Final Technical Report

Award Number DE-FG02-99ER25376

Probabilistic and Numerical Techniques in the Study of Statistical Theories of Turbulence

Richard S. Ellis and Bruce Turkington
Department of Mathematics and Statistics
University of Massachusetts
Amherst, MA 01003

DOE Patent Clearance Granted

KPH Worsak

9.9.03

Mark P. Dvorscak

(630) 252-2393

E-mail: mark.dvorscak@ch.doe.gov

Office of Intellectual Property Law

DOE Chicago Operations Office

Date

1 Papers Published or in Preparation

We list the papers published or in preparation during the period May 1, 1999–April 30, 2002 of the award.

- (1) M. Boué, P. Dupuis, and R. S. Ellis. Large deviations for small noise diffusions with discontinuous statistics. *Prob. Th. Relat. Fields* 116:125–149, 1999.
- (2) C. Boucher, R. S. Ellis, and B. Turkington. Spatializing random measures: doubly indexed processes and the large deviation principle. *Ann. Prob.* 27:297–324, 1999.
- (3) R. S. Ellis. The theory of large deviations: from Boltzmann's 1877 calculation to equilibrium macrostates in 2D turbulence. *Physica D* 133:106–136, 1999.
- (4) B. Turkington. Statistical equilibrium measures and coherent states in two-dimensional turbulence. *Commun. Pure Appl. Math.* 52:781–809, 1999.
- (5) C. Boucher, R. S. Ellis, and B. Turkington. Derivation of maximum entropy principles in two-dimensional turbulence via large deviations. *J. Stat. Phys.* 98:1235–1278, 2000.
- (6) R. S. Ellis, K. Haven, and B. Turkington. Large deviation principles and complete equivalence and nonequivalence results for pure and mixed ensembles. *J. Stat. Phys.* 101:999–1064, 2000.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

- (7) R. Jordan, B. Turkington, and C. L. Zirbel. A mean-field statistical theory for the nonlinear Schrödinger equation. *Physica D* 137:353–378, 2000.
- (8) R. Jordan and B. Turkington. Statistical equilibrium theories for the nonlinear Schrödinger equation. *Contemporary Mathematics* 283:27-39, 2001. *Proceedings of the AMS-IMS-SIAM Joint Summer Research Conference on Dispersive Wave Turbulence, June 2000.*
- (9) B. Turkington. A statistical equilibrium model of zonal shears and embedded vortices in a Jovian atmosphere. *Proceedings of the IUTAM Symposium – Advances in Mathematical Modeling of Atmosphere and Ocean Dynamics, University of Limerick, Ireland, July 2000*, pp. 271–278, 2001.
- (10) B. Turkington, A. Majda, K. Haven, and M. DiBattista. Statistical equilibrium predictions of jets and spots on Jupiter. *Proc. Nat. Acad. Sci. USA* 98:12346–12350, 2001. Article posted at URL <http://www.pnas.org/cgi/content/full/98/22/12346>.
- (11) R. S. Ellis, K. Haven, and B. Turkington. Nonequivalent statistical equilibrium ensembles and refined stability theorems for most probable flows. *Nonlinearity* 15:239–255, 2002.
- (12) R. S. Ellis, K. Haven, and B. Turkington. Analysis of statistical equilibrium models of geostrophic turbulence. *J. Appl. Math. Stoch. Anal.* 15:341–361, 2002.
- (13) M. Boué, D. Hernández-Hernández, and R. S. Ellis. Large deviations for a random walk model with state-dependent noise. Accepted for publication in *SIAM J. Control and Optimization*, 2003.
- (14) R. S. Ellis, R. Jordan, P. Otto, and B. Turkington. A statistical approach to the asymptotic behavior of a class of generalized nonlinear Schrödinger equations. Submitted for publication, 2003.
- (15) B. Turkington and Z. Liang. Undulating coherent structures in two-dimensional turbulence: a quasi-equilibrium approach. Technical report, Univ. of Mass. Amherst, 2003.
- (16) B. Turkington and N. Whitaker. Direct numerical simulations and statistical theories of coherent structures in two-dimensional turbulence. Technical report, Univ. of Mass. Amherst, 2003.

