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Diagnostics for advanced laser acceleration experiments

Candidato
A. Misuri

Relatori
Chiar.mo Prof. A. Giulietti

Chiar.mo Prof. W. Leemans

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Introduction

The first proposal for plasma based accelerators was suggested in 1979 by Tajima and Dawson [1]. Since then there has been a tremendous progress both theoretically and experimentally. The theoretical progress is particularly due to the growing interest in the subject and to the development of more accurate numerical codes for the plasma simulations (especially Particle In Cell codes). The experimental progress follows from the development of multi-terawatt laser systems based on the chirped-pulse amplification technique. These efforts have produced results in several experiments world-wide [2], [3] with the detection of accelerated electrons of tens of MeV. The peculiarity of these advanced accelerators is their ability to sustain extremely large acceleration gradients. In the conventional radio frequency linear accelerators (RF linacs) the acceleration gradients are limited roughly to 100 MV/m; this is partially due to breakdown which occurs on the walls of the structure. The electrical breakdown is originated by the emission of the electrons from the walls of the cavity. These electrons cause an avalanche breakdown when they reach other metal parts of the RF linacs structure.

Moreover, plasmas can sustain electron plasma waves with a non relativistic wavebreaking field ($E_0 = cm_e\omega_p/e$) of the order of $E_0 = 100$ GV/m which is approximately three orders of magnitude greater than the one obtained in conventional RF linacs. The aim of this work is to show two diagnostics (i.e. an interferometer and an electron spectrometer) which have been projected and partially realized by the *L'OASIS* (Laser Optics and Acceleration System Integrated Studies) group at the Lawrence Berkeley National Laboratory (U.S.A.) in collaboration with the ILIL (Intense Laser Irradiation Laboratory) group CNR-Pisa. The primary intention of the work is the study of the accelerated electrons created focusing a femtosecond laser pulse on a subsonic gas jet.

The first chapter of the thesis is about some basic theory of the advanced acceleration schemes. Then the second and third chapters are about the plasma

diagnostics based on interferometry. The fourth and fifth chapters are about the diagnostics of the electron beam accelerated by the laser; in particular the electron optics and magnetic field properties. It has to be noticed that projecting a magnetic spectrometer for electron bunches produced by laser wakefield is a very delicate and complex matter. The electron bunches produced have an energy spread of 100% and this is a novel problem to face with in the electron spectroscopy history.

Chapter 1

Advanced acceleration schemes

1.1 Plasma wave generation

Different accelerators configurations have been developed to create the plasma wave needed to accelerate the electron bunches. These can be first classified by the drive beam used to generate the plasma wave:

1. electron beam (Plasma WakeField Accelerators)
2. laser beam (Plasma Beat Wave Accelerators, Laser WakeField Accelerators, Self-modulated Laser WakeField Accelerators)

In general a three dimensional solution of the models which describe these configuration is possible only in the linear regime. In the non linear regime an analytical solution has been found only in one dimension (1-D), while in the three dimension (3-D) non linear regime the use of numerical codes is usually required. The physical forces which drive the wakefields (i.e. space-charge and ponderomotive forces) and the mathematical models used to describe wakefield generation can be briefly discussed as follows [4].

Now we will briefly show some of the most widely investigated plasma-based accelerators.

1.1.1 Plasma Wakefield Accelerators

“Plasma Wakefield Accelerators” (PWFA) are called all the plasma based accelerators in which the plasma wave is driven by one or more electron beams. The

wakefield can be excited if the electron beam is relativistic and if it terminates in a time t_f shorter than ω_p^{-1} .

The space-charge forces associated with the drive electron beam is the origin of the plasma wave in the electron driven accelerators. When the electron beam propagates into a uniform plasma, $\eta = \eta_0$ (η is the plasma electron density), the beam density, η_b , generates a space-charge potential via Poisson's equation:

$$\nabla^2 \phi = k_p^2 \left(\frac{\eta}{\eta_0} + \frac{\eta_b}{\eta_0} - 1 \right)$$

where $k_p = \omega_p/c$ and $\omega_p^2 = \frac{4\pi e^2}{m} \eta(r)$ is the *plasma frequency*. The resulting space-charge force $F_{sc} = -m_e c^2 \nabla \phi$ can drive a plasma wakefield. Consider a long uniform relativistic electron beam of density $\eta_b \ll \eta_0$ and radius $r_b \gg \lambda_p$. The space-charge potential within the beam, $r < r_b$, is $\phi_{sc} = r^2 k_b^2 / 4$ where $k_b^2 = 4\pi e^2 / m_e c^2$. The plasma electrons will respond so as to cancel the space-charge potential of the beam i.e., the perturbed plasma density is $\delta\eta = -\eta_b$. If the electron beam terminates in a time short compared to ω_p^{-1} a plasma wakefield of the form $\delta\eta = \eta_b \sin k_b(z - ct)$ is generated. The axial electric field of the wake behind the beam is given by $\frac{\partial E_z}{\partial z} = -4\pi e \delta\eta$, i.e. $E_z = \left(\frac{4\pi e \eta_b}{k_p} \right) \cos k_p(z - ct)$. The peak amplitude of the wake is $E_{max} = (\eta_b / \eta_0) E_0$. Recent studies [8] showed that the transformer ratio, R_t (that is the ratio of the electron energy gained in the wakefield to the initial drive beam energy, $R_t = \frac{\delta E}{E_b}$), is limited to $R_t \leq 2$ in the linear regime. In the non linear regime[9] it has been shown that $R_t > 2$.

In a set of experiments by Rosenzweig *et al.* [10] in which wakefields were driven by a single electron bunch with an energy of 21 MeV a duration of 7 ps and a total charge of 4 nC in a plasma of density $(0.4 - 7) \times 10^{12} \text{ cm}^{-3}$ and of length 33 cm, the maximum energy gain observed was 0.2 MeV and a maximum accelerating gradient of 5 MeV. In Japan [11] some experiments have used high-energy RF accelerators, i.e. electron bunches energy of 500 MeV, length of 10 ps and charge of 10 nC. The plasma used was 1 meter long and with a density of 10^{12} cm^{-3} . In this case the maximum energy gain observed was 30 MeV (i.e an accelerating gradient of 30 MeV/m).

1.1.2 Plasma Beat Wave Accelerators

The Plasma Beat Wave Accelerators (PBWA) was first proposed by Tajima and Dawson[1]in 1979. This scheme was also the first one to be tested experimentally

because at that time the ultrashort and ultrahigh energy pulse laser technology was not available. In the PBWA two long (hundreds of ps), mutually coherent, laser pulses of frequencies ω_1 and ω_2 are used to resonantly excite a plasma wave. This is done by appropriately adjusting the laser frequencies and the plasma density such that the resonance condition $\omega_1 - \omega_2 \approx \omega_p$ occurs.

In the classical experiments done by Clayton *et al.*[12] was observed electron acceleration using two lines of a CO_2 laser. In particular[13] with a plasma density of $9 \times 10^{15} \text{ cm}^{-3}$ it was observed a 28 MeV energy gain, using a 2 MeV injected electron beam (i.e. an accelerating gradient of 2.8 GV/m). A number of experiments were performed elsewhere. They have shown that a severe limit to an effective acceleration is provided by the difficulty of setting an uniform and stable plasma medium, and by parametric instabilities (mostly Raman scattering) which deplete the laser pump energy.

1.1.3 Laser Wakefield Accelerators

In the Laser Wakefield Accelerators (LWFA) a single short ($\lesssim 1 \text{ ps}$) ultrahigh intensity ($\gtrsim 10^{18} \text{ W/cm}^2$) laser pulse drives a plasma wave. As said the wakefield is driven most efficiently when the laser pulse length, $L = c\tau_L$, is approximately equal to the plasma wavelength (i.e. $L \approx \lambda_p$).

In laser-driven plasma-based accelerators, wakefields are driven via the ponderomotive force. The ponderomotive force can be derived by considering the electron momentum equation in the *cold electron fluid limit*, the ion motion can be neglected due to the extremely short pulse duration: $\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{p} = -e[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$. Then the electric and magnetic fields of the laser can be written as: $\mathbf{E} = -\partial \mathbf{A} / \partial ct$ and $\mathbf{B} = \nabla \times \mathbf{A}$, where the vector potential of the laser is polarized predominantly in the transverse direction, $\mathbf{A} = A_0 \cos(kz - \omega t) \mathbf{e}_\perp$.

In the linear 3-D regime the wakefield generation can be examined using the cold fluid equations (Poisson equation, the continuity equation and the momentum equation). For example if we consider, $|a| \ll 1$, the normalized intensity of the driving laser beam ($|a| = e|A|/m_e c^2$), and $\delta\eta/\eta_0 \ll 1$, the perturbed density of the plasma wave, we have that the plasma wave generated in an initially uniform plasma is described by [6]:

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2 \right) \frac{\delta\eta}{\eta_0} = c^2 \nabla^2 \frac{a^2}{2} \quad (1.1)$$

the solution to the equation 1.1 is:

$$\frac{\delta\eta}{\eta_0} = \frac{1}{\omega_p} \int_0^t \sin \omega_p(t-t') c^2 \nabla^2 \frac{a^2}{2} dt' \quad (1.2)$$

These two equations describe plasma waves generated at the frequency ω_p and are valid far from the *wavebreaking*, $E \ll E_0$.

The wavebreaking gives the maximum electric field amplitude of the plasma wave. In 1-D linear approximation, the problema can be discussed starting from an electric field which has the form $E_z = E_{max} \sin \omega_p(z/v_p - t)$ where $v_p \approx c$ is the phase velocity. The peak field amplitude can be very high and can be estimated from Poisson's equation: $\nabla E = 4\pi e(\eta_0 - \eta_e)$. A simple estimate for the E_{max} is given by assuming all of the plasma electrons are oscillating with a wavenumber $k_p = \omega_p/c$. This gives $(\omega_p/c)E_{max} = 4\pi e\eta_0$ (in fact the maximum field is reached when all the electrons have been expelled), and $E_0 = 4\pi e c \eta_0 / \omega_p$ is the nonrelativistic wavebreaking field. More accurately 1-D relativistic cold fluid equations gives that the maximum amplitude is[5]: $E_{WB} = \sqrt{2(\gamma_p - 1)}E_0$, where $\gamma_p = (1 - v_p/c^2)^{-1/2}$. Moreover if warm plasma is considered a reduction in the 1-D wavebreaking field occurs, instead wavebreaking in 3-D regime has not been thoroughly investigated, at our knowledge.

The analysis of the solution 1.2 indicate that wakefields will be generated most efficiently when the envelope scale length of the normalized laser intensity a^2 is of the order of the plasma wavelength $\lambda_p = 2\pi c/\omega_p$. For the laser-driven wakefields the electric field of the wake behind the driven beam can be calculated[7] finding that the radial extent of the wake is on the order of the laser spot size.

It has to be noticed that a nonlinear description of the plasma is necessary if $E_{max} = \sqrt{2(\gamma_p - 1)}E_0 \gg E_0$ (where $\gamma_p = (1 - v_p/c)^{-1/2}$) i.e. if the plasma wave velocity, v_p , highly relativistic. In this case the precise value of L which maximizes the wake amplitude will depend on the shape of the axial pulse profile. For example considering a circular polarized laser pulse with a Gaussian pulse profile, $a^2 = a_0^2 e^{-\xi^2/L^2}$ ($\xi = z - ct$), the wake behind the pulse ($\xi^2 \gg L^2$) is given by[14]:

$$\frac{E_{max}}{E_0} = \frac{\sqrt{\pi}a^2}{2} k_p L e^{\frac{k_p^2 L^2}{4}} \quad (1.3)$$

were it has been assumed $a^2 \ll 1$. Equation 1.3 explicitly shows the dependence of the wake amplitude on the pulse length L . In particular the wake

amplitude achieves a maximum value of $E_{max}/E_0 = a_0(\pi/2e)^{1/2} \approx 0.76a_0^2$ when $L = \lambda_p/\pi\sqrt{2}$. Expressing the pulse length in terms of the full width half maximum we have that the maximum of the wake is reached when: $L_{FWHM} = 0.37\lambda_p$.

Nakajima *et al.*[15] observed electron acceleration in LWFA experiments with the use of a 1-ps Nd:glass laser, 8 TW, on a plasma of density $3.5 \times 10^{15} \text{ cm}^{-3}$. The electrons were injected in the plasma with energies near 1 MeV and accelerated to energies $> 5 \text{ MeV}$, estimates based on simulation imply an acceleration gradient of 0.7 GV/m.

1.1.4 The Pisa group experiment at L.O.A.

As example of LFWA experiment can be considered the recent experiment performed by the ILIL group at the LOA laboratory. Here the linearly p-polarized beam of the Ti:Sapphire laser of *Laboratoire d'Optique Appliquée* (Salle Jaune), 1 J in 35 fs at 815 nm wavelength, was focused by an off-axis F/5 parabola on a $1.0 \mu\text{m}$ thick, $500 \mu\text{m}$ wide plastic foil target, at an angle of incidence was of 20 degrees. The laser intensity distribution in the focal spot was gaussian, with a 50% of the total laser energy within a circle of $\approx 4 \mu\text{m}$ diameter resulting in a laser intensity on target of $I_L \approx 8 \times 10^{19} \text{ W/cm}^2$. The electron density peak of the plasma created was $\approx 4 \times 10^{19} \text{ e/cm}^3$. Then the number of electrons emitted coaxial with the laser was measured with a charge collector having an acceptance of ≈ 7 degrees and placed after a $10 \mu\text{m}$ quartz plate(necessary for optical measurements). The quartz plate had as side-effect to cut-off electrons of energy below 0.2 MeV thus the measured charge was of 0.2 nC per shot. In particular most of the electrons was found inside an aperture of 10^{-3} sterad, with energies up to 40 MeV. Moreover 3-D numerical simulation validate a reliable method to generate ultra-short and ultra-collimated electron bunches.

