed. Reynolds stresses were calculated. The structure of the bubble wake was delineated, and the influence of the bubble on the flow field was also determined. These experimental data can be used to understand the evolution and the complex phenomena of turbulent bubbly flows.

In Phase III, the single bubble dynamics study performed and presented in Phase II report was extended to a dilute bubbly flow investigation. The developed and validated PTV and SIV techniques were also improved for this task. New equipment was acquired to increase the resolution of the measurements. A test facility for studying bubbly flows with a wide range of Reynolds numbers was constructed. The test facility and experimental setup are shown. The experimental conditions are described, and sample images of a two-phase turbulent bubbly flow are shown in figures 1-5. A proposed data analysis methodology is presented in figures 6 and 7. The obtained experimental data can be used to validate the turbulence models utilized in computational fluid dynamics and best-estimate thermal hydraulic codes.

PHASE III

Objective: The first objective of phase III was to construct a test facility to perform turbulent bubbly flow PIV three-dimensional measurements.

Accomplishments: A schematic of the flow and Particle Image Velocimetry (PIV) systems is shown in figure 1. The illumination source is a Twin Nd:YAG laser. Each of the laser rods fires at a frequency of 30 Hz. The laser delivers about 350 mJ per pulse, at a wavelength of 532 nm. The pulse width is about 7 ns. The two-dimensional liquid velocity fields are acquired by a 1016×1016 pixel high resolution CCD camera. The camera runs at 30 Hz in continuous mode, but it has the triggered double exposure capability, which allows for studying high velocity fields. In the triggered double exposure mode, the camera can acquire 60 frames per second at full resolution, under certain conditions.

Two medium resolution CCD cameras, here referred as to shadow cameras A and B respectively, are employed to study the dispersed phase. These cameras have a resolution of 640×480 pixel at 30 Hz. The SIV technique is used to track the bubbles and determine bubbles' shapes. The illumination sources are red LED panels. These panels use high luminance single LEDs, and are home made. A schematic of the camera system arrangement is shown in figure 2. The shadow cameras are in a perpendicular position, respect to each other, so three-dimensional reconstruction of the bubble motion and shape are possible, as done with the single rising bubble in Phase II.

The images acquired by the cameras are sent to high performance imaging boards, and then to the RAM of personal computers. The laser, CCD cameras and LEDs system is synchronized by a high accuracy pulse generator. The software required to run the experiments is in the LABVIEW environment, and home developed.

The flow facility allows to study bubbly flows from stagnant water to turbulent Reynolds number of about 21000. The volumetric quality ranges from 0 to 7.4%. The bubble size can be controlled by varying the water flow inlet through the bubble generator, as it can be seen in figure 1. Pictures of the experimental setup before and during laser illumination and image acquisition are shown in figure 3. Example PIV images without and after image enhancement can be seen in figure 4. The simultaneous images acquired by the shadow cameras are shown in figure 5.

Objective: Develop a methodology to analyze the PIV and SIV images.

Accomplishments: the proposed methodology for data analysis from PIV and SIV images are shown in figures 6 and 7, respectively..

Other Accomplishments: Two Ph.D. Dissertations we completed under partial support of this project. Details of the project can be found in these two Dissertations.

- Schmidl, William Daniel 1999 Three-Dimensional Experimental Investigation of the Two-Phase Flow Structure in a Bubbly Pipe Flow. Ph.D. Dissertation. Texas A&M University, College Station, TX.
- Ortiz-Villafuerte, Javier 1999 Three-Dimensional Experimental Investigation of the Shape and Dynamics of a Rising Bubble in Stagnant Water with Particle Tracking Velocimetry. Ph.D. Dissertation. Texas A&M University, College Station, TX.

Since most of the results obtained during this project come from second Dissertation, this has been attached to this final report.

In addition, several papers were presented and published during the project.

REFEREED JOURNAL PUBLICATIONS

1. Hassan, Yassin A.; Ortiz-Villafuerte, Javier & Todd, Donald R. 2001, "Velocity Measurements in Bubbly Flows with Particle Tracking Velocimetry," *Flow Visualization & Processing*, Vol. 8, pp. 243-252, 2001.
2. To appear in special issue of *Journal of Flow Visualization & Image Processing*.
3. Ortiz-Villafuerte, Javier; Hassan, Yassin A. & Schmidl, William D. 2001 Rocking Motion, Trajectory and Shape of Bubbles Rising in Small Diameter Pipes," *Experimental Thermal and Fluid Science* 25 1-2, 43-53.
4. Hassan, Yassin A.; Ortiz-Villafuerte, Javier & Schmidl, William D. 2001, "Three-Dimensional Measurements of Single Bubble Dynamics in a Small Diameter Pipe Using Stereoscopic Particle Image Velocimetry," *International Journal of Multiphase Flow* 27 5, 817-842.
5. Ortiz-Villafuerte, Javier; Hassan, Yassin A. & Schmidl, William D. 2000, "Three-Dimensional PTV Study of the Surrounding Flow and Wake of a Bubble Rising in a Stagnant Liquid," *Experiments in Fluids* 29 7, S202-S210.
6. Hassan, Yassin A.; Schmidl, William D. & Ortiz-Villafuerte, Javier 1999, "Three-Dimensional Bubbly Flow Measurement Using PIV," *Journal of Visualization* 1 3, 291-301.

OTHER REFEREED PUBLICATIONS (transactions, workshops, conferences)

1. Ortiz-Villafuerte, Javier; Hassan, Yassin A.; Schmidl, William D.; Sánchez-Silva, Florencio; Todd, Donald R. & Yilmaz, Fatma 2001 Three-Dimensional Turbulence Measurements in Bubbly Two-Phase Flows with Particle Image Velocimetry. *4th International Conference on Multiphase Flow (ICMF-2001)*, New Orleans, LA, U.S.A., May 27 - June 1, 534.
2. Todd, Donald R.; Ortiz-Villafuerte, Javier; Schmidl, William D.; Hassan, Yassin A. & Sánchez-Silva, Florencio 2001 Analysis of Bubbly Flow Using Particle Image Velocimetry. *Ninth International Conference on Nuclear Engineering (ICONE-9)*, Nice, France, April 8-12, ICONE-766.
3. Ortiz-Villafuerte, Javier; Todd, Donald R.; Schmidl, William D. & Hassan, Yassin A. 2000 Turbulence Measurements in Bubbly Two-Phase Flows with Particle Image Velocimetry. *ANS Winter Meeting*, Washington, DC, U.S.A., November 12-16, ANS Transactions, 380-381.

4. Ortiz-Villafuerte, Javier; Schmidl, William D. & Hassan, Yassin A. 2000 Three-Dimensional Stereo Measurements in Bubbly Pipe Flows with Particle Image Velocimetry. *International Mechanical Engineering Congress & Exposition (IMECE'00)*, Orlando, FL, U.S.A., November 5-10.
5. Todd, Donald R.; Hassan, Yassin A. & Ortiz-Villafuerte, Javier 2000 Flow Visualization in Bubbly Flow Regimes Using Particle Tracking Velocimetry. *9th International Symposium on Flow Visualization (9ISFV)*, Edinburgh, Scotland, U.K., August 22-25, 390.
6. Ortiz-Villafuerte, Javier; Schmidl, William D. & Hassan, Yassin A. 2000 Particle Tracking Velocimetry Measurements of the Flow Field Generated by a Rising Bubble in a Small Diameter Pipe. *ASME Fluids Engineering Division Summer Meeting (FEDSM'00)*, Boston, MA, U.S.A., June 11-15, FEDSM2000-11322.
7. Ortiz-Villafuerte, Javier; Schmidl, William D. & Hassan, Yassin A. 2000 Three-Dimensional Study of Single Bubble Dynamics and the Surrounding Flow Field in a Small Diameter Pipe by Using Particle Tracking Velocimetry. *8th International Conference on Nuclear Engineering (ICONE-8)*, Baltimore, MD, U.S.A., April 2-6, ICONE-8635.
8. Ortiz-Villafuerte, Javier; Schmidl, William D. & Hassan, Yassin A. 1999 Experimental Study of the Transient Turbulence generated by a Bubble Using PIV. *International Mechanical Engineering Congress and Exposition (IMECE'99)*, Nashville, TN, U.S.A., November 14-19, Proceedings of the ASME HTD 364-4, 297-304.
9. Ortiz-Villafuerte, Javier; Hassan, Yassin A. & Schmidl, William D. 1999 Study of the Bubble Shape Using the PIV Shadow Technique. *ANS Winter Meeting*, Long Beach, CA, U.S.A., November 14-18, ANS Transactions 81, 344-347.
10. Hassan, Yassin A.; Ortiz-Villafuerte, Javier & Schmidl, William D. 1999 Three-Dimensional Bubbly Flow Measurements Using Stereoscopic Particle Image Velocimetry. *Ninth International Topical Meeting on Nuclear Thermal-Hydraulics (NURETH-9)*, San Francisco, CA, U.S.A., October 3-8.
11. Ortiz-Villafuerte, Javier; Schmidl, William D. & Hassan, Yassin A. 1999 Three-Dimensional PTV Experimental Study of the Wake of a Bubble Rising in a Liquid. *Third International Workshop on Particle Image Velocimetry (PIV'99)*, Santa Barbara, CA, U.S.A., September.
12. Hassan, Yassin A.; Schmidl, William D.; Ortiz-Villafuerte, Javier & Scharf, Jordan R. 1999 Analysis of the Three-Dimensional Structure of a Bubble Wake Using PIV and Galilean Decomposition. *33rd National Heat Transfer Conference (NHTC'99)*, Albuquerque, NM, U.S.A., August 15-17, NHTC99-308.

13. Hassan, Yassin A.; Schmidl, William D.; Ortiz-Villafuerte, Javier & Scharf, Jordan R. 1999 Stereoscopic PIV Study of a Rising Bubble in Stagnant Liquid. *3rd ASME/JSME Joint Fluids Engineering Conference (FEDSM99)*, San Francisco, CA, U.S.A., July 18-22, **FEDSM99-7266**.
14. Ortiz-Villafuerte, Javier; Schmidl, William D. & Hassan, Yassin A. 1999 Three-Dimensional Experimental Study of the Wake of a Bubble Rising in Stagnant Liquid. *ANS Annual Meeting*, Boston, MA, U.S.A., June 6-10, ANS Transactions **80**, 302-305.
15. Hassan, Yassin A.; Ortiz-Villafuerte, Javier & Schmidl, William D. 1999 Dynamics of the Wake of a Rising Single Bubble in Stagnant Liquid. *Two-Phase Flow Modeling and Experimentation 1999 Vol. 2*. Editors: Celata, G. P.; Di Marco, P. & Shah, R. K. Edizioni ETS, Pisa, Italy, 1079-1084.
16. Hassan, Yassin A.; Ortiz-Villafuerte, Javier; Schmidl, William D. & Scharf, Jordan R. 1999 Measurements of a Rising Single Bubble in Stagnant Liquid. *2nd Pacific Symposium on Flow Visualization and Image Processing (PSFVIP-2)*, Honolulu, HI, U.S.A., May 16-19, **PF156**.
17. Ortiz-Villafuerte, Javier; Hassan, Yassin A. & Schmidl, William D. 1998 Bubble Shape Identification Using Shadow Particle Image Velocimetry Technique. *ANS Winter Meeting*, Washington, DC, U.S.A., November 15-19, ANS Transactions **79**, 357-360.

Figure 1. Schematic of the flow and PIV systems.

Figure 2. Schematic of the camera system arrangement.

(a)

(b)

Figure 3. Experimental setup. a) Before data acquisition. b) During data acquisition.

Figure 4. Typical PIV images. a) Original. b) Enhanced.

Figure 5. Typical SIV images. Original and enhanced shadow images.
a) Images from shadow camera A. b) Images from shadow camera B.

Figure 6. Methodology for PIV data analysis.

Figure 7. Methodology for SIV data analysis.

TRANSIENT THREE-DIMENSIONAL EXPERIMENTAL INVESTIGATION OF THE
SHAPE AND DYNAMICS OF AN AIR BUBBLE IN STAGNANT WATER IN A SMALL
DIAMETER PIPE WITH PARTICLE TRACKING VELOCIMETRY

A Dissertation

by

JAVIER ORTIZ VILLAFUERTE

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ABSTRACT

Three-Dimensional Experimental Investigation of the Shape and Dynamics of a Rising Bubble in Stagnant Water with Particle Tracking Velocimetry. (December 1999)

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Chair of Advisory Committee: Dr. Yassin A. Hassan

The Particle Tracking Velocimetry technique has been used for a three-dimensional, transient, experimental study of a single bubble dynamics in a restricted medium. The three-dimensional velocity field was reconstructed via stereoscopic matching of two-dimensional images. A hybrid tracking technique has been used to determine the flow around a bubble. The development of the Shadow Particle Image Velocimetry allowed studying the bubble shape and rotation. An accurate estimate of the bubble dimensions, orientation, trajectory, and velocity and acceleration of a bubble rising in water, was obtained.

The flow around and within the wake of the bubble was determined from ensemble averaging instantaneous velocity fields. The ensemble average operation was performed by considering a conditional sampling technique. The conditional ensemble averaging was performed for specified bubble trajectories. It was found that bubbles rising close to the wall generate more turbulence, and the disturbances induced in the liquid reach further downstream, when compared to bubbles rising along the pipe core.

The bubble Reynolds number was in the range from 350 to 700. Regarding the bubble motion, it was found that the inclusion of the disturbed flow field in the bubble motion equation generates a scattering of the data for the drag and lift coefficients. The wall influence on these coefficients was introduced through the velocities and accelerations of the liquid and the bubble. The results indicate that the presence of the seed particles in the liquid have an influence on the bubble velocity and bubble shape. The instantaneous drag coefficient did not delineate a trend with respect to the rotation parameter; however, it shows a behavior similar to the standard drag

curve as function of the Reynolds number. The average drag coefficient values are 0.90 and 0.98 for the bubble trajectories along the pipe core and close to the pipe wall, respectively. No trend for the instantaneous lift coefficient values as a function of the Reynolds number and rotation parameter was observed. The average lift coefficient for the bubble trajectories rising along the pipe center and close to the pipe wall are of values of 0.37 and 0.44, respectively.

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CHAPTER I

INTRODUCTION

Nuclear power plants have become more economically competitive, and the risk of severe accidents has significantly decreased, since safety issues have the highest priority. The new generation of light water reactors (LWRs), both pressurized water reactors (PWRs) and boiling water reactors (BWRs), have incorporated passive safety systems (Taylor 1989; Fisher 1990; Golay & Todreas 1990). These include gravity-driven cooling systems, and heat-removal from the reactor vessel through such physical processes as natural circulation, condensation, evaporation, etc. However, there is still a lack of knowledge of various phenomena occurring in the reactor, and the coupling relationships between nuclear and thermohydraulic parameters. Consequently, empirical or semiempirical correlations are still used to model some aspects of reactor behavior, in particular, when events involve two-phase flows. This approach is due to the lack of accurate expressions that exist to model the reactor under such conditions. In general, for accident analysis, computational codes give a reasonable representation of qualitative behavior, but the quantitative values may have high uncertainties. It is clear that energy, mass, and momentum transport and transfer between phases have to be investigated based on first principles.

The interaction between the two phases in a bubbly flow is a phenomenon not completely understood and still needs to be studied in detail. This particular flow is important not only in the nuclear industry, but also in the chemical, petroleum, and medical industries, among others. The turbulence phenomena in bubbly flow regimes needs to be better understood to improve predictions of the flow behavior and the heat and momentum transfer characteristics. A clear understanding of these physical processes has a direct impact in developing and improving many engineering systems.

For dilute bubbly flows, in which bubble interaction is at a minimum, the turbulent motion of the liquid generated by the bubble is yet to be completely understood. Dilute bubbly flows are an extension of a single bubble dynamics. Therefore, improvements in the description and modeling

of a single bubble motion, the flow field around the bubble, and the dynamical interactions between the bubble and the flow will consequently improve bubbly flow modeling. The understanding of the physical phenomena will have far-reaching benefits in upgrading the operation and efficiency of current processes and in supporting the development of new and innovative approaches.

Most of the work performed in single bubble dynamics study does not consider the wall influence on bubble motion and shape. In many practical applications, such as steam generators, pressurizers, and nuclear reactor fuel bundles, bubbles rise in narrow spaces, where the solid structure influence cannot be ignored. The need of studying the influence of wall on bubble rise velocity, shape, drag, mass and lift coefficients is apparent.

Some of the results and references related to the work to be developed in this investigation are presented in the following sections. This chapter closes presenting the objectives and questions to be answered in this experimental study. An overview of the next chapters is also included.

1.1. Bubbly flow studies

Many theoretical, numerical, and experimental investigations about two-phase bubbly flows exist in the scientific literature. They cover a wide range of subjects, such as single bubble and droplet dynamics, turbulent bubbly flow calculations, heat and mass transfer, bubbly flow modeling, etc.

The mathematical modeling of the two-phase flow equations of mass, momentum, and energy has been presented by Ishii (1975), Drew (1983), Kashiwa & Gore (1991), Drew & Passman (1999), among other authors. Wijngaarden (1976) studied the interaction between bubbles in liquid. Biesheuvel & Wijngaarden (1984) derived equations of mass and motion for dilute bubbly flow. Kataoka *et al.* (1998) determined the turbulence spectrum in bubbly flow. They found that the turbulence spectra in the inertial sub-range is proportional to $-8/3$ power of the wave number, compared to the $-5/3$ power for single phase. This result is in agreement with experimental data by Lance & Bataille (1991).

It is now well known that the void fraction (the ratio of the volume of the dispersed phase to the volume of the dispersed phase plus the continuous phase volume) peaks near the wall in co-current flow, for low void fraction. As the void fraction increases the transition from bubbly to

slug flow occurs, and then the void fraction peaks in the center, although smaller peaks close to the wall still occur. Void coring also happens in counter-current flow. It is also known that bubbles of less than about 3 or 4 mm of equivalent diameter travel close to the container wall, while bigger bubbles tend to rise along the center (Tran-Cong *et al.* 1998). These phenomena of phase distribution have been associated with turbulence in the liquid phase, and with the lift forces acting on the bubbles. The phase distribution phenomena have been studied by Drew and Lahey (1982), Žun (1988), Lahey (1990), and Lopez de Bertodano *et al.* (1990), among others.

The study of the turbulence structure in a two-phase bubbly flow is one of the problems in which experimental, numerical and theoretical work is being extensively done nowadays. It is now considered that the turbulence in two-phase flow has two different sources: one is the turbulence generated in the continuous liquid phase, and the other is the turbulence induced by the movement of the bubbles in the flow. This last agitation due to the bubbles has been called pseudo-turbulence (Lance & Bataille 1991). Understanding of bubble-induced turbulence is still lacking. For homogeneous potential bubbly flows, the Biesheuvel & Van Winjgaarden's (1984) results for the pseudo-turbulence stress tensor are widely used.

Theofanous & Sullivan (1982) measured turbulence in dispersed two-phase bubbly pipe flow. They also proposed a relationship to predict turbulence levels. In other experimental investigation, Liu & Bankoff (1993) found that under certain conditions, bubbles present in the flow could reduce the fluid turbulence to a lower level than that corresponding to a single-phase flow. These researchers detected this phenomenon at high liquid flow rates and low void fractions. In turbulent bubbly flows, bubbles modify both production and dissipation of turbulent kinetic energy (Serizawa *et al.* 1975; Wang *et al.* 1987; Besnard *et al.* 1991; Kataoka & Serizawa 1991; Liu & Bankoff 1993; Kataoka *et al.* 1993; Nakoryakov *et al.* 1996). Serizawa & Kataoka (1994) have proposed a bubble exchange model to predict the turbulent momentum transport term associated with bubbles. Bubble wakes transport some turbulent kinetic energy to the liquid (Morel & Bestion 1997).

1.2. Single bubble dynamics

Most of the two-phase bubbly flow systems employed in industry consist of bubbles rising in swarms. Considering the dynamics of a single bubble and its associated wake, however, can make a first approach to the complex interaction phenomena between bubbles and wakes. This is so because many of the assumptions made in the modeling of two-phase flow are directly related

to phenomena involving single bubbles. Frequently, drag, added mass, and lift coefficients used in bubbly flow analysis are those measured or computed for single bubbles. The need of information about parameters for single bubbles then becomes apparent.

Clift *et al.* (1978) presented a complete summary of hydrodynamic information from experimental, theoretical, and numerical studies about single particles (bubbles, drops, or solid particles). Heat and mass transfer data were also presented. For fluid particles, the measurements and correlations of terminal velocities in pure and contaminated systems, the formation and shedding of wakes in the surrounding fluid, and the internal circulation in the particles were discussed. Furthermore, particle behavior in turbulent flows, unsteady analysis of particle motion, and formation and breakup of fluid particles were also considered. Fan & Tsuchiya (1990) presented a review of hydrodynamics of bubbles. They addressed the phenomena associated with bubble wakes in liquids and liquid-solid suspensions. Results from theoretical, numerical, and experimental studies were shown. Further, the authors described the phenomena associated with wakes for spherical-cap bubbles. Crowe *et al.* (1998) presented the multiphase phenomena from both Eulerian and Lagrangian approaches, for single and groups of particles. These researchers presented methods to numerically solve the conservation equations.

Although the three references just mentioned summarize most of the work done on single particles, many questions remain unanswered and more investigations are being performed. The equation of motion for single particles presents the problem of accurately describing the forces acting on the particles. Even for small spherical particles moving on rectilinear paths, the question of including or not, and how to include, unsteady forces in the motion equation is unclear. Michaelides (1997) presented a review of transient equation of motion for particles. It is generally accepted that the Basset or history force is negligible in gas-liquid systems for Reynolds (Re) numbers higher than about 100 (Park *et al.* 1995; Domgin *et al.* 1998). Park *et al.* (1995) found that both steady and unsteady forces cannot be accurately predicted in contaminated systems. A similar problem occurs during the measurement of the drag coefficient. In fact, the cause of the discrepancies found in the literature for drag coefficient of bubbles rising in stagnant water is considered due to different levels of contaminants in the water.

Lift coefficients are difficult to measure and compute because the lift force is usually smaller than the drag force acting on a bubble (Sridhar & Katz 1995; Takagi & Matsumoto 1998). For bubbles at high Re , the commonly used value of the lift coefficient is 0.5. This value is obtained using potential flow theory. Magnaudet *et al.* (1995) found that the same value of 0.5 for the lift

coefficient can be used too in an accelerated flow. However Kurose *et al.* (1998) claimed that the lift coefficient strongly depends on the shear rate. This is contrary to Magnaudet *et al.*'s result. Recently, computations of the lift coefficient in a turbulent boundary layer by Tran-Cong *et al.* (1998) showed values up to about 3.5.

Regarding the flow surrounding a bubble, Miksis *et al.* (1982) computed the bubble shape and the surrounding fluid flow field. They considered potential flow and high Re , so the viscosity effects are restricted to the boundary layer of the bubble. A reformulation of the motion equations as an integro-differential system of equations was derived. Ryskin & Leal (1984) considered a similar problem, but they also included a study of flow separation at the bubble surface. They solved the system of equations with a finite-difference scheme, and for Re from 1 to 200, and Weber (We) numbers up to 20. They concluded that the onset of flow separation depends generally on both Re and We . Tomiyama's group (Tomiyama 1998) and Tryggvason's group (Esmaeeli & Tryggvason 1999) have considered computations for single bubbles and arrays of bubbles. Tsuchiya & Fan (1988) have carried out experimental studies of the wake for spherical-cap bubbles. Lunde & Perkins (1997), Hassan *et al.* (1998) and Brücker (1999), among others, studied the wake of ellipsoidal bubbles.

The pseudo-turbulence generated by single bubbles has been investigated by Hassan *et al.* (1998), Ellingsen & Riso (1998), Rivière & Cartellier (1998) and Suzuki *et al.* (1999). Suzuki *et al.* (1999) also presented a correlation for the size of the boundary layer of a bubble. Hassan *et al.* (1998) also performed a transient study of the measured kinetic energy within the test volume. The measurements were carried out in a small-diameter pipe, consequently, wall influence was noticeable.