2 Success of Our Graduate Student, Kyle Haven

Kyle Haven received his Ph.D. from the University of Massachusetts in May 2001, working under the direction of Richard S. Ellis and Bruce Turkington. Kyle's exemplary dissertation [7] consisted of his contributions to our currently supported DOE research project on statistical theories of turbulence. The quality of his work led to his obtaining postdoctoral offers from five prestigious institutions; he accepted the offer from the Courant Institute of Mathematical Sciences at New York University.

One of the main factors that allowed Kyle to carry out the outstanding research in his dissertation was the support by our current DOE grant. In addition, for one month during the summer of 1999, Kyle participated in the summer student program in the T7 group at Los Alamos National Laboratory, where he obtained valuable exposure to a variety of problems and approaches in statistical theories of turbulence.

3 Overview of the Research Project

The complex systems that we studied are governed by nonlinear partial differential equations that excite a wide range of spatial and temporal scales. Our methodology was to model these systems via the ideas of equilibrium statistical mechanics, to derive reduced descriptions of the statistical models using modern probabilistic techniques, and to compute the predicted behavior of the system using numerical methods applied to the reduced description. Each stage of this passage—nonlinear partial differential equation to statistical model to reduced description—decreases the computational complexity of the problem. Our main contribution was to synthesize statistical modeling, mathematical analysis, and numerical methods in a general framework and to apply this synthesis to complex physical phenomena arising in a number of areas including fluid, dispersive wave, and geophysical turbulence. As we outline in the next section, our research settled a number of key questions in statistical theories of turbulence and related areas.

Each of the two PI's brought unique expertise to the joint research effort. Professor Ellis is a recognized expert in mathematical statistical mechanics and probability theory, especially the theory of large deviations. His book *Entropy, Large Deviations, and Statistical Mechanics* is a standard reference on large deviations and applications of the theory to the statistical mechanics of spin systems [2]. Professor Turkington's expertise lies in mathematical fluid dynamics, including the analysis of nonlinear partial differential equations and the use of numerical methods. His numerical method [17, 18], developed in collaboration with N. Whitaker, is the most widely used tool in the computation of maximum-entropy, coherent states in two-dimensional turbulence.

4 Major accomplishments

We made fundamental advances in a number of problems arising in statistical equilibrium theories of turbulence. The theoretical insights and techniques developed in items (a), (b), and (c) have a stunning real-world application in item (d), which concerns the

prediction of jetstreams and vortices in the Jovian atmosphere. In addition, the same set of ideas and methods have been used in item (e) to give new insights into the formation of solitary waves in a turbulent wave system.

- (a) **Maximum entropy principles.** In our paper [1] we analyze in a unified framework the Miller-Robert continuum model of equilibrium states in an ideal fluid [11, 12, 13, 14] and a modification of that model due to Turkington [15]. The maximum entropy principles that govern these models are rigorously derived by a new method based on the theory of large deviations.
- (b) **Equivalence and nonequivalence of ensembles.** In our paper [3] we give a complete analysis of the equivalence and nonequivalence of the microcanonical, canonical, and mixed ensembles at the level of equilibrium macrostates for a large class of models of turbulence. Our main results analyze the equivalence and nonequivalence in terms of concavity properties of finite-dimensional entropy functions.
- (c) **Nonlinear stability of flows.** In our paper [4] we refine the well known Arnold stability theorems by proving the nonlinear stability of steady mean flows for the quasi-geostrophic potential vorticity equation in the case when the ensembles are nonequivalent. Numerical computations implemented for geostrophic turbulence over topography in a zonal channel demonstrate that nonequivalence of ensembles occurs over a wide range of the model parameters and that physically interesting equilibria are often omitted by the canonical model.
- (d) **Geophysical application.** The theories developed in items (a), (b), and (c) are applied in [16] to predict the emergence and persistence of coherent structures in the active weather layer of the Jovian atmosphere. The paper [16] is of fundamental importance because it is the first work in which sophisticated statistical theories are synthesized with actual observational data from the Voyager and Galileo space missions.
- (e) **Nonlinear dispersive waves.** Using an asymptotically exact mean-field theory, we demonstrated in [8, 9] for a class of nonlinear Schrödinger equations that the self-organization of solutions into a ground-state solitary wave immersed in fine-scale fluctuations is a relaxation into statistical equilibrium. We also validated these theoretical predictions quantitatively by comparing them with the results of direct numerical simulations. In [6] we prove a large deviation principle that is a mathematically rigorous statement of the explanation of the observed formation of large-scale coherent structures within small-scale wave turbulence given in [9].