1.1.5 Self-Modulated LWFA

In the Self-Modulated LWFA a single short ($\approx 1 \text{ ps}$) ultrahigh intensity ($\gtrsim 10^{18} \text{ W/cm}^2$) laser pulse is used to excite the wake. The main difference with the LFWA method is that the plasma densities are higher i.e. the the pulse length, L , is long compared to the plasma wavelength, λ_p . Moreover the laser power P is larger than the *critical power* P_c , $P_c \approx 17\omega^2/\omega_p^2 \text{ GW}$ for relativistic guiding (see section 1.4). In this high regime the pulse is axially modulated at the plasma

frequency (via self-modulation instability [16]). Associated with the modulated pulse structure there is a large amplitude, resonantly driven plasma wave.

In principle the advantages of the self-modulated LWFA over the standard LWFA are simplicity and enhanced acceleration. “Simplicity” because several conditions are not needed, as the matching condition ($L \approx \lambda_p$). “Enhanced acceleration” follows by several reasons:

1. the self-modulated LWFA operates at higher density, hence a larger wake-field will be generated.
2. since $P > P_c$ the laser pulse will tend to focus to higher intensity, thus increasing a_0 and E_z
3. relativistic optical guiding allows the modulated pulse structure to propagate for several Rayleigh lengths¹, thus extending the acceleration distance.

The disadvantages of the self-modulated LFWA can be summarized as:

1. at higher densities the laser pulse group velocity ($\approx$ the plasma wakefield phase velocity) decreases and, hence *electron detuning* (see the next section) from the plasma wakefield can limit the acceleration distance.
2. diffractions due to the modulated pulse structure.

Modena *et al.*[17] demonstrated the acceleration of self-trapped electrons to energies ≥ 44 MeV (limit of the detector) using a 1-ps, 25-TW, 5×10^{18} W/cm² laser pulse in a plasma of density 1.5×10^{19} cm⁻³. Estimates based on the detuning length imply an acceleration gradient >100 GV/m.

1.2 Electron detuning

When we have analyzed the electron acceleration we did not take consider the finite velocity of the plasma wake in comparison to the electron velocity i.e. it

¹The Rayleigh length is the longitudinal size of the focal spot for a Gaussian laser beam in the vacuum. If the minimum of the spot size, r_0 , is in $z=0$ then the spot size $r_s(z)$ varies according to:

$$r_s = r_0 \left(1 + \frac{z^2}{Z_R^2} \right)^{1/2}$$

where Z_R is the Rayleigh length.

can occur that the electrons outrun the plasma wave and move in a decelerating phase region of the plasma wave. This limits the energy gain of the electrons in the plasma wave and is commonly referred to as *electron phase detuning*.

In particular for the plasma wakefield accelerators the phase velocity is equal to the drive electron beam velocity ($v_p = v_b$); for the laser-wakefields accelerators it is equal to the group velocity of the laser ($v_p = v_g$). If the group velocity in 1-D is $v_g = c(1 - \omega_p^2/\omega^2)^{1/2}$ (i.e. $\gamma_g = (1 - v_g^2/c^2)^{1/2} = \omega/\omega_p$) the 3-D correction gives $\gamma_g = (\omega_p^2/\omega^2 + 2c^2/\omega^2 r_0^2)^{-1/2}$ that is for tightly focused laser pulses this 3-D correction can significantly limit the group velocity. An important parameter is the detuning length defined as the length the electron must travel before it phase slips by one-half of a period with respect to the plasma wave; for an highly relativistic electron the detuning distance is given by: $L_d \approx \frac{\lambda_p}{1 - \frac{v_p^2}{c^2}}$. In particular for laser driven accelerators this length has to be compared with the Rayleigh length, Z_R . The shorter length between L_d and Z_R limits the maximum energy gain reachable with a fixed acceleration gradient. In fact these lengths express the maximum extent over which the electron bunch will experience the acceleration gradient.

1.3 Laser-plasma instabilities

As said the laser propagation distance in a plasma is an important characteristic of the performance of a laser driven accelerator. The laser plasma instabilities can limit the laser propagation distance and degrade the performance of the accelerator. Here we will take in exam only the instabilities which are relevant to the laser-driven accelerators.

1.3.1 Stimulated Raman Scattering

Stimulated Raman Scattering involves the interaction of a light wave with an electron plasma wave. It consists of the decay of the pump laser field, of frequency ω_0 and wave vector $\mathbf{k}_0$, respectively, into an electron plasma wave ($\omega, \mathbf{k}$) and two “daughter” light waves, namely a Stokes wave ($\omega_0 - \omega, \mathbf{k}_0 - \mathbf{k}$) and an anti-Stokes wave ($\omega_0 + \omega, \mathbf{k}_0 + \mathbf{k}$). From which we have the *Backward Raman Scattering* (BRS) and the *Forward Raman Scattering* (FRS).

In the BRS the pump wave ($\omega_0, \mathbf{k}_0$) decays into a plasma wave ($\omega, \mathbf{k}$) and a

backward scattered wave $(\omega_0 - \omega, \mathbf{k}_0 - \mathbf{k})$, where $\omega \approx \omega_p$ and $k \approx 2k_0$. In laser plasma accelerators BRS is significant for a number of reasons:

1. For difference pump intensities we have
 - (a) At *low pump laser intensities* the spectrum of the backscattered radiation can be used to determine $\omega - \omega_p$ and, hence, used as diagnostics of the plasma density.
 - (b) At *high pump laser intensities* the backscattered spectrum broadens so that $\omega - \omega_p$ can no longer be distinguished. BRS can erode the back of the pulse, then the energy is being transported out of the pulse.
 - (c) At *very high pump laser intensities* simulations indicate that BRS erodes the front portion of a long pulse in the high-density self-modulated regime. In the body of the pulse BRS is suppressed, possibly due to the plasma heating.
2. The BRS can be one of the reasons of the trapping of the electron in the plasma wake; in fact it can heat the background plasma electrons and let them reach the minimum energy required to be trapped by the plasma wake².

In the FRS the plasma wave $(\omega, \mathbf{k})$ and the scattered waves $(\omega_0 + \omega, \mathbf{k}_0 + \mathbf{k})$ propagate parallel (or nearly parallel) to the pump wave. The phase velocity of the associated plasma wave is $v_p \approx c$ hence this wave can be used to accelerate the electrons. The FRS instability has been used as basis for the explication of the self-modulated LFWA in which the single long laser pulse becomes modulated via FRS and drives a large amplitude plasma wave. For this reason the onset of FRS in an experiment is a valuable test to judge the transition from a “pure” wakefield to a self-modulated wakefield regime.

1.3.2 Self-Modulation Instabilities

In the 1-D limit pulse modulation can occur via forward Raman scattering, recently, a formalism has been developed[18] to describe the 3-D evolution laser pulse in plasmas, including the effects of the diffraction, relativistic and channel

²If the energy of the electron is too low it slips backward through the plasma wave.

guiding, finite pulse duration and coupling to the self-consistent plasma wave generated by the pulse structure. This formalism has been used to describe a class of “whole beam” instabilities, which includes self-modulation[16]. In this formalism equations are derived to describe the evolution of the local laser pulse spot size, $x_s(\xi, t)$, and the local laser pulse centroid, $x_c(\xi, t)$, where the transverse profile of the laser field is assumed to be a Gaussian of the form $a \sim \exp[-(x - x_c)^2/x_s^2]$ and the y-profile is similarly defined. Using this formalism the self-modulation instability consists of a periodic “sausaging” of the laser spot size x_s . In particular the process of envelope self-modulation can be understood by considering a long laser pulse ($L \gg \lambda_p$) with $P = P_c(1 - \Delta n/\Delta n_c)$ on which a finite wakefield exists. The plasma density modulation of the wake is of the form $\delta n = \delta n_0(r) \cos(k_p \xi)$, which modifies the plasma refractive index, as indicated by [19]. This density oscillation acts on the laser pulse as an axially periodic density channel. In regions of a local density channel (i.e. $\partial \delta n / \partial r > 0$) the radiation focuses. In regions where $\partial \delta n / \partial r < 0$ the diffraction is enhanced. This causes the laser pulse envelope to become modulated at λ_p , which subsequently enhances the growth of the plasma wave, and the process proceeds in a highly nonlinear manner. The final result can be a fully self-modulated laser pulse, composed of a series of laser “beamlets” of length $\approx \lambda_p/2$, which can remain optically guided over several Rayleigh lengths.

1.4 Plasma channeling

As we have seen, one of the most severe limit on the energy gain in a homogeneous plasma (especially in the LWFA regime) is laser diffraction. Reaching the high intensities needed for plasma wave excitations, on the order of 10^{18} W/cm², requires that the laser pulse must be focused to spot sizes on the order of several laser lengths, i.e. the high plasma electric fields are confined to distances on the order of the Rayleigh length. Thus increase the length over which the laser beam is focused is a difficult and important task, here we will examine the laser guiding in plasma channel as a mean to extend the acceleration length.

Similar to conventional fiber optics, the transverse profile of the laser beam are determined by the transverse profile of the guide, i.e. the plasma density profile. There are two main types of channel approximations that have been considered in the literature: the parabolic and the hollow channel. The parabolic channel has a cylindrically symmetric and parabolic density profile with a minimum density on

axis. The hollow channel refers to a cylindrically symmetric step plasma density profile, where the density inside the channel is zero.

Plasma channeling is based on the principle of the refractive guiding. Refractive guiding can occur when the radial profile of the index of refraction, $n(r)$, exhibits a maximum on axis, i.e. $\partial n/\partial r < 0$. As example we consider a parabolic channel, here the channel density can be expressed (in the 1-D) limit as: $\eta(r) = \eta_0 + \Delta\eta r^2/r_0^2$ where r_0 determines the curvature of the channel plasma density profile. For large amplitude waves (i.e. variations in the electron mass occurs) the refractive index can be expressed by $n(r) \approx 1 - (\omega_p^2/2\omega^2)(\eta(r)/\eta_0\gamma(r))$ (where ω is the laser frequency and $\omega_p^2/\omega^2 \gg 1$). Then considering the effects given by the plasma density channel the refractive index, in the limits $a_0 \ll 1$ and $|\Delta\eta_p/\eta_0| \ll 1$, is given by[20]:

$$n(r) = 1 - \frac{\omega_p^2}{\omega^2} \left(1 - \frac{a^2}{2} + \frac{\Delta\eta_p}{\eta_0} + \frac{\delta\eta_w}{\eta_0} \right) \quad (1.4)$$

where $\Delta\eta_p = \Delta\eta r^2/r_0^2$ and a wake response is included in $\delta\eta_w/\eta_0$. An equation describing the evolution of the laser spot size, r_s , can be derived by analyzing the paraxial wave equation with an index of refraction given by 1.4. Assuming the laser pulse is approximately Gaussian, $a_0 \propto \exp(-2r^2/r_s^2)$, and ignoring the wake, the normalized spot size, $R = r_z/r$ (r is the initial spot size) evolves according to 1.4 as:

$$\frac{d^2 R}{dz^2} = \frac{1}{Z_R^2 R^3} \left(1 - \frac{P}{P_c} - \frac{\Delta\eta}{\Delta\eta_c} R^4 \right) \quad (1.5)$$

where $P \propto a^2 r_s^2$ is the laser power and it is independent of z in the paraxial approximation, Z_R is the Rayleigh length, P_c is the critical power for self focusing ($P_c \approx 17(\omega/\omega_p)^2$), $\Delta\eta_c = (\pi r_e r_0^2)^{-1}$ is the critical depth and $r_e = e^2/m_e c^2$ the classical electron radius. The terms on the right side of 1.5 represent vacuum diffraction, relativistic self-focusing and channel guiding, respectively.

In the limit $\Delta\eta_p = 0$ (i.e. in absence of the channel) it can be shown that relativistic guiding of a long pulse occurs when laser spot satisfies $P \geq P_c$. On the other hand, in the limits $P/P_c \ll 1$ and $a_0 \ll 1$ it can be shown that a parabolic density channel ($\Delta\eta_p = \Delta\eta r^2/r_0^2$) can guide a Gaussian laser pulse if the channel satisfies: $\Delta\eta \geq \Delta\eta_c$.

Chapter 2

The laser system

In this chapter, after a small introduction on the parameters that rule a chirped pulse, we will discuss the laser system (see fig. 2.1) and the properties of the laser beam. It is valuable to notice that the femtosecond laser systems have had their development after the 1990 when Sibbet actually did the first self-modelocked Ti:sapphire[21] laser. The use of the Ti:Sapphire was highlighted by P.F. Moulton[22] in the 1986 when he showed the important characteristic that leaded to the use of that material as an active material:

- Large gain band (700 nm to 1100 nm)
- High termical conductivity (45 W/mK at 300 K)
- High damage threshold (10 J/cm^2)

Then using the CPA techniques allowed the amplification of the femtosecond laser to energies of few Joule.

Figure 2.1: Schematic of the laser system.

2.1 Chirp parameters

Let us consider a gaussian pulse of the form: $E_{in}(t) = E_0 e^{\frac{-t^2}{\tau^2}} e^{i[\omega_0 t + \psi(t)]}$ then in the spectral domain we will have:

$$G_{in}(\omega) = \int_{-\infty}^{+\infty} E_{in}(t) e^{-i\omega t} dt = g_{in}(\omega) e^{i\eta(\omega)}$$

where $g_{in}(\omega)$ is the spectral amplitude and $\eta(\omega)$ is the spectral phase. So the pulse after dispersion-compression processes can be written as:

$$G_{out}(\omega) = G_{in}(\omega) S(\omega) = g_{in}(\omega) s(\omega) e^{i[\eta(\omega) + \sigma(\omega)]}$$

where $S(\omega)$ is the transfer function of the system considered and of all the optical elements of the system (for example the gratings and the lenses of a stretcher). Thus the output gaussian pulse is the Fourier transformation of the function $G_{out}(\omega)$:

$$E_{out}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} G_{out}(t) e^{i\omega t} dt = e_{out}(\omega) e^{i\varphi(\omega)}$$

So the output spectral phase $\varphi(\omega)$ can be developed in Taylor series (centered in ω_0 , central frequency):

$$\varphi(\omega) = \varphi(\omega_0) + \varphi'(\omega_0)(\omega - \omega_0) + \frac{1}{2!} \varphi''(\omega_0)(\omega - \omega_0)^2 + \frac{1}{3!} \varphi'''(\omega_0)(\omega - \omega_0)^3 + \dots$$

the first three terms of the series are called:

- $\varphi'(\omega_0)$ =Group Delay (GD)
- $\varphi''(\omega_0)$ =Group Velocity Dispersion (GVD)
- $\varphi'''(\omega_0)$ =Third Order Dispersion (TOD)

The importance of these terms is clear if we consider the dispersion and we call it $T(\omega) = \frac{d\varphi}{d\omega}$, then its development in series (still centered at ω_0) is:

$$T(\omega) = T(\omega_0) + T'(\omega_0)(\omega - \omega_0) + \frac{1}{2!} T''(\omega_0)(\omega - \omega_0)^2 + \dots$$

It follows from the definition of $T(\omega)$ that:

$$T(\omega) = \varphi'(\omega_0) + \varphi''(\omega_0)(\omega - \omega_0) + \frac{1}{2!} \varphi'''(\omega_0)(\omega - \omega_0)^2 + \dots$$

That is if the GVD and the TOD are non-null the dispersion introduced by the optical system considered will change respectively linearly and with the square of the frequency. So in order to obtain a very short pulse we have, in principle, to nullify all the terms introduced by the optical materials inside the system. Actually the terms higher than the fourth are very hard to nullify.