1.3. Bubble shape analysis and three-dimensional reconstruction

Bubble shapes and sizes, and their variations, are important parameters. They reflect the dynamic evolution of the gas/liquid interface due to the pressure changes of the bubble and the surrounding fluid. Moreover, the interfacial area is an important parameter for heat and mass transfer between bubbles and the surrounding fluid. The interfacial area values are determined by knowing the bubble surface shape and bubble dimensions. A bubble's rise velocity and path are known to be dependent on its shape and dimensions.

Clift *et al.* (1978) presented a generalized graphical correlation of fluid particle shapes, in terms of the Eötvös (Eo), Morton (M), and Re dimensionless numbers. Furthermore, wall effects

on the motion and particle shape were also considered. Bhaga & Weber (1981) studied shapes and terminal velocities of bubbles rising in viscous liquids. They also presented a graphical correlation for the shape of fluid particles similar to that one of Clift *et al.* (1978). Fan & Tsuchiya (1990) discussed characteristics of rising bubbles, such as shape, terminal velocity, drag, paths, orientation, and effects of wake on bubble motion. Karamanev (1994) has proposed a semianalytical equation describing the single bubble rise velocity in a quiescent, contaminated liquid for any shape and size of a bubble. Karamanev's equation requires that the internal circulation has no effect on the rise velocity, and that the drag coefficient of the bubble can be computed on basis of its real shape and size. Kataoka & Serizawa (1990) performed measurements of interfacial area concentration. In their study, they derived an equation relating bubble diameter to the interfacial area concentration. Kataoka *et al.* (1986) found that the interfacial area concentration has a peak near the wall in bubble flow, consistent with the results for void fraction.

Among many numerical studies of bubble shape and motion, Tomiyama *et al.* (1993) considered bubble motion in stagnant liquid as a function of M and Eo numbers. Their two-dimensional results for bubble shape showed similar shapes to Clift *et al.*'s (1978) three-dimensional graphical correlation. Manga & Stone (1995) studied the deformation caused by hydrodynamics interactions between bubbles. Takagi & Matsumoto (1998) performed a study of the forces acting on the bubble. The study of bubble trajectory has been experimentally considered by Saffman (1956), Mercier *et al.* (1973), Lunde & Perkins (1997), and Brücker (1999), among others.

The shape of an object boundary contains important information. For example, shapes of fluid particles reflect the dynamic interactions between the fluid particle and its surrounding environment. Several methods exist to detect or identify and, if necessary, reconstruct object shapes. Moreover, if its shape can identify an object, it is much easier only to store and work with its boundary. Geometric curves and surfaces are widely used in computer, robot, or machine vision for object recognition, since many objects are combinations of such geometric models. For arbitrary shapes, parametric curves are also used for identification and reconstruction processes. For example, Bezier and spline curves are well known. Recently, rational Gaussian curves and surfaces have attracted attention (Goshtasby 1993). Another detection method maps the original shape into a parametric space. A good example of this method is the Hough transform. The Hough Transform (HT) is a method to identify curves by

exploiting the duality between points on a curve and the parameters of such a curve (Ballard 1981). The HT decomposes a boundary into its constituent shape primitives. These primitive curves have associated parameters, which are used to build a symbolic representation of the shape under identification (Leavers 1992).

Shape detection of fluid particles is often performed by visual inspection of photographic plates, since the resolution of the photographic film is much higher than digital recording systems. For the latter, image processing is a tool to improve the resolution of the image. Computer vision techniques, such as thresholding, noise removal, edge detection, etc., are part of the image processing. Ellis *et al.* (1992) used image processing to measure the geometry of hydrogen bubbles and their dynamic growth. To determine the overall shape of the bubbles, they used a fifth-order Bezier curve.

In three-dimensional PIV measurements, a bubble image could show only fragments of the bubble, and not the whole two-dimensional shape, and/or show up a different shape from the actual one. For such cases, it is clear that a reconstruction process is necessary. Even when the two-dimensional shape is well defined, parameters such as orientation and dimensions still need to be computed. Shadow techniques satisfy the need of acquiring the whole two-dimensional shape of an object. For an air-water system a Shadow Particle Image Velocimetry (SPIV) provides an image of the shadow of the whole air bubble, in a plane, which can be combined with images from other cameras to determine the three-dimensional shape and dimensions of the bubble.

Most of the implementations of HT involve straight-line detection. When the object to be recognized and reconstructed is symmetric, for example circles, ellipses, etc., the dynamic generalized Hough transform (DGHT) algorithm (Leavers 1992) has been proven to be particularly useful. Although not always direct, all the parameters associated with a curve are easily computed with the DGHT algorithm. For an ellipse, the DGHT will determine the center of coordinates, the angle of rotation, and the value of the two semiaxes. This algorithm, further, can be used when occlusion of objects appears on the image. For distorted images of the bubble, the DGHT algorithm can be used to reconstruct first a two-dimensional shape, and then, through stereo matching with an image from another camera, the three-dimensional shape of the bubble.

1.4. Three-dimensional particle image velocimetry and tracking algorithms

Analytic and numeric solutions to the conservation laws are generally restricted to ideal flows and simple geometry, due to the nonlinearity of the equations, the complex interactions between phases, and the lack of accurate closure relationships. Therefore, it is readily apparent that there is a need for accurate experimental measurements that provide data for bubbly flows. This information is useful for testing new theoretical models, and developing or improving existing computer constitutive models.

It is well known that turbulent flows are three-dimensional. For two-phase flows, the interaction between phases is also three-dimensional. Consequently, any flow measurement technique considered for two-phase flow investigations has to be able to simultaneously determine the three velocity components. Ishii & Theofanous (1998) have edited a special issue about some advanced experimental techniques currently employed for two-phase phenomena investigation.

In single bubble dynamics and dispersed bubbly flows, the velocity fields contain many different velocity scales. For example, just in front of the bubble, the liquid velocity is of the same order of magnitude as that of the bubble. In regions several bubble diameters away from the bubble, however, the liquid velocity field may not detect or feel the influence of the presence of the bubble. The various time and length scales involved in single bubble dynamics imply that full-field information is needed to observe the flow behavior. Point measurement techniques can provide information at various time scales, but their spatial resolution is limited. On the other hand, flow visualization measurement techniques can provide information at any point in the measurement zone. However, with present electronic technology, the duration of the measurement is limited due to requirements for large storage and fast data handling.

Particle image velocimetry (PIV) is a nonintrusive flow measurement technique that yields a two-dimensional velocity field. This technique is a very efficient tool since it can obtain both qualitative and quantitative spatial information about the flow being studied. Transient information is also available since a series of pictures of the same area under study can be taken in a very short time. In recent years, PIV has become a standard tool for fluid mechanics studies, and the number of papers in which this technique is used has grown exponentially. Reviews about PIV techniques have been presented by Adrian (1991), Buchhave (1992) and Grant (1997). Grant (1994) edited a special issue about selected papers on PIV.

Particle image velocimetry has been extended for three-dimensional measurements. Images from at least two cameras are acquired. Then a stereo pair matching method is performed to determine the three-dimensional position of a fluid particle in the viewing volume under study

(Kasagi & Nishino 1991; Maas *et al.* 1993; Prasad & Adrian 1993; Costes *et al.* 1994; Hassan *et al.* 1998). A three-dimensional reconstruction from orthogonal views was performed by Adamczyk & Rimai (1988). For multiphase investigations, PIV can also be used if the different phases can be distinguished. Philip *et al.* (1994) used fluorescent seeds and special filters, by letting one camera to capture both phases and other camera in conjunction with the filter only capture the fluorescent seed images. Hassan *et al.* (1998) also differentiated between the phases by simply considering the size of the images.

With PIV, the velocity is measured by recording the displacement of microscopically small neutral density particles. Two or more short light pulses fired with a known time separation illuminate these tracer particles, embedded in a volume of the flow. The images appear with spacing proportional to the local velocity vector. The problem, then, is to track and extract the velocity information quickly and accurately from the pattern. Different tracking methods may be used to process the data. These include techniques such as cross-correlation (Hassan *et al.* 1992; Yamamoto *et al.* 1995), particle tracking velocimetry (Nishino *et al.* 1989; Malik *et al.* 1993; Wernet and Pline 1993), Spring Model (Okamoto *et al.* 1995), etc. Recently, new algorithms based on pattern recognition are becoming popular; among them neural networks, genetic algorithms, and fuzzy logic techniques seem to have good potential for particle tracking. There are different kinds of neural networks (NN) used in PIV. Grant & Pan (1995) used Kohonen algorithm, while Hassan & Philip (1997) used an Adaptive Resonance Theory (ART2) technique. Yoon *et al.* (1997) employed a genetic algorithm for particle tracking. Fuzzy Logic techniques have been employed by Wernet (1993).

1.5. Objectives

The purpose of this investigation was to provide experimental data of the shape and dynamic behavior of a single bubble rising in stagnant water in a small-diameter pipe. The interaction of the bubble with its surrounding fluid was studied. The structure of the wake flow was also investigated. Ensemble averages under the condition of what trajectory the bubble follows in its rising path were used to determine mean quantities. Fluctuating components were obtained by Reynolds decomposition.

Some of the questions that this experimental investigation tries to answer are:

- how much energy the bubble brings into the test zone,
- how long it takes the viscosity and wall friction to dissipate the energy in the test volume,

- how much turbulence the bubble induces,
- how the structure of the bubble wake is,
- how far upstream and downstream the bubble-induced liquid agitation is felt,
- what the magnitude of the different forces acting on the bubble is, and
- what the pipe wall influence on drag and lift coefficients is.

The overview of the following chapters is next. In Chapter II, the experimental setup and the camera calibration procedure are presented. Chapter III shows the application of the hybrid tracking technique, which is a combination of the ART2 NN and Spring Model. This approach increased the number of velocity vectors, and the determination of the three-dimensional position of the tracer particles through a stereoscopic technique. The implementation of the SPIV technique and its application for bubble shape identification is presented in Chapter IV. The description of the DGHT algorithm for two-dimensional bubble shape reconstruction for both PIV and SPIV images, and the combination of the bubble parameters to determine the three-dimensional shape of the bubble are also shown in Chapter IV. In Chapter V the description of both the mean and turbulent flow field upstream and downstream the bubble is shown. The wake structure is also analyzed in this chapter. An uncertainty analysis of the measured velocity field is also presented in Chapter V. The forces acting on the bubble and the determination of drag and lift coefficients are shown in Chapter VI. Also, Chapter VI presents an analysis of the bubble shape. Finally, Chapter VII presents a summary of this experimental investigation, the conclusions, and recommended future work.

CHAPTER II

EXPERIMENTAL SETUP AND CAMERA CALIBRATION

An experimental facility was constructed to conduct measurements of the interaction between a rising air bubble and its surrounding fluid in a pipe flow. The experimental set up consisted of a flow system, an optical system, and a data acquisition system. All these are described in detail next. Then the mathematical model used to calibrate the cameras is described. The uncertainties associated with the calibration process are presented.

2.1. Experimental setup

A schematic of the experimental setup and the camera configuration is shown in figure 1. Air bubbles of average size 3-mm spherical-equivalent diameter were injected into a 12.7-mm i.d., 15.9-mm o.d., 1.3-m long Plexiglas pipe. The flow was seeded with small polystyrene tracer particles. The density of these particles is 1.05 g/cm^3 and their diameter is $40 \text{ }\mu\text{m}$. The particles need to be small enough to effectively follow the flow, and large enough to reflect a sufficient amount of light to be detected by the cameras. Throughout the measurements, it was assumed that the tracer particles are homogeneously distributed over the whole viewing volume, including those regions close to the pipe wall. This could be confirmed by observing the distribution of the particle images in the viewing volume. The measurements were carried out at a location of about $L / D = 66$, where L is the length from pipe inlet to measurement zone, and D the pipe inner diameter. In general, bubble terminal velocity is reached just a few bubble diameters after bubble release. In the case of this experimental study, once the bubble reached the viewing volume it would have gained a velocity equal to the terminal velocity. However, under certain conditions, such as unsteady, non-uniform flows, bubble terminal velocity may not reach steady state.

A water filter removed any contaminants exceeding a diameter of $5 \text{ }\mu\text{m}$, prior to the commencement of the experiment and the addition of the tracer particles. Enclosing the test section in a rectangular Plexiglas box filled with water, see figure 1a, reduced the pipe curvature effect. The Plexiglas box has the same refractive index as that of the pipe. In addition, as explained later, refraction effects were included in the calibration parameters.

The optical elements of the set up included an argon ion laser, an acoustic optic modulator

Figure 1. a) Facility setup and camera configuration; and b) some relevant dimensions are shown.

(AOM), a beam splitter, mirrors, and a multimode fiber with a fiber coupler. The light source was the argon ion laser. It has a maximum power output of 5 W, comprising 7 frequency groups. The 514-nm frequency was selected for illumination because it delivered the maximum power output (about 2 W). This frequency corresponds to green light. The laser beam passed through the AOM (its rise time is less than 200 ns), which chopped the continuous laser light at intervals of 16.67 ms. The laser pulse width was 250 μ s. Then, the light was directed to a beam splitter, which divided the beam into two parts. Each part was directed to a separate fiber coupler. The light beam was transmitted to the viewing volume through two 400- μ m i.d. multimode fibers. These two fibers were placed at the top of the tube to illuminate the viewing volume with cones of light. The images (size, intensity, and distribution on the image) of the tracer particles indicated that the viewing volume was adequately illuminated. Four Charged Couple Device (CCD) cameras were utilized in this experimental investigation. As shown in figure 1, one of the cameras was only used for bubble shape analysis. For this analysis, forward scattering was utilized. Light Emitting Diode (LED) units emitting red light coupled with a diffusing screen were used to illuminate the test volume. A red filter was placed in front of the Shadow camera to detect the red light images only.

Several optical elements were attached to the CCD cameras to improve the accuracy of the measurements. Such attachments were: a close-up lens, a telephoto lens, a 15-mm extension ring, a 2 $\times$ range extender, an intensifier tube, a 25-mm lens, and a 10-mm extension ring. The close-up lens was mounted on the front of the telephoto zoom lens. This is a macro zoom lens of focal length 18-108 mm, f 2.5. The purpose of adding the close-up lens and the 2 $\times$ -range extender was to enlarge the magnification factor of the whole lens assembly. The 15-mm extension ring adjusted a focused image through the lens onto the intensifier tube. The 10-mm extension ring focused the image produced by the intensifier tube onto the camera's CCD detector array. The intensifier tube is encapsulated in a protective case, and is operated by a 3 V DC. It has a glass window, and contains a high-efficiency GaAs photocathode with a luminous gain of 12000. Both gain and brightness of the intensifier can be adjusted for better performance. The whole camera assembly can be seen in figure 2.

The CCD cameras have a resolution of 640 $\times$ 480 pixels, when run in the RS-170 frame interlace mode at 30 frames/s. By running the cameras in field mode, the resolution is reduced by half in the CCD array Y-direction. Thus, the camera's resolution is only 640 $\times$ 240 pixels; however, the framing rate is doubled to 60 frames/s. The cameras were run in field mode to take

Figure 2. Camera assembly.

advantage of the higher framing rate, therefore allowing for the study of higher velocity flows. The inclusion of an image intensifier did not deteriorate the resolution of our images because the image intensifier has at least 2 million microchannels, where the incoming light rays increase their intensity, and its output format (2/3") is bigger than our 640×480 CCD array (8.64 mm × 6.48 mm). When the cameras are run in field mode there are odd and even fields. However, the particle image sizes were, in average, approximately 10 pixels in area, and only the centroid of individual particle images are used for the tracking process. The effect of the 1-pixel difference from odd and even lines, then, falls within the calibration error, as explained later.

Once the cameras were triggered, the images were exposed onto the CCD array, and simultaneously stored in 4 frame grabber boards, also called imaging boards, installed in two personal computers (2 imaging boards per computer). Each imaging board is able to store up to 4 megabytes of data, or equivalently 27 sequential image frames. These 27 frames, from each camera, were then transferred to the computer's hard drive. Then all the sets of 27 frames were transferred to workstations, where the data analysis was performed. The imaging boards had also the function of controlling all the signal system: timing of laser pulses, camera triggering, and data acquisition.

2.2. Camera calibration

Camera calibration is the technique from which camera parameters are determined, by using set of image points with known world coordinates. Such parameters include camera position in

Figure 3. The pinhole camera. A point P in the 3D world coordinate system is projected into an image point I , located at the interception of the line joining the point P and the camera center of projection C through the camera image plane Π .

the world coordinate system and orientation. Also, internal camera parameters, such as focal length, or principal point distance, lens distortion parameters, etc. can be computed through camera calibration.

2.2.1. The pinhole camera model and the direct linear transformation method

In order to construct a mathematical model relating the image points and the three-dimensional world positions of an object, a camera model is needed. One of the most widely used camera models is the so called *pinhole* model, in which the camera is modeled by its optical center, or principal point, C and its image plane Π , see figure 3. The pinhole camera is an idealized device for describing the light rays passing through the camera optical center, or center of projection, and falling into a solid angle determined by the camera image plane. Now, since light travels along straight lines, each image point corresponds to a well defined light ray passing

through the pinhole, and coming from an object in the three-dimensional world. This transformation of a three-dimensional (3D) world point into a two-dimensional (2D) image is known as perspective projection.

Although perspective projection is non-linear, a series of linear transformations can relate a point P of world coordinates (x, y, z) to an image point I of coordinates (u, v) , if projective coordinates are used. The linear transformations consist of translation and rotation of the projective center of the camera as a rigid body, followed by a perspective transformation (Gonzales & Wintz 1987; O'Hern *et al.* 1997).

Let $I^* = (s\ u, s\ v, s)^t$ be the projective coordinates of I , and $(x, y, z)^t$ be the coordinates of P , then we can write

$$\begin{bmatrix} s\ u \\ s\ v \\ s \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}, \quad [1]$$

where $\mathbf{T}$ is a 3×4 matrix, generally called the perspective matrix. $\mathbf{T}$ is defined up to a scale factor, so it needs a constraint to be specified uniquely. The simplest constraint is to set one of the $\mathbf{T}$ elements to a constant different than zero. In the particular case that P is in the focal plane of the camera, that is, the line CP is parallel to the image plane Π , $s = 0$ and then the image coordinates are undefined.

Equation [1] can be rewritten in its components to arrive to a relationship between image and world coordinates for a given point as:

$$u = \frac{t_{11}x + t_{12}y + t_{13}z + t_{14}}{t_{31}x + t_{32}y + t_{33} + 1}, \quad [2]$$

$$v = \frac{t_{21}x + t_{22}y + t_{23}z + t_{24}}{t_{31}x + t_{32}y + t_{33} + 1}, \quad [3]$$

where it was assumed $s \neq 0$, and set $t_{34} = 1$.

Equations [2] and [3] simply, implicitly, reflect the collinearity condition, which states that

the point P , its corresponding image point I , and the camera center of projection C lie on a straight line. Thus, they do not consider the refraction effect, when light ray travels through different physical media, neither any kind of lens distortion, and image digitization effects, if electronic equipment is used in the acquisition and storage of the images (Maas *et al.* 1992; Maune 1975). Equations [2] and [3] have 11 unknowns, combined, so at least six non-coplanar image-point coordinates and their corresponding world-point coordinates are needed to solve for the eleven t_{ij} 's. Once the latter are computed, all camera parameters can be determined, although the equations to solve will be non-linear.

Equations [2] and [3] are the same derived by Abdel-Aziz & Karara (1971). These equations are denoted as the Direct Linear Transformation (DLT) method. When the DLT method was refined to allow for the computation of lens distortion parameters, film deformation, etc., it still was referred as DLT (Marzan & Karara 1975). Based on Marzan and Karara's experimental investigations and those from other researchers (Bopp & Krauss 1978; Karara 1980), it was found that in most cases only the first radial distortion coefficient was needed to model lens distortion and film deformation effects. The inclusion of the first radial distortion coefficient κ in [2] and [3] is as follows:

$$u + (u - u_0) \kappa r^2 = \frac{t_{11}x + t_{12}y + t_{13}z + t_{14}}{t_{31}x + t_{32}y + t_{33}z + 1}, \quad [4]$$

$$v + (v - v_0) \kappa r^2 = \frac{t_{21}x + t_{22}y + t_{23}z + t_{24}}{t_{31}x + t_{32}y + t_{33}z + 1}, \quad [5]$$

where u_0 and v_0 are the image coordinates of the center of projection, and $r^2 = u^2 + v^2$. This system of equations can be solved by a non-linear least squares method, or by iteration. Again, at least six non-coplanar image-point coordinates and their corresponding world-point coordinates are needed to solve for the twelve unknowns, the eleven t_{ij} 's and κ .

2.2.2. Calibration data

One of the major problems on calibration is that of the different physical media through which the light rays travel in an actual experimental set up. When all of the surfaces to be crossed, from one medium to other, by a ray of light are planar, a mathematical model similar to

that used by Maas *et al.* (1992) can be implemented. For curved surfaces, however, the problem is much more complex, since the normal to such surfaces changes at every surface point, in the most general case. Specifically for pipe geometry, the normal is a function of the angle of the incident ray. Then, the equations describing the collinearity condition should be extended to consider the change of refraction index for different physical media.

Some alternatives exist to reduce the refraction effect, overall for curve surfaces. For example, the use of liquids with refraction index close to, or same as, that of a thin-wall container of the fluid under research, has been used to help to reduce, and sometimes minimize, the refraction effect (Chen & Fan 1992).

Instead of extending the model for the collinearity condition, some researches have preferred the option of calibrating under the same conditions in which the actual experiment would be carried out (Tsao *et al.* 1995; Hassan *et al.* 1998). In this way, the refraction effects are included into the eleven transformation coefficients of the DLT method. This alternative presents the problem of constructing a test, or calibration, set up which must be placed exactly on the same position where the actual experimental set up would be located.

In this experimental investigation, a rigidly supported tank full of water was used to maintain the pipe on a fixed position. The tank had two circular holes, one at the bottom and one at the top, of the same outside diameter of the pipe, see figure 1a. Since the tank could not move, and the pipe had to go through the two holes, other pipe of the same dimensions and material could be used as test set up, and then the transformation coefficients t_{ij} 's, and κ if used, computed so, could be used for the actual experimental data. The test pipe was also full of water to accurately simulate the actual experimental conditions.

The calibration procedure started with setting a coordinate system. Then a calibration grid was constructed by printing tiny circular black dots of 0.2 mm of diameter on a sheet of paper. The dot pitch was 1.27 mm on both horizontal and vertical directions. The calibration grid had a total of 24 dots, arranged in a matrix-like pattern of 3 columns and 8 rows. The grid was glued to a thin plastic slab, which could be moved forwards, backwards or sideways by a micrometer. The accuracy of the movement of the micrometer is less than 0.01 mm.

Then, the approximate location of the center of the pipe was achieved by using the views of the two orthogonal cameras, the Center and the Shadow cameras, according to the notation used in figure 1. The Center camera was that seeing the XZ world plane, and the Shadow camera was that seeing the YZ world plane. To determine the center of the pipe on the images of these two

cameras, it was first necessary to obtain the location of the pipe walls on the images. Due to the very high zoom of the camera lens used in this experiment, the dot pitch distances were basically constant across the pipe images, and then the middle point between the wall positions it was considered to be the center of the pipe on each respective world plane. The calibration grid was placed such that the center column coincides with the pipe center on the XZ plane, and the front of the slab was at the center of pipe on the YZ plane.

Once the origin of coordinates was set, the calibration grid was moved forwards 8 times, at intervals of 0.5 mm each time. Then, the grid was repositioned at the origin, and then moved backwards 5 times at intervals of 0.5 mm too. At some positions close to the center of the pipe, the calibration grid was moved sideways, once to the left and once to the right, 1.27 mm (one dot pitch distance). For each of all positions of the calibration grid, images were taken with all 4 cameras (obviously, the shadow camera only sees the edges of the calibration grid, so only the Y world direction can be calibrated in this way) used in the experiment. In this way, calibration data all over the viewing volume were gathered. A total of 624 calibration points (26 images) were acquired, but due to a digitization problem in one of the cameras, the images of top row of the calibration grid was not considered, and then 546 points were actually used as calibration data.

Figure 4 shows the 4 images, one from each camera, of the calibration grid, when the grid was placed at the origin. The images have been negated and enhanced for better visualization. The average area of a calibration point is about 70 pixel.