References

- [1] C. Boucher, R. S. Ellis, and B. Turkington. Derivation of maximum entropy principles in two-dimensional turbulence via large deviations. *J. Stat. Phys.* 98:1235–1278, 2000.

- [2] R. S. Ellis. *Entropy, Large Deviations and Statistical Mechanics*. New York: Springer-Verlag, 1985.
- [3] R. S. Ellis, K. Haven, and B. Turkington. Large deviation principles and complete equivalence and nonequivalence results for pure and mixed ensembles. *J. Stat. Phys.* 101:999–1064, 2000.
- [4] R. S. Ellis, K. Haven, and B. Turkington. Nonequivalent statistical equilibrium ensembles and refined stability theorems for most probable flows. *Nonlinearity* 15:239–255, 2002.
- [5] R. S. Ellis, K. Haven, and B. Turkington. Analysis of statistical equilibrium models of geostrophic turbulence. *J. Appl. Math. Stoch. Anal.* 15:341–361, 2002.
- [6] R. S. Ellis, R. Jordan, P. Otto, and B. Turkington. A statistical approach to the asymptotic behavior of a class of nonlinear Schrödinger equations. Submitted for publication, 2003.
- [7] K. Haven. *Large Deviation Principles and Complete Equivalence and Nonequivalence Results for Microcanonical and Canonical Ensembles with an Application to Geophysics*. Ph.D. dissertation, Univ. of Mass. Amherst, 2001.
- [8] R. Jordan and B. Turkington. Statistical equilibrium theories for the nonlinear Schrödinger equation. *Contemporary Mathematics* 283:27–39, 2001. *Proceedings of the AMS-IMS-SIAM Joint Summer Research Conference on Dispersive Wave Turbulence, June 2000*.
- [9] R. Jordan, B. Turkington, and C. L. Zirbel. A mean-field statistical theory for the nonlinear Schrödinger equation. *Physica D* 137:353–378, 2000.
- [10] A. J. Majda, D. W. McLaughlin, and E. G. Tabak. A one-dimensional model for dispersive wave turbulence. *J. Nonlinear Sci.* 7:9–44, 1997.
- [11] J. Miller. Statistical mechanics of Euler equations in two dimensions. *Phys. Rev. Lett.* 65:2137–2140, 1990.
- [12] J. Miller, P. Weichman, and M. C. Cross. Statistical mechanics, Euler’s equations, and Jupiter’s red spot. *Phys. Rev. A* 45:2328–2359, 1992.
- [13] R. Robert. A maximum-entropy principle for two-dimensional perfect fluid dynamics. *J. Stat. Phys.* 65:531–553, 1991.
- [14] R. Robert and J. Sommeria. Statistical equilibrium states for two-dimensional flows. *J. Fluid Mech.* 229:291–310, 1991.
- [15] B. Turkington. Statistical equilibrium measures and coherent states in two-dimensional turbulence. *Commun. Pure Appl. Math.* 52:781–809, 1999.

- [16] B. Turkington, A. Majda, K. Haven, and M. DiBattista. Statistical equilibrium predictions of jets and spots on Jupiter. *Proc. Nat. Acad. Sci. USA* 98:12346–12350, 2001. Article posted at URL <http://www.pnas.org/cgi/content/full/98/22/12346>.
- [17] B. Turkington and N. Whitaker. Statistical equilibrium computations of coherent structures in turbulent shear layers. *SIAM J. Sci. Comput.* 17:1414–1433, 1996.
- [18] N. Whitaker and B. Turkington. Maximum entropy states for rotating vortex patches. *Phys. Fluids A* 6:3963–3973, 1994.