2.2 The oscillator

The femtosecond laser system used in our experiment at the Lawrence Berkeley National Laboratory is based on Kerr-lens modelocking in Ti:Sapphire crystal of the oscillator. The active material is pumped with 4W of 532nm cw radiation from a diode pumped intracavity doubled Nd:YAG laser. It is to notice, in fact, the absorption band of the Ti:Sapphire has a range from 450nm to 550nm (as shown in fig. 2.2). Then a couple of prisms will avoid dispersion effects due to the nonlinear effects. A scheme of the cavity is shown in the figure 2.2.

Figure 2.2: a) Cavity scheme of the Ti:Sapphire b) Absorption emission band of Ti:sapphire crystal.

2.2.1 Optical Kerr effect

The well known Kerr effect is due to the non-linear effects that appear in a crystal subject to an intense laser beam irradiation. In fact the macroscopic polarization can be written as:

$$\mathbf{P} = \epsilon_0 \chi^{(1)} \mathbf{E} + \chi^{(2)} \mathbf{E}^2 + \chi^{(3)} \mathbf{E}^3 + \dots \quad (2.1)$$

where $\chi^{(1)}$ is the linear magnetic susceptibility, $\chi^{(2)}$ and $\chi^{(3)}$ are nonlinear magnetic susceptibility. The second order term could create a frequency duplication effect but in the material (as the Ti:Sapphire) with axial-symmetry this effect is negligible. So the polarization can be limited to the first two terms of the development:

$$\mathbf{P} = \epsilon_0 \chi^{(1)} \mathbf{E} + \chi^{(2)} \mathbf{E}^2 + \chi^{(3)} \mathbf{E}^3 + \dots$$

the nonlinear refraction index can be expressed by:

$$n = n_0 + n_2 \langle \mathbf{E}^2 \rangle \quad (2.2)$$

then the refraction index is proportional to the intensity of the electric field, this behavior is called optical Kerr effect.

2.2.2 Optical Kerr effect modelocking

When the laser pulse has gained enough energy it will experience, inside the active material, a nonlinear refraction index that changes with the intensity. So if we consider a gaussian profile of the beam the intensity will be greater at the center than at the tails; that is the nonlinear refraction index will be greater at the center and will be smaller and smaller going towards the tails. As result the crystal will behave as a focusing lens (see fig.2.3).

Figure 2.3: Kerr effect on a gaussian beam inside a dielectric material.

So the optical Kerr effect modelocking operates in the following manner: once that the continuous wave (cw) operation point has been found the cavity is detuned from its optimum by shortening the effective cavity length. Thus the focusing properties of the Kerr effect will let the crystal act as a focusing lens. Since the strength of Kerr effect is intensity dependent, a train of short pulses yields a much higher magnitude Kerr lens than a constant amplitude cw radiation, for the same average laser power. The result is that the cavity is forced in a modelocking

regime. The nonlinear refraction index shown in eq.2.2 will introduce also an unwanted effect: the laser pulse propagating inside a dispersive medium (such as glass or Sapphire) lengthens and its frequency changes from front to back, i.e. acquires a frequency chirp. This is called group velocity dispersion (GVD). This can be easily seen considering an electric field inside a nonlinear medium:

$$E(z, t) = A(z, t)e^{i\omega_0[(n_0+n_2I)\frac{z}{c}-t]}$$

it follows that there is a phase difference introduced by the non linearity given by $\delta\phi = \omega_0 n_2 I \frac{z}{c}$, this introduces a frequency shift given by:

$$\delta\omega = -\frac{d\delta\phi}{dt} = -\omega_0 n_2 \frac{z}{c} \frac{dI}{dt}$$

which causes a frequency modulation that will 1)increase the spectral width and2) increase the pulse duration. If the former effect could decrease the time duration of the pulse (the length of the pulse is bound to the number of the oscillating modes as: $\delta\tau \approx \frac{1}{M\delta\nu}$, where M is the number of the oscillating modes) the latter could lead to an appreciable pulse-lengthening so the latter effect has to be compensate. A prism pair provides the required effect. Let us see the behavior of the four prisms shown in fig. 2.4 that well represent the prism pair and the mirror used in the cavity.

Figure 2.4: Behavior of four prisms used for dispersion compensation.

The first prism divides the pulse in its different spectral components, each of them will enter in different points of the second prism, then they will pass through different thickness of glass. The smaller wavelengths are the most deviated by the first prism, so they will travel mostly in the air; thus greater is the wavelength

fewer is the air they pass through and, instead, they will pass by thicker glass. The result is that the smaller wavelengths are delayed with respect to the bigger one. The last two prism are in a symmetrical positions with respect to the first ones (that's why we can use the mirror in the cavity) and they will compensate the dispersion and reconstruct spatially the different wavelengths. With this system the GVD may be exactly compensate (to the second order) provided the distance between the prisms is chosen appropriately. Moreover moving one of the prism it is possible to change the amount of glass the laser pulse propagates through, the output pulse length (spectral bandwidth) of the oscillator can be changed. Note, that pulse length control could also be accomplished by a spectrum-limiting slit inserted near the second prism.

The oscillator in this configuration lases at about 800nm which produces a 90MHz train of pulses (≈ 20 fs) with an average power of about 0.25W. The spectral bandwidth (hence pulse length) is controllable from about 10nm to 60nm with an intracavity slit or a prism pair (see fig. 2.5).

Figure 2.5: Two typical spectra with the slit next to the prism closed to obtain the largest and the smallest bandwidths.

Before amplification we have to stretch the pulse to avoid damage to the amplifiers.

focal distances ($F_1 = F_2 = F$) we find that the length called z is smaller than zero. This follows by the fact that the system L_1 - L_2 makes the distance between the grating G_2 and G'_1 (image of the grating G_1) negative. Now since the relation between z and the GVD can be expressed by[23]:

$$\frac{d^2\varphi}{d\omega^2} = -2k\beta^2 z$$

where k is the wave number, z is the length cited before and β can be expressed as:

$$\beta = \frac{\lambda^2}{2\pi c d \cos \theta_2}$$

where θ_1 is the angle of incidence and θ_2 can be evaluated by the equation of the diffraction gratings: $\sin \theta_1 + \sin \theta_2 = \frac{\lambda}{d}$. Then it follows that:

- $z < 0$, GVD;0 the pulse is stretched, i.e. the smallest wave lengths follow the biggest ones.
- $z > 0$, GVD;0 the pulse is compressed, i.e. the greatest wave lengths follow the smallest ones.

So since in our case the z is smaller than zero the device will work as a stretcher. The measurements of the time duration was taken after the regenerative amplifier (since the process could modify the spectra) and the result is given in the figure 2.8. As shown the stretched pulse duration changes between ~ 100 ps to ~ 250 ps.

2.4 Amplification

The stretched pulse is injected in a regenerative amplifier then in a three pass preamplifier and then in a 2+2 pass power amplifier which brings the power to 10 TW.

2.4.1 Regenerative amplifier

The pulse changes its height and polarization (from p-horizontal to s-vertical) passing through a periscope then it enter in the regenerative amplifier (fig. 2.9) using the reflection on the crystal; in fact the changing in the polarization allows

Figure 2.8: Two typical spectra with about the largest and the smallest bandwidths achieved in the system.

the pulse to be mostly reflected by the surface of the crystal instead of being transmitted. Now since the pump laser generates 11W average power at 1KHz repetition rate and the repetition rate of the oscillator is $\sim 90\text{MHz}$, one pulse out of 9×10^4 needs to be selected. This is done with a quarter wave plate and the input Pockel's cell. When the cell is *off* the polarization is switched by the quarter-wave plate from s to circular then the pulse is reflected by the mirror again in the plate; the polarization is changed from circular to p, so that the pulse is amplified a first time and then come back through the quarter wave plate so that the polarization switch again to s. Finally the pulse is reflected by the surface of the crystal. Each pulse on 9×10^4 pulses the Pockel's cell is switched *on* after the pulse has rotated from s to p so the couple quarter wave plate and Pockel's cell (*on*) doesn't change the polarization and the pulse can be amplified. After a number of round trips inside the cavity, usually 15-20, the trapped pulse experiences a gain of over 10^6 , bringing the energy up to about $\sim 1 - 1.3 \text{ mJ}$. Then changing the trapped pulse polarization again (with a quarter wave voltage applied to the output Pockel's cell) and using the output coupler plate the pulse is injected in the three pass amplifier (the spectrum bandwidth of the pulse has been shown in figure 2.8).

Figure 2.9: Regenerative amplifier schematic.

2.4.2 Three pass preamplifier

The output of the regenerative amplifier is injected in the three pass preamplifier. The beam is deviated by the mirror M_0 and then reflected three times in the preamplifier (fig.2.10) before it is redirected to the last amplifier.

Figure 2.10: Three pass preamplifier schematic.

The Ti:sapphire in the cavity of the preamplifier is pumped by 380 mJ of 532 nm radiation from a commercial Q-switched, frequency doubled Nd:YAG laser operating at 10Hz. In this configuration the pulse obtained has an energy of 50-55 mJ.

2.4.3 2+2 pass amplifier

Here the pulse passes (through the reflections shown in the fig. 2.11) two times inside the active material (Ti:sapphire) then about 50% of the pulse energy is deviated towards an experiment on plasma channeling [24]. The remaining part of the beam is redirected other two times inside the amplifier. The first beam reaches an energy of about 100 mJ instead the second an energy of about 400-500mJ and it is directed to the vacuum compressor. The Ti:sapphire in this amplifier is pumped from both sides with two 9mm diameter beams containing up to 950 mJ energy (per side) in a 7-8 ns long pulse at 532nm.

Figure 2.11: 2+2 pass amplifier.

2.5 Vacuum compressor

To achieve the required peak pulse power we have to compress the stretched pulse again. The energy stored in the beam is so high that if we compress the pulse in the air we would ionize it, so from now on we will work in vacuumed chambers. The laser beam is compressed by the two holographic diffraction gratings (1200 gr/mm, 90% absolute reflection efficiency at 800 nm) scheme shown in figure 2.12.

The pulse injected is chirped by the first pass in the two gratings but it is also space dispersed so a mirror is placed after the two gratings to be reflected backward at the gratings with a horizontal offset and spatially recombined; so the total frequency dispersion is double of the dispersion due to a couple of gratings. The pulse duration of the pulse can be adjusted by changing the grating distance; in fact a simple model for the compressor gives the GVD as expressed by $\tau = \frac{P}{c}$,

Figure 2.12: Vacuum compressor schematic.

where P can be calculated as follows and it is the trajectory ABC (shown in 2.12) followed by the different wavelengths in the compressor:

$$P = \frac{L_g}{\cos \theta} [1 + \cos(\gamma - \theta)]$$

where γ is the incident angle and θ the reflected one, that can be calculated by: $\sin \gamma + \sin \theta = \frac{\lambda}{d}$ and d is the period of the gratings (where $\frac{1}{d}$ is the number of gratings per mm), λ is the incident wave length.

Figure 2.13: Image of the laser spot of 6 microns.

The pulse produced has 500 mJ in 50 fs, the length was measured using a frequency resolved optical gating system (FROG). The spectra was close to a gaussian envelope with a band width of $\sim 22 \sim 25$ nm and focal diameter of ~ 6 micron as FWHM. The focal spot was obtained with an off-axis parabola with

a focal length of 30 cm, the incident beam size is ≈ 7.5 cm so the diffraction limited focal spot (assuming an incident Gaussian beam with a FWHM of ≈ 4 cm) is $\approx 2 \mu\text{m}$ (FWHM). In figure 2.13 is shown an image of the spot magnified with an equivalent plane set up.

Chapter 3

Plasma diagnostic

The aim of this chapter is to show, first in theory then with the help of experimental data, the suitability of the use of a Mach-Zehnder interferometer as diagnostic instrument to analyze the plasma characteristics. In particular the interferometer chosen is a Folded wave Mach-Zehnder in which the probe beam is splitted after it has passed in the plasma (usually the probe beam is splitted in two arms before it passes in the medium we want investigate, that is located in one of the arms in which the probe has been splitted); then the two beams will interfere giving the interferograms and with them the phase shifts. So first of all it is given a simple model of the plasma to preview the phase shift due to the plasma.

3.1 Plasma parameters

We will assume that the plasma behaves as two fluids made of electrons and ions [25][26]. In particular since the plasma acceleration phenomenon lasts only few plasma periods the ions are considered as an uniform stationary background. With this assumption the dielectric function of the plasma is:

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

from which the refractive index can be obtained:

$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \tag{3.1}$$

where $\omega_p^2 = \frac{4\pi e^2}{m}\eta(r)$ is the *plasma frequency* and $\eta(r)$ is the electron density (that is assumed to have a cylindrical symmetry), so:

$$n = \sqrt{1 - \frac{\eta(r)}{\eta_c}} \quad (3.2)$$

where $\eta_c = \frac{m\omega_L^2}{4\pi e^2}$ (where m and e are the charge and the mass of the electron respectively and ω_L the plasma frequency) is the *critical density*. From the equations 3.1 and 3.2 it is clear that an electromagnetic wave can propagate in a plasma only if its frequency is below the plasma frequency, i.e. if the plasma density is smaller than the critical density. In our experiment the critical density is $\eta_c \approx 1.71 \times 10^{21} \text{ e/cm}^3$, on the other hand the gas jet density it is assumed to be $\gtrsim 10^{19} \text{ e/cm}^3$ which implies that even with the full ionization of the helium gas the plasma will be under critical.