2.2.3. *Computation of transformation coefficients*

Once all the calibration images were acquired, they were fed into a computer program that determined the centroid of the calibration spots. The program is called FINDSPOTS, and it is written in FORTRAN 77 (Blanchat 1992). The program computes the spot centroid to subpixel accuracy, by calculating the area of the spots that surpass a predetermined gray-level threshold. The output of the program is the u and v coordinates of the centroid of the calibration points, which along with their corresponding world coordinates provide the necessary information to compute the transformation coefficients.

To compute the transformation coefficients in [2] and [3], this system of equations was set in matrix form. The system was solved by the subroutine DGLSS, which is part of NETLIB. NETLIB is a software repository, freely available through internet, and it is maintained basically

Figure 4. The four simultaneous images of the calibration grid, when placed at the origin of the coordinate system. (a) Image from the Center camera; (b) image from the Left camera; (c) image from the Right camera; and (d) image from the Shadow camera.

by AT&T Bell Laboratories, The University of Tennessee and Oak Ridge National Laboratory.

DGLSS is a least squares subroutine that solves undetermined or overdetermined systems of linear equations, by performing a QR factorization of the input matrix, using Householder transformations. In matrix form, the system of equations to solve is

$$\begin{bmatrix}
 x_1 & y_1 & z_1 & 1 & 0 & 0 & 0 & 0 & -u_1 x_1 & -u_1 y_1 & -u_1 z_1 \\
 0 & 0 & 0 & 0 & x_1 & y_1 & z_1 & 1 & -v_1 x_1 & -v_1 y_1 & -v_1 z_1 \\
 x_2 & y_2 & z_2 & 1 & 0 & 0 & 0 & 0 & -u_2 x_2 & -u_2 y_2 & -u_2 z_2 \\
 0 & 0 & 0 & 0 & x_2 & y_2 & z_2 & 1 & -v_2 x_2 & -v_2 y_2 & -v_2 z_2 \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 x_N & y_N & z_N & 1 & 0 & 0 & 0 & 0 & -u_N x_N & -u_N y_N & -u_N z_N \\
 0 & 0 & 0 & 0 & x_N & y_N & z_N & 1 & -v_N x_N & -v_N y_N & -v_N z_N
 \end{bmatrix}
 * \begin{bmatrix}
 t_{11} \\
 t_{12} \\
 t_{13} \\
 t_{14} \\
 t_{21} \\
 t_{22} \\
 t_{23} \\
 t_{24} \\
 t_{31} \\
 t_{32} \\
 t_{33}
 \end{bmatrix}
 = \begin{bmatrix}
 u_1 \\
 v_1 \\
 u_2 \\
 v_2 \\
 \cdot \\
 \cdot \\
 \cdot \\
 \cdot \\
 \cdot \\
 \cdot \\
 \cdot \\
 \cdot \\
 u_N \\
 v_N
 \end{bmatrix}, \quad [6]$$

with $N = 546$.

To the end of quantifying the effect κ , the system [4] and [5] was also solved. For this case the transformation parameters t_{ij} 's computed by DGLSS were used as initial guesses in the non-linear system, in which κ was initially set to zero. Then, the t_{ij} 's and κ were fed into the subroutine DNLS1E, which is a least squares subroutine that uses a modified Levenberg-Marquardt algorithm to solve N non-linear functions on M variables. DNLS1E is also part of NETLIB. DNLS1E requires the non-linear system to be input as an external function.

2.2.4. Calibration error

Once all the t_{ij} 's, and κ , were determined, an error analysis was performed. The final results are shown in table 1. Observe that the inclusion of κ did not affect the results, so it was decided to consider only with the equations [2] and [3], to simplify and accelerate calculations that would

Table 1. Camera calibration errors

error [pixel]	Left camera		Center camera		Right camera	
	DLT	DLT + κ	DLT	DLT + κ	DLT	DLT + κ
	$\kappa = -1.6 \times 10^{-12}$		$\kappa = -1.6 \times 10^{-12}$		$\kappa = -3.7 \times 10^{-12}$	
rms u	1.2	1.2	1.0	1.0	1.4	1.4
max u	3.5	3.5	3.3	3.3	3.8	3.8
rms v	0.9	0.9	1.0	1.0	1.0	1.0
max v	2.8	2.8	3.2	3.2	5.5	5.5

be performed later. The null influence of κ was due, mainly, to the high value of the focal length (high zoom) of the telephoto lens. To better quantify the calibration error, it was necessary to compare the rms errors against the average area of a typical seed particle image. Such average value was about 40 pixel, and the minimum area for any image spot of a seed particle was 9 pixel. Then, if an image spot is considered as a square, the spot will have a side of 3 pixel, value that was higher than any of the rms errors, for any camera.

One of the major advantages in positioning the cameras on the chosen experimental set up was that the orthogonal cameras (Center and Shadow) would provide directly two out of the three world coordinates of the seed particles and/or the bubbles. The other two cameras would basically help on the estimation of the out of plane coordinate. To actually achieve such positioning it was necessary to prove that the v coordinates (z world coordinates) coincided on all the cameras, and that the image plane of the orthogonal cameras was parallel to the world plane they were seeing. The calibration results showed that the maximum difference between any camera, at any calibration point, for the v coordinates was less than 3 pixel, for the same world point, naturally. This maximum difference was between the center and the left cameras.

Also, for the Center camera, the maximum error in the X direction, on any calibration point, when moving the calibration grid in the Y direction, was less than 4 pixel. This happened at $X = -1.27$ mm. With respect the Z direction, the maximum error for any calibration point was less than 2 pixel. It is safe, then, to say that the image plane of the Center camera was parallel to the XZ plane. For the Shadow camera the results of calibration are very similar to those of the center

camera. In the Z direction, the maximum error was less than 2 pixel; and, in the Y direction the maximum error was less than 5 pixel. This maximum difference was at $Y = -2.0$ mm. Therefore, the Shadow camera image plane could be considered parallel to the YZ plane.

In all above cases, the calibration error was mainly due to the change of illumination on the calibration grid, when it moved to different positions. The calibration results, then, confirmed that the zoom of the telephoto lens was the same for all cameras, and that the desired positioning of the cameras was effectively achieved.

CHAPTER III

HYBRID PARTICLE TRACKING VELOCIMETRY AND THREE-DIMENSIONAL VELOCITY FIELD RECONSTRUCTION

Although many two-phase flow measurement techniques exist in the chemical, petroleum and nuclear industries (Ishii & Theofanous 1998), and many of the different patterns in a two-phase flow have been studied, there still exists a lack of information and understanding of the physical phenomena. Basic hydrodynamics, mixing, mass and heat transport have not been described based on first principles. Traditionally, design and scale-up of the systems are largely based on empirical models. In these models, the design parameters are largely evaluated by correlations that are applicable to narrow operating ranges. A good knowledge of the structure of the flow, including the behavior of the individual phases and the pattern of the flow, is important in areas as safety of the nuclear power plants and efficient design of other industrial systems.

A brief description of the physical fundamentals of the particle image velocimetry flow measurement technique is presented in this chapter. Then, a short description of the Adaptive Resonance Theory 2 Neural Network (ART2 NN) and the Spring Model tracking techniques, and their combination to form the hybrid tracking technique used in this investigation, is shown. Finally, the method to determine the third dimension from a stereo pair of images is described.

3.1. Pulsed-light velocimetry

Pulsed-light velocimetry (PLV) is a flow-field measurement technique, in which the images of small markers freely flowing in the fluid under study are acquired at two, or more, different times, by pulsing some light source (Adrian 1991). The motion of the markers is then a measure of the motion of the fluid under study, and, consequently, a whole flow field composed of many local velocities $\mathbf{u}$, at different locations in the fluid, can be estimated from:

$$\mathbf{u}(\mathbf{x}, t) = \frac{\Delta \mathbf{x}(\mathbf{x}, t)}{\Delta t}, \quad [7]$$

where $\Delta \mathbf{x}$ is the marker position change in the time interval Δt .

The markers used to follow the fluid motion can be solid particles, gas bubbles, liquid droplets, or patches of molecules, among others. The images of these markers are acquired by photographic cameras and/or electronic cameras, and are recorded on photographic film, couple charged device (CCD) detector arrays, holographic plates, etc. Depending on the kind of markers to be used in the measurement, PLV is divided into two categories: particulate markers, and molecular markers (Adrian 1991). Those measurement techniques using particles as markers are grouped into the particle image velocimetry measurement (PIV) techniques.

Adrian (1991) identified three different operational modes for the PIV technique, depending on the mean number of concentration of scattering particles per unit volume: laser-speckle mode, particle-tracking mode, and high-image-density PIV mode. In the laser-speckle mode the image plane is overcrowded with the images of the tracer particles, and then particle overlapping generates random interference patterns known as laser speckle. The technique that measures the displacement of the laser speckle is known as laser-speckle velocimetry (LSV). When the particle-tracking mode, or low-image-density PIV mode, is employed, the particle concentration is low, and then particle overlapping is not probable. The particles, in general, are sparsely distributed on the image plane, allowing for tracking of individual particles. Here, the measurement of particle displacement is known as particle-tracking velocimetry (PTV). The high-image-density PIV mode occurs in between LSV and PTV. It is standard nowadays to refer to the high-image-density PIV mode simply as PIV.

When compared against other flow measurement techniques, PIV and PTV have the advantage of being non-intrusive and to provide full-field information. Then, the spatial resolution of these techniques is high. On the contrary, laser-Doppler anemometry (LDA) and hot wire techniques produce information on a single point in the measurement volume. Moreover, hot wire can introduce undesirable physical disturbance in the flow that could lead to measurement errors. On the other hand, the temporal resolution of LDA and hot wire are superior to PIV and PTV.

3.2. Particle image velocimetry

Particle image velocimetry is a full-field, non-intrusive velocity measurement technique that can obtain both qualitative and quantitative spatial and temporal information about a 2D flow velocity field. With PIV, the velocity is measured by recording the displacement of microscopically small neutral-density particles. Two or more short light pulses fired with a

known time separation illuminate these tracer particles, embedded in a volume of the flow. The images appear with spacing proportional to the local velocity vector. Therefore PIV is capable of producing instantaneous velocity maps. The problem, then, is to track and extract the velocity information quickly and accurately from the pattern.

It is well known that turbulent flows are 3D. For two-phase flows, the interaction between phases is also 3D. Consequently, any flow measurement technique considered for two-phase flow investigations has to be able to simultaneously determine the three velocity components. The extension of PIV to perform 3D measurements has been successfully achieved by acquiring Images from at least two cameras with different view angles. Then a stereo pair matching method is performed to determine the 3D position of a fluid particle in the viewing volume under study (Kasagi & Nishino 1991; Maas *et al.* 1993; Prasad & Adrian 1993; Costes *et al.* 1994; Hassan *et al.* 1998). Adamczyk & Rimai (1988) performed a 3D reconstruction from orthogonal views. For multiphase investigations, PIV can also be used if the different phases can be distinguished. Philip *et al.* (1994) used fluorescent seeds and special filters, by letting one camera to capture both phases and other camera in conjunction with the filter only capture the fluorescent seed images. Hassan *et al.* (1998) also differentiated between the phases by simply considering the size of the images. Transient information is also available since a series of pictures of the same area under study at many time steps can be taken.

Measurements performed with automated digital PIV are limited on the temporal aspect by the storage capability of the imaging boards and the RAM and hard drive of the computers. The spatial aspect is limited by the resolution, pixel size and array format, of the CCD cameras. These two limitations, however, are being overcome with the use of new technologies.

3.3. Hybrid particle tracking technique

Different tracking methods may be used to process the data. These include techniques such as cross-correlation (Hassan *et al.* 1992; Yamamoto *et al.* 1995), particle tracking velocimetry (Nishino *et al.* 1989; Malik *et al.* 1993; Wernet & Pline 1993), Spring Model (Okamoto *et al.* 1995), etc. Recently, new algorithms based on pattern recognition are becoming popular; among them neural networks, genetic algorithms, and fuzzy logic techniques seem to have good potential for particle tracking. There are different kinds of neural networks used in PIV. Grant & Pan (1995) used a Kohonen NN, while Hassan & Philip (1997) used an ART-2 NN technique. Yoon *et al.* (1997) employed a genetic algorithm for particle tracking. Fuzzy Logic techniques

Figure 5. General process to determine the three-dimensional flow field.

have been employed by Wernet (1993).

The general process to determine the 3D flow field from the acquisition of the 2D images by the CCD cameras is outlined in figure 5. The recorded images are directly digitized via framegrabbers. Prior to employing the tracking routines, an initial processing stage is employed. The gray scales and the particle center of gravity are estimated. Then, a 2D hybrid tracking (in u , v coordinates) of the seed particle images is used. The need of a hybrid technique arises from the different flow patterns observed. The two tracking techniques used in this experimental study were an ART2 NN (Hassan & Philip 1997), and the Spring Model (Okamoto *et al.* 1995). The ART2 NN is an algorithm that self-organizes stable recognition codes (clusters) in real time in response to arbitrary sequences of input patterns, one at a time (Carpenter & Grossberg 1987). The Spring Model is a pattern recognition technique, which allows for tracking in rotating and shear flows.

In this hybrid-tracking algorithm, the particles are initially tracked with the ART2 NN, and then with the Spring Model. The ART2 NN tracks particles that present an alignment in four consecutive frames. Therefore, it allowed to perform Lagrangian measurements of the velocity

and acceleration of the liquid. Lagrangian measurements are required in the study of the forces acting on the bubble, as shown in Chapter VI. The ART2 was more successful in yielding velocity vectors mainly in regions away from the bubble. On the other hand, the Spring Model is a frame-to-frame tracking algorithm, which tracks a group of particles in two consecutive frames. The inclusion of this technique is necessary because it is not probable that the tracer particles will have the alignment property in the areas of turbulence, and in the bubble wake. As expected, the Spring Model yielded more velocity vectors in the wake of the bubble, and in areas of high turbulence. Once the tracking is performed, the data is combined, and repeated vectors are filtered out.

3.4. Matching of velocity vectors from stereo images

Before the three components of the position vector of any particle can be determined it is necessary to obtain information from two different views of the same particle. The criteria and algorithm to match velocity vectors from the stereo images are described next.

The process started by selecting a 2D vector tracked by the ART2 NN from four consecutive images (only two when the tracking algorithm employed is the Spring Model) acquired by the Center camera (CC). Then, the corresponding match vector was looked for among all the 2D vectors similarly tracked from images acquired by, say, the Right camera (RC). To consider a vector from RC being a possible match, it had to satisfy the following:

- The difference in the magnitude, in pixels, of the velocity in the v -direction, z -world coordinate, was less than a given tolerance ε_z , that is,

$$|\Delta v_C - \Delta v_R| \leq \varepsilon_z. \quad [8]$$

Observe that this is a direct consequence of that the three PIV cameras were positioned such that the v -coordinates coincide, as demonstrated by the camera calibration results, see Chapter II.

- The x -image coordinate of the possible match vector on the RC, u_R , was in a bounded range given by

$$u_{R \min} - \varepsilon_x \leq u_R \leq u_{R \max} + \varepsilon_x, \quad [9]$$

Figure 6. The variation of u for the PIV cameras against its world position, x , at $y = 0$. Also shown the results of application of a linear regression to the data points. See text for notation.

where ϵ_x is a tolerance. $u_{R \min}$ and $u_{R \max}$ depend on the y -world coordinate (the depth), and were determined as follows.

The x -world coordinate of the starting point of the 2D velocity vector tracked from images acquired by the CC was computed from the u_C and its corresponding linear regression performed with data from the CC, as shown in the figure 6. Observe that figure 6 can be used here because it was demonstrated in Chapter II that the image plane of the CC was practically parallel to the XZ-world plane.

Once x was calculated, the y range in which the starting point of the 2D velocity vector could fall into was calculated from

$$y^* = \pm \sqrt{r_0^2 - x^2}, \quad [10]$$

where r_0 is the radius of the pipe. Then, x was also used to compute its corresponding

u_{R0} , at $y = 0$, from the corresponding linear regression performed with data from images from the RC, as shown in figure 6.

From figures 7a and 7b it can be seen that there was practically a linear dependence between the depth (y) and u_R and u_L for both the RC and the LC, respectively. Further, the values of the slopes were very close, for each respective camera. In fact, the standard deviation from the average value of the slopes was 0.262 and 0.277 for the RC and the LC, respectively. The average values of the slopes, say s_R and s_L , were -10.981 and 10.610 for the RC and the LC, respectively. The linear relation between y and u_R , and y and u_L , is a consequence of the high value of the focal length of the cameras (high zoom). s_R and u_{R0} , s_L and u_{L0} respectively, are the parameters defining completely a straight line, where the independent variable is y and the dependent variable is u . By evaluating this equation at y^* and $-y^*$, the $u_{R \min}$ and $u_{R \max}$, $u_{L \min}$ and $u_{L \max}$ respectively, in which the vector to match was located could be determined, for each respective camera.

All 2D-velocity vectors tracked from images acquired by the RC, LC respectively, that satisfied the two previous conditions were considered candidate match vectors to the originally chosen vector from the CC. To determine the real match vector, if any, the *epipolar geometry* (Ayache 1991; Mass *et al.* 1993) constrain was applied to every candidate vector.

The epipolar geometry constraint, in brief, indicates that for any point on an image, its corresponding match point on the corresponding stereo image lies on a line crossing the stereo image, but the precise location of the matching point on the line is unknown. The epipolar geometry is a very important condition used in robot and computer vision for the computation of the third coordinate from two 2D scenes, and it will not be covered here. The interested reader is referred to Ayache (1991) for a full description, and computational aspects. The epipolar constraint was employed here as follows. The distance between each of the component points, starting and end points, of the candidate velocity vector and its corresponding epipolar line were calculated. That candidate vector whose component points are closer to their epipolar lines was considered the winning matching vector. In the case of two or more vectors having one point close to its epipolar line, but the other point not so close, the vector whose magnitude in the v -component was closest to the reference vector from the CC was considered the winning vector.

Figure 7. The variation of u respect to the depth, y , at different x positions. a) Data from the Right Camera; and b) data from the Left Camera. Also shown, the results of applying a linear regression to the data points.

Observe that in this process a matching of the 2D vectors was performed, and not individual particles. Therefore, the possibility of mismatching particles was highly reduced. Also, there was an important reduction of computational time, since there was no need to go through the whole process for all the particles on the images. Actually, only the starting point of the vector was used to look for candidate matches. Furthermore, when there was only one candidate vector, or none, the epipolar constraint was not necessary to be computed.

3.5. Determination of the three-dimensional position of the tracer particles

Once a 2D vector was matched between a pair of the three cameras, the 3D position of each individual seed particle was computed. For this calculation of the (x, y, z) world coordinates of the particle, the image coordinates of the particle from two cameras, say CC and RC, and the calibration coefficients for each camera are used in the perspective projective equations, [2] and [3]. The system of equations is casted as

$$\begin{bmatrix} t_{11R} - u_R t_{31R} & t_{12R} - u_R t_{32R} & t_{13R} - u_R t_{33R} \\ t_{21R} - v_R t_{31R} & t_{22R} - v_R t_{32R} & t_{23R} - v_R t_{33R} \\ t_{11C} - u_C t_{31C} & t_{12C} - u_C t_{32C} & t_{13C} - u_C t_{33C} \\ t_{21C} - v_C t_{31C} & t_{22C} - v_C t_{32C} & t_{23C} - v_C t_{33C} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} u_R - t_{14R} \\ v_R - t_{24R} \\ u_C - t_{14C} \\ v_C - t_{24C} \end{bmatrix}. \quad [11]$$

This overdetermined system of equations was solved by least squares, using the same algorithms described in Chapter II. The system is similar for the combination of the CC and the LC. The system is applied to every single particle belonging to a vector that could be matched between the CC and the RC, CC and LC respectively.

A schematic showing the whole process from tracking 2D vectors to the 3D position reconstruction is presented in figure 8. In this figure the 2D tracking correspond to the ART2 NN. Only two frames would be shown for the tracking with the Spring Model.

Figure 8. 3D stereoscopic reconstruction technique.

CHAPTER IV

BUBBLE SHAPE IDENTIFICATION AND RECONSTRUCTION

The behavior of a bubble moving in a liquid differs from that of a solid particle. The bubble shape can change due to the action of hydrodynamic forces. The interaction between rising gas bubbles and the surrounding fluid determines the shape of the bubble and the extent of the disturbance to the surrounding flow field. Instantaneous bubble shapes and sizes are important because they reflect the dynamic changes of their pressures inside the bubbles and in the surrounding fluid. Consequently, the velocity field changes of the surrounding fluid are also expressed. Bubble shape and dimensions play an important role in heat and mass transfer between the continuous and dispersed phases, since they determine the interfacial area available for such phenomena. Bubble volume and shape also affect the rise velocity and drag coefficient.

Experimental investigations have shown that small air bubbles are spherical or slightly ellipsoidal, and their rising motions are rectilinear. When their equivalent diameters increase, the bubble shapes will change to ellipsoidal (although not necessarily symmetric), and they will move along a zigzag or helical paths. If the equivalent diameter increases again, the bubbles will have a spherical-cap shape, and their motion becomes rectilinear again. The new trends in computational fluid mechanics (CFD) show that bubble shape could be predicted via direct numerical simulation from first principles, assumptions as potential flow, and some empirical correlations (Tomiya 1998). To make such predictions reliable, experimental results are needed to provide systematic evaluation of surface interface response. The data available on bubble size and shape is by no means complete. Lui & Bankoff (1993) for example, have provided a comprehensive set of data on void fraction profiles and bubble chord length, but unfortunately no data on interfacial area concentration and bubble shape. Kataoka & Serizawa (1990) have provided interfacial area concentration data without bubble size and shape information.

The detailed information of bubble shape and size can only be obtained through visualization techniques. In this chapter, a technique that can reveal the full 2D projection of the shapes of bubbles rising in a liquid is presented. This technique is denoted the Shadow Particle

Image Velocimetry (SPIV). In conjunction with PIV, the measurement technique employed herein is a hybrid technique to identify the bubble shapes and velocity vectors.

4.1. Bubble detection in a bubbly flow

Multiphase flows can be easily studied by PIV, if the image of each phase is distinguishable. Several methods have been reported to separate the images resulting from different phases. The use of fluorescent tracers in the continuous phase as particle seeds is one way of solving this problem. With the aid of optical filters, one may separate the radiation scattered by fluorescent and neutral particles. In this process, one camera captures both phases. Another camera is equipped with a filter of the same wavelength of scattered radiation from the particles. Thus, it only captures the seed particle images. Then, by subtracting the images from both cameras the bubbles can be distinguished (Philip *et al.* 1994). This method of distinguishing phases also has the advantage that the camera equipped with the filter can identify the tracer particle images that overlap with the bubble images. Another method for identifying the phases considers the different image sizes or the gray levels of the seed particles and bubbles. The large difference in size and optical characteristic between tracer seeds and bubbles facilitates the flow phase discrimination (Hassan *et al.* 1998). This approach can be employed with or without applying the mask technique to a digital PIV record such that the images of particles that are either smaller or larger than a predetermined threshold value disappear from the image (Gui *et al.* 1996).

In this study, the size of bubble image was easily distinguishable from the images of the seeds as seen in figure 9a. From images like this one, at least the semiaxis of the bubble on the Z-direction can be determined, if an ellipsoidal shape is assumed for the bubble, or the radius, if a spherical shape is assumed. It also can provide the position of the center of the bubble in the Z-direction. Calculating the middle point between the two extremes of the bubble can estimate the center position. However, the uncertainty of the parameters can be significant. This is due to the strong reflection of the laser light on the bubble's surface does not allow for seeing the actual bubble boundary. On the X-direction, no reliable information can be inferred, since the bubble's edge is not visible. The position of the center of the bubble on the X-direction can be approximated by the u -coordinate of the centroid of the largest spot that is part of the bubble (the spot at top). Actually, this was the method used to determine the position of the bubble on the images acquired by the PIV cameras, and then permitting for an automated process to

Figure 9. Typical simultaneous images acquired by the PIV and Shadow cameras when the bubble is present in the test volume. a) Image from the Center camera. It shows the XZ world plane; and b) image from the Shadow camera. It shows the YZ world plane. These two images allow for the three-dimensional reconstruction of the bubble shape.

determine bubble trajectories. Although the positions of the centroids of the bubble are approximations, it was found to be very close to the actual positions determined with the SPIV technique. Consequently, the averages of the flow fields performed under the condition that the bubble followed a particular trajectory were not affected, as explained later.