As shown, the refractive index is a function of the plasma density so it is still to verify that the changing in the refractive index is big enough to cause a clearly visible shift of the fringes for a given optical path of the interferometric probe; in fact we will (as shown in the follow) create fringes without gas or plasma in the path of the probe beam and, then, we will analyze the shift of the fringes due to the plasma in comparison with the ones obtained in the vacuum condition.

Let us consider a probe which has an $800nm$ wavelength and propagates orthogonal to the plasma symmetry axis (see figure 3.1); another reference probe has the same optical path of this but will go through the vacuum. The interference between the two probes will give the interferograms. Moreover it is assumed that the probe isn't deviated by the plasma (i.e. no strong density gradients occur).

With these approximations we obtain that the phase shift of the probe passing through the plasma (with refractive index n_2 and inside a vacuum background) is given by:

$$\delta\phi = \frac{2\pi}{\lambda_{probe}} \int_{-\xi_0}^{\xi_0} (1 - n_2) d\xi$$

using the fact that the refractive index of the plasma is ($n_2 \approx 1 - \frac{\eta(r)}{2\eta_c}$) we have:

$$\delta\phi = \frac{2\pi}{\lambda_{probe}} \int_{-\xi_0}^{\xi_0} \frac{\eta(r)}{2\eta_c} d\xi \quad (3.3)$$

Figure 3.1: Schematic of the plasma with the probe beam (the 'main beam' location identify the symmetry axis).

Now assuming a cylindrical symmetry ¹ of the plasma and changing in cylindrical coordinates, ($r = \sqrt{\xi^2 + y^2}$ as shown in fig.3.1) we obtain:

$$\delta\phi = \frac{2\pi}{\lambda_{probe}\eta_c} \int_{y_0}^R \eta(r) \frac{r dr}{\sqrt{r^2 - y^2}} \quad (3.4)$$

It has to be noticed that, because of the cylindrical symmetry, the phase difference has been considered the same for $y > 0$ and for $y < 0$, and so only one has been considered. At this point, using the Abel inversion, it is straightforward to find out the electron density (which is the information we look for):

$$\eta(r) = -\frac{\eta_c \lambda_{probe}}{\pi^2} \int_r^R \frac{d\delta\phi}{dy} \frac{dy}{\sqrt{y^2 - r^2}} \quad (3.5)$$

If we suppose a constant plasma density ($\eta(r) = \eta_0$) the equation 3.3 gives the fringe shift ($\delta\Phi$):

$$\delta\Phi = \frac{\delta\phi}{2\pi} = \frac{1}{\lambda_{probe}\eta_c} \xi_0 \eta_0$$

so the maximum fringe shift occurs when the probe beam passes for $y = 0$ that is when $\xi_0 = R$, here (assuming a full ionization of the gas and a constant density over a radius of 200 microns, as actually will be) the fringe shift will be: $\delta\Phi_{max} \approx 1.4$ of a fringe. The meaning of this fringe shift is only a *mean behavior*,

¹At the Intense Laser Irradiation Laboratory has been developed a novel technique which generalizes the Abel inversion to a non-axisymmetric density distribution [27]

that is the fringe shift at the center of the plasma is bigger then at the boundaries and so what we described is a mean of these shifts.

Using this simple model has been possible to estimate that the expected fringe shift can allow an interferometric study of our plasma. The resulting fringes will be analyzed in the follow using two techniques: the Fast Fourier Transform (FFT)-based and the Continuos Wavelet Transform (CWT) -based.

3.2 FFT and CWT concepts.

The interferogram analysis done is computer based and makes use of the well known algorithm to extract the phase map of the interferogram [29]. The algorithm can be summarized as follows. An interferogram can be seen as:

$$I(\xi, y) = B(\xi, y) + V(\xi, y) \cos[\omega_0 \xi + \phi(\xi, y)] \quad (3.6)$$

where ω_0 is the fringe spatial frequency in the ξ direction; $\phi(\xi, y)$ the phase information we want to determine; $B(\xi, y)$ represent the background variations of the interferogram and $V(\xi, y)$ is the fringe visibility. Then it follows that equation 3.6 can be written as:

$$I(\xi, y) = B(\xi, y) + V'(\xi, y)e^{i\omega_0 \xi} + \overline{V'}(\xi, y)e^{-i\omega_0 \xi} \quad (3.7)$$

where $V' = \frac{V(\xi, y)}{2}e^{i\phi(\xi, y)}$ and $\overline{V'}$ is the complex conjugate of V' . At this point there are two ways to proceed, the customary way using the *FFT* or using the novel technique *CWT*.

3.2.1 Fast Fourier Transform

The Fast Fourier Transform is the optimized version (for speed) of the Discrete Fourier Transform used to calculate the Fourier Transform of discrete functions. In our case we will explain the concepts only using the Fourier transform i.e. considering analytical functions without taking in account the numerical processes to obtain them from the discrete data. Using this method we can extract the phase difference information from the interferogram, filtering out the effects due to the visibility of the fringes and to the background noise; in fact performing the Fourier transform of equation 3.7 we obtain (for each y):

$$I(\omega, y) = \widehat{B}(\omega, y) + \widehat{V}'(\omega - \omega_0, y) + \widehat{\overline{V}'}(\omega - \omega_0, y) \quad (3.8)$$

So since the spatial variations $B(\xi, y)$, $V'(\xi, y)$ and $\phi(\xi, y)$ happen at smaller frequencies than the frequency ω_0 their Fourier transformed functions are separated by ω_0 .

Figure 3.2: Fourier spectra separation and selection of one of this spectra with its translation schematic.

Because of this separation one of these terms can be taken in exam (see figure 3.2) without the effects of the others: $\phi(\xi, y)$ is the imaginary part of $\log(V'(\xi, y))$ (where $V'(\xi, y)$ is the inverse Fourier transform of $V'(\omega - \omega_0, y)$ that is $V'(\omega - \omega_0, y)$ translated in zero):

$$\log(V'(\xi, y)) = \log\left(\frac{V(\xi, y)}{2}\right) + i\phi(\xi, y)$$

$\phi(\xi, y)$ obtained in this way (i.e. numerically) is determined up to a factor 2π which has to be added or subtracted to avoid unphysical phase jumps.

3.2.2 Continuous Wavelet Transform

Here we will give an overview of the Continuous Wavelet Transform technique, for a more detailed description we suggest to refer to [31], [30]. The Fourier transform uses a base of functions made by oscillating terms $e_\omega = e^{-i\omega\xi}$ so that:

$$\widehat{I}_y(\omega) = \langle e_\omega | I \rangle = \int_{-\infty}^{+\infty} e^{-i\omega\xi} I(\xi, y) d\xi$$

Instead of using infinitely oscillating functions e_ω , the CWT method use a family of functions obtained as translations and dilatations of a compact-support function called “Mother Wavelet”:

$$\psi_{a,b}(\xi) = \frac{1}{a} \psi \left(\frac{\xi - b}{a} \right)$$

(where $[a, b] \in \Re$ $a > 0$). The set of all the $\psi_{a,b}(\xi)$ defines the base over which the signal will be decomposed. The choose of the Mother wavelet is quite free (with restrictions shown in [30]) and must be adapted to the information that should be extracted from the signal. As the Fourier coefficients, the CWT coefficients are defined as:

$$W_I(a, b) = \langle \psi_{a,b}(\xi) | I \rangle$$

that is:

$$W_I(a, b) = \int_{-\infty}^{+\infty} \frac{1}{a} \overline{\psi} \left(\frac{\xi - b}{a} \right) I(\xi, y) d\xi. \quad (3.9)$$

Moreover, with the help of the Fourier transform, we can rewrite 3.9 in a form suitable for fast numerical computations:

$$\widehat{W}_I(a, b) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \widehat{\psi}(a\omega) e^{i\omega b} \widehat{I}(\omega, y) d\omega.$$

The inverse wavelet transform exists if the Mother wavelet has zero mean and the simpler procedure is the so called ”Morlet inversion formula”:

$$I(\xi, y) = \frac{1}{k_\psi} \int_0^{+\infty} W_I(a, b) \frac{da}{a}$$

where it has been assumed that:

$$k_\psi = \int_0^{+\infty} \overline{\widehat{\psi}(a, \Xi)} \frac{da}{a}$$

is finite non zero and independent on Ξ . Between all the possible bases one has been chosen:

$$\psi(\xi) = e^{-i\omega_0 \xi} e^{-\left(\frac{\xi}{\tau}\right)^2} \quad (3.10)$$

where ω_0 and τ control the peak frequency and the width of the function. The function 3.10 defines the so called “Morlet base” and it is widely used when the local frequency - and so the phase- of the signal has to be measured.

An important concept in CWT analysis is the one of the *ridge*, which is a function which assigns to the parameter b the corresponding a for which the *energy* (the norm in L^2 of each wavelet) is maximum. This function represents the subset of the CWT map where most of the “energy” is contained. So following the behavior of this function can be assigned the frequency which yield most of the energy in each point of the interferometer avoiding the possibility of misunderstanding a numerical structure for an actual signal (see the case of a filament [30]). In figure 3.3 is shown the ridge function of an interferogram. Once the frequencies are determined the analysis is similar to the FFT method since we will select the range of the frequencies we are interested (as shown in figure 3.2) and then using the inverse CWT we obtain the phase shift map.

Figure 3.3: A ridge function, the white line, as from an interferogram obtained with a Folded wave Mach-Zehnder, the *number of voice* is linked to the frequency and depends on the sampling chosen.

3.3 Folded Wave Mach-Zehnder

Once the suitability of an interferometer has been proved we decided to use a Folded Wave Mach-Zehnder. The Mach-Zehnder interferometer is usually used to analyze the characteristics of the gas jets, in particular the folded wave Mach-Zehnder has been used in previous experiments and it has given very good results [28]. First of all we will show the setup of the experiment then the setup of the folded wave Mach-Zehnder.

3.3.1 Experiment Setup

As seen in the previous chapter the main pulse is a 10 TW pulse with 10 Hz repetition rate. This pulse is focused using an off-axis parabola with a focal

length of 12–13 in (≈ 30 cm), so assuming a gaussian beam with a spot diameter of ≈ 2.5 in (≈ 6 cm) we hold that the focal spot size is given by $2 * F * \lambda \approx 6.5 \mu m$ [where $F = (\text{focal length}) / (\text{spot diameter})$]. Before the full focalization is occurred the beam is reflected by a flat mirror on the gas jet. As shown by the figure 3.4 the loss of the mirror M_0 is used as probe beam and delayed by three mirrors. So the diameter of the probe beam is about the same of the main beam which implies it is much bigger than the region with plasma, the gas jet is about 1.2 mm wide.

Figure 3.4: Set up of the experiment and delay line for the Mach-Zehnder.

3.3.2 Interferometer Setup

The Folded-Wave Mach-Zehnder as the name suggests is a modified Mach-Zehnder interferometer; the modification allows to place all the interferometer setup outside the vacuum chamber. The probe pulse passes through the plasma after ≈ 10 ps the main beam has passed then it is divided by the beam splitter (labeled as D_1 in figure 3.5) in two beams: the first goes through the periscope S_1 where the image of the plasma (formed by the lens L_1) rotated upside-down is formed on the ccd; then the second beam passes through the periscope S_2 and is formed, without changing its orientation, on the ccd. Then the two beams are recombined on the ccd in such a way that the part of the beam that has traversed the plasma interferes with the part of the other beam which hasn't and the other way round. The result are two images of the plasma, at this point one of them is selected and analyzed. It has to be noticed that in the future development of

the system there is the possibility of an asymmetric magnification of the plasma using two cylindrical lenses but in the follows only symmetric magnifications are considered.

Figure 3.5: Folded-Wave Mach-Zehnder interferometer setup schematic, the arrows represent the orientation of the interaction of the probe beam with the plasma.

The alignment part of the interferometer is very important since the probe pulse lasts few femtosecond. The two arms of the interferometer should have the same optical path whit a precision of microns. So first of all the alignment has been done without gas jet and with a continuos laser pulse to obtain the *no-signal fringe* (that is the fringes without the plasma or the gas in the vacuum chamber and due to a non perfect parallelism between two mirrors in the two arms of the interferometer). Then the femtosecond laser has been used. The periscope S_1 has been adjusted (changing the distance from the beam splitter D_1 using a micrometric screw) as far as the no-signal fringes are obtained again.

3.4 Interferogram analysis

The interferograms we will show have been taken at low power of the main pulse (≈ 1 TW) this because the side scattered laser blinded the interferometer and no interferogram could be taken at full power (10 TW). Anyway this problem will be overcome when the probe pulse will be frequency doubled and the side

scattered laser by the plasma could be filtered. The consequence of the power reduction will be a partial ionization of the gas jet with a smaller density of the plasma and so a smaller fringe shift in the interferogram.

We are showing three interferograms (shown in figure 3.7, 3.9, 3.11 with the Region Of Interaction (ROI) used to perform the analysis) taken in three different positions of the gas jet in comparison with the focus location of the main laser; in fact the gas jet can be moved upstream and downstream (with respect to the main laser beam direction). This degree of freedom has been very useful since the accelerated electron bunches production has been observed to be very sensitive to it (most of the accelerated electrons has been produced for the focus location at $\approx 1500 \mu m$ upstream). In particular in the first interferogram the focus location is upstream ($1248 \mu m$ before the center of the gas jet); in the second one it is almost in the center of the gas jet ($312 \mu m$ after the gas jet); then in the last one the focus location is down stream ($1872 \mu m$ after the gas jet). The use of the continuous wavelet and the Abel inversion has given the density profile shown in figure 3.7, 3.9, 3.11. The density of the plasma is below the expected $2 \times 10^{19} e/cm^3$ which we would obtain if we have had full ionization, and it is instead $\approx 1.4 \times 10^{19} e/cm^3$.