4.2. *The shadow particle image velocimetry flow visualization technique*

Three-dimensional analysis is required for bubbles' complicated shape and motion. This is due to the complex 3D (spirals or zigzags) bubble trajectories that do not remain in the light sheet (as in 2D PIV measurements) for a long duration. For 3D PIV measurements, a cone of light illuminated the viewing volume via two optic fibers, see figure 1. In this approach, the bubble images obtained showed only fragments of the bubble, and not the whole 2D projection shape, as shown in figure 9a. Sometimes, the bubble image shape even delineated a different shape from the natural bubble shape. This effect can be due to the change of scale and also to perspective distortion of the lens action, the lack of adequate illumination, or the intense light

reflection from the bubble.

To measure the bubble's shape, the PIV system was supplemented with a red shadow image technique. The shadow is produced by the reflected red light rays from light emitting diodes (LEDs), located opposite to one of the digital cameras, the S camera in figure 1. The light intensity from the LEDs is low, compared to the laser light. A red filter was placed in front of the S camera lens to only capture the bubble shadow. This high pass filter blocked the intense green light reflected by the bubbles. The corresponding shadow image of the rising bubble in figure 9a is presented in figure 9b. Both, figures 9a and 9b can be considered typical images.

4.3. Two-dimensional bubble shape reconstruction

It is clear that a bubble shape reconstruction process is necessary when 3D PIV measurements are performed. Even when the 2D shape is well defined, as provided by the SPIV technique, parameters such as orientation and dimensions still need to be computed. Several shape-identification and reconstruction techniques for distorted or incomplete images exist. They range from simply matching the image to a geometrical curve or surface, to the use of parametric curves or surfaces (as an example, Bezier and spline curves), and to the mapping of the original shape into a parametric space (such as the Hough transform). In the present study, a 2D image reconstruction process has been developed to determine both shapes and dimensions of the bubbles. This reconstruction method is based on the Dynamic Generalized Hough Transform (DGHT) algorithm, which is presented in detail by Leavers (1992). The DGHT algorithm is particularly useful when the object to be recognized and reconstructed is symmetric (e.g., circles, ellipses, etc.). Then, a 3D reconstruction can be achieved by combining two or more images obtained from cameras at different view angle.

Shape detection of fluid particles is often performed by visual inspection of photographic plates, since the resolution of the photographic film is much higher than digital recording systems. For the latter, image processing is a tool to improve the quality of the image. Computer vision techniques, such as thresholding, noise removal, edge detection, etc., are part of the image processing. The image analysis process of an image is divided into several steps (Hassan *et al.* 1998). The main steps of the reconstruction procedure were applied to figures 9a and 9b, and are described next.

- First, the images were expanded back to 640×480 format. This step was necessary because to achieve a framing rate of 60 Hz the resolution of the cameras was dropped by

half on the Y-direction. The algorithm used to expand the images consists in introducing a new line of pixels, which is a copy of the previous line of pixels. Figures 9a and 9b are images already expanded back.

- The second step applies a predetermined threshold value to the gray-scale digital image. Consequently, a binary image is obtained. The threshold operator is applied here because the processing algorithms are easier to implement using the resulting binary image (Davies 1990). The threshold value depends on the brightness of the image. In this step, most (and sometimes even all) of the noise (seeds included) can be dismissed, since usually the bubble image is much brighter than the seeds. Then, the resulting image is inverted. The result of this step can be seen in figure 10a and 11a, for the figures 9a and 9b, respectively
- Once the image is thresholded, a wide range of binary imaging operations becomes possible. These operators are applied on the resulting images from the previous operation. The third step consists of applying a size filter, which includes reducing and expanding algorithms. Here, the spot images are reduced. This reduction can be repeated for a predetermined number of times, so small spot images vanish. Afterwards, the remaining spot images are re-expanded until their original size and shape are restored (Davies 1990).
- The fourth step removes the remaining noise through a salt-and-pepper filter. This noise, basically due to electronic noise, appears as a light spot on a dark background or a dark spot on a light background. This filter fills small holes inside the big images, and also smoothes the object boundaries. The result of using the size and the salt-and-pepper filters is shown in figures 10b and 11b.
- The fifth step in this approach consists of detecting the edges of the bubble image boundaries. Figures 10c and 11c present the results of applying the edge-detection operator to figures 10b and 11b, respectively
- The last operation consists of determining the points that can be used as connectivity points. This step is critical; if the total number of connectivity points is small or of bad quality, any detection or reconstruction method will introduce a high degree of uncertainty in the estimated values of the required parameters. The connectivity points to be used in the reconstruction algorithm are shown in figures 10d and 11d. Clearly the images acquired by the shadow camera yield many and good-quality connectivity

Figure 10. Bubble identification and reconstruction process for figure 9a. a) Binary image after applying the threshold and inversion operator; b) resulting image after applying size and salt-and-pepper operators to a; c) application of edge-detection operator to b; d) connectivity points to be used in the DGHT algorithm; e) the ellipse that best fits the connectivity points in d; and f) overlay of e and the original image. Also shown, the rotation angle with respect to the z-axis.

Figure 11. Bubble identification and reconstruction process for figure 9b. a) Binary image after applying the threshold and inversion operator; b) resulting image of applying size and salt-and-pepper operators to a; c) application of edge-detection operator to b; d) connectivity points to be used in the DGHT algorithm; e) the ellipse that best fits the connectivity points in d; and f) overlay of e and the original image. Also shown, the rotation angle with respect to the z-axis.

points. After these points are identified, the DGHT algorithm is applied.

4.3.1. The dynamic generalized Hough transform algorithm

The Hough transform decomposes a boundary into its constituent shape primitives. In principle, for any analytically given curve, the DGHT can be used to compute all of the parameters associated with the curve (Leavers 1992). In the present case, we will apply the DGHT algorithm for detection of the ellipsoidal shape, since it was already assumed that the bubble would have an ellipsoidal shape according to the values of the parameters Re , Eo and M . In this analysis, the equation for an ellipse is written as (Leavers 1992)

$$x^2 + y^2 - A(x^2 - y^2) - 2Bxy - Cx - Dy - E = 0. \quad [12]$$

Randomly choosing five of the connectivity points, one can solve for A , B , C , D , and E , by utilizing, for example, Gauss elimination with back propagation. The ellipse parameters are estimated after each iteration. The five parameters A , B , C , D , and E , are related to the ellipse parameters through

$$\theta = \frac{1}{2} \arctan \left(\frac{B}{A} \right) \quad [13]$$

$$e = \sqrt{\frac{\cos 2\theta - A}{\cos 2\theta + A}} \quad [14]$$

$$y_0 = \frac{1}{2} \frac{D(1-A) + BC}{1 - A^2 - B^2} \quad [15]$$

$$x_0 = \frac{1}{2} \frac{C + 2y_0 B}{1 - A} \quad [16]$$

$$a = \sqrt{\frac{(1 + e^2) \left(E + x_0 \frac{C}{2} + y_0 \frac{D}{2} \right)}{2e^2}} \quad [17]$$

$$b = ae \quad [18]$$

where θ is the rotation angle of the ellipse from the horizontal axis; e is the eccentricity (the ratio of the semiminor to semimajor axis); x_0 and y_0 are the center coordinates of the ellipse; a is the semiaxis in the X-direction; and b is the semiaxis in the Y-direction.

Although effective stopping criteria to end the calculation cycle for the DGHT algorithm is known, here it was simply iterated for a number of times. For the case of images from the PIV cameras, 150000 iterations were performed, while 10000 iterations were performed for the images from the shadow camera. These numbers for the iteration process were chosen by considering the average number of the connectivity points available, as well as the quality of such points. It should be mentioned that between 30 and 50% of the iterations for the PIV images did not yield values that satisfied the ellipse equation [12]. The shadow images successfully satisfied equation [12] about 90% of the iterations. For each parameter, the value with the highest frequency corresponds to the value that best satisfies the ellipse equation.

4.3.2. Accuracy of the reconstruction

After the iteration process finishes, the computed parameters of the ellipse can be used to draw the ellipse that best fits the connectivity points employed in the DGHT algorithm. Figures 10e and 11e show the ellipse that best satisfied the connectivity points given in figures 10d and 11d, respectively. Once the ellipse parameters are known, one can visually overlay the reconstructed ellipse over the original image for a fast comparison of the reconstruction process accuracy. Figures 10f and 11f present an overlay of the reconstructed ellipse and the original images 9a and 9b. The results are very encouraging; the agreement between the reconstructed ellipse and the original image is very good even for the PIV image. These images demonstrated the capabilities of the DGHT to be used in the study of bubbly flow images.

Histograms can be constructed for each parameter. The width of the histogram is related to the standard deviation, so it is a direct measurement of the uncertainty associated with the determined parameter. A well-defined, isolated peak indicates that the parameter has a good statistical value. When the distribution (the histogram) shows multiple or wide peaks, the quantity or quality of the connectivity points is insufficient or bad. Histograms of θ , x_0 and a are showed in figures 12 and 13, for the PIV and shadow images analyzed here, respectively. Clearly, the shadow image produced narrow and sharp peaks for x_0 and a ; while these

Figure 12. Histograms of three of the parameters of the ellipse that best fits the bubble in figure 9a, computed through the DGHT. a) Rotation angle; b) center of the ellipse on the x-direction; and c) value of the semiaxis on the x-direction.

Figure 13. Histograms of three of the parameters of the ellipse that best fits the bubble in figure 9b, computed through the DGHT. a) Rotation angle; b) center of the ellipse on the x-direction; and c) value of the semiaxis on the x-direction.

parameters from the PIV image show wider and bimodal peaks, as a consequence of the quality of the connectivity points used. For the PIV image, it should be mentioned that in the Y-direction the parameters produced well-defined peaks, because more information was available in the direction.

With respect to the rotation angle, both images yielded small and wide peaks. This, however, is not a failure of the DGHT, but a consequence of the very close values of the semiaxes. In other words, when a and b are close to each other, the ellipse becomes closer to a circle, and therefore the rotation angle becomes more undefined.

The DGHT algorithm offers some advantages over other reconstruction techniques. It yields all five parameters associated with the bubble, so dimensions, shape and travel path can be determined. Other techniques, simply interpolate, and sometimes extrapolate, between the boundary points available for the reconstruction. Although this can give a closer match to the real shape of the bubble, the dimensions and 3D movement will have to be determined by other means. Memory storage and computation time in the DGHT algorithm are low. When occlusion of objects exists the DGHT algorithm has also been proven to get good results. The primary disadvantage of this scheme is the need for an *a priori* analytical expression for the curve to be detected. Thus, for a very distorted bubble, the best that the DGHT algorithm can achieve is the approximation of the actual shape to a given curve.

4.4. Three-dimensional bubble shape reconstruction

Once the parameters of the ellipse are known for both the PIV and the shadow images, it is necessary to compute these parameters in world coordinates. Observe that the world coordinates can be easily calculated in this case, because the PIV center and the shadow cameras are parallel to the XZ and YZ planes, respectively, as demonstrated in Chapter II. The process is very similar process to that of the 3D reconstruction of the position of the tracer particles, shown in Chapter III. The calibration curves for calculating world coordinates from image coordinates are shown in figure 14.

After combining the parameters of ellipses from each view, a 3D reconstruction was achieved. The parameters of this bubble were the following. The centroid of the bubble was located at $X_0 = -0.16$ mm, $Y_0 = -1.99$ mm, and $Z_0 = 3.86$ mm. The values of the semiaxes were: on the X-direction, $a = 1.24$ mm; on the Y-direction, $b = 1.46$ mm; and on the Z-direction, $c = 1.39$ mm. The rotation angles about the Y and the X axes were $\alpha = -4$ and $\beta = 25$ degrees,

(a)

(b)

Figure 14. Calibration curves for computing world coordinates from image coordinates. a) Variation of y against u on images from the Shadow camera; and b) variation of z against v for images from both the Shadow and the Center cameras.

respectively. These angles are measured positive from the z -axis, and are positive counter-clockwise. The spherical-equivalent radius of the bubble was 1.36 mm. The scale factors used for the calculations were 38.8 pixel/mm, 39.7 pixel/mm, and 39.5 pixel/mm on the X-, Y-, and Z-directions, respectively.

Figures 15 presents three different views of the 3D reconstructed bubble. Experimental data for air bubbles rising in stagnant water indicate that for a bubble with a spherical-equivalent radius of the computed dimensions, the shape of the bubble should be very close to an sphere. Observe that the bubble reconstructed here is very close to a spheroid.

Finally, the error associated with the reconstruction algorithm presented was always less than 3 pixel (0.08 mm). This maximum error is obviously for the PIV image, on the X-direction. For the localization of the centroid the error is smaller, but it will be considered as 3 pixel in the error analysis calculations.

Figure 15. Three different views of the three-dimensional reconstruction of the bubble shown in figure 9, after combining information from the two-dimensional reconstructions shown in figures 10e and 11e.

CHAPTER V

FLOW AROUND A RISING BUBBLE AND WITHIN ITS WAKE

The previous chapters have dealt with the development of a methodology capable of providing transient, three-dimensional information about the flow field generated by a freely rising bubble in a liquid. Such methodology is based on the measurement capabilities of the PIV technique. Furthermore, with the aid of the SPIV technique, an accurate quantitative and qualitative description of the bubble shape fluctuation, orientation, and trajectory can be obtained. In this chapter, the experimental results concerning the flow field upstream and downstream of an ellipsoidal air bubble rising in stagnant water, in a small-diameter pipe are presented and discussed. The wall influence on the development and behavior of the liquid flow is particularly addressed. The information obtained from the SPIV measurements is presented in the next chapter.

This chapter first presents an analysis of the uncertainties associated to the PIV measurements, and then the error propagation in the calculations of average and fluctuating quantities, vorticity, and kinetic energy. Then a brief theoretical background of the flow around bubbles is presented. Finally, the experimental data is presented and discussed.

5.1. Uncertainty analysis of PIV measurements and error propagation

Flow visualization techniques cannot resolve the whole time and length scales due to camera resolution and limited frame rate. In our measurements, limits on the minimum and maximum range of the 2D tracks were set after analyzing many of the acquired pictures. It was found that $4 \text{ pixels} \leq u \leq 25 \text{ pixels}$ (38.80 pixels/mm), and $4 \text{ pixels} \leq v \leq 40 \text{ pixels}$ (19.56 pixels/mm) were the optimum values for the 2D tracking of the velocity vectors. In terms of the velocity units, the ranges are $6.19 \text{ mm/s} \leq V_u \leq 38.66 \text{ mm/s}$, and $12.27 \text{ mm/s} \leq V_v \leq 122.72 \text{ mm/s}$.

The uncertainty in the velocity measurements is a direct consequence of a combination of errors from three main sources. These are the localization of the centroid of the seed particle images and their displacements in the 2D images, the determination of the 3D position of the seed particles, and the uncertainty of how well the tracer particles follow the actual fluid motion. These uncertainties are considered in detail next.

5.1.1. The Stokes number

Estimation of how closely the seed particles follow the flow, or how fast the seed particles respond to the flow changes, can be calculated utilizing the particle Stokes number. This parameter can be calculated as the ratio of the relaxation time of the seed particle to the characteristic time of the flow. The particle relaxation time, τ_p , also known as the momentum or velocity response time, is the time needed by a seed particle to reach $e^{-1} U$, if the particle was initially at rest. Here U is the fluid velocity. The time τ_p can be computed from

$$\tau_p = \frac{(2 \rho_p + \rho_f) d_p^2}{36 \mu_f}, \quad [19]$$

where ρ is the density, d the seed tracer diameter, μ the viscosity, and the subscript p refers to particle properties, and f to the fluid properties. In this experiment, the relaxation time of the seed was calculated to be 140 μ s. The fluid time scale, τ_f , is the time needed by the fluid to be advected a characteristic length L , that is,

$$\tau_f = \frac{L}{U}. \quad [20]$$

For the characteristic time of the flow, an equivalent diameter of the bubble of 3 mm was used as the characteristic length, and the maximum measurable liquid velocity. The resultant time was 24 ms. Then, the Stokes number, τ_p / τ_f , was calculated to be 5.72×10^{-3} . Therefore the 40 μ m seed particles closely followed the fluid motion changes, and therefore did not significantly contribute to the total error.

5.1.2. Accuracy of two-dimensional localization of particle centroid

Adrian (1986) determined the error associated in localizing the 2D particle-image centroids, and their displacements between two or more consecutive frames. For the continuous laser employed in this experiment, the uncertainty in the time interval comes from the laser pulse width, (250 μ s), so it had a negligible effect on the total error of the velocity measurements. The error in locating the centroid displacements depends on the optical parameters of the camera,

lenses, and laser, and the algorithm for finding the particle-image centroid. The FINDSPOTS computer program (Blanchat 1992) used has a subpixel accuracy of about 95%, and the magnification of the system used in the experiment allowed for even better accuracy. The error in determining the centroid displacement was less than 1% of the total error.

5.1.3. Uncertainty of three-dimensional particle position and instantaneous velocity

The main error source in the velocity measurements was the calculation of the three world coordinates of the seed particles. For this type of error, the calibration is the main source, since it was done by using a least squares method, as explained in Chapter II. To calculate the error associated with each world coordinate, [11] was used to compute the 3D positions of all the calibration points. Then, the computed values were compared against the known positions of the calibration points. The root mean square (r.m.s.) error in each direction σ_x , σ_y , and σ_z , respectively, was 26.4 μm in the X-direction; 96.1 μm in the Y-direction; and 23.2 μm in the Z-direction. In terms of the velocity, the r.m.s. error for each component, σ_u , σ_v , and σ_w , respectively, was: 1.58 mm/s in the x-component; 5.77 mm/s in the y-component; and 1.39 mm/s in the z-component. These values are estimated for a framing rate of 60 Hz. It can be concluded that the error associated with small magnitude velocity vectors can be significant.

5.1.4. Error propagation

If the errors associated to a measurement are small and symmetric around zero, the standard deviation for any quantity s , σ_s , derived from, say, three measurements is given by

$$\sigma_s^2 = \left(\frac{\partial s}{\partial r_1} \right)^2 \sigma_{r1}^2 + \left(\frac{\partial s}{\partial r_2} \right)^2 \sigma_{r2}^2 + \left(\frac{\partial s}{\partial r_3} \right)^2 \sigma_{r3}^2, \quad [21]$$

where r_1 , r_2 , and r_3 are the values of the measurements, and σ_{r1} , σ_{r2} , and σ_{r3} , are their respective associated errors. This equation is known as the *error propagation formula*, and it will be used for the calculations of the errors of the quantities derived from the velocity measurements and their associated errors.

For the calculation of the velocity gradients, the central finite difference scheme, given by

$$\frac{\partial u_i}{\partial r} \approx \frac{u_{i+1} - u_{i-1}}{2\Delta r}, \quad [22]$$

which has a second order accuracy, was used. The error for the gradient of the u velocity component, for example, can be computed by observing that the function s , in this case, is an addition of measurements, so using [21],

$$\sigma_{ur} = \sqrt{(\sigma_u^2 + \sigma_u^2)}, \quad [23]$$

where it has been assumed that the σ_u is the same at every point. Then, $\sigma_{ur} = 2.23$ Hz. For the rest of the components, $\sigma_{vr} = 8.16$ Hz, and $\sigma_{wr} = 1.97$ Hz.

The vorticity vector is given by

$$\boldsymbol{\omega} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \mathbf{k}. \quad [24]$$

Thus, the error for the vorticity vector components was estimated through equations similar to [23], that is,

$$\sigma_{\omega x} = \sqrt{(\sigma_{wy}^2 + \sigma_{vz}^2)}. \quad [25]$$

Therefore $\sigma_{\omega x} = 8.39$ Hz, $\sigma_{\omega y} = 8.16$ Hz, and $\sigma_{\omega z} = 1.97$ Hz.

For the calculation of the error in the average [21] yields, for the $\bar{u}$ velocity component, as an example,

$$\sigma_{\bar{u}} = \frac{\sigma_u}{\sqrt{n}}, \quad [26]$$

where n is the number of samples utilized in the average operation. The expressions for $\bar{v}$ and $\bar{w}$ are similar. The error associated to the mean kinetic energy,

$$\overline{K} = \frac{1}{2} m (\overline{u}^2 + \overline{v}^2 + \overline{w}^2), \quad [27]$$

where m is the mass of the liquid in motion, can be computed from

$$\sigma_{\overline{K}} = m \sqrt{\frac{1}{n} (\overline{u}^2 \sigma_u^2 + \overline{v}^2 \sigma_v^2 + \overline{w}^2 \sigma_w^2)}. \quad [28]$$

The fluctuating components of the velocity vector are calculated by using Reynolds decomposition. The u' fluctuating component of the instantaneous velocity can be obtained from

$$u' = u - \overline{u}. \quad [29]$$

The error for the fluctuating components of the velocity can be calculated from

$$\sigma_{u'} = \sqrt{\sigma_u^2 + \sigma_{\overline{u}}^2} = \sqrt{\frac{n+1}{n}} \sigma_u, \quad [30]$$

and similarly for the other fluctuating components. Finally, the turbulent kinetic energy,

$$k = \frac{1}{2} m (u'^2 + v'^2 + w'^2), \quad [31]$$

has an error associated given by

$$\sigma_k = m \sqrt{\frac{n+1}{n} (u'^2 \sigma_u^2 + v'^2 \sigma_v^2 + w'^2 \sigma_w^2)}. \quad [32]$$

5.2. Theoretical background

Newton's second law of motion states that the product of the mass and the acceleration experienced by a body equals the total external force acting on the body, that is

$$m \frac{D\mathbf{u}}{Dt} = \mathbf{F}, \quad [33]$$

where $\mathbf{F}$ is the sum of all external forces, and

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \quad [34]$$

is the substantial, or substantive, acceleration. This is the acceleration following the body (the Lagrangian approach), and contains a local contribution, for unsteady flow, and a contribution due to the convection of the flow.

Consider now a fluid element, and apply [33] to it. Then, when the fluid element is in motion two kinds of external forces act on it. One is a gravitational force, which acts throughout the mass of the fluid element. The other force acts on the fluid element surface. This force is composed of the pressure and friction forces. If the fluid is incompressible and Newtonian with density, ρ , and viscosity, μ , being constant, and the components of $\mathbf{F}$ are considered, [33] can be written as

$$\rho \frac{D\mathbf{u}}{Dt} = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{u}, \quad [35]$$

which are known as the Navier-Stokes equations (NSE).

The continuity equation for an incompressible flow,

$$\nabla \cdot \mathbf{u} = 0, \quad [36]$$

and the NSE, [35], are sufficient to solve for the unknowns $\mathbf{u}$ and p , provided appropriate boundary conditions are applied.

For a system with two phases, one of them being continuous and the other dispersed, the continuity and the NSE need to be solved for both phases. Interfacial relationships are then required to close the problem. Consider a solid particle in a flow stream. The requirement of no flow through the particle's surface implies that the normal and tangential flow velocities are zero relative to the solid particle velocity. The tangential velocity condition is known as the *non-slip*

condition, while the normal velocity condition is known as the *kinematic* condition.

The difference between fluid and solid particles cannot be neglected in the calculations of the velocity field. In a viscous fluid particle there exists an internal velocity field, or internal circulation, and the velocity at different locations on the particle can be different, for example there might be a velocity difference at the center and the surface of the fluid particle. For a fluid particle in a continuous fluid, additional boundary conditions are required to completely close the continuity and NSE for each phase. These additional boundary conditions are that the normal and tangential stresses must be balanced at the interface. These balances of stresses reflect the Newton's third law of motion. The stress is a consequence of the pressure field around the particle, and the tangential stress is due to shearing on the surface of the fluid particle.

By taking the curl of [34], the vorticity transport equation can be obtained as

$$\frac{D \boldsymbol{\omega}}{Dt} = \frac{\partial \boldsymbol{\omega}}{\partial t} + \mathbf{u} \cdot \nabla \boldsymbol{\omega} = \boldsymbol{\omega} \cdot \nabla \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega}. \quad [37]$$

This equation states that the change in local vorticity plus the convection of vorticity by the flow equal the stretching or compression of the vorticity by the flow plus a diffusion term due to viscous friction. This equation is frequently used to describe the circulatory motion inside the wake of a bluff body. The wake is the region downstream a body in a flow, where there exists vorticity. The rotational flow inside a wake is due to the magnitude difference of the pressure and velocity between the outer and interior regions of the wake. For two-dimensional axisymmetric flows, the first term on the right hand side of [37] is equal to zero, which, in such a case, means that the distribution of vorticity is due only to viscous diffusion.

Analytic solutions for the flow around rigid and fluid particles of spherical shape are only available for $Re < 1$ ($Re = \rho U d / \mu$, where U is the free stream velocity and d the particle's diameter). For intermediate Re , numerical solutions exist, but usually limited to steady and axisymmetric flows. In these studies, the most common approach is to use the potential flow theory, limiting the boundary layer to a very small area surrounding the spherical particle. The boundary layer theory has been applied with success only for high Re ($Re > 3000$). For lower Re , the main difficulties are that the predicted velocity and pressure distributions are comparable to the experimental data up to 20 to 30 degrees from the front stagnation point on the particle's surface (Clif *et al.* 1978).

The study of the turbulence structure in a two-phase bubbly flow is one of the problems in which experimental, numerical and theoretical work is being extensively done nowadays. It is now considered that the turbulence in two-phase flow has two different sources: one is the turbulence generated in the continuous liquid phase, and the other is the turbulence induced by the movement of the bubbles in the flow. This last agitation due to the bubbles has been called pseudo-turbulence (Lance & Bataille 1991).

Most of the difficulties faced by the experimental and numerical communities when studying turbulence structure are the large range of time and length scales existing in turbulent flows. Consider, for example, the case of a single bubble rising in water. In this case all the liquid turbulence is pseudo-turbulence. The ratio of the velocity magnitude for positions close to the bubble to the points several bubble diameters away from the bubble's surface is orders of magnitude. This implies stiff differences in the time and length scales along at such given positions. Further, the wake of the bubble causes flow fluctuations that reach up several bubble diameters.

The most common approach to describe the flow field around a rising bubble is the potential flow. For dispersed bubbly flow, the influence of each bubble on the liquid motion is applied by using the superposition principle. Potential flow fails in describing the flow in the wake of the bubble, so it is clear that the superpositioning effects will not provide an accurate prediction of a bubbly flow.

The Reynolds stress tensor transport equation for a two-phase system can be written as:

$$\frac{D}{Dt}(\overline{\mathbf{u}'\mathbf{u}'}') = \text{Diff}(\overline{\mathbf{u}'\mathbf{u}'}') + \mathbf{P} + \Phi - \varepsilon + \Pi, \quad [38]$$

where the left hand side of the equation represents the convection of the stress components by the mean flow. On the right hand side presents the diffusion term, Diff; the production, $\mathbf{P}$, due to liquid shear; the distribution of the stresses, Φ , due to pressure fluctuations; the dissipation term, ε ; and an interfacial term, Π ; respectively. This equation takes into account the liquid turbulence in the two-phase flow momentum equations. It is assumed that the stress tensor is composed of the single-phase liquid turbulence, denoted here by (l) , and the bubble's pseudo-turbulence, denoted by (b) . Consequently, it can be written as:

$$\overline{u'_i u'_j} = \overline{u'_i u'_j}^{(l)} + \overline{u'_i u'_j}^{(b)} . \quad [39]$$

These turbulent correlations are considered to have significant importance in bubble migration, and therefore in phase distribution.

For homogeneous potential bubbly flows, the Bieshevel & van Winjgaarden's (1984) results are widely used. They demonstrated that the pseudo-turbulence stress tensor is given by

$$\overline{u'_i u'_j}^{(b)} = \frac{3}{20} \alpha \| \mathbf{U}_r \|^2 \delta_{ij} + \frac{1}{20} \alpha \overline{\mathbf{U}_r \mathbf{U}_{rj}} , \quad [40]$$

where α is the void fraction, and the relative velocity between the liquid and bubble is given by

$$\mathbf{U}_r = \mathbf{u}_l - \mathbf{u}_b . \quad [41]$$

For the turbulence of the liquid, a widely used approach is the k - ϵ model, and variations of it.

5.3. Nodalization and conditional sampling

In this experimental investigation, 81 sets of data were acquired. Each set consisted of 27 frames resulting in a total of 2187 frames for the experiment. Figure 16 shows the distribution of the bubble centroids for the whole experiment on the XY plane. This figure clearly shows the anisotropy of the bubble paths during rising. The bubbles were released at the pipe center.

The data sets were divided into groups having similar bubble trajectories to perform conditional sampling. A similar trajectory is that in which the bubble, in its rising path, within the viewing test volume, passes through the same region (locations) in at least 3 of 4 frames, in which the bubble was present in the viewing volume. The nodalization scheme for the bubble position in the viewing volume is illustrated in figure 17a. The horizontal plane was divided into five regions. The circle segment number 1 is at the center of the pipe, with a radius of 2.25 mm. The annulus region was divided into four segments, 2, 3, 4, and 5, as shown in the figure. The axial nodalization consists of four segments, each with a size of $\Delta Z_b = 2.75$ mm. For the liquid phase data, the measurement volume is divided into a 3D grid of $9 \times 9 \times 9$ in the X-, Y-, and Z-

Figure 16. Projection of the distribution of the bubble centroids for the experiment of 240 images on the XY plane. The anisotropy of the bubble paths during rising within the test volume should be noted. The bubbles were released at the pipe center.

Figure 17. Nodalization of the test section. (a) Gas; and (b) liquid phases. For the air bubble, the nodalization has been established considering an average equivalent bubble diameter of 3 mm, and its location is determined by its centroid.

directions, respectively. The total number of the cells is 729, although only 621 fall inside the pipe. The size of the node for ΔX and ΔY is equal to 1.41 mm and for $\Delta Z = 1.22$ mm. Figure 17b illustrates the nodalization used for the analysis of the liquid phase data. The distribution of various trajectories was as follows: 17.3% of the bubbles followed trajectory 111; 2.5% of the rising paths were trajectory 222; 3.7% of the bubbles traveled trajectory 333; 19.8% had bubble trajectory 444; and 18.5% had trajectory 555. The rest of the rising paths were a combination of trajectories.

Once the velocity fields were determined for each separated bubble trajectory, and in order to better quantify the wall influence in the structure of the flow, and to obtain more samples for the averaging operation, the nodalization scheme for bubble position in the test volume was then considered of consisting of only two zones in the horizontal plane. Namely, one zone close to the wall as regions 2, 4, and 5, and a zone far from the wall as region 1, see figure 17a. The value of the radius dividing both regions was set to 2.25 mm (same as above). This value was chosen as optimum after studying the bubble's trajectory for all of the data sets. The regions in the pipe annulus were all rotated to coincide to the third quadrant ($x < 0$ and $y < 0$) in the horizontal plane, assuming symmetry of the flow fields, so an average for all trajectories close to the wall could be performed. The bubble trajectory close to the pipe wall consequently occurred 44.5% of the experiment.

An example of bubble trajectory 111 is depicted in figure 18. This figure clearly indicates that the bubble trajectories were not straight paths for this experiment. The actual trajectory of the bubble in figure 18 is 1112. A similar figure for a trajectory close to the wall is shown in figure 19. For this figure, the actual trajectory is 4445. The Reynolds number of the bubbles, $Re_b = \rho_l U_b d_b / \mu_l$, was in a range from about 350 to 700. At this range, a bubble freely rising in stagnant water exhibits oscillating motion along the rising path, and shape change. The predicted shape is ellipsoidal, although not necessarily symmetric. The motion is expected to be helical and/or zigzag, and even with a rocking motion. The helical paths along with the rocking motion, and the ellipsoidal shape, were observed in this experiment with both the combined PIV and shadow images, as it can clearly be seen in figures 18 and 19.

By combining the bubble data for a specified trajectory, we obtained an average flow field, for each instant in time. The conditional average velocity was obtained through

Figure 18. Example of bubble trajectory at the center of the pipe. The two-dimensional projections of the trajectory from the Center and Shadow cameras, and the three-dimensional bubble reconstructions are shown. Actual trajectory is 1112.

Figure 19. Example of bubble trajectory along the pipe wall. The two-dimensional projections of the trajectory from the Center and Shadow cameras, and the three-dimensional bubble reconstructions are shown. Actual trajectory is 4445.

Figure 20. Total number of vectors used for the averaging operation at each time step for both bubble trajectories. $t = 0$ is the first frame in which the bubble appears in the viewing volume.

$$\bar{u}(x, y, z, t) = \frac{1}{n} \left\{ \sum_{i=1}^n u_i(x, y, z, t) \begin{array}{l} \text{specified} \\ \text{bubble} \\ \text{trajectory} \end{array} \right\}. \quad [42]$$

The total number of vectors for each time step for the trajectories in the pipe center and close to the wall is shown in figure 20. Clearly, from figure 20, only the time interval from $t = -75$ ms to about $t = 375$ ms can be considered to have enough vectors to yield reliable results. The averaged flow field was interpolated in the regions in which no velocity vectors were found. The interpolation algorithm is based on the Hardy scheme (Hardy 1990; Blanchat 1992), which performs a biquadratic interpolation.

5.4. Mean flow field description

Due to the three-dimensionality and unsteadiness of the flow behavior, 3D plots of all parameters are needed to better understand of the flow structure. Figures 21a to 24a present the

distribution of the mean kinetic energy for the four frames in which the bubble is present in the viewing volume, for the bubble trajectories rising along the pipe center. These frames correspond to $t_p = 0$ ms, $t_p = 16.67$ ms, $t_p = 33.3$ ms, and $t_p = 50.0$ ms, respectively. Similarly, figures 21b to 24b show the distribution of the magnitude of the vorticity in the test volume. Observe that the areas of higher mean kinetic energy coincide with the areas of higher vorticity magnitude, in general. This is explained as follows. For this bubble trajectory, at the instant of the bubble presence in the viewing volume, practically all of the liquid flow is restricted to a space from about one bubble diameter, d_b , to two d_b between the bubble boundary and the pipe wall, in the horizontal plane. Due to the incompressibility of the water, the flow experiences acceleration to compensate for the smaller flow area available. The areas surrounding the bubble, therefore, are the ones where the mean kinetic energy and vorticity reach higher values, as clearly seen in the respective figures. The flow closer to the bubble travels upstream surrounding the bubble's primary wake, and collides at the end of the wake, close to the center of the pipe. This collided flow generates a flow barrier, which decelerates any flow passing through it. It should be noted that a symmetric flow is not expected. This is because the bubble did not rise in a straight trajectory. In addition, ellipsoidal shape and rocking motion were observed. Similar results were observed for the trajectories close to the pipe wall. That is, the higher values of the mean kinetic energy and vorticity magnitude are found close to the bubble, see figures 25a to 28a, and 25b to 28b, respectively. The flow in the radial direction apart from the bubble with a distance more than one bubble diameter is practically undisturbed, reducing its magnitude as it gets closer to the pipe wall.

The influence zone of the bubble is different for these two trajectories. For trajectory along the pipe core, the bubble influence reaches downstream to an average distance of two bubble diameters, from the bubble's top. This can be seen in figure 21. For the bubble trajectories close to the pipe wall, the bubble influence downstream on the flow field reaches greater distances in comparison with the cases of bubbles rising along the pipe core. As it can be seen in figure 25, the downstream influence reaches to an average of 3 d_b . This is so because the increase of the acceleration of the liquid flow between the bubble surface and the pipe wall is limited due to the wall friction effect. This friction overcomes the momentum transfer from the bubble to the liquid. Consequently, the bubble pushes up liquid that experiences this resistance. Subsequent liquid layers are then pushed further up.

The bubble's primary wake for trajectories along the pipe center extends 3 d_b upstream, from

Figure 21. Distribution of a) mean kinetic energy, in μJ ; and b) vorticity magnitude, in Hz, at $t_p = 0$ ms. Bubble rising along the pipe center.

Figure 22. Distribution of a) mean kinetic energy, in μJ ; and b) vorticity magnitude, in Hz, at $t_p = 16.67$ ms. Bubble rising along the pipe center.

(a)

(b)

Figure 23. Distribution of a) mean kinetic energy, in μJ ; and b) vorticity magnitude, in Hz , at $t_p = 33.33$ ms. Bubble rising along the pipe center.

Figure 24. Distribution of a) mean kinetic energy, in μJ ; and b) vorticity magnitude, in Hz, at $t_p = 50.0$ ms. Bubble rising along the pipe center.

Figure 25. Distribution of a) mean kinetic energy, in μJ ; and b) vorticity magnitude, in Hz , at $t_p = 0$ ms. Bubble trajectory close to the pipe wall.

Figure 26. Distribution of a) mean kinetic energy, in μJ ; and b) vorticity magnitude, in Hz , at $t_p = 16.67$ ms. Bubble trajectory close to the pipe wall.

Figure 27. Distribution of a) mean kinetic energy, in μJ ; and b) vorticity magnitude, in Hz, at $t_p = 33.33$ ms. Bubble trajectory close to the pipe wall.

Figure 28. Distribution of a) mean kinetic energy, in μJ ; and b) vorticity magnitude, in Hz , at $t_p = 50.0$ ms. Bubble trajectory close to the pipe wall.

the bubble's bottom. The primary wake seems to be closed and not turbulent. This could not be completely determined because of the lack of resolution in the measurements. The primary wake length for trajectories close to the wall is extended to a distance of $3 d_b$. However, it was distorted and of smaller volume than that of the case of bubble trajectory along the pipe core. It was not possible to determine whether the wake is closed or open. The flow barrier was also generated but was restricted to an area of the bubble size.

Recent experimental investigations (Lunde & Perkins 1997; Brücker 1999) have shown that for spiraling bubbles two parallel counter-rotating vortex filaments are attached to the bubble's rear. The wake of zigzagging (moving only in a plane) and rocking (non-coplanar motion) bubbles, instead, is intermittent, and consists of hairpin vortices. These are shed from the bubble surface with more or less regular frequency, although the vortex shape depends on the bubble size. These results are applicable, however, only to situations where wall influence is negligible. As mentioned before, the rocking and spiraling bubble motion was predominant in this experiment. Therefore, a combination of continuous trailing wake and vortex shedding is expected. However, due to the averaging process, in which the bubble rocking motion was not considered, the wake structure cannot be completely described. The results obtained here seem to indicate the wake structure associated with a spiraling bubble, more than a rocking one, at least for the primary wake. For the far wake, it seems more probable that the irregular vortex structures observed are originated from vortex shedding, and then associated to rocking bubble motion. The vortex structures shed from the bubble's surface do not spread because of the wall. Further, the constant interaction of these vortex spots with the wall, between themselves, and with the mean and turbulent flow, distorted the circulatory motion.

5.5. Turbulent flow field description

It should be noted that the spatial resolution of the present measurements close to the bubble's thin boundary layer is not enough to resolve the large velocity gradient close to the bubble surface. The location of the high turbulence spots was studied in detail to determine their behavior with regards to the mean flow. It was noted that the turbulence spots are usually at the same distance from the bubble. Therefore, such turbulence spots are being continuously generated as the bubble rises up. However, the turbulent kinetic energy is not always the same at such spots. Figures 29 to 32 show the distribution of the turbulent kinetic energy at $t_p = 33.33$ ms, for the two bubble trajectories studied here. It can be deduced from the figures that when the

Figure 29. Distribution of turbulent kinetic energy, in μJ , at $t_p = 0$ ms. a) Bubble trajectory along the pipe center; and b) bubble trajectory close to the pipe wall.

(a)

(b)

Figure 30. Distribution of turbulent kinetic energy, in μJ , at $t_p = 16.67$ ms. a) Bubble trajectory along the pipe center; and b) bubble trajectory close to the pipe wall.

(a)

(b)

Figure 31. Distribution of turbulent kinetic energy, in μJ , at $t_p = 33.33$ ms. a) Bubble trajectory along the pipe center; and b) bubble trajectory close to the pipe wall.

Figure 32. Distribution of turbulent kinetic energy, in μJ , at $t_p = 50.0$ ms. a) Bubble trajectory along the pipe center; and b) bubble trajectory close to the pipe wall.

bubble travels close to the pipe wall, it generates more turbulence than for the case when the bubble trajectory is along the center of the pipe. Plots for the turbulence intensities and Reynolds stresses are shown in figures 33 to 38. These plots are for trajectories along the pipe core and close to the pipe wall, at $t_p = 33.33$ ms.

The turbulent intensities $\sqrt{u'^2}$ and $\sqrt{w'^2}$ are extended farther upstream, when the bubble is rising close to the wall. For the case of $\sqrt{v'^2}$, it reaches about the same distance for both bubble trajectories. The presence of the turbulent intensities, downstream of the rising bubble, is farther when bubble trajectory is close to the wall. The magnitudes of $\sqrt{u'^2}$ and $\sqrt{w'^2}$ are higher for bubble trajectory close to the pipe wall; but the opposite happens in the pipe core for $\sqrt{v'^2}$. These results are in agreement with the characteristics of the flow.

5.6. Transient analysis of the kinetic energy

Figure 39 shows the behavior of the mean, turbulent and total kinetic energy through the whole measurement, for both bubble trajectory cases studied here. In this figure, time step 0 corresponds to the first frame in which the bubble is present in the viewing volume. Observe that on the moments in which the bubble is present in the test zone, the mean kinetic energy in the volume, and consequently the total kinetic energy, reaches its maximum. The sharp increase and decrease of energy in the volume are observed at the first frame (time $t_p = 0$ ms) in which the bubble is the test zone, and the second frame after the bubble has departed the viewing volume (time $t_a = 16.67$ ms), respectively. It can also be seen in figure 39 that the mean kinetic energy is higher when the bubble rises along the center of the pipe. This is because practically all the flow is accelerated due to reduced flow area available. In the case of the bubble close to the pipe wall, the flow away from the bubble does not reach the same high acceleration as in the case of the bubble rising along the pipe center, as explained above.

With respect to the turbulent kinetic energy, once the bubble and its primary wake have left the test zone, the close-to-the-wall bubble trajectories generate more turbulence. This could be explained by considering vortex shedding. It is known that vortex shedding is reduced, and even suppressed, when the distance between a bluff object and the container wall is small (Bosch *et al.* 1996). Since the vortex shedding may not be periodic in this case (due to the combination of rocking and spiraling bubble motion), and the vortex strength could be lower, the surrounding

Figure 33. Turbulence intensity $\sqrt{u'u'}$ in mm/s. The data presented are for $t_p = 33.33$ ms after the bubble enters the viewing volume. (a) Bubble trajectory along the pipe core; and (b) bubble close to the pipe wall.

Figure 34. Turbulence intensity $\sqrt{v'v'}$ in mm/s. The data presented are for $t_p = 33.33$ ms after the bubble enters the viewing volume. (a) Bubble trajectory along the pipe core; and (b) bubble close to the pipe wall.

Figure 35. Turbulence intensity $\sqrt{w'w'}$ in mm/s. The data presented are for $t_p = 33.33$ ms after the bubble enters the viewing volume. (a) Bubble trajectory along the pipe core; and (b) bubble close to the pipe wall.

Figure 36. Reynolds stresses $\overline{u'v'}$ in mm^2/s^2 . The data presented are for $t_p = 33.33$ ms after the bubble enters the viewing volume. (a) Bubble trajectory along the pipe core; and (b) bubble close to the pipe wall.

Figure 37. Reynolds stresses $\overline{v'w'}$ in mm^2/s^2 . The data presented are for $t_p = 33.33$ ms after the bubble enters the viewing volume. (a) Bubble trajectory along the pipe core; and (b) bubble close to the pipe wall.

Figure 38. Reynolds stresses $\overline{u'w'}$ in mm^2/s^2 . The data presented are for $t_p = 33.33$ ms after the bubble enters the viewing volume. (a) Bubble trajectory along the pipe core; and (b) bubble close to the pipe wall.

Figure 39. Transient behavior of the kinetic energy in the test volume. Time 0 corresponds to when the bubble is first present in the viewing volume.

turbulent flow in the wake can easily distort the vortex pattern. This indicates that the bubble's wake is turbulent for such trajectories close to the wall. In the case of the bubble rising along the center of the pipe, the vortex shedding from the bubble's surface is more periodic, that in the case when the bubble rocks. Thus, the turbulence generated in both cases is due to interaction between flow, which after colliding with the wall is redirected towards the pipe center, and the vortex structures arriving into the volume.

It is important to note that the total kinetic energy is practically the same when the bubble is in the viewing volume for both bubble trajectories. The maximum is reached at the third time step the bubble is present in the test zone ($t_p = 33.33$ ms). This maximum is about $70.0 \mu\text{J}$. The results indicate that bubbles of similar size bring the similar amount of momentum into the test volume. To get a better idea of how much energy is brought by the bubble in the test volume, consider the velocity magnitude corresponding to $70.0 \mu\text{J}$. The speed of the liquid in the test volume would be about 33.0 cm/s. This is about 50% higher than the speed of the bubble alone.

It is also interesting to note that the decay of the kinetic energy is practically the same for both trajectories. The difference exists because more turbulence is generated by the bubbles rising close to the wall, as explained before. After the primary wake completely leaves the measurement zone ($t_a = 33.33$ ms), the transient behavior of the total kinetic energy is practically constant. This indicates that the decay constant is more than $4.0 \mu\text{J/s}$.

The previous results allow for deducing some important conclusions about the use of potential flow theory in the description of the flow around a bubble and within its wake when wall influence is important. Potential flow can only be applied for regions two d_b downstream of the bubble, when the bubble rises along the center of the pipe. For the case of a bubble rising close to the wall, potential flow can be used three d_b downstream, and at least one d_b away of the bubble surface in the radial direction. Within the bubble's wake, several small zones of high vorticity and turbulence were observed during all the time steps after the bubble left the viewing volume. Therefore, potential flow theory should not be considered to describe the flow within the wake of a bubble, especially when wall influence is important.

CHAPTER VI

SHAPE, TRAJECTORY AND FORCES ACTING ON A BUBBLE RISING IN A SMALL DIAMETER PIPE

Bubble characteristics are of considerable importance in determining performance in various two-phase flows, for example in gas absorption, chemical reactors, and boiling heat transfer. The bubble dynamic results are useful in extending the knowledge of bubble behavior in two-phase systems and in providing data to test flow models and correlations. A bubble shape cannot be completely predicted unless all the physical variables pertinent to the bubble rise phenomenon are taken into account. Shape and trajectory of a bubble moving in a liquid are the consequence of the forces acting on it, and the surrounding flow field. The motion of bubbles rising in stagnant water has been experimentally studied extensively since the 50's. In most cases, wall influence was not considered in the results. The study of the motion of bubbles in restricted containers has focused on the measurements of terminal velocities and steady drag coefficients. In the last decade, numerical simulations have been performed to compute the trajectories of particles in non-uniform and unsteady flows, although the models used are very restrictive in most cases. The theoretical analysis mainly concern with the accurate description of the forces acting on the bubble, and how to include them in the motion equation.

In this chapter, the results concerning the shape, trajectory and drag and lift forces for an ellipsoidal bubble rising in quiescent water, in a small diameter pipe, are presented, compared against known results in the literature, and discussed.

6.1. Theoretical background

When an air bubble is released in water, it will rise up due to buoyancy only, if no other external forces exist on the system. Depending on the bubble dimensions, it will attain a determined shape and trajectory. It is commonly accepted that bubbles of less than about 1 mm of equivalent diameter are spherical and have a linear rising trajectory. Bubbles larger than 6 mm of equivalent diameter follow a straight path too, but their shape is close to a spherical-cap. In the range in between, 2 and 6 mm, bubbles are ellipsoidal. They are not necessarily symmetric, or wobble. The trajectory can be a zigzag, spiral, and/or a combination of those two paths.

Rocking motion is frequently observed.