Figure 3.6: Interferogram and in the white rectangle the ROI (focus at $1248 \mu m$ *before* the gas jet).

Figure 3.7: Two dimensional (on the left) and three dimensional (on the right) plasma density map when the laser has been focused $1248 \mu m$ *before* the gas jet; the plasma density is expressed in critical plasma density units (i.e. $1.7 \times 10^{21} e/cm^3$).

Figure 3.8: Interferogram and in the white rectangle the ROI(focus at $312 \mu m$ after the gas jet).

Figure 3.9: Plasma density map when the laser has been focused $312 \mu m$ after the gas jet; the plasma density is expressed in critical plasma density units (i.e. $1.7 \times 10^{21} e/cm^3$).

Figure 3.10: Interferogram and in the white rectangle the ROI (focus at $1872 \mu m$ after the gas jet).

Figure 3.11: Plasma density map when the laser has been focused $1872 \mu m$ after the gas jet; the plasma density is expressed in critical plasma density units (i.e. $1.7 \times 10^{21} e/cm^3$).

3.5 Conclusions

The suitability of a Folded Wave Mach Zehnder with femtosecond pulses has been demonstrated. The probe pulse was blinded by the sidescattered light when the laser worked at full power i.e. the needing of a frequency doubled probe has been found. In the most recent setup this feature will be implemented introducing a new frequency doubled laser pulse in the vacuum chamber.

The use of the Continuous wavelet method has permitted to obtain a good description of the plasma density in three position of the laser beam focus. This has underlined the correspondence between the focus and the density distribution of the plasma created, this correspondence will be surely widely investigated in future experiments.

The use of the continuous wavelet based method has been preferred since it has been demonstrated [30] to be more robust and more accurate in producing phase-shift maps (especially where small structures -such as filaments- have to be investigated) than the FFT based method.

Chapter 4

Basic electron optics theory

The purpose of this chapter is to explain the theory which has been used to reach a first design of the electron spectrometer.

The first order optics explained in this chapter is a powerful tool which gives very reliable results.

The main idea is to divide the magnetic field in two components: radial (B_r) and axial (B_z); then we assume that the axial magnetic field is constant along a reference trajectory (i.e. the trajectory will be a circle arc inside the magnet). Around this *main trajectory* we assume a constant magnetic field or a magnetic field which varies with a constant gradient (*field index* as defined after). The behavior of the radial part it will be linked to the behavior of the axial field by a power series, then we will assume the “paraxiality” of the electron beam which let us linearize the equations of their motion in the angles.

4.1 Magnetic field index

The magnetic field is plotted in cylindrical coordinates: radius r , azimuth θ and z (Fig.4.1). It is then possible to describe the field B in the form:

$$B = \frac{K}{r^n} \quad (4.1)$$

where K is a constant and n is the so-called *field index*. What we will assume is that the field index will be constant over the whole region of interest. By differentiation, with K and n fixed:

Figure 4.1: Ideal magnet and cylindrical coordinates for the axial and radial stability analysis. The magnetic field swelling out from the gap is also shown.

$$n = -\frac{r}{B} \frac{dB}{dr} \quad (4.2)$$

so that n is proportional to the radial gradient of the field. Now what we search for is the right value of the field index to achieve a double focusing spectrometer. For *double focusing* we mean that the focus for the azimuthal and for the axial plane coincide. So making an analysis of the stability of the axial and of the radial trajectories we'll find the condition for such a coincidence.

4.1.1 Axial stability

Let us consider a magnetic field that weakens radially and an electron, of charge $q = -e$, which rotates with radius r in the midplane between the poles of the magnet; the magnetic field (see Fig. 4.1) weakens radially. The field lines *bulge* outward and, although on the median plane the field is entirely axial, there is a radial component B_r at regions above and below which increases with increasing axial displacement z measured from the mid-plane.

It is legitimate to express this z -dependence in a power series with coefficients evaluated at the mid-plane where their values are constant for a given radius, i.e.:

$$B_r = z \left(\frac{dB_r}{dz} \right)_{z=0} + \frac{z^2}{2} \left(\frac{d^2 B_r}{dz^2} \right)_{z=0} + \dots \quad (4.3)$$

For small values of z only the first term need to be retained, which implies that B_r varies linearly with z in this approximation. In our case we have that $\nabla \times \mathbf{B} = 0$ because there is no current between the two poles, therefore:

$$\left(\frac{dB_r}{dz} \right)_{z=0} = \left(\frac{dB_z}{dr} \right)_{r=0} \quad (4.4)$$

but the $\mathbf{B}$ field in the mid-plane is all directed along the z -axis so that $(\mathbf{B}_r)_{z=0} = 0$ and $(\mathbf{B})_{z=0} = B_z \hat{\mathbf{z}}$, that is the radial field is approximately (at the first order):

$$B_r = z \frac{dB}{dr} \quad (4.5)$$

and from the definition of the field index we find:

$$B_r = -nB \frac{z}{r} \quad (4.6)$$

Assuming that the field lines don't bulge too much (i.e. $\frac{dB}{dz} \ll \frac{dB}{dr}$) in the region of interest the above approximation is valid and assuming the B field oriented in the positive sense of the z -axis, we have ($\omega = eB/\gamma m$):

$$\frac{d^2 z}{dt^2} = -n\omega v \frac{z}{r} \quad (4.7)$$

where v is the speed at which the particle moves in the radial plane, $v = \omega r$, then we arrive at the Kerst-Serber equation for axial betatron motion:

$$\frac{d^2 z}{dt^2} = -n\omega^2 z \quad (4.8)$$

solving this equation we find the law that rules the oscillation of the particle in the axial plane during his motion in the machine:

$$z = z_m \sin(n^{1/2} \omega t) \quad (4.9)$$

So in one revolution the particle does:

$$\nu_z = \frac{n^{1/2} \omega}{\omega} = n^{1/2} \quad (4.10)$$

oscillations.

The fact that the particle oscillates is due to the fact that the orbit is stable, i.e. if a particle is moved by the main trajectory it will be forced to go back to it. This occurs if $n > 0$. Otherwise the orbit is unstable.

4.1.2 Radial stability

Now we consider the motion in the mid-plane between the two poles. Where the equation of motion, using polar coordinates, is:

$$m\gamma \frac{d^2 r}{dt^2} - \frac{m\gamma v^2}{r} = -qvB \quad (4.11)$$

of course a solution of this equation is $r = \text{constant}$, and this is the case if the particle is launched tangentially in the orbit with just the right velocity. But in general the electrons will have a divergence and an energy spread in a way that some of them will not have the *right momentum*. We need to find the conditions for which the orbits with a given r are stable. For *radially stable orbit* we mean that if an electron is moved away from the main path of a length much smaller than its radius of curvature it will tend to return to its former path. So assuming that an electron is moved by a distance x from its radius of curvature r_e we obtain:

$$m\gamma \frac{d^2(r_e + x)}{dt^2} - \frac{m\gamma v^2}{r_e + x} = -qvB_x \quad (4.12)$$

where B_x is the field at $r = r_e + x$. Developing in Taylor's expansion the second term of 4.12 (since $x \ll r_e$) and using the fact that $r_e = \text{const}$ (with an assigned momentum) we obtain:

$$m\gamma \frac{d^2 x}{dt^2} - \frac{m\gamma v^2}{r_e} \left(1 - \frac{x}{r_e}\right) = -qvB_x \quad (4.13)$$

but $m\gamma v^2/r_e = qvB_e$ where B_e is the magnetic field in the main path, that is at $r = r_e$, so:

$$m\gamma \frac{d^2 x}{dt^2} - qvB_e \left(1 - \frac{x}{r_e}\right) = -qvB_x \quad (4.14)$$

For small displacement or small bending of the magnetic field we can express B_x in term of a Taylor expansion: $B_x = B_e + xdB/dr + \dots$. Then

$$m\gamma \frac{d^2 x}{dt^2} - qvB_e \left(1 - \frac{x}{r_e}\right) = -qv(B_e + x \frac{dB}{dr}) \quad (4.15)$$

that is (since $\omega = \frac{qB_e}{\gamma m}$ and $\frac{r_e}{B_e} \frac{dB}{dr} = n$):

$$\frac{d^2 x}{dt^2} + (1 - n)\omega^2 x = 0 \quad (4.16)$$

This is the Kerst-Serber equation for radial motion. As for the previous formula we can solve this equation and we find:

$$x = x_m \sin(n^{1/2}\omega t) \quad (4.17)$$

$$\nu_x = \frac{(1-n)^{1/2}\omega}{\omega} = (1-n)^{1/2} \quad (4.18)$$

where ν_x are the oscillations did by the particle in one revolution.

From the precedent discussion about axial and radial stability it is clear that we need that both the motions, in the radial and in the axial plane, are stable to achieve a *double focusing device*. It means that during the passage of the electrons in the magnetic field they effectuate the same portion of a wavelength of both radial and axial motion. We impose that $\nu_z = \nu_x$ (eq.s 4.10 and 4.18) then the stability condition in terms of the field index is:

$$n = \frac{1}{2} \quad (4.19)$$

4.1.3 Radial oscillation of the electron beam

In the precedent analysis we considered an electron beam with momentum p following a circular path r_e . We need to understand, now, the behavior of those electrons with momentum $p + dp$. This difference in momentum brings also a difference in orbit and, then, in magnetic field, since the magnetic field is a function of the radius, as shown in eq. 4.1. So the equation 4.12 becomes:

$$m(\gamma + d\gamma)\frac{d^2(r_e + x)}{dt^2} - \frac{m(\gamma + d\gamma)v^2}{r_e + x} = -q(v + dv)(B_e + dB) \quad (4.20)$$

where B_e is the magnetic field on the main path, associated to electrons with momentum p .

At this point performing some approximations as we have done in the previous paragraphs we arrive to the equation:

$$\frac{d^2x}{dt^2} + (1-n)\omega^2 x = r_e\omega^2 \frac{dp}{p} \quad (4.21)$$

Now, using the formalism developed by J.J.Livingood [33], we divide by v^2 ; then, remembering that $\omega/v = 1/r_e$ and we assume as L a coordinate along the main trajectory (as shown in fig. 4.2), that is $L = vt$, we obtain:

Figure 4.2: Azimuthal coordinate that runs along the main trajectory between the two poles of the magnet.

$$\frac{d^2x}{dL^2} + (1 - n)\frac{x}{r_e^2} = \frac{1}{r_e} \frac{dp}{p} \quad (4.22)$$

the solution of this equation is:

$$x = a \sin\left(\frac{\delta L}{r_e}\right) + b \cos\left(\frac{\delta L}{r_e}\right) + \left(\frac{r_e}{\delta}\right) \left(\frac{dp}{p}\right) \quad (4.23)$$

where $\delta = (1 - n)^{1/2}$, from which the radial slope is:

$$x' = \left(\frac{dx}{dL}\right) = a \frac{\delta}{r_e} \cos\left(\frac{\delta L}{r_e}\right) - b \frac{\delta}{r_e} \sin\left(\frac{\delta L}{r_e}\right) \quad (4.24)$$

and taking as initial conditions $x = x_1$, $x' = x'_1$ when $L = 0$, we obtain that $a = r_e x'_1 / \delta$ and $b = x_1 - (r_e^2) dp/p$. Assuming that the trajectory inside the magnet of a particle with momentum p is circular we have that the total path of the electron inside the magnet is: $L = r_e * \theta$ (θ is the so called *bending angle*). Noticing that the momentum of an ion doesn't change passing through a magnet, we can write the matrix $\mathbf{M}_{\text{bend}}$ for the electrons transmission inside a magnetic field, which is defined by the relation:

$$\mathbf{X} = \mathbf{M}_{\text{bend}} \mathbf{X}_1 \quad (4.25)$$

we find

$$\mathbf{M}_{\text{bend}} = \begin{pmatrix} \cos \phi & \frac{r_e}{\delta} \sin \phi & \frac{r_e}{\delta^2} (1 - \cos \phi) \\ -\frac{\delta}{r_e} \sin \phi & \cos \phi & \frac{1}{\delta} \sin \phi \\ 0 & 0 & 1 \end{pmatrix} \quad (4.26)$$

for

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_1' \\ \frac{dp}{p} \end{pmatrix}$$

where

$$\phi = \frac{\delta L}{r_e} = (1 - n)^{1/2} \theta$$

(n is the field index as in eq. 4.2) This means that if we want to consider the whole trajectories of the electrons we must add the passage in the field-free region, that changes only the displacement without changing the slope, so assuming that the electrons are at a distance S_0 from the entrance of the magnet and that they continue their trajectories for a distance equal to S_1 , the full transfer relation is:

$$\mathbf{X} = \mathbf{M}_{\text{drift}}(S_1) \mathbf{M}_{\text{bend}} \mathbf{M}_{\text{drift}}(S_0) \mathbf{X}_0 \quad (4.27)$$

where

$$\mathbf{M}_{\text{drift}}(S) = \begin{pmatrix} 1 & S & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (4.28)$$

evaluating the displacement and the slope of the electrons that pass through the magnet, multiplying out the above matrix, we find out that¹:

$$x_1 = ax_0 + bx_0' + c \frac{dp}{p} \quad (4.29)$$

$$x_1' = dx_0 + ex_0' + f \frac{dp}{p} \quad (4.30)$$

where a,b,c,d,e,f are the coefficients that come from the calculation. Now we can find the distance (S_1) between the image point and the output of the magnet; in fact imposing that the image point it is, for a given momentum, where all the

¹The existence of this solution is guaranteed by the fact that all the determinants of all the equations is bigger than zero.

particles that started from x_0 will arrive, no matter the slope they had at the beginning:

$$b = \frac{r_e}{\delta} \sin \phi + S_0 \cos \phi - S_1 \left(\frac{S_0}{r_e} \delta \sin \phi - \cos \phi \right)$$

and setting it equal to zero:

$$S_1 = \frac{\frac{r_e}{\delta} \sin \phi + S_0 \cos \phi}{\frac{S_0}{r_e} \delta \sin \phi - \cos \phi} \quad (4.31)$$

S_1 is the distance between the image point and the output of the magnet. Moreover a is the magnification of the spectrometer if we consider $x_1 = S_{1x}$ and $x_0 = S_0$, i.e.:

$$M_x = \frac{-1}{(S_0/r_e) \delta \sin \phi - \cos \phi} \quad (4.32)$$

4.1.4 Axial oscillation of the electron beam

The same analysis done for the radial motion can be done for the axial motion and in this case all the calculations are the same as far as we change few parameters as show in the table below.