6.1.1. Bubble shape

Bubble shape information, although still somewhat qualitative, is presented graphically in terms of three dimensionless groups: the Reynolds number, the Eötvös number, and the Morton number (Grace 1973; Bhaga & Weber 1981). The Eötvös number, Eo , is given by

$$Eo = \frac{g \Delta\rho d_e^2}{\sigma}, \quad [43]$$

where g is the magnitude of the gravity vector, $\Delta\rho$ is the difference between the bubble's medium density and liquid fluid density, d_e is the spherical equivalent diameter of the bubble, and σ is the surface tension of the liquid fluid. The Morton number, M , is given by

$$M = \frac{g \mu^4 \Delta\rho}{\rho^2 \sigma^3}, \quad [44]$$

where the liquid fluid has viscosity μ , and density ρ .

Most of the experimental and numerical studies of single bubble dynamics are performed with the assumption that the container walls have negligible influence on the behavior of the bubble. According to Clift *et al.* (1978) for wall effect $\lambda \leq 0.6$ ($\lambda = d_e / D$, where D is the pipe diameter) and $Eo < 40$, some empirical correction factors, depending on the bubble's Reynolds number, Re_b , and λ can be used for predictions of terminal velocities and drag coefficients. The bubble Reynolds number, Re_b , is given by

$$Re_b = \frac{U_b d_e}{\nu}, \quad [45]$$

where ν is the kinematic viscosity of the liquid. Regarding the bubble shape, for intermediate Re_b , the observation indicated that bubbles tend to be flat in the perpendicular direction of motion, and the wall tends to elongate the bubble along the flow direction. Therefore, the final

resultant shape would be close to a spheroid. The change in shape, however, induces changes in the shedding frequency, and consequently, the complexity of the flow structure increases.

Among the numerical studies of bubble shapes and trajectories, Tomiyama (1998) considered bubble motion and shape in stagnant liquid as a function of M and Eo numbers. In his 3D simulation, an agreement with Grace's graphical correlation for the bubble shape was obtained. The bubble trajectory was studied in two dimensions only. The results showed that for a given M , an increase in Eo leads the bubble to follow a zigzag path, with a change in the shape from spheroidal to wobbling. This behavior is also confirmed by Grace *et al.* (1976).

6.1.2. The equation of motion

The trajectory of a particle is computed by integrating the momentum, or motion equation. A recent review of the equation of motion for particles, bubbles, and droplets has been presented by Michaelides (1997). The equation of motion of a particle moving in a uniform fluid can be written as (Magnaudet 1997)

$$m_p \frac{d \mathbf{v}}{dt} = m_p \mathbf{g} + m_f \left(\frac{D \mathbf{u}}{Dt} - \mathbf{g} \right) + \mathbf{F}_p, \quad [46]$$

where the subscripts p and f refer to the particle and fluid, respectively, m is the mass, $\mathbf{g}$ is the gravity vector, $\mathbf{u}$ is the velocity of the fluid in the absence of the particle (the undisturbed velocity), $\mathbf{v}$ is the velocity of the particle, and $\mathbf{F}_p$ are all the hydrodynamic forces resulting from the presence of the particle in the flow. The second term in the right hand side is the force exerted by the flow on the volume of fluid occupied by the particle. The substantial derivatives for both the particle and the fluid are given by [34]. When the velocity $\mathbf{u}$ is non-uniform, the so-called Faxen force needs to be included into [46]. This force is proportional to $\nabla^2 \mathbf{u}$.

The contributions to the total hydrodynamic force $\mathbf{F}_p$ come from the drag force $\mathbf{F}_D$, the lift force $\mathbf{F}_L$, the added-mass (virtual mass) force $\mathbf{F}_M$, and the Basset or history force, $\mathbf{F}_B$, that is

$$\mathbf{F}_p = \mathbf{F}_D + \mathbf{F}_L + \mathbf{F}_M + \mathbf{F}_B. \quad [47]$$

$\mathbf{F}_B$ is significant for very small Re . Domgin *et al.* (1998) showed that $\mathbf{F}_B$ is only 1.4% of the

total force on a bubble for $Re = 311$. Therefore, for freely rising air bubbles in water, F_B is usually neglected.

When the expressions for each component of F_p are introduced in [46], without considering Faxen and Basset forces, the equation of motion can be rearranged to look like

$$V_b (\rho_b + \rho_l C_M) \frac{d \mathbf{v}}{dt} = V_b \rho_l (1 + C_M) \frac{D \mathbf{u}}{Dt} + V_b (\rho_b - \rho_l) \mathbf{g} + \frac{1}{2} C_D \rho_l A_b \|\mathbf{U}_r\| \mathbf{U}_r + V_b \rho_l C_L (\mathbf{U}_r \times \boldsymbol{\omega}) , \quad [48]$$

where the subscripts b and l are associated to the bubble and liquid, respectively. C_M , C_D , and C_L , are the added mass, drag and lift coefficients, respectively. V and A are the volume and projected area of the bubble, ρ the density, and $\mathbf{U}_r$ the relative velocity between the two phases given by [41]. Using the current notation, $\mathbf{U}_r$ is given by

$$\mathbf{U}_r = \mathbf{u} - \mathbf{v} , \quad [49]$$

and

$$\boldsymbol{\omega} = \nabla \times \mathbf{u} . \quad [50]$$

The components of $\boldsymbol{\omega}$ are given by [24]. The substantial acceleration of the liquid, the relative velocity and the cross product in the lift force are all evaluated at the center of the bubble (as if the bubble was not there).

6.1.3. Nature of the forces acting on the bubble

The left-hand side of [48] represents the bubble mass plus the liquid added mass times the bubble acceleration. On the right hand side appears firstly the force resulting from the pressure gradient and the shear stress of the liquid, when the bubble is not in the liquid. Next, it's the buoyancy force; then, the drag force; and finally, the lift force.

The virtual or mass force is an unsteady force due to the acceleration of the relative velocity between the bubble and the liquid. It is the force needed to accelerate the bubble surrounding

liquid. The liquid acceleration is produced by work done due to bubble/liquid interaction. The added mass coefficient C_M has a value of 0.5 for spherical particles, according to potential flow theory. However, C_M is actually a tensor called added mass matrix, or induced inertia tensor (Brennen 1995). The elements of the added mass matrix are determined from steady potential flows and/or the steady translation movement of the particle in the fluid. For spheres, the added mass matrix contains non-zero values only across the diagonal. These values equal 0.5 only for spheres embedded in infinite media. For non-spherical particles, however, all the elements of the added mass tensor are non-zero. This implies that the amount of liquid mass displaced depends on the shape of the bubble, as it is expected. For an ellipsoid, for example, the values depend on the aspect ratio of the ellipsoid.

For particles moving in non-linear trajectories in unsteady flows, a correction to C_M is introduced through the acceleration parameter (Crowe *et al.* 1998), or acceleration number (Michealides 1997), or acceleration modulus (Clift *et al.* 1978), Ac , which is defined by

$$Ac = \frac{\|\mathbf{U}_r\|^2}{d_e \left\| \frac{d\mathbf{U}_r}{dt} \right\|}. \quad [51]$$

The corrected value of C_M is given by (Michealides 1997)

$$C_M = 1.05 - \frac{0.066}{0.12 + Ac^2}. \quad [52]$$

Observe that as Ac tends to zero, C_M tends to 0.5. Recent results from numerical simulations indicate, however, that C_M equals 0.5 for spherical particles for a wide range of Re and Ac (Magnaudet 1997). Also, other experimental results have proven that C_M still is close to 0.5 even for distorted bubbles. Therefore, $C_M = 0.5$ is widely used in bubbly flow analysis.

The drag force is one of the well studied forces acting on any kind of fluid particle. The drag coefficient C_D is accurately calculated through empirical correlations, which only depend on Re (Clift *et al.* 1978; Fan & Tsuchiya 1990). These correlations, however, yield average values, which can be far from the actual instantaneous values, because of unsteadiness of the flow, the

oscillation of the particle projected area, and/or the particle's acceleration. In fact, the average C_D is usually determined through the expression for the drag force in [48] assuming that the particle already travels at terminal velocity without the presence of lateral (lift) forces. Therefore, the magnitude of the drag force equals the buoyancy force value. Results have shown that depending on the average operation used to analyze the experimental data, C_D can be considerably different (Hunt *et al.* 1992).

Another problem regarding C_D is the contamination of the systems in which the particle flows. For air bubble of about 2 to 4 mm d_e rising in water, for example, the terminal velocity is reduced in contaminated systems, and the drag coefficient is increased. This *contaminated* C_D is actually closer to the C_D of a rigid particle at the same Re, and usually is calculated through the correlation for rigid particles. The contamination of the bubble surface brings out other phenomena that complicate the analysis of the flow field. As the surface of the bubble gets contaminated it affects its surface tension, σ , and the shear stress, then the internal circulation is suppressed. There is an enhancement of boundary layer separation, and then vortex shedding. As mentioned above, wall influence on the drag coefficient is introduced by the empirical wall factor λ .

The lift force is the most difficult one to measure or compute. This is so because it is generally smaller than the other interacting forces. Its importance comes from the role it can play on bubble migration, and then phase distribution, because it is normal to the direction of motion of the particle. According to Batchelor (1967), the lift force arises due to the vorticity generation at a rigid surface. As mentioned above, bubbles moving in liquids with non-high purity behave close to a rigid particle. The vorticity can also be generated because of shear in the liquid. Further, the liquid velocity gradient may induce a rotation of the bubble. The uneven pressure distribution surrounding the rocking bubble yields a lateral force on the bubble. This force is called Saffman lift force. For some authors, however, the lift force arises from the Saffman effect only when there is no particle rotation. Even if the bubble would not rotate, the pressure field can induce lateral movement to the bubble. This Saffman force, without rotation, usually appears in the equation of motion as the only component of the total lift force. This is the case shown in [48] for the lift force.

The Magnus effect is the lift force due to the rotation of the particle. In this case the lateral force arises from the pressure difference between the sides of the particle. This is due to the different velocities at each side, generated by the rotation. The particle may be spinning because

of collisions with other particles or the wall. Rotation also may be induced by the shear in the liquid flow. The Magnus effect is rarely included in the equation of motion of the particles because there is no information available about the rotation of the particles. An expression for this force is given by (Crowe *et al.* 1998)

$$\mathbf{F}_{\text{Mag}} = \frac{1}{2} \rho_l \|\mathbf{U}_r\| C_{\text{LR}} A_b \left(\frac{\mathbf{U}_r \times \boldsymbol{\omega}_r}{\left\| \boldsymbol{\Omega} - \frac{1}{2} \boldsymbol{\omega} \right\|} \right), \quad [53]$$

where C_{LR} is the rotational lift coefficient and $\boldsymbol{\omega}_r$ is the difference between the angular velocity of the particle $\boldsymbol{\Omega}$, and half the liquid vorticity $\boldsymbol{\omega}$, that is

$$\boldsymbol{\omega}_r = \boldsymbol{\Omega} - \frac{1}{2} \boldsymbol{\omega}. \quad [54]$$

$\boldsymbol{\omega}_r$ can be considered as the relative rotation motion of the particle with respect to the fluid.

It should be noted that the lift coefficient C_L in [48] is only due to shear, and has been called the pure straining lift coefficient C_{LS} (Magnaudet 1997). The total lift coefficient C_L is the sum of C_{LS} and C_{LR} , and then the total lift force is the sum of the contribution from the flow shear plus the rotation of the particle. For large Re , $C_L = 0.5$, obtained from potential inviscid flow, is widely used. When one of the lift forces is not considered in the equation of motion, the lift coefficient of the other force is assumed to equal C_L . As pointed out by Crowe *et al.* (1998), C_L remains unclear for intermediate Re .

Finally, the buoyancy force is due to the density difference between the bubble and the liquid.

6.2. Accuracy of measurements

The uncertainties in determining both the shape and the forces acting on the bubble come from the accuracy of the DGHT algorithm presented in Chapter IV. The value obtained for the error was 0.08 mm (3 pixel), and it corresponded to the measurements in the X-direction (the images from the center camera). This value is the error associated to the value each of the

semiaxes a , b , and c , and also for the centroid of the bubbles.

Following the error analysis presented in Chapter V, the maximum errors of the velocity components of the bubble are 4.8 mm/s, and consequently 288.0 mm/s² for the acceleration components. The errors of the acceleration of the liquid, calculated from the velocity components given in Chapter V, are 94.8 mm/s² in the X-direction, 346.2 mm/s² in the Y-direction, and 83.4 mm/s² in the Z-direction. Although these values are of the order of the velocity and acceleration of the bubble, it should be noted that the bubble travels at an order of magnitude faster.

To determine the error associated to the calculation of the forces acting on the bubble, and consequently on the calculations of C_D and C_L , [48] can be casted as

$$V_b \rho_l \left[\frac{\rho_b}{\rho_l} \frac{d \mathbf{v}}{dt} + C_M \left(\frac{d \mathbf{v}}{dt} - \frac{D \mathbf{u}}{Dt} \right) - \frac{D \mathbf{u}}{Dt} \right] = V_b (\rho_b - \rho_l) \mathbf{g} + \left[\frac{1}{2} C_D \rho_l A_b \|\mathbf{U}_r\| \mathbf{U}_r \right]_{\uparrow} + \left[\frac{1}{2} C_L \rho_l A_b \|\mathbf{U}_r\| \mathbf{U}_r \right]_{\perp}. \quad [55]$$

The left hand side represents the total inertia of the system. On the right hand side the expression for the lift force in [48] has been changed to the common expression used in aerodynamics for the total lift. This expression allows for an easier calculation of the lift coefficient. The parallel and perpendicular symbols are introduced to stress that the drag force acts parallel to $\mathbf{U}_r$, while the lift is normal to it.

The maximum error on the calculation of the inertia force comes from the y-components of the acceleration terms. By combining the error from both the bubble and the liquid accelerations, by using [25], the error in the Y-direction is 450.3 mm/s². The error in the X-direction is 303.2 mm/s²; and 299.8 mm/s² for the Z-direction. The error in the calculation of the bubble is about 0.4 mm³.

The substantial accelerations above are easily computed in a Lagrangian reference frame. This is an advantage of the particle tracking techniques. This also justifies the need of a hybrid tracking technique in these situations. The ART2 NN tracked the seed particles during 4 consecutive frames, which is the same number of frames in which the bubble appeared in the viewing volume, in general. Therefore, the Lagrangian approach can be used for both the liquid

and the bubble simultaneously.

6.3. *Experimental results and discussion*

In this experiment 81 bubbles were released, one at a time, in quiescent water, in a 12.7 mm i.d. pipe. The time interval between each release was of few minutes. It is known that it takes less than a second for the disturbances generated by the bubble to vanish, so the flow was totally stagnant when the next bubble was released. Each camera captured the image of the bubble only for 3, or 4, or in very rare events 5 frames, out of the total 27 frames for each bubble release. In general, the bubble could be seen in 4 frames; however, only three frames from the Center and Shadow cameras could be used for the 3D reconstruction method shown in Chapter IV. The following results are those from bubbles that could be three-dimensionally reconstructed.

Following the nodalization scheme for the bubble presented in Chapter V (figure 17a), the data were divided according to the bubble trajectory. In this case a similar trajectory is specified for the bubbles which rose up in similar fashion at least two consecutive frames out of the three frames. Although this condition seems too weak for the motion of the bubbles in the test volume, it should be considered that the lateral motion of the bubbles was nicely continuous. That is, when a bubble moved from a region into other, the distance traveled was small and the trajectory was smooth enough that the bubble was considered to be rising in a similar trajectory. On the other hand, if a bubble moved in different regions and did not satisfy the condition of staying at least two consecutive time steps in one of the nodal zones, then it did not belong to the same trajectory. Also, bubbles crossing more than two boundaries during its rising path were not part of a similar trajectory.

Few bubbles traveled in regions 2 and 3 (see figure 16) so they were not considered for the data analysis. The final trajectory distributions considered for data analysis were: 24 bubbles traveled through trajectory 111; 18 bubbles rose up in trajectory 444; and 15 bubbles followed trajectory 555. Combinations of all trajectories, and for those close to the wall were also included in the averaging operations. The combination of all trajectories (111, 444, and 555) is referred to as trajectory *all*, while the combination of trajectories close to the wall (444 and 555) is denoted as trajectory *out*.

6.3.1. *Bubble shape and velocity analysis*

The Reynolds number of the bubbles, Re_b , was in the range from 350 to 700. At this range, a

Table 2. Bubble dimensions

Bubble Trajectory	a and σ_a [mm]	b and σ_b [mm]	c and σ_c [mm]	d and σ_d [mm]
111	1.24	1.46	1.34	2.69
	0.10	0.08	0.07	0.12
444	1.26	1.43	1.36	2.70
	0.10	0.09	0.07	0.13
555	1.27	1.45	1.38	2.73
	0.09	0.09	0.09	0.13
out	1.27	1.44	1.37	2.71
	0.09	0.09	0.08	0.13
all	1.25	1.45	1.36	2.70
	0.10	0.08	0.08	0.13

bubble freely rising in stagnant water presents oscillations in shape and rising path. The predicted shape is ellipsoidal, although not necessarily symmetric. The motion is expected to be helical and/or zigzag, and even a rocking motion. The ellipsoidal shape, however, would be observed in systems with high-purity working fluids, and with negligible wall influence. Clift *et al.* (1978) considered that for $0.12 < \lambda < 0.6$, and $Re_b > 100$, walls can cause elongation of bubbles in the vertical direction, so for the ellipsoidal shape, the result would tend to change to spheroidal shape again.

At least 3 images of a bubble need to be three-dimensionally reconstructed in order to calculate the three components of the acceleration vector. The acceleration was assigned to the second of the 3D reconstructions of a bubble (the one in the middle). Such second reconstruction corresponded in 93% of the experiment to the third frame ($t_p = 33.33$ ms), in which the bubble appeared in the test volume. Table 2 shows the average and standard deviation from the mean for each different trajectory, and combinations of trajectories, of the values of the semiaxes and spherical equivalent diameter determined from the instantaneous reconstructions.

From Table 2, it can be concluded that the shape of the bubbles was oblate spheroid. The minimum and maximum eccentricity were $e = b/c = 1.05$ for trajectory 555, and $e = b/a = 1.18$ for trajectory 111, respectively. Observe that the largest dimension was always in the Y-direction (b in here), followed by the semiaxis in the Z-direction (c), and the smallest semiaxis was in the X-direction (a). From Table 2, the wall effect factor was $\lambda = 0.21$, and $Re_b > 100$ in all cases, so the wall influence could be noticed. Since all bubbles were of the same size, approximately, it was not possible to determine the role of the volume on the rise path. It is well known that bubbles of less than 3 mm in diameter tend to collect close to the wall. In this experiment almost all bubbles were smaller than 3 mm of d_e , and the trajectories were 42% along the pipe core, so no tendency was observed for this experiment condition.

Another feature to note is that bubbles rising along the pipe center had a maximum value of the semimajor axis, while the bubbles rising close to the wall had larger semiaxes in the X and Z directions. This is in agreement with the fact that the wall influence is to elongate the dimensions of the semiaxes parallel to the wall, while it diminishes the dimension of the semiaxis perpendicular to the wall. The maximum error for any value in Table 2 is 6.5%, and it corresponds to the average value of a for trajectory 111 (0.08/1.24). Because of the magnitude of this error, the wall influence results presented are still tendencies (clear tendencies though). It will be necessary to reduce the error to 1 pixel (0.03 mm) to draw definite conclusions. It should be noted that the standard deviations σ of each parameter in Table 2 are not their associated errors, but only the spread of data from the calculated mean value shown. The actual errors were presented in the previous section.

In the present study, the bubbles rose in a helical path.. Actually, the prediction for rising path is based on direct experimental observations. The rocking motion was also observed, with both the PIV and shadow cameras, as shown in figures 18 and 19. Figures 18 and 19 are typical samples of shapes and trajectories found in the measurements, so they can be used as examples. The ellipsoidal shape of a bubble is the result of the pressure difference inside and on the surface of the bubble. This pressure difference increases with higher Re_b . During a PIV measurement, seed particles are needed to track the liquid motion; and therefore the system is contaminated. Contaminants can induce changes in the physical properties of the fluids, and/or can agglomerate on the bubble surface. It is known that contaminants tend to harden the bubble interface, tending to make the bubble to act as a rigid particle. Thus the bubble is not allowed to laterally expand.

For a bubble rising in a helical or zigzag path, the frequency of contact with the wall is high, when the radius of the spiral trajectory that the bubble would follow in an infinite medium is larger than the radius of the pipe in which it actually is moving. The collision induces a decrease in the lateral dimensions, and an increase in the vertical ones. Therefore, by combining the effects of the contaminants and the collision frequency with the pipe wall, the final shape should be closer to a spheroid (an oblate spheroid actually) than to an ellipsoid. This was the case, as deduced from figures 18 and 19, and the data shown in Table 2.

Another way to study bubble shape and trajectory is through the dimensionless numbers associated to the bubble. Grace (1973) and Bhaga & Weber (1981) have presented maps of shape regimes for single bubbles rising in Newtonian liquids based on Re_b , M , and Eo . The maps, however, excluded any data where wall influence could be noticed. The map of the regimes given by Grace (1973) predicts a transition between spherical to ellipsoidal bubbles for $Re_b = 350$, and from ellipsoidal to wobbling for $Re_b = 700$. Table 3 shows the average and standard deviations of the dimensionless numbers Re_b , M , and Eo associated to the second reconstructed bubble, and the bubble speed. The value of the velocity is the magnitude of the three-dimensional velocity vector, which was used to compute the dimensionless numbers. It is important to address the shape of a fluid particle which allows for physical analysis of the forces acting on the bubble. This is due to that Re_b is the ratio of magnitude of the inertial forces to the viscous forces. M only involves physical properties of the system under study, so it is commonly used as a parameter, which for the air-water system is $M = 2.56 \times 10^{-11}$. A more general expression for M , than the one given in [44], is presented by Fan & Tsuchiya (1990). Eo is the ratio of the pressure of the liquid on the bubble due to gravity to the surface tension pressure. The Weber dimensionless number, We , given by

$$We = \frac{\rho_l U_b^2 d_e}{\sigma}, \quad [56]$$

is the ratio of the liquid dynamic pressure to the surface tension pressure. The Tadaki number, Ta , is frequently used in the calculation of the aspect ratio, and the frequency of oscillation of the bubbles in their rising path. Ta is given by

Table 3. Speed and dimensionless parameters of the bubble

Bubble Trajectory	U and σ_U [mm/s]	Re_b and σ_{Re}	Eo and σ_{Eo}	We and σ_{We}	Ta and σ_{Ta}
111	193.2	526.7	0.97	1.39	1.93
	20.5	66.2	0.09	0.32	0.24
444	181.0	491.8	0.98	1.22	1.80
	20.0	61.7	0.10	0.29	0.23
555	195.7	539.4	1.00	1.45	1.98
	18.4	59.3	0.09	0.29	0.21
out	187.7	513.5	0.99	1.33	1.88
	20.4	64.3	0.09	0.31	0.24
all	190.0	519.0	0.98	1.35	1.90
	20.5	64.9	0.09	0.31	0.24

$$Ta = Re_b M^{0.23}. \quad [57]$$

A frequently used correlation to compute the aspect ratio in 3D contaminated systems is (Fan & Tsuchiya 1990)

$$\frac{h}{b} = [0.81 + 0.206 \tanh(2(0.8 - \log Ta))]^3, \quad [58]$$

where h is the maximum vertical dimension (c). Then, the eccentricity $e = 1.10$, for $Ta = 1.93$. Thus the difference between e from [58] and $e = b / c$ from Table 2 is only between 1 to 5%. Other correlations were also used to calculate the eccentricity; they predicted shapes from spherical to ellipsoidal with e up to 1.16. This value is at most 11% higher than $e = b / c$ from Table 2. These other correlations, however, assume that there exists symmetry around the vertical axis of the bubble, and that wall influence is negligible. No information was found to compare the eccentricity for $e = b / a$. Since only average values are shown in Table 2, for the

whole range of Re_b in this experiment, the oblate spheroidal shape determined from the analysis of the images of the bubble is in agreement with Grace's and Bhaga & Weber's maps.

Table 3 shows that the inertia of the bubble is 2 orders of magnitude higher than the viscous forces, which are due to the low viscosity of the water. The pressure due to gravity is balanced by the superficial tension pressure, while the dynamic pressure of the liquid is from 30 to 40% higher than that due to the surface tension. The dominant force here is the inertia, but the system cannot be completely considered as inviscid.

Regarding the velocity of the bubbles, Table 3 shows that those bubbles traveling in trajectories close to the pipe wall are slower than those following a path in the pipe core. This is expected, because of the wall friction, which opposes to the bubble motion. The data for trajectory 555 is however unexpected. It seems that those bubbles in region 5 were actually closer to the pipe center than to the pipe wall. As with the bubble shape, the contaminants and wall affect the velocity of the bubble. The rigidity of the bubble's surface, because of contaminants present in the water, suppresses the internal circulation and decreases the rising velocity of the bubble (Clift *et al.* 1978). It also enhances vortex shedding from the boundary layer.

In an infinite medium, Grace's and Bhaga & Weber's shape maps predict $Re_b \approx 800$ for $Eo = 1$ and $M = 2.56 \times 10^{-11}$. The average values of Re_b shown in Table 3 are lower with a factor of about 30 to 40%. Observe that the errors of the velocity in Table 3 are less than 3%. The experimental data for air bubbles rising in contaminated water given by Fan & Tsuchiya (1990) and Clift *et al.* (1978) however show that Re_b is actually lower than that predicted by the maps of Grace and Bhaga & Weber. Fan & Tsuchiya show that the terminal velocity in an infinite medium $U_{T\infty}$ is about 210 mm/s for $d_e = 2.7$ mm, while Clift *et al.* give $U_{T\infty} \approx 180$ mm/s, for the same d_e and $Eo = 1$. Thus, $Re_b = 486$ from Clift *et al.*'s data, while $Re_b = 567$ from Fan & Tsuchiya's data. This implies that the shape maps should be considered more as qualitative representation than a reliable quantitative information source, at least in this case.

If a bubble has terminal velocity in an infinite medium $U_{T\infty}$, this will be reduced to U_T , because of wall influence according to (Clift *et al.* 1978)

$$\frac{U_T}{U_{T\infty}} = (1 - \lambda^2)^{3/2}. \quad [59]$$

In this experiment $\lambda = 0.21$, so the velocity decrease is only about 7%. The corresponding Re_b for Fan & Tsuchiya's data is 529.0, and 453.4 for Clift *et al.*'s data. In the present experiments, however, the bubbles always had a significant acceleration while crossing through the viewing volume, so terminal velocity was never reached. If, anyway, it is assumed that [59] is valid for the instantaneous velocities measured, the difference between Re_b in Table 3 and Re_b from Fan & Tsuchiya's data is less than 8%. This result shows that the tracer particles added to the water follow the data for contaminated systems, in the average results. These contaminants, however, can be responsible for the spread of the data around the mean.

Another aspect that needs consideration is that [59] is strictly valid for a fluid particle traveling along the pipe center. As shown above, bubbles rise slower as they get closer to the wall, but [59] cannot predict such velocity decrease. The data for the trajectory 444 in Table 3 shows a velocity decrease of about 7%, with respect to those for trajectory 111. In computational simulation of bubble dynamics, it is usually introduced a force to stop the bubbles from penetrating walls (Tomiyama 1998). This force, known as wall force or lubrication force, includes the diameter of the bubble, the distance to the wall and the terminal velocity. An empirical factor, the wall force coefficient, is introduced to consider different fluid systems.

6.3.2. Drag and lift forces acting on the bubble

The drag and lift forces are computed here by using the method outlined by Shridar & Katz (1995). First, note that [55] can be represented as

$$\mathbf{F}_r = \mathbf{F}_i - \mathbf{F}_g = \mathbf{F}_D + \mathbf{F}_L, \quad [60]$$

where $\mathbf{F}_i$ is the total inertia force, and $\mathbf{F}_g$ is the buoyancy force. This equation simply shows that the drag and lift forces are balanced by the inertia and buoyancy forces. Since $\mathbf{F}_D$ is parallel to the relative velocity $\mathbf{U}_r$, it can be computed from the projection of $\mathbf{F}_r$ in the direction of $\mathbf{U}_r$. Such projection is simply the dot product between $\mathbf{F}_r$ and $\mathbf{U}_r$, that is,

$$\mathbf{F}_D = \frac{\mathbf{F}_r \cdot \mathbf{U}_r}{\|\mathbf{U}_r\|}. \quad [61]$$

The explicit expressions for the components of F_D are given by

$$F_{Dx} = \frac{F_{rx} U_{rx} + F_{ry} U_{ry} + F_{rz} U_{rz}}{\|U_r\|} \sin \theta \cos \varphi, \quad [62]$$

$$F_{Dy} = \frac{F_{rx} U_{rx} + F_{ry} U_{ry} + F_{rz} U_{rz}}{\|U_r\|} \sin \theta \sin \varphi, \quad [63]$$

$$F_{Dz} = \frac{F_{rx} U_{rx} + F_{ry} U_{ry} + F_{rz} U_{rz}}{\|U_r\|} \cos \theta, \quad [64]$$

where

$$\theta = \arctan \left(\frac{\sqrt{U_{rx}^2 + U_{ry}^2}}{U_{rz}} \right), \quad [65]$$

and

$$\varphi = \arctan \left(\frac{U_{ry}}{U_{rx}} \right). \quad [66]$$

And then the lift force components can be calculated from

$$F_L = F_r - F_D. \quad [67]$$

Once the magnitudes of the drag and lift forces are known, the respective coefficients can be computed from their expressions given in [55], that is,

$$C_D = \frac{\|\mathbf{F}_D\|}{\frac{1}{2} \rho_l A_b \|\mathbf{U}_r\|^2}, \quad [68]$$

and

$$C_L = \frac{\|\mathbf{F}_L\|}{\frac{1}{2} \rho_l A_b \|\mathbf{U}_r\|^2}. \quad [69]$$

The drift and drag were computed using [60] to [67]. Then the drag and lift coefficients were calculated from [68] and [69]. The required Lagrangian velocities and accelerations were computed by tracking the bubble and tracer particles motion in three consecutive frames, as shown in figures 18 and 19. The velocity and acceleration of the liquid were interpolated to the centroid of the bubble using data from the whole test volume. These data were instantaneous measurements, that is, before the ensemble average operation used in Chapter V.

Figure 40 shows the results of the drag coefficient for all bubble trajectories as function of Re_b for two conditions. First, the full symbols present the drag coefficient under the assumption of $\mathbf{u} = \mathbf{0}$ and $Ac = 0$. The open symbols, on the other hand, present the drag coefficient by using [55]. In addition, Ac was included through C_M , given by [52]. Figure 40 shows an important result. The inclusion of the disturbed liquid flow field spread data. The full symbols, on the contrary, show a nice continuous trend, similar to that of the standard drag curve. This is in agreement with what was presented in the theory section, where it was stated that the flow field of the liquid should be undisturbed (on the absence of the bubble). Shridar & Katz (1995) used only liquid data from 2 to 4 d_b away from the bubble, in order to ensure that the actual undisturbed flow was used in the computations.

In this experiment all the liquid in the test volume was set in motion at the time the bubble was present, and thus it was disturbed. As just mentioned, the velocity and acceleration of the liquid used in the computations were from data of the whole test volume, including the wake. The strong oscillations of the drag coefficient in figure 40, for this case, are therefore due to the inclusion of the disturbed flow in the computations. Figures 21 to 28 show that only those regions about 2 d_b from the bubble's top can be considered undisturbed. In such regions both

Figure 40. Drag coefficient as function of Re_b . Full symbols present data where $\mathbf{u} = \mathbf{0}$ and $Ac = 0$ in [55]. Open symbols present data where both $\mathbf{u}$ and Ac are included in [55]. See text for explanation.

energy and vorticity are very small, so the assumption $\mathbf{u} = \mathbf{0}$ is justified. In these regions the velocity magnitude is about 3 mm/s, while the acceleration has a magnitude from 100 to 300 mm/s². In fact, setting $\mathbf{u} = \mathbf{0}$ only introduces an error to the computation of the drag and lift of about 10%. This is so because there is an order of magnitude of difference between bubble and liquid velocity and acceleration. This difference of magnitude is reflected in the magnitude of the inertia force. Of the three terms that contribute to the inertia force, the term that most contributes is the term involving C_M . This term is at least an order of magnitude higher than the other two terms, so the error introduced by neglecting the flow field velocity only affects this term. This error must be added to that due to uncertainties in the velocity and acceleration of the bubble. The total error estimation for the drag and lift coefficients is 15 to 20%, when $\mathbf{u} = \mathbf{0}$ and $Ac = 0$.

Table 4 shows averages and standard deviations of the bubble acceleration magnitude and

Table 4. Bubble acceleration magnitude and drag coefficient

Bubble Trajectory	a_c and σ_{ac} [mm/s ²]	C_{D1} and σ_{CD1}	C_{D2} and σ_{CD2}	C_D and σ_{CD}
111	2775.3	0.90	0.87	0.90
	1098.9	0.68	0.57	0.20
444	2886.7	1.04	1.00	1.07
	670.4	0.49	0.39	0.34
555	2699.2	0.47	0.57	0.86
	734.5	0.27	0.24	0.20
out	2801.5	0.78	0.80	0.98
	695.5	0.49	0.39	0.30
all	2790.5	0.83	0.83	0.94
	879.0	0.58	0.47	0.26

coefficients from 3 different computations using [55], for each bubble trajectory. The drag coefficient C_{D1} results from the inclusion of both $\mathbf{u}$ and A_c in the computations; C_{D2} includes $\mathbf{u}$, but $A_c = 0$; and C_D assumes $\mathbf{u} = \mathbf{0}$ and $A_c = 0$. Observe that $A_c = 0$ implies $C_M = 0.5$. The results for the drag coefficient in Table 4 again show that the inclusion of the disturbed liquid flow in the computations yields higher standard deviations, even when $A_c = 0$. Table 4 also shows that bubbles rising close to the wall have higher drag as a consequence of the velocity reduction, although the magnitude of the error does not allow for the quantification. The results to be presented next are those in which $\mathbf{u} = \mathbf{0}$ and $A_c = 0$.

Figure 41 shows the drag coefficient versus the Reynolds number of the bubble computed through [68], by a widely used correlation, and assuming that the bubble reached terminal velocity. The expression of C_D at terminal velocity can be shown to be

$$C_D = \frac{4}{3} \frac{d_e}{U_b^2} g. \quad [70]$$

Figure 41. Drag coefficient comparison.

The terminal velocity condition implies that only the buoyancy force balances the drag. The correlation, for rigid particles but frequently used for fluid particles, between C_D and Re is given by

$$C_D = \frac{24}{Re} \left(1 + 0.15 Re^{0.687} \right). \quad [71]$$

An important result from figure 41 is that the experimental results for C_D using force balance were very close to those when C_D was calculated through [70]. In fact, the difference in the average C_D was from 5 to 10%. In all cases [70] yielded higher average values than force balance. The difference may be explained by recalling that the bubbles were not traveling at terminal velocity, so the inertia force has a contribution to the results. Another source for the difference may be due to the projected areas used in each case, because [70] assumes an

spherical particle, while in the force balance computations, the measured semiaxes were used to calculate the area. Observe, anyway, that the error associated to C_D is higher than the difference in values.

A bubble rising in spiral trajectory changes orientation. Experimentally it has been found that the frequency of orientation change is the same as that of the oscillatory motion of the bubble. The change in orientation is the angular velocity of the bubble or rotation. Bubble rotation, as explained before, induces a lateral force to the bubble, the Magnus effect. The rotation parameter R is the ratio of the angular velocity of the bubble to its translation motion, that is

$$R = \frac{\sqrt{U_\alpha^2 + U_\beta^2}}{U_b}, \quad [72]$$

where

$$U_\alpha = a \frac{\Delta\alpha}{\Delta t} \quad [73]$$

and

$$U_\beta = b \frac{\Delta\beta}{\Delta t}. \quad [74]$$

The change in orientation is given by $\Delta\alpha$ in the XZ plane, and $\Delta\beta$ in the YZ plane, see figures 18 and 19. Observe that there is no information available on the XY plane, so the rotation of the bubble in that plane is missing in [72].

Table 5 shows the average and standard deviation of the rotation parameter and the oscillation frequency from two equations. The oscillation frequency f is related to the Strouhal number Sr . Fan & Tsuchiya (1990) show that the f can be calculated from

Table 5. Bubble rotation parameter and oscillation frequency

Bubble Trajectory	R and σ_R	f_1 and σ_{f1} [Hz]	f_2 and σ_{f2} [Hz]
111	0.13	6.5	5.7
	0.10	0.6	1.2
444	0.22	6.6	5.6
	0.17	0.6	1.2
555	0.21	6.5	5.7
	0.15	0.6	1.2
out	0.22	6.6	5.4
	0.15	0.7	1.1
all	0.18	6.6	5.5
	0.14	0.6	1.1

$$Sr = f \frac{d_e}{U_b}, \quad [75]$$

where Sr can be computed from

$$Sr = 0.100 C_D^{0.734} \quad \text{for } C_D \leq 2, \quad [76]$$

or

$$Sr = 0.16 \left(1.0 - \frac{0.57}{Ta} \right)^2 \quad \text{for } Ta < 8. \quad [77]$$

In Table 5, f_1 uses [76] and f_2 uses [77]. The average values of R clearly show that the wall

Figure 42. Drag coefficient as function of the bubble rotation parameter.

induces rotation to the bubble. The angular velocity is about 40% higher close to the wall. The average rotation can be computed using the values of R in Table 5, bubble speed in Table 3, and a and b in Table 2. Respect the oscillation frequency, f_1 is very close to the value reported for contaminated system of 7.7 Hz, for a 2 mm d_e bubble (Fan & Tsuchiya 1990). In this experiment d_e was about 2.7 mm, so it agrees with the fact that as d_e increases f decreases. The oscillation frequencies in Table 5 are for infinite mediums. Observe that a comparison with the actual oscillation frequency in this experiment is not possible, because the bubble only appears in 4 consecutive frames, which is equivalent to only 50 ms.

Figure 42 shows the influence of R in C_D . the data show no clear tendency, and simply concentrate about the mean value given in Table 4. This result is in agreement with other experimental results that showed no influence of the spinning of a particle in the drag coefficient (Crowe *et al.* 1998), although only rigid particles at high Re were used in such experiments.

Figure 43 shows the variation of the lift coefficient as function of Re , while figure 44 shows the variation respect R . There is no clear trend in the data in figures 43 and 44. Attempts to fit the data with polynomial regressions yielded regression coefficients of about 0.6. The data for

Figure 43. Lift coefficient as function of Reynolds number.

Figure 44. Lift coefficient as function of rotation parameter.

C_L is spread around 0.4. The theoretical value obtained from potential flow, and also found in some numerical studies, is 0.5. This value is for spherical particles in inviscid fluid, although it has been found to very close for ellipsoidal bubbles. The difference between the theoretical value and the present experimental results arises from the bubble shape, the wall influence, and the rocking motion of the bubble. The oblate spheroid shape indicates an asymmetry of the pressure field around the bubble. In addition, the rocking motion of the bubble implies that the bubble experience different velocity at different parts. Consequently, the pressure field changes to accommodate the velocity variations across the bubble. These pressure field variations induce lateral forces on the bubble. The lift force that an ellipsoidal bubble feels, therefore, is different than that exerted over a spherical bubble.

Observe that the measured lift force, and therefore the lift coefficient, has contributions from the Saffman and Magnus effects. The first effect comes from the shear flow generated by the flow acceleration around the bubble plus the flow deceleration because of wall friction. The Magnus force arises from the rocking motion of the bubble. The average drag force for all the bubbles rising in the trajectory in the pipe core was 9.2×10^{-5} N, while for the trajectory close to the wall the average drag force was 9.3×10^{-5} N. The lift forces were smaller, as expected. For the trajectory close to the wall, the average lift force was 4.0×10^{-5} N, while for the trajectory in the pipe center was 3.4×10^{-5} N. The average lift force is at least 40% smaller than the drag force, so the oscillatory trend of the data is directly related to the uncertainty in the measurements. Even when the measurement errors are considered, it is clear from the figures that an average value will not yield accurate predictions of the lift force.

Finally, Table 6 shows the results obtained for the average and standard deviation of the lift coefficient for three different computations. The values of the lift coefficients in Table 6 were computed in the same conditions as those in Table 4 for the drag coefficients, that is, C_{L1} was calculated similarly to C_{D1} , C_{L2} was calculated similarly to C_{D2} , and C_L was calculated similarly to C_D . The results in Table 6 again show that the introduction of the disturbed flow makes the data to oscillate around the mean values shown.

An important result in Table 6 is that the lift coefficient is higher for the bubbles rising close to the wall. Obviously, a bubble close to the wall should have experienced a higher lift force than one in the pipe core. The difference in the average values is in the range of the measurement errors, though.

Table 6. Bubble lift coefficient

Bubble Trajectory	C_{L1} and σ_{CL1}	C_{L2} and σ_{CL2}	C_L and σ_{CL}
111	0.60	0.54	0.37
	0.32	0.31	0.25
444	0.67	0.50	0.48
	0.35	0.28	0.23
555	0.66	0.45	0.38
	0.38	0.24	0.14
out	0.68	0.59	0.44
	0.37	0.34	0.20
all	0.65	0.47	0.41
	0.35	0.26	0.22

CHAPTER VII

CONCLUSIONS

This chapter summarizes the main results of the present experimental investigation, and presents the needed improvements to obtain high-accuracy measurements.

The Particle Tracking Velocimetry technique has been used for a three-dimensional, transient experimental study of single bubble dynamics in a restricted medium. The three-dimensional velocity field was reconstructed via stereoscopic matching of two-dimensional vectors from two camera images at different view angles. A hybrid tracking technique has been shown to be particularly effective to determine the flow around a bubble. The Spring Model tracking algorithm was successfully applied to describe the flow within the bubble's wake, while the ART-2 Neural Network was used to track the tracer particles in the liquid for four consecutive frames, mainly in regions away from the bubble. The ART-2 Neural Network provided data that were used for the Lagrangian evaluation of the velocity and acceleration required in the analysis of the forces acting on the bubble. The development of a Shadow Particle Image Velocimetry allowed the bubble's shape study, and an accurate measurement of the dimensions, orientation, trajectory, and velocity and acceleration of a bubble rising in water.

This study also has shown that the use of algorithms generally employed in computer vision would provide a great tool for flow visualization measurement techniques. Such algorithms include the Dynamic Generalized Hough Transform for bubble shape identification and reconstruction; the image processing techniques that allow for an accurate detection of boundaries and centroids of objects; and the epipolar geometry constrain, which is needed for the determination of the three world coordinates of a point inferred from the two stereo images.

In this investigation, 81 bubbles were sequentially released in stagnant water in a small-diameter pipe. The time interval between the release of the bubbles was in the order of few minutes. This time separation was enough to obtain quiescent flow between successive bubble injection. The flow around the bubble and within the wake was determined from ensemble averaging of instantaneous velocity fields. The ensemble average operation was performed by considering a conditional sampling technique. The condition of the averaging was that of specific bubble trajectory. The beginning of the trajectory is started once the bubble entered the

viewing volume. The measurement volume was divided into $9 \times 9 \times 9$ cartesian grid for the liquid nodalization scheme. The total number of cells was 729, although only 621 fall inside the pipe. The different bubble trajectories were determined by dividing the horizontal cross sectional plane into 5 zones, one of them positioned at the pipe center. The Z-direction was divided in 4 regions.

The velocity fields were determined for each specific bubble trajectory. In order to better quantify the wall influence on the flow structure, and to obtain more samples for the averaging operation, the flow field was assumed symmetrical for each nodal region that had the similar bubble trajectory. Therefore, these flow fields were combined. The data of the regions in the pipe annulus were added to the third quadrant ($x < 0$ and $y < 0$) in the horizontal plane, due to the assumption of the symmetry of the flow field. A reasonable number of samples for the average for bubble trajectories close to the wall could be performed.

The bubble Reynolds number was in the range from 350 to 700. In the present study, the bubbles rose in a helical path, in agreement with the map of Clift *et al.* (1978). Rocking motion was also observed, with both the PIV and Shadow cameras. The ellipsoidal shape of a bubble is a result of the pressure difference inside and outside of the bubble. By combining the influence of the seed particles and the collision frequency with the pipe wall, the final bubble shape was an oblate spheroid. The largest dimension was always in the Y-direction, followed by the semiaxis in the Z-direction, and the smallest semiaxis was in the X-direction. It is noted that bubbles rising at the pipe center zone had the maximum value of the semimajor axis, while the bubbles rising close to the wall had larger semiaxes in the X and Z directions. This is in agreement with the fact that the wall influence is to elongate the dimensions of the semiaxes parallel to the wall, while it diminishes the dimension of the semiaxis perpendicular to the wall. The average eccentricity of the bubbles was found to be well predicted by a correlation available in the literature.

Regarding the bubble motion, it was found that the inclusion of the flow disturbance in the bubble motion equation generates scattering of the data for the drag and lift coefficients.

In Chapter I, a list of the objectives of this experimental study was presented. Each objective accomplishment is addressed next.

How much energy does the bubble bring into the test volume? For the two bubble trajectories studied, bubbles rising along the pipe center and others close to the wall, the total kinetic energy in the test volume shows similar characteristics. A sharp increase in the total

kinetic energy exists at the first time step the bubble enters the viewing volume ($t_p = 0$ ms). The maximum is reached at the third time step the bubble is present in the test zone ($t_p = 33.33$ ms). This maximum is about $70 \mu\text{J}$. The decrease of the total kinetic energy is noticeable at the first time step the bubble has departed the measurement volume ($t_a = 0$ ms). After the third time step bubble departed the viewing volume, at $t_a = 33.33$ ms, the behavior is rather constant. This time also indicated the departure of the primary wake from the viewing volume. The bubble trajectory close to the wall shows higher total kinetic after $t_a = 33.33$ ms.

The higher total kinetic energy of the bubble rising close to the wall is a result of the turbulent kinetic energy being generated. The mean kinetic energy is higher for the bubble trajectory at the pipe center until $t_a = 33.33$ ms. After that time both trajectories have similar values of the mean kinetic energy.

Larger mean kinetic energy is produced by bubbles rising along the pipe core, (before $t_a = 33.33$ ms), due to the acceleration of the liquid surrounding the bubble to compensate the reduction of flow area available. However, in the case of the bubble trajectory close to the wall, the liquid more than one bubble diameter away from the bubble in the horizontal plane, at same height, does not accelerate as much.

How long does it take for the viscosity and wall friction to dissipate the energy in the viewing volume? After the primary wake completely leaves the measurement zone ($t_a = 33.33$ ms), the transient behavior of the total kinetic energy is practically constant for both bubble trajectories. This indicates that the decay constant is more than $4.0 \mu\text{J/s}$.

How much turbulence does the bubble induce during its rising path? The bubbles rising close to the wall generate more turbulent kinetic energy. When the bubble is within the test volume, the turbulent kinetic energy is less than the mean until $t_a = 16.67$ ms. At this period the turbulent kinetic energy is about 40% of the total for the trajectory close to the wall; while it is 35% for the trajectory along the pipe center. After $t_a = 33.33$ ms, more than 70% of the total kinetic energy in the measurement volume is the contribution of turbulent kinetic energy.

How is the structure of the wake? The rocking and spiraling bubble motion was predominant in this experiment. Therefore, a combination of continuous trailing wake and vortex shedding is expected. However, due to the averaging process and low spatial resolution, the wake structure could not be accurately described. The results obtained here seem to indicate the wake structure associated to a spiraling bubble, more than a rocking one, particularly for the primary wake. For

the far wake, it is probable that the irregular vortex structures observed were originated from vortex shedding, and then associated to rocking bubble motion. The vortex structures shed from the bubble's surface did not spread for a long distance due to the wall effect. Further, the interactions of these vortex spots with the wall, wake/wake interactions, and wake/mean flow interactions, distort the circulatory motion.

The primary wake of a bubble rising in the pipe core seems to be closed and not turbulent. This could not be completely determined because of the lack of resolution in the measurements. For the bubble trajectory close to the wall, the primary wake was distorted and of smaller volume than that of the case of bubble trajectory along the pipe core. It was not clear to conclude whether the wake is closed or open.