Radial Parameter to replace	with the Axial
$\delta = \sqrt{1-n}$	$\delta' = \sqrt{n}$
$\phi = \delta\theta = \sqrt{1-n}\theta$	$\psi = \delta'\theta = \sqrt{n}\theta$
x_o, x'_o	z_o, z'_o

Performing these changes we obtain an equation for the axial image point:

$$S_{1Z} = \frac{\frac{r_e}{\delta} \sin \psi + S_{0Z} \cos \psi}{\frac{S_{0Z}}{r_e} \delta \sin \psi - \cos \psi} \quad (4.33)$$

where $\psi = n^{1/2}\theta$, then if $n = 1/2$ we have $\psi = \phi$ and, of course, $S_{0Z} = S_0$ so that $S_{1Z} = S_1$, that is the radial and the axial image coincide as previewed.

4.2 Edge effects

4.2.1 Conventions

1. It is customary to define *positive* the angle between the particle beam and the outward normal, at the point where the electrons meet the magnet, if it is outside the orbit. Instead if the angle is inside the orbit it is considered *negative* (see fig. 4.3).

Figure 4.3: Conventions for the angles.

2. The radial magnetic field is zero on the midplane between the poles.
3. An azimuthal coordinate called L that runs along the pattern of the electron (see fig. 4.2).

4.2.2 Axial Focusing

Now we consider an electron beam coming in the magnet with an angle u positive and at a distance z above the median plane; this implies that the lines of force of the magnetic field bulge outward in such a way that there are two components of the magnetic field: one axial (B_a) and one radial B_h , see fig. 4.4).

Figure 4.4: Magnetic field at the edge of the magnet

The equations of the motion of an electron passing through the magnet edge will be:

$$\frac{d^2 z}{dt^2} = -\frac{qv}{m} B_h \sin u$$

and taking L along the electron trajectory in such a way that $v = \frac{dL}{dt}$ we obtain (since $\frac{d^2 z}{dt^2} = \frac{dv_z}{dL} \frac{dL}{dt} = \frac{dv_z}{dL} v$):

$$\frac{dv_z}{dt} = -\frac{q}{m} B_h \sin u \quad (4.34)$$

integrating on a rectangular path lying in the vertical plane, shown in fig. 4.5, we obtain:

Figure 4.5: Path along which has been integrated.

$$(v_z)_{L_1} - (v_z)_{L_2} = -\frac{qv}{m} B_0 \tan u$$

where L_1 and L_2 are, respectively, the azimuthal coordinate inside (where lines of force of the magnetic field are parallel to z axis) and outside (where the magnetic field is assumed to be zero). Since $v_z = \frac{dz}{dt} = \frac{dz}{dL} \frac{dL}{dt} = \frac{dz}{dL} v$ we have:

$$\left(\frac{dz}{dL} \right)_{L_1} - \left(\frac{dz}{dL} \right)_{L_2} = -\frac{q}{mv} B_0 z \tan u \quad (4.35)$$

The equation 4.35 gives the relation between the vertical slope of the orbit outside and inside the magnet, this means that it gives the edge properties of the magnet; that is the edge has vertical *focusing* properties if the slope “downstream” at L_2 should be less than slope “upstream” at L_1 i.e. $u > 0$. Otherwise if the angle u were negative, the action would be diverging.

A parallelism between our case and the properties of thin lenses can be outlined if we introduce a focal distance f_z as:

$$f_z = \frac{z}{\tan \phi} \quad (4.36)$$

Figure 4.6: The angle ϕ used.

ϕ is the angle shown in fig 4.6. Since $\tan\phi$ is given by the derivative of the trajectory we obtain that the focal distance for the axial motion is:

$$f_z = -\frac{r_e}{\tan u} \quad (4.37)$$

This gives the evidence of the behavior of the edge effect as a thin lens focusing ($r_e = Mv/B_0q$).

The trouble is that an actual *fringing* field is not a step-function (as for a thin lens) but it is a more smooth function that produces a changing in slope but also in the z position (as seen previously).

One important correction can be done to take in account is the fact that the fringing field isn't a step function, introducing the function Ψ as follows [34]:

$$\Psi = \frac{KG}{r_e} \left(\frac{1 + \sin^2 u}{\cos u} \right) \quad (4.38)$$

Where:

$$K = \sqrt{\frac{2B_0}{G} \frac{1}{[Br_e]}}$$

and $B_0 = B_z(0)$ is the central field, $[Br_e]$ is the rigidity, G is the gap (K can vary between 1/2 and 1). then the axial focal distance is:

$$f_z = -\frac{r_e}{\tan(u - \Psi)} \quad (4.39)$$

4.2.3 Radial Focusing

An analysis similar to the one done in the chapter 4.2.2 and assuming that a paraxial beam we reach a formula for the focal distance for the radial motion which is:

$$f_r = \frac{r_e}{\tan u} \quad (4.40)$$

The important consequence of the equations 4.37 and 4.40 is that when the angle u is positive the effect is an axial focusing and radial defocusing. In this second case has been found out that the shape of the fringing field doesn't affect the radial focal properties of the magnet.

We can write a matrix as we have done in the previous case, but this time we take in account both the planes (radial and axial) together:

$$\mathbf{M}_{\text{edge}}(u) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{\tan u}{r_e} & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{\tan(u-\Psi)}{r_e} & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (4.41)$$

From now on the vector base is given by:

$$\mathbf{X} = \begin{pmatrix} x \\ x' \\ z \\ z' \\ \frac{dp}{p} \end{pmatrix}$$

where both x' and z' are the derivative of x and z respect to the main trajectory called L ($x' = \frac{dx}{dL}$, $z' = \frac{dz}{dL}$); these two derivatives are, in practice, the angle between the trajectory considered and the main trajectory.

4.3 Conclusions

We have found the tools we need to perform a first order analysis of the electron beam inside a magnet, first order theory in most cases results to be very good and close to the actual behavior of an electron beam moving inside a magnetic field.

The matrix $\mathbf{M}_{\text{bend}}$ which describes the transport through the magnet follows from 4.26 and from the table given in the paragraph 4.1.4:

$$\mathbf{M}_{bend} = \begin{pmatrix} \cos \phi & \frac{r_e}{\delta} \sin \phi & 0 & 0 & \frac{r_e}{\delta} (1 - \cos \phi) \\ -\frac{\delta}{r_e} \sin \phi & \cos \phi & 0 & 0 & \frac{1}{\delta} \sin \phi \\ 0 & 0 & \cos \psi & \frac{r_e}{\delta'} \sin \psi & 0 \\ 0 & 0 & -\frac{\delta'}{r_e} \sin \psi & \cos \psi & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (4.42)$$

where $\phi = \delta\theta$, $\psi = \delta'\theta$ and θ is the bending angle (and $\delta = \sqrt{1-n}$ $\delta' = \sqrt{n}$).

Using the matrix given in 4.28 and 4.41 plus the one just written we have that the overall transport is given by:

$$\mathbf{X}_1 = \mathbf{M}_{drift1} \mathbf{M}_{edge1} \mathbf{M}_{bend} \mathbf{M}_{edge0} \mathbf{M}_{drift0} \mathbf{X}_0 \quad (4.43)$$

As said the first order technique gives a reliable description of the optical proprieties of a magnet, for a more deep description it is necessary to perform numerical calculations. Anyway the complexity and their reduced velocity impose to perform a first order analysis to find out the working point parameters. In the numerical simulations a code solves the equations of motion of the particles inside a magnetic field which can be the interpolation of a measured magnetic field or a model given by a code which solved the Maxwell's equations.

Figure 4.7: Geometry used by the particle tracing code.

4.4 Example of particle tracing code

Our example is based on the actual setup we had during a LWFA experiment, with the plasma, the source of the electrons, at a fixed distance from the center

Figure 4.8: Axial magnetic field between the poles of the electro-magnet using a current of 60 A. The diameter of the poles is 100 mm and the gap between them is 75 mm. The poles are located between -50 mm to 50 mm.

of the magnet (as shown in Fig. 4.7).

The poles of the magnet (a GMW-mod.3472-70) are flat and round, their diameter is 100 mm the gap between them is 75 mm and the distance of the gas jet from the center of the magnet is 760 mm. The first purpose of the magnet in such a position was of deviating the electron beam from its trajectory giving us, in this way, a former idea of the energy of the electrons. A measure of the axial magnetic field between the poles in such a set up gives the field shows in Fig.4.8.

In Fig. 4.8 is well shown how a gap comparable with the diameter of the poles causes a distortion of the magnetic field; in fact an ideal magnet with infinite flat poles gives a constant magnetic field between the poles, so in an actual magnet we should have such a field at least in a small region in the center of the poles.

Figure 4.9: Ray tracing for electrons coming from a point-source at 0.76 m from the center of the poles

In our case this doesn't happen because of the saturation of the poles, this is due to the high current in the coils necessary to achieve the same magnetic field that we should achieve with a lower current if the poles were closer. Even if it wasn't our first purpose we decided to perform a ray tracing in this configuration using a simple two dimensional model, in which we assumed a radial symmetry and we used a magnetic field with a parabolic shape that fits the actual field in the central part of the poles. The model uses a Fortran program for the solution of the Newton's equations ($\omega = qB_z/\gamma m$):

$$\frac{d^2x}{dt^2} = \omega v_y \quad (4.44)$$

$$\frac{d^2y}{dt^2} = -\omega v_x, \quad (4.45)$$

Figure 4.10: Same set up of the Fig. 4.9 but with the central ray *over* the middle of the poles and, on the right, zoom on the image-point for electrons of 18, 20 and 22 MeV (20 MeV $\pm$ 1%)

which are solved with 4th-order Runge-Kutta method and an accuracy of 0.1 mm. At this point we can easily see the trajectory of an electron beam that goes through a magnetic field with a non uniform profile (in our case parabolic), this allows us to underline these aspects:

- electrons with bigger radius of curvature must see a bigger field otherwise they are defocused (see Fig. 4.9 electrons with 100 MeV)
- Even if the electrons see a magnetic field that increase with the radius of curvature (as for the electrons with energies of 10 MeV in Fig.4.9) there is an aberration effect (due to the shape of the poles and to the magnetic field itself).

In this example has been used a rough analytical model for the magnetic field since we needed only a rough idea of the behavior of the electron beam. Anyway this example shows very well how a numerical code works.

Changing the y position of the point of entrance of the electrons in the magnetic field we improve the focusing properties, as we can see in the Fig.4.10, but the resolution is very low. In fact looking at the electrons with energies of 20 MeV it is clear that we have improved the focusing but there are still some aberration effects that grow for the electrons away from the main beam (see Fig. 4.10). Another negative aspect is that the image point is very close to the poles of the magnet so that the noise it should be bigger than if it was far away.

Chapter 5

Project of a double focusing spectrometer.

As seen in the previous chapter we want to achieve a double *focusing spectrometer* (i.e. the axial and radial foci coincide), this request came from the fact we want to obtain the highest energy density on the detector surface. In this chapter we will see the engineering problems we had to face in projecting the new spectrometer (called “H-pacman”) and its optical features. The name “H-Pacman” comes from the original magnet called “pacman” for the shape (a C-magnet) and the color (yellow) of its yoke, then modified to an H magnet.

Our experience with the magnetic field behavior drove us to a particular kind of magnet which uses the edge focus effect shown before instead of a gradient effect. The reason can be found in some experimental explanations:

- The shaping of the poles is only a two dimensional problem, in fact we want that the field index to be equal to zero. A two dimensional pole face is, moreover, easier to build that is cheaper to build.
- It is possible a control on the optical properties of the magnet using field clamps (extra return pattern, as we will see in the following) at the entrance and at the output of the magnet. Instead in a magnet which bases its optical properties on a field gradient it is impossible to modify the field index effect.

Figure 5.1: Old and new (dashed) setup for the oasis experiments.

5.1 Set up of the experiment.

The set up of the future experiments and the results of the previous ones are the starting point for the project of the spectrometer.

The setup of the experiments is shown in fig. 5.1. In particular, the setup in the rectangular vacuum chamber is the running experiment; in the future experiments more lasers than in the picture will focus on the gas-jet for plasma channeling and injection purposes; moreover a new smaller chamber will be inserted. This small chamber will contain more diagnostic for the laser beam; the last chamber will keep the electron beam (deviated by the magnet) under vacuum until it reaches the detector (the phosphor screen in the figure).

The magnet will be, at least, at a distance of 83 cm (distance between the gas jet and the end of the rectangular chamber) because this is the necessary room for the diagnostic, as the FROG system or the ICT.

Now we need more data about the electrons we will analyze, this data could come from the previous experiments:

1. The expected angular divergence is of ± 40 mrad
2. The size of the source of the electrons is expected as small as the focal spot i.e. $6 \mu\text{m}$

3. The energy dispersion is 100%: 1 to 40 MeV
4. Total expected electronic charge: 5 nC

These data give us the limits in which we will move. Anyway we have to take in account also a theoretical prevision that says we will reach energies up to 100 *MeV* with a lower energy dispersion and a smaller angular divergence.

The most important parameter is the upper energy limit we can obtain, which will give us the intensity of the magnetic field.

From these data we have also an hint on the gap that our magnet should have; the gap should be large enough to let most part of the beam to go through but it shouldn't be too big to force us to use a magnet too big or with a non uniform magnetic field. So the gap is given by:

$$\text{Gap} = 2\alpha S_0 \approx 8 \text{ cm}$$

where α is the divergence, S_0 is the distance between the entrance of the magnet and the source of the electron (about 1 meter).

5.2 Wedge magnet

As starting point we took a standard wedge-pole C-dipole magnet in use as ion wedge magnet at the Berkeley cyclotron (shown in fig. 5.2).

We choose that magnet for its characteristics which are already close to what we need; in fact the gap is 7.5 cm and the shape of the poles have been drawn to give almost double focusing properties to the magnetic field.

Figure 5.2: These are part of the drawings of the so called “pacman” magnet.

Another feature of this magnet which is important for our purposes is the bending angle; in this case is 90 degrees. The importance of the bending angle is bound to the detection part; in fact a bending angle of few degrees deviates the electron trajectory only few degrees from the original -straight- trajectory. This means that on the detector will arrive both deviated electrons but also eventually scattered electrons (i.e. noise).

We want to investigate electrons with energies up to 100 MeV and we want to bend them with a bending radius which can fit in the poles of the magnet:

$$r_e = \frac{L}{2 \sin \frac{\gamma}{2}} \quad (5.1)$$

which correspond to the trajectory shown in Fig. 5.3

Figure 5.3: Main trajectory in the “pacman” midplane, all units in cm.

We can calculate the magnetic field we need, using the formula of the so called *rigidity*, Br_e [35]):

$$Br_e = \frac{p}{0.2998}$$

where the units are Tesla for the magnetic field, meters for the bend radius and GeV for the momentum. This tells us that the magnetic field should be around 0.8 T for electrons with momentum of 0.1 GeV and radius $r_e = 0.4$ m.

Now we have to investigate the magnetic field effects on the structure of the magnet i.e. if any change has to be performed on the yoke of the magnet to obtain the magnetic field wanted and which will be the “quality” of the magnetic

field. In fact the working point of the original magnet was ≈ 0.4 T which is half of the magnetic field we want.

To perform this kind of check we used the POISSON code which is based on a Runge Kutta algorithm. The general idea is that the program will use a mesh (created by the AUTOMESH code) to solve the Maxwell's equations with the Runge Kutta algorithm of a two dimensional model of the magnet we are investigating. As result it will give the magnetic field expected inside and outside the magnet depending on the iron and current (that flows in the coils) used for the simulation.

Figure 5.4: Here it is shown how we have simulated a C-magnet. The white part of the magnet is the one the POISSON code simulates.

Using the data we have on the C-magnet (as the iron we want to use, *steel 1010*, the number of turnings, the dimensions of the yoke) we can find out the starting boundary condition for POISSON. One remark has to be done before we continue, the code simulates only one quarter of the entire magnet assuming cylindrical or cartesian symmetry. This implies that it simulates an H-magnet, anyway the magnetic properties are exactly the same the only difference is that we have to “split” the vertical part of the yoke in two (see Fig.5.4) then the magnetic field obtained has the same behavior. The needing of splitting the yoke is that otherwise the program would expect a vertical return path which is the double (for symmetry reasons) of what actually it is. In particular we used as input file the file shown in Appendix A whose solution is shown in fig. 5.5 and the results will be discussed in the following.

Figure 5.5: This is the output of the poisson program using the file shown in appendix A.

5.3 Engineering Issues

Using the POISSON code we found the behavior of the magnet around our working point. It was immediately clear that saturation effects occurred for currents lower the one we needed to reach to obtain a magnetic field of 8 kG (for 100 MeV electrons).

The clearest way to discover saturation effects is to vary the current inside the coils and to plot the magnetic field (in the gap) versus the current. This plot shows clearly the behavior of the magnetic field. If the relationship between the current and magnetic field is linear we are in a regime in which we can assume the ideal condition $B = nI\mu_0$ (n = number of turnings/m, I = current and $\mu = 4\pi 10^{-7} \text{ A}^{-1}$); that is the magnet is working properly. If the relationship is non-linear it implies that a saturation effect is occurring. In fig.5.6 a linear fit is shown by the line plotted and the dots represents the simulated data (magnetic field versus current), so it is clear that there is saturation at higher currents.

Since the iron used (*steel 1010*) for the magnet (then also for the simulation) saturates at magnetic field of the order of 2 T the saturation observed can not be due to the core of the yoke. It is, instead, due to the saturation of the corners of the poles. Here the lines of force of the magnetic field are more dense and the magnetic field reaches higher values than elsewhere. Increasing the size of the yoke we reduce the density of the lines of force of the magnetic field around the corners, deleting the saturation in these points.

Then performing the same analysis with the yoke increased by $\approx 4 \text{ cm}$ in both axis (horizontal and vertical) we find that the saturation disappears (fig. 5.7).

It has to be noticed that the yoke was increased of 4 cm in the 2-D model used

Figure 5.6: The non-linearity of the magnetic field versus current in the original “pacman”.

Figure 5.7: The linearity of the magnetic field versus current in the modified “pacman”: H-Pacman.

for the simulation. As previously explained the code simulates an H-magnet so if we were interested in the C-magnet configuration (as originally was “pacman”) we should have increased the size of the vertical part of the yoke of 4 cm more (i.e. the yoke would have been increased 4 cm in the horizontal part and 8 cm in the vertical). But since the shape of the yoke it is not a problem we decided to use an H-magnet.

Another important engineering issue is represented by the current flow in the coils.

The current we need is around 400 A which implies that a normal copper bending with external cooling (as the one used in the previous version of the magnet) would melt the internal part of the windings before we reached the field wanted, destroying the magnet.

The solution can be found using a different kind of cables, *the hollow cables*. In this case the water cooling effect is due to the water that flows inside the hollow in the cable. The ratio between the size of the hollow and of the copper walls, the number of turnings, the current and the water pressure are the main parameters. All these parameters are taken in account in the MATHCAD program (developed by Matthaeus Leitner and shown in appendix B) which gives the temperature rise due to the current flow. The result is very good in fact the temperature rise obtained using 8 layers of 8 turns each (see fig 5.8) is of few degrees¹ (≈ 17 degrees).

Figure 5.8: Here there is how it will look like a section of a coil.

Now we have to look at the magnetic field profile generated by the H-Pacman as described before. The magnetic field has two “critic” points we have to examine:

- where the particles will pass inside the gap. ($\rightarrow$ harmonic analysis)
- the entrance and the output of the magnet. ($\rightarrow$ fringing field analysis)

5.3.1 Harmonic Analysis.

The harmonic analysis is the custom way to investigate the magnetic field behavior; this method is based on a harmonic development of the vector potential $\mathbf{A}(x, y)$ thought as the real part of a complex function $F(z)$, in a region defined by the radius of normalization r_{norm} .

Under this hypothesis we find that the magnetic field is of the form [36]:

¹It has to be underlined that a sustainable temperature rise has to be lower than 20 degrees.

$$B_x - iB_y = \sum_{m=0}^{\infty} B_m e^{im\phi}$$

where

$$B_m = -\frac{(m+1)}{r_{norm}} \left(\frac{r}{r_{norm}} \right)^m (b_{m+1} - ia_{m+1})$$

the terms a_n , b_n come from the harmonic analysis of the vector potential and they are numerically calculated by Poisson. In particular the radius of normalization defines the area which is analyzed, so it has to be, at least, the same size as radial extension of the electron beam which will pass inside the gap of the magnet. Thus if the harmonic analysis is *positive* it will guarantee that the electrons will observe a constant magnetic field in the analyzed region. We say *positive* an harmonic analysis if the magnetic field has the properties we expected it should have i.e. non bulging lines of force inside the gap. In particular we have to check that: $a_0 \gg b_0$ and $a_0 \gg a_m \forall m \geq 1$. In fact in this case the lines of force of the magnetic field will be almost all along the “y-axis” that in this case is directed orthogonal to the poles. We have performed this check corresponding to a current of 300 A (a maximum field of 6.39 kG) and a radius of normalization of $r_{norm} = 3$ cm we found the results shown in the following table:

m	a_n	b_n
1	-1.9159E+04	0.0000E+00
3	-2.1577E-03	0.0000E+00
5	1.8103E-03	0.0000E+00
7	1.4972E-04	0.0000E+00
9	3.0995E-06	0.0000E+00
11	3.5904E-05	0.0000E+00
13	-2.1855E-05	0.0000E+00
15	2.1024E-05	0.0000E+00
17	-2.6880E-05	0.0000E+00
19	1.1043E-05	0.0000E+00

the two conditions just written are well satisfied, i.e. the magnetic field obtained has a behavior that is similar to the one we have supposed in the theory shown in Chapter 4. Thus the electrons will see a constant magnetic field with non bulging lines of force i.e. a field index $n = 0$.

5.3.2 Fringing Field.

Now we have to analyze the fringing field (par. 4.2.2), that is we have to optimize the Poisson model we have done before. In fact we are interested in the most accurate analysis of the fringing field and the customary way to do that is to move the iron yoke aside, as shown in fig. 5.9. Other changes were made as explained in the follow.

The axial size of the yoke has been increased to avoid the saturation of the iron in this region, but all the lines of force of the magnetic field go back in the vertical part of the yoke (see fig. 5.9). We didn't change the high of the magnet (instead of moving the coil) since this kind of changing could have affect the fringing field as well².

Figure 5.9: This is the new layout of the POISSON model we used to analyze the fringing field behavior; it is also shown a typical shape of the *field clamps*.

Now, since we are interested in the magnetic fringing field we have to consider the use of the field clamps.

The use of the field clamps *improve* the magnetic field profile at the entrance and at the output, i.e. the magnetic field is closer to the model of the magnetic field used in our theoretical analysis. A long fringing field, in fact, would worsen the optical properties of the magnet. If we plot together the magnetic field profile with and without field clamps we see a different behavior, the magnetic field generated by the magnet with field clamps fall down quicker than the one without field clamps (as shown in the fig. 5.10). Then the use of the field clamps

²It has to be noticed that this kind of model simulates in a proper manner only the magnetic field on the left part of the magnet shown in fig.5.9 since the the yoke of the magnet itself has been changed and the right coil has been moved.

Figure 5.10: The two plots show the behavior of the fringing field with and without the field clamps.

permit a control on the profile of the fringing field which is responsible of the axial focusing of the electron beam. Thus, if necessary, moving the field clamps the axial focus position can be adjusted.

5.4 Optical Properties

As shown in chapter4 the optical properties are determined by some parameters:

- bend radius (r_e)
- bending angle (θ)
- field index (n)
- input and output angles (u_1, u_2)

These are fixed parameters once the geometry of the poles has been defined. Now we have, anyway, to take in account another aspect that is that the bending angle is strongly linked to the area where the magnetic field can be considered constant. Considering a constant magnetic field inside the magnet is a too generic condition, in fact the experience shows that the magnetic field extents outside the gap with, sometimes, long tails. To take in account the displacement due

to the fringing field (non-constant field) we introduced an approximation called *effective pole* [33]. This approximation consists in considering a region with a constant magnetic field which is proportional to the area of the pole (see fig. 5.3). The proportionality between the actual pole and the effective pole is given by:

$$R = \frac{\int_0^{+\infty} B_z(r) dr}{B_z(0) r_{actual}} \approx 1.136$$

where r_{actual} is the actual length of the pole used to simulate the magnetic field in POISSON. So using the effective pole (bigger than the real one) and the bending radius found before ($r_e = 0.4$ m) the bending angle turns out to be 90° , as already seen before. In fact the bending angle is still given by the eq.5.1 but this time $L = 56.75$ cm which implies, as said, $\theta = 90^\circ$.

The field index has been chosen to be zero, so this parameter is already fixed.

5.4.1 Double focusing

At this point we have to find out the input and output angles to achieve the double focusing properties we want to have.

We have to use the equations that come up solving the system given in eq.4.43 and imposing that all the electrons with the same energy that start from the source will arrive in the same final point (it doesn't matter the slope they had at the beginning). In this way we find the distance of the two object points from the edge of the effective pole in the axial plane:

$$S_{1z} = \frac{S_0 r_e}{2S_0 \tan(u - \Psi) - r_e} \quad (5.2)$$

and in the radial plane:

$$S_{1r} = \frac{r_e + S_0 \tan u}{\frac{S_0}{r_e} - \tan u - \frac{S_0}{r_e} \tan^2 u} \quad (5.3)$$

where all the parameters has the same meaning they had in the chapter 4. Now we can impose that the two object points coincide, so we will reach the double focusing feature and find the angle u at which the beam should enter and leave the magnet.

A first solution has been found with the approximation of a step function fringing field (i.e. with the function $\Psi = 0$) and it gives an angle of $u = 26.68^\circ$. But calculating the function Ψ we obtain $\Psi = 0.24976^\circ$ which implies $S_{1z} = 67.3 \text{ cm}$ and $S_{1r} = 66.1 \text{ cm}$, i.e. a shift of the location of the two foci of $\approx 1 \text{ cm}$. So using the parameter $\Psi = 0.24976^\circ$ we obtain a corrected u :

$$u = 26.78^\circ$$

Which makes the two object points coincide in single point. In particular a valuation of the resolution $R_p = \frac{\Delta p}{p}$ of the spectrometer can be done. In fact the resolution can be found as[33]:

$$R_p = \frac{s_{scr} - M_x w_0}{-2M_x r_e (S_0/r_e) [\sin \theta + \tan u (1 - \cos \theta)] + 1 - \cos \theta}$$

where: s_{scr} is the resolution of the detector, w_0 the size of the electron source (comparable with the laser waist), M_x is the magnification of the spectrometer which can be found from the matrix 4.43 as it was done for the equation 4.32 and it gives:

$$M_x = \frac{-1}{(S_0/r_e) [\sin \theta - 2(\tan u) \cos \theta - \tan^2 u \sin \theta] - \cos \theta - \tan u \sin \theta}$$

Then evaluating the magnification we find that $M_x \approx -0.7$ and assuming a very good resolution of the detector, $10 \mu\text{m}$, we found a resolution which is extremely high: $\approx 5 \times 10^{-5}$. Numerical analysis performed by G. Dugan[37] for a similar and smaller spectrometer assured a resolution of 5×10^{-3} .

5.5 Conclusion

The use of the linear theory of the optical magnetic dipoles has been a valid tool to outline the main characteristics of the magnetic spectrometer we want to build. Moreover the specialized POISSON code allowed us to achieve a first project. More numerical analysis will be required to obtain a full description of the optical proprieties of the magnet. Anyhow the main characteristics of the spectrometer (the high resolution and the double focusing proprieties) will be maintained and pursued with small changing of the shape of the poles. On the other hand nothing, in principle, will be modified for what concern the coils and the yoke. Then a first design of the spectrometer can be given, as in figure 5.11.