For the bubbles rising along the pipe core, the flow closer to the bubble travels upstream surrounding the bubble's primary wake, and tilts and collides at the end of the wake, close to the center of the pipe. This collided flow generates a flow barrier zone, which decelerates any flow passing through it. The flow barrier was also generated in the case of bubbles rising close to the wall but was restricted to a zone of an area on the order of the bubble size. Also, the flow in the radial direction apart from the bubble with a distance more than one bubble diameter is practically undisturbed, reducing its magnitude, as it gets closer to the pipe wall.

How far upstream and downstream the bubble is the bubble-induced agitation felt? The influence zone of the bubble is different for the two trajectories. For trajectory along the pipe core, the bubble influence reached downstream to an average distance of two bubble diameters, measured from the bubble's top. For the bubble trajectories close to the pipe wall, the bubble influence downstream reached a longer distance. This average distance was about three bubble diameters. The increase in the distance is due to acceleration increase of the liquid flow between the bubble surface and the pipe wall, which is limited with the wall friction effect. This friction overcomes the momentum transfer from the bubble to the liquid. Consequently, the bubble pushes up liquid that experiences this resistance. Subsequently, liquid layers are then pushed further up.

The bubble's primary wake for trajectories along the pipe center extends three bubble diameters upstream, from the bubble's bottom. The primary wake length for trajectories close to the wall is extended to a distance of about three bubble diameters.

What is the magnitude of the different forces acting on the bubble? Only the drag and lift forces were evaluated in this study, because one of the objectives is to determine the drag and lift

coefficients. In addition, the results presented here were obtained under the assumption of the liquid flow field was quiescent, and without the inclusion of the acceleration parameter. It is noted that the measured lift force, and therefore the lift coefficient, has contributions from the Saffman and Magnus effects. The first effect exhibits from the shear flow generated by the flow acceleration around the bubble plus the flow deceleration because of wall friction. The Magnus force arises from the rocking motion of the bubble. The average drag force for the bubbles rising in the trajectory in the pipe core was 9.2×10^{-5} N, while for the trajectory close to the wall the average drag force was 9.3×10^{-5} N. The lift forces were smaller, as expected. For the trajectory close to the wall, the average lift force was 4.0×10^{-5} N, while for the trajectory along the pipe center was 3.4×10^{-5} N.

What is the pipe wall influence on the drag and lift coefficients? The wall influence on these coefficients is actually introduced through the velocities and accelerations of the liquid and the bubble. The results show that the seed particle presence in the water has a significant influence on the velocities of the bubbles, due to the changes of the fluid properties (seeds contaminate the water). By using data for contaminated systems, there was a difference of only about 8% with the measured average bubble velocities. This value can be attributed to direct wall influence, and it very closely agrees with the 7% predicted from a commonly used correlation.

In this investigation, the instantaneous drag coefficient did not show any obvious trend with respect to the rotation parameter. This phenomenon has been also found for large and small Reynolds numbers in other experiments. The instantaneous drag coefficient shows a similar behavior with the Reynolds number as the standard drag curve. The results presented next were obtained under the assumption of the liquid flow field was quiescent, and without the inclusion of the acceleration parameter. The drag coefficient for the bubble rising in the pipe core was 0.90; while it was 0.98 for the trajectory close to the wall. These results reflect the decrease of velocity due to wall friction. However, these values are higher than the values obtained from different correlations commonly used in the literature. It was surprising to find out that the drag coefficient calculated using the terminal velocity condition was indeed very close to that computed through force balance. The difference in values was from 5 to 10%. The bubble terminal velocity was not achieved in this experiment.

Regarding the lift coefficient, no trend was found as function of the Reynolds number and rotation parameter, of the instantaneous lift values. For the trajectory close to the wall it has an average value of 0.44. For the bubble rising in the pipe center, the value was 0.37. The

theoretical value commonly used in the literature is 0.5. This value is for spherical particles in inviscid fluid. The difference between the theoretical value and the present experimental results arises from the bubble shape, the wall influence, and the rocking motion of the bubble. The oblate spheroid shape indicates an asymmetry of the pressure field around the bubble. In addition, the rocking motion of the bubble implies that the bubble experience different velocity at different locations of the bubble surface. Consequently, the pressure field changes to accommodate the velocity variations across the bubble. These pressure field variations induce lateral forces on the bubble. The lift force that an ellipsoidal bubble experiences is different from that exerted over a spherical bubble.

Observe that the values given above are an average. The instantaneous values were both higher and smaller than the average. This confirms that the pressure field depends on the instantaneous shape and translation and angular velocities of the bubble.

In general, there still exist some aspects that need investigation. It is clear that higher resolution is required to describe the flow surrounding the bubble. Particularly, it is necessary to determine the structure of the flow close and within the bubble's boundary layer, and inside the wake. Moreover, it is recommended to study the bubble Reynolds number within a small range, to better quantify the drag and lift coefficients for ellipsoidal bubbles. Clearly, a large sample of the instantaneous data is needed to determine the accurate mean and turbulent patterns. The liquid and the bubble grids of the test volume should be increased. The bubble trajectory needs to be followed longer, so the average operations yield more accurate data, and Lagrangian statistics can be achieved.

REFERENCES

- Abdel-Aziz, Y. I. & Karara, H. M. 1971 Direct linear transformation from comparator coordinates into object space coordinates in close range photogrammetry. *Proc. of the ASP/UI Symposium on Close-Range Photogrammetry*, 1-18.
- Adamczyk, A. A. & Rimai, L. 1988 Reconstruction of a 3-dimensional flow field from orthogonal views of seed track video images. *Experiments in Fluids* **6**, 380-386.
- Adrian, R. J. 1986 Multi-point optical measurements of simultaneous vectors in unsteady flow - a review. *Int. J. Heat Transfer Fluid Flow* **7** 2, 127-145.
- Adrian, R. J. 1991 Particle-imaging techniques for experimental fluid mechanics. *Annual Rev. Fluid Mech.* **23**, 261-304.
- Ayache, N. 1991 *Artificial Vision for Mobile Robots: Stereo Vision and Multisensory Perception*. The MIT Press, Cambridge, MA.
- Ballard, D. H. 1981 Generalizing the Hough transform to detect arbitrary shapes. *Pattern Recognition* **13** 2, 111-122.
- Batchelor, G. K. 1967 *An Introduction to Fluid Dynamics*. Cambridge University Press, Cambridge.
- Besnard, D. C., Kataoka, I. & Serizawa, A. 1991 Turbulence modification and multiphase turbulence transport modeling. *Turbulence Modification in Multiphase Flows*. (Editors Michaelides, E. E., Fukano, T. & Serizawa, A.) ASME FED **110**, 51-57.
- Bhaga, D. & Weber, M. E. 1981 Bubbles in viscous liquids: shapes, wakes and velocities. *J. Fluid Mech.* **105**, 61-85.
- Biesheuvel, A. & Wijngaarden, L. V. 1984 Two-phase flow equations for a dilute dispersion of gas bubbles in liquid. *J. Fluid Mech.* **148**, 301-318.
- Blanchat, T. K. 1992 Development of pulsed laser velocimetry techniques for measurement of two-phase interfacial drag in a horizontal stratified flow. Ph.D. Dissertation, Texas A&M University, College Station, TX.
- Bopp, H. & Krauss, H. 1978 Extension of the 11-parameter solution for on-the-job calibrations of non-metric cameras. *International Archives of Photogrammetry* **22** 5, 7-12.
- Bosch, G., Kappler, M. & Rodi, W. 1996 Experiments on the flow past a square cylinder placed near a wall. *Experimental Thermal and Fluid Science* **13**, 292-305.

- Brennen, C. E. 1995 *Cavitation and Bubble Dynamics*. Oxford University Press, New York, NY.
- Brücker, C. 1999 Structure and dynamics of the wake of bubbles and its relevance for bubble interaction. *Phys. Fluids* **11** 7, 1781-1796.
- Buchhave, P. 1992 Particle image velocimetry - Status and trends. *Experimental Thermal and Fluid Science* **5**, 586-604.
- Carpenter, G. & Grossberg, S. 1987 ART-2: self-organization of stable category recognition codes for analog input patterns. *Applied Optics* **26** 3, 4919-4930.
- Chen, R. C. & Fan L. S. 1992 Particle image velocimetry for characterizing the flow structure in three-dimensional gas-liquid-solid fluidized beds. *Chemical Engineering Science* **47**, 3615-3622.
- Clift, R., Grace, J. R. & Weber, M. E. 1978 *Bubbles, Drops and Particles*. Academic Press, New York, NY.
- Costes, S. V., Okamoto, K. & Hassan, Y. A. 1995 Development of a three-dimensional particle image velocimetry algorithm and analysis of synthetic and experimental flow in three-dimensions. *Proc. of the Seventh International Symposium on Flow Visualization*. (Editor Crowder, J.) Begell House, New York, NY, 758-763.
- Crowe, C, Sommerfeld, M. & Tsuji, Y. 1998 *Multiphase Flows with Droplets and Particles*. CRC Press, Boca Raton, FL.
- Davies, E. R. 1990 *Machine Vision: Theory, Algorithms, Practicalities*. Academic Press, San Diego, CA.
- Domgin, J. F., Huilier, D. G. F., Karl, J. J, Gardin, P. & Burnage, H. 1998 Experimental and numerical study of rigid particles, droplets and bubbles motion in quiescent and turbulent flows - Influence of the history term. ICMF'98-219, on CD.
- Drew, D. A. & Lahey, R. T. 1982 Phase-distribution mechanisms in turbulent low-quality two-phase flow in a circular pipe. *J. Fluid Mech.* **117**, 91-106.
- Drew, D. A. 1983 Mathematical modeling of two-phase flow. *Ann. Rev. Fluid Mech.* **15**, 261-291.
- Drew, A. D. & Passman, S. L. 1999 *Theory of Multicomponent Fluids*. Springer, New York, NY.
- Ellingsen, K. & Risso, F. 1998 Measurements of the flow field induced by the motion of a single bubble. ICMF'98-286, on CD.
- Ellis, K. A., Fahidy, T. Z. & Pritzker, M. D. 1992 Application of computer vision to bubble detection at a gas-evolving electrode. *Chemical Engineering Science* **47** 13/14, 3623-3630.

- Esmaeeli, A. & Tryggvason, G. 1999 Direct numerical simulation of bubbly flows. Part 2. Moderate Reynolds number arrays. *J. Fluid Mech.* **385**, 325-358.
- Fan, L. S. & Tsuchiya, K. 1990 *Bubble Wake Dynamics in Liquids and Liquid-Solid Suspensions*. Butterworth-Heinemann, Stoneham, MA.
- Fisher, A. 1990 Next generation nuclear reactors. Dare we build them? *Popular Science* **April**, 68-77.
- Golay, M. W. & Todreas, N. E. 1990 Advanced light-water reactors. *Scientific American* **April**, 82-89.
- Gonzalez, R. C. & Wintz, P. 1987 *Digital Image Processing*. Addison-Wesley, Reading, MA.
- Goshtasby, A. 1993 Design and recovery of 2-D and 3-D shapes using rational Gaussian curves and surfaces. *International Journal of Computer Vision* **10** 3, 233-256.
- Grace, J. R. 1973 Shapes and velocities of bubbles rising in infinite liquids. *Trans. Inst. Chem. Eng.* **51**, 116-120.
- Grace, J. R., Wairegi, T. & Nguyen, T. H. 1976 Shapes and velocities of single drops and bubbles moving freely through immiscible liquids. *Trans. Inst. Chem. Eng.* **54**, 167-173.
- Grant, I. 1994 (Editor) *Selected Papers on Particle Image Velocimetry*. SPIE Milestone Series Volume MS 99, Bellingham, WA.
- Grant, I. & Pan, X. 1995 An investigation of the performance of multi layer, neural networks applied to the analysis of PIV images. *Experiments in Fluids* **19**, 159-166.
- Grant, I. 1997 Particle image velocimetry: a review. *Proc. Instn. Mech. Engrs.* **211** C, 55-76.
- Gui, L. & Merzkirch, W. 1996 Phase separation of PIV measurements in two-phase flow by applying a digital mask technique. *ERCOFTAC Bull.* **30**, 45-48.
- Hardy, R. L. 1990 Theory and applications of the multiquadratic-biharmonic method. 20 years of discovery. 1968-1988. *Computers Math. Applic.* **19** 8/9, 163-208.
- Hassan, Y. A., Blanchat, T. K., Seeley Jr., C. H. & Canaan, R. E. 1992 Simultaneous velocity measurements of both components of a two-phase flow using particle image velocimetry. *Int. J. Multiphase Flow* **18**, 371-395.
- Hassan, Y. A. & Philip, O. G. 1997 A new artificial neural network tracking technique for particle image velocimetry. *Experiments in Fluids* **23**, 145-154.
- Hassan, Y. A., Schmidl, W. & Ortiz-Villafuerte, J. 1998 Investigation of three-dimensional two-phase flow structure in a bubbly pipe flow. *Meas. Sci. Technol.* **9**, 309-326.

- Hunt, J. R. C., Perkins, R. J. & Fung, J. C. H. 1994 Review of the problems of modeling disperse two-phase flows. *Multiphase Science and Technology. Volume 8 Two-Phase Flow Fundamentals*. (Editors Hewitt, G. F., Kim, J. H., Lahey, Jr., R. T., Delhaye, J. M. & Zuber, N.) Begell House, New York, NY, 595-643.
- Ishii, M. 1975 *Thermo-Fluid Dynamic Theory of Two-Phase Flow*. Eyrolles, Paris.
- Ishii, M. & Theofanous, T. G. (Guest Editors) 1998 Advanced multiphase flow instrumentation and measurement techniques. *Nuclear Engineering and Design* **184** (2-3), 161-460.
- Karamanev, D. G. 1994 Rise of gas bubbles in quiescent liquids. *AIChE Journal* **40** 8, 1418-1421.
- Karara, H. M. 1980 Non-metric cameras. *Developments in Close Range Photogrammetry - 1*. Chap. 4. (Editor Atkinson, K. B.) Applied Science Publishers LTD, London, 63-80.
- Kasagi, N. & Nishino, K. 1991 Probing turbulence with three-dimensional particle image velocimetry. *Experimental Thermal and Fluid Science* **4**, 601-612.
- Kashiwa, B. A. & Gore, R. A. 1991 A four equation model for multiphase turbulent flow. *Turbulence Modification in Multiphase Flows*. (Editors Michaelides, E. E., Fukano, T. & Serizawa, A.) ASME FED **110**, 23-27.
- Kataoka, I., Ishii, M. & Serizawa, A. 1986 Local formulation and measurements of interfacial area concentration in two-phase flow. *Int. J. Multiphase Flow* **12** 4, 505-529.
- Kataoka, I. & Serizawa, A. 1990 Interfacial area concentration in bubbly flow. *Nuclear Engineering and Design* **120**, 163-180.
- Kataoka, I. & Serizawa, A. 1991 Bubble dispersion coefficient and turbulent diffusivity in bubbly two-phase flow. *Turbulence Modification in Multiphase Flows*. (Editors Michaelides, E. E., Fukano, T. & Serizawa, A.) ASME FED **110**, 59-66.
- Kataoka, I., Serizawa, A. & Besnard, D. C. 1993 Prediction of turbulence suppression and turbulence modeling in bubbly two-phase flow. *Nuclear Engineering and Design* **141**, 145-158.
- Kataoka, I., Besnard, D. C. & Serizawa, A. 1998 Analysis of turbulence spectra in bubbly flow. ICMF'98-242, on CD.
- Kurose, R., Misumi, R. & Komori, S. 1998 Drag and lift forces acting on a spherical bubble in a linear shear flow. ICMF'98-133, on CD.
- Lahey Jr., R. T. 1990 The analysis of phase separation and phase distribution phenomena using two-fluid models. *Nuclear Engineering and Design* **122**, 17-40.

- Lance, M. & Bataille, J. 1991 Turbulence in the liquid phase of a uniform bubbly air-water flow. *J. Fluid Mech.* **222**, 95-118.
- Leavers, V. F. 1992 *Shape Detection in Computer Vision Using the Hough Transform*. Springer-Verlag, London.
- Liu, T. J. & Bankoff, S. G. 1993 Structure of air-water bubbly flow in a vertical pipe - I. Liquid mean velocity and turbulence measurements. *Int. J. Heat Mass Transfer* **36** 4, 1049-1060.
- Lopez de Bertodano, M., Lee, S.-J., Lahey Jr., R. T. & Drew, D. A. 1990 The prediction of two-phase turbulence and phase distribution phenomena using a Reynolds stress model. *Journal of Fluids Engineering* **112**, 107-113.
- Lunde, K. & Perkins, R. J. 1997 Observations of wakes behind spheroidal bubbles and particles. ASME FEDSM'97-3530, on CD.
- Maas, H. G., Gruen, A. & Papantoniou, D. 1993 Particle tracking velocimetry in three-dimensional flows. Part I. Photogrammetric determination of particle coordinates. *Experiments in Fluids* **15**, 133-146.
- Magnaudet, J., Rivero, M. & Fabre, J. 1995 Accelerated flows past a rigid sphere or a spherical bubble. Part 1. Steady straining flow. *J. Fluid Mech.* **284**, 97-135.
- Magnaudet, J. J. M. 1997 The forces acting on bubbles and rigid particles. ASME FEDSM'97-3522, on CD.
- Malik, N. A., Dracos, Th. & Papantoniou, D. 1993 Particle tracking velocimetry in three-dimensional flows. Part II: Particle tracking. *Experiments in Fluids* **15**, 279-294.
- Manga, M. & Stone, H. A. 1995 Collective hydrodynamics of deformable drops and bubbles in dilute low Reynolds number suspensions. *J. Fluid Mech.* **300**, 231-263.
- Marzan, G. T. & Karara, H. M. 1975 A computer program for direct linear transformation solution of the colinearity condition, and some applications of it. *Proc. of the Symposium on Close-Range Photogrammetric Systems*, 420-476.
- Maune, D. F. 1975 Photogrammetric self-calibration of scanning electron microscope. *Proc. of the Symposium on Close-Range Photogrammetric Systems*, 335-357.
- Mercier, J., Lyrio, A. & Forslund, R. 1973 Three-dimensional study of the nonrectilinear trajectory of air bubbles rising in water. *Journal of Applied Mechanics* 650-654.
- Michaelides, E. E. 1997 Review – The transient equation of motion for particles, bubbles, and droplets. *Journal of Fluids Engineering* **119**, 233-247.
- Miksis, M. J. & Vanden-Broeck, J. M. 1982 Rising bubbles. *J. Fluid Mech.* **123**, 31-41.

- Morel, C. & Bestion, D. 1997 Study about turbulence modeling in steam water two-phase flows. ASME FEDSM'97-3684, on CD.
- Nakoryakov, V. E., Kashinsky, O. N., Randin, V. V. & Timkin, L. S. 1996 Gas-liquid bubbly flow in vertical pipes (Data bank contribution). *Journal of Fluids Engineering* **118**, 377-382.
- Nishino, K., Kasagi, N. & Hirata, M. 1989 Three-dimensional particle tracking velocimetry based on automated digital image processing. *Journal of Fluids Engineering* **111**, 384-391.
- O'Hern, T. J., Torczynski, J. R., Shagam, R. N., Blanchat, T. K., Chu, T. Y., Tassin-Leger, A. L. & Henderson, J. A. 1997 Optical diagnostics for turbulent and multiphase flows: particle image velocimetry and photorefractive optics. SANDIA REPORT SAND94-2589, Albuquerque, NM.
- Okamoto, K., Hassan, Y. A. & Schmidl, W. D. 1995 New tracking algorithm for particle image velocimetry. *Experiments in Fluids* **19**, 342-347.
- Park, W. C., Klausner, J. F. & Mei, R. 1995 Unsteady forces on spherical bubbles. *Experiments in Fluids* **19**, 167-172.
- Philip, O., Schmidl, W. & Hassan, Y. 1994 Development of a high speed particle image velocimetry technique using fluorescent tracers to study steam bubble collapse. *Nuclear Engineering and Design* **149**, 375-385.
- Prasad, A. K. & Adrian, R. J. 1993 Stereoscopic particle image velocimetry applied to liquid flows. *Experiments in Fluids* **15**, 49-60.
- Rivière, N. & Cartellier A. 1998 Bubble-induced agitation at moderate particle Reynolds numbers. ICFM'98-475, on CD.
- Riskin, G. & Leal L. G. 1984 Numerical solution of free-boundary problems in fluid mechanics. Part 2. Buoyancy-driven motion of a gas bubble through a quiescent liquid. *J. Fluid Mech.* **148**, 19-35.
- Saffman, P. G. 1956 On the rise of small air bubbles in water. *J. Fluid Mech.* **1** 3, 249-274.
- Serizawa, A., Kataoka, I. & Michiyoshi, I. 1975 Turbulence structure of air-water bubbly flow – II. Local properties. *Int. J. Multiphase Flow* **2**, 235-246.
- Serizawa, A. & Kataoka, I. 1994 Dispersed Flow-I. *Multiphase Science and Technology. Volume 8 Two-Phase Flow Fundamentals*. (Editors Hewitt, G. F., Kim, J. H., Lahey, Jr., R. T., Delhay, J. M. & Zuber, N.) Begell House, New York, NY, 125-194.
- Sridhar, G. & Katz, J. 1995 Drag and lift forces on microscopic bubbles entrained by a vortex. *Phys. Fluids* **7** 2, 389-399.

- Suzuki, Y., Aritomi, M., Takeda, Y. & Mori, M. 1999 Liquid flow structure surrounding bubbles in bubbly flows. ICONE-7163, on CD.
- Takagi, S. & Matsumoto, Y. 1998 Numerical study of the forces acting on a bubble and particle. ICMF'98-285, on CD.
- Taylor, J. J. 1989 Improved and safer nuclear power. *Science* **244**, 318-235.
- Theofanous, T. G. and Sullivan, J. 1982 Turbulence in two-phase dispersed flows. *J. Fluid Mech.* **116**, 343-362.
- Tomiyama, A., Zun, I., Sou, A. & Sakaguchi, T. 1993 Numerical analysis of bubble motion with the VOF method. *Nuclear Engineering and Design* **141**, 69-82.
- Tomiyama, A. 1998 Struggle with computational bubble dynamics. ICMF'98-004, on CD.
- Tran-Cong, S., Marié, J. L. & Perkins, R. J. 1998 Experimental study of the bubble motion in an upward turbulent boundary layer. ICMF'98-682, on CD.
- Tsao, R., Jones, S. A., Giddens, D. P., Zarins, C. K. & Glagov, S. 1995 An automated three-dimensional particle tracking technique for the study of modeled arterial flow fields. *Journal of Biomechanical Engineering* **117**, 211-218.
- Tsuchiya, K. & Fan, L. S. 1988 Near-wake structure of a single gas bubble in a two-dimensional liquid-solid fluidized bed: vortex shedding and wake size variation. *Chemical Engineering Science* **43** 5, 1167-1181.
- Wang, S. K., Lee, S. J., Jones Jr, O.C. & Lahey Jr, R. T. 1987 3-D turbulence structure and phase distribution measurements in bubbly two-phase flows. *Int. J. Multiphase Flow* **13** 4, 327-343.
- Wernet, M. P. & Pline, A. 1993 Particle displacement tracking technique and Cramer-Rao lower bound error in centroid estimates from CCD imagery. *Experiments in Fluids* **15**, 295-307.
- Wernet, M. P. 1995 Fuzzy logic particle tracking velocimetry. NASA TM-106194, Lewis Research Center, Cleveland, OH.
- Wijngaarden, L. V. 1976 Hydrodynamic interaction between gas bubbles in liquid. *J. Fluid Mech.* **77** 1, 27-44.
- Yamamoto, F., Iguchi, M., Wada, A. & Koketsu, M. 1995 Mathematical fundamentals of the binary image cross-correlation method for 2-D and 3-D PTV. *Proc. of the International Workshop on PIV, Fukui'95*, 79-90.

- Yoon, C., Hassan, Y. A., Ortiz-Villafuerte, J. & Schmidl, W. D. 1996 Genetic algorithm tracking technique for particle image velocimetry and comparison with other tracking models. *ASME HTD* **333**, 381-387.
- Žun, I. 1988 Pressure drop prediction for bubbly flow based on local turbulent characteristics. *Particulate Phenomena and Multiphase Transport Vol. 5* (Editor Veziroglu, T. N.), 207-225, Hemisphere, Washington, DC.