Figure 5.11: Drawing of the H-pacman spectrometer (upper and side view); the units are in centimeters.

The high stability of the laser system shown in chapter 2 guarantees that reliable spectra can be obtained also with electrons having 100% energy spread. In fact, since the bending radius is linked to the electron momentum and to the magnetic field, varying the magnetic field shot-by-shot the analyzed momentum will change and consequently the portion of the spectrum analyzed. It has to be noticed that the spectrometer is designed in order to filter out the electrons having bending radii different from a given value, independent from the value of B . Another relevant feature is that the large gap will guarantee the possibility of analyzing the whole beam without limiting the size of the entering beam as it happens in most of the spectrometers.

APPENDIX A

Here is the POISSON code input file used as starting point for the analysis of the magnetic field properties performed on the model of the PACMAN magnet.

```
    based on L'OASIS spectrometer
$ REG CONV=1.000000, ;Conversion factor (in cm/unit) for length units.
KPROB=0,
DX=0.3,
XMAX=46.275000,
YMAX=28.871000,
XMIN=0.000000,
YMIN=0.000000,
NPOINT=7,
mode=0 $
$ PO X=0.000000, Y=0.000000 $
$ PO X=28.035000, Y=0.000000 $
$ PO X=46.275000, Y=0.000000 $
$ PO X=46.275000, Y=28.871000 $
$ PO X=0.000000, Y=28.871000 $
$ PO X=0.000000, Y=3.7500000 $
$ PO X=0.000000, Y=0.000000 $
$ REG MAT=1, CUR=-6400.000000, NPOINT=5 $ !Dimension of the coils and
current inside the coils
$ PO X=19.000000, Y=5.6880000 $ ! (64 turns * 100 Ampere)
$ PO X=28.035000, Y=5.6880000 $
$ PO X=28.035000, Y=12.778000 $
$ PO X=19.000000, Y=12.778000 $
$ PO X=19.000000, Y=5.688000 $
$ REG MAT=2, NPOINT=12 $ !Dimension of the yoke
```

\$ PO X=0.000000, Y=3.750000 \$
\$ PO X=18.365000, Y=3.750000 \$
\$ PO X=19.000000, Y=4.386000 \$
\$ PO X=19.000000, Y=5.688000 \$
\$ PO X=19.000000, Y=12.778000 \$
\$ PO X=28.035000, Y=12.778000 \$
\$ PO X=28.035000, Y=5.688000 \$
\$ PO X=28.035000, Y=0.000000 \$
\$ PO X=46.275000, Y=0.000000 \$
\$ PO X=46.275000, Y=28.871000 \$
\$ PO X=0.000000, Y=28.871000 \$
\$ PO X=0.000000, Y=3.750000 \$

APPENDIX B

Here is the three pages of MATHCAD program used to choose the correct water cooled bindings to use with the projected magnet (the temperature rise should be lower then 20 degrees). In particular it has been used the water pressure in the building in which the magnet will operate and all the characteristics coming from the magnet design.

<p>Conductor Input:</p> <p>CopperSideLength := 0.34·in (excl. insulation)</p> <p>HoleDiam := 0.184·in</p> <p>CornerCut := 0.03·in</p> <p>InsulationThickness := 0.0324in</p> <p>Total Conductor Size including Insulation:</p> <p>ConductorSideLength := CopperSideLength + InsulationThickness·2</p> <p>CopperArea := $\text{CopperSideLength}^2 - \left(\frac{\text{HoleDiam}}{2}\right)^2 \cdot \pi - 2 \cdot \text{CornerCut}^2$</p> <p>ConductorArea := $\text{ConductorSideLength}^2$</p> <p>FillingFactor := $\frac{\text{CopperArea}}{\text{ConductorArea}}$</p> <p>Conductor Data Summary:</p> <p>ConductorSideLength = 0.404·in (incl.insulation)</p> <p>ConductorArea = 0.163·in² (incl.insulation)</p> <p>CopperArea = 0.087·in²</p> <p>FillingFactor = 53.432 %</p> <p>1</p>	<p>Pancake Input:</p> <p>PancakeCurrent := 400·A T := 60</p> <p>NumberOfWindingsRadial := 8</p> <p>MeanPancakeLength := 1.9614 m</p> <p>Pancake Electrical Properties:</p> <p>PancakeConductorLength := MeanPancakeLength·2·NumberOfWindingsRadial</p> <p>CopperResistivity := 0.00267·10⁻⁶·ohm·in·(T + 234) at T deg C</p> <p>PancakeResistanceAt40DegC := $\text{CopperResistivity} \cdot \frac{\text{PancakeConductorLength}}{\text{CopperArea}}$</p> <p>PancakePowerConsumption := PancakeResistanceAt40DegC·PancakeCurrent²</p> <p>PancakeVolts := $\frac{\text{PancakePowerConsumption}}{\text{PancakeCurrent}}$</p> <p>PancakeRequiredInsulation := PancakeVolts</p> <p>Pancake Data Summary:</p> <p>PancakeConductorLength = 31.382·m</p> <p>PancakeResistanceAt40DegC = 0.011·ohm</p> <p>PancakeCurrent = 400·A</p> <p>PancakeVolts = 4.448·V</p> <p>PancakePowerConsumption = 1779.4·W</p> <p>PancakeRequiredInsulation = 4.448·V</p> <p>2</p>
---	---

Cooling Water Input:

Calculation Of Temperature Rise:

$$\text{BuildingWaterPressure} := 90 \text{ psi}$$

$$\text{WaterBackPressure} := 40 \text{ psi}$$

$$\text{PressureDrop} := \text{BuildingWaterPressure} - \text{WaterBackPressure}$$

$$\text{PancakeHeadLoss} := \frac{\text{PressureDrop}}{\text{PancakeConductorLength}}$$

$$\text{PancakeHeadLoss} = 48.562 \frac{\text{psi}}{100 \text{ ft}} \quad \text{HoleDiam} = 4.674 \text{ mm}$$

$$\text{WaterFlow} := 0.4 \frac{\text{gal}}{\text{min}} \quad \text{From LBNL Design Data 10A}$$

$$\text{PancakeTempRise} := \frac{3.8 \cdot \text{PancakePowerConsumption}}{\text{WaterFlow}} \left(\frac{1}{\text{kW min}} \right) \cdot \text{K}$$

Water flow should be turbulent to maximize cooling through exchange of water flow layers.
Water velocity should not exceed 20ft/sec to minimize wear.

Pancake Cooling Requirements Summary:

$$\text{BuildingWaterPressure} = 90 \text{ psi}$$

$$\text{WaterBackPressure} = 40 \text{ psi}$$

$$\text{PressureDrop} = 50 \text{ psi}$$

$$\text{PancakeConductorLength} = 102.961 \text{ ft}$$

$$\text{PancakeHeadLoss} = 48.562 \frac{\text{psi}}{100 \text{ ft}}$$

$$\text{WaterFlow} = 0.4 \frac{\text{gal}}{\text{min}}$$

$$\text{PancakeTempRise} = 16.904 \cdot \text{K}$$

Bibliography

- [1] T.Tajima and J.M. Dawson *Phys. Rev. Lett.* vol. 43, p.267, (1979).
- [2] W.P. Leemans, P. Volfbeyn, K.Z. Guo, S. Chattopadhyay, C.B. Schroeder, B.A. Shadwick, P.B. Lee, J.S. Wurtele and E. Esarey *Ph. Of Plasmas*, vol. 5, p.1615, n. 5 (1998).
- [3] D. Giulietti, M. Galimberti, A. Giulietti, L. A. Gizzi, P. Tomassini, M. Borghesi, V. Malka, S. Fritzler M. Pittman and K. Taphou, *Production of ultra-collimated bunches of multi-MeV electrons by 35-fs laser pulses propagating in exploding-foil plasmas* submitted for publication *Ph. Of Plasmas* (2002).
- [4] E. Esarey, P. Sprangle, J. Krall and A. Ting *IEEE Trans. Plasm. Sc.*, vol. 24, No. 2, p.252 (1996).
- [5] A.I. Akhiezer and R.V. Polovin *Sov. Phys. JETP*, vol.3, p.696 (1956).
- [6] P. Chen, J.M. Dawson, R.W. Huff and T. Katsouleas *Ph. Rev. Lett.*, vol.54, p.693 (1985).
- [7] P. Chen, J.J. Su, T. Katsouleas, S. Wilks and J.M. Dawson *IEEE Trans. Plasma Sci.*, vol.PS-15, p. 218 (1987).
- [8] R.D. Ruth, A.W. Chao, P.L. Morton and P.B. Wilson, *Part. Acc.*, vol. 17, p. 171, (1985).
- [9] J.B. Rosenzweig *Phys. Fluids*, vol.58, p.555 (1987).
- [10] J.B. Rosenzweig, P. Schoessow, B. Cole, W. Gai, R. Konecny, S. Mtingwa, J. Norem, M. Rosing and J. Simposon, *Phys. Fluids B*, vol.2, p.1376 (1990).

- [11] H. Nakanishi, A. Enomoto, A. Ogata, K. Nakajima, D. Whittum, Y. Yoshida, T. Ueda, T. Kobayashi, H. Shibata S. Tagawa, N. Yugami and Y. Nishida, *Nucl. Inst. Meth.*, vol.A328, p.596 (1993).
- [12] C.E. Clayton, C. Joshi, C. Darrow, D. Umstadter and F.F. Chen *IEE J. Nucl. Sci. NS-32*(5), p.3551 (1985).
- [13] C.E. Clayton, M.J. Everett, A. Lal, D. Gordon, K.A. Marsh and C. Joshi *Phys. Plasmas*, vol.1, p.1753, (1994).
- [14] P. Sprangle, E. Esarey, A. Ting and G. Joyce *Appl. Phys. Lett.*, vol.53, p. 2146 (1988).
- [15] N. Nakajima, T. Kawakubo, H. Nakanishi, A. Ogata, Y. Kitagawa, R. Kodama, K. Mima, H. Shriaga, K. Suzuki, K. Yamakawa, T. Zhang, Y. Kato, D. Fisher, M. Downer, T. Tajima, Y. Sakawa, T. Shoji, N. Yugami and Y. Nishida *Proc. AIP Conf. Advanced Acc. Concepts*, vol.335, p.145, P. Schoessow, Ed. New York: Amer. Inst. Phys. (1995).
- [16] N.E. Andrev, V.I. Kirsanov and L.M. Gorbunov *Phys. Plasmas*, vol.2, p.2573 (1995).
- [17] A. Modena, Z. Najmudin, A.E. Dangor, C.E. Clayton, K.A. Marsh, C. Joshi, V. Malka, C.B. Darrow, C. Danson, D. Neely and F.N. Walsh *Nature*, col.337, p.606 (1995).
- [18] E. Esarey, J. Krall and P. Sprangle *Phys. Rev. Lett.*, vol 72, p. 2887 (1994).
- [19] T. Katsouleas, C. Joshi, J.M. Dawson, F.F. Chen, C.E. Clayton, W.B. Mori, C. Darrow and D. Umstadter “*Plasma Accelerators*” in *Proc. A/P Conf. Laser Acceleration Particles*, vol.130, p. 63, Amer. Inst. Phys. (1985).
- [20] P. Volfbeyn, W.P. Leemans, J.S. Wurtele and T.J. Greytak “*Experimental Studies of Laser Guiding and Wake Excitation in Plasma Channels.*” *Ph.D. in Phys. M.I.T.* (1998).
- [21] D.E. Spence, P.N. Kean and W.Sibbet *Opt. Lett.* Vol.16, No.42 (1991).
- [22] P.F. Moulton *J. Opt. Soc. Am. B*, Vol.3, No.1 (1986).

- [23] C. Le Blanc *Principes et realisation d'une source laser terawatt femtoseconde basee sur le saphir dope au titane. Caraterisation des impulsions produites et demonstration du regime au niveau de 10^{18} W/cm², Ph.D. thesis, june 17 1993, Ecole Polytechnique.*
- [24] P. Volfbeyn, E.Esarey and W.P.Leemans *Ph. of Pl. Vol 6, No.5, p.2269 (1999).*
- [25] W. L. Kruer *The Physics of Laser Plasma Interactions.*
- [26] F.F. Chen *Introduction to Plasma Physics, Plenum Press N.Y. and London (1974).*
- [27] P. Tomassini, A.Giulietti *Opt. Comm. Vol.199, p.143 (2001).*
- [28] W.P. Leemans, P. Volfbeyn, K.Z. Guo, S. Chattopadhyay, C.B. Schroeder, B.A. Shadwick, P.B. Lee, J.S. Wurtele and E. Esarey *Ph. of Pl. Vol. 5, No.5, p.1615 (1998).*
- [29] M. Takeda, H. Ina and S. Kobayashi *Opt. Soc. Am., Vol. 72, No.1,p.156 (1982).*
- [30] P. Tomassini, M. Borghesi, M. Galimberti, A. Giulietti, D. Giulietti, O. Willi and L.A. Gizzi *Appl. Opt. vol. 40, p.6561 (2001).*
- [31] M. Holschneider *Wavelet: An analysis tool, Clarendon Press-Oxford (1995).*
- [32] J. J. Livingood *Principles of Cyclic Particle Accelerators, D.Van Nostrand Company inc. (1961).*
- [33] J. J. Livingood *The Optics of Dipole Magnets, Academic Press (1969).*
- [34] A. Wu Chao, M. Tigner *Accelerator Physics and Engineering, World Scientific (1999).*
- [35] K.L.Brown, D.C.Carey, Ch. Iselin and F. Rothacker *Transport A Computer Program For Designing Charged Particle Beam Transport Systems, CERN 73-16 (1973) & CERN 80-04 (1980)..*
- [36] J. H. Billen, L. M. Young *POISSON SUPERFISH Documentation.*
- [37] G. Dugan, A. Misuri and W.P. Leemans *LBNL report,identifying number 799570.*