Parity Non-Conservation in Proton-Proton Elastic Scattering

V.R. Brown¹, B.F. Gibson², J.A. Carlson², and R. Schiavilla³

¹ Massachusetts Institute of Technology, Cambridge, MA 02139, and University of Maryland, College Park, MD 20742-4111 USA

² Los Alamos National Lab, Los Alamos, NM 87545, USA

³ Old Dominion University, Norfolk, VA 20742, USA

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Abstract. The parity non-conserving longitudinal asymmetry in proton-proton (pp) elastic scattering is calculated in the lab-energy range 0-350 MeV using contemporary, realistic strong-interaction potentials combined with a weak-interaction potential comprised of ρ - and ω -meson exchanges as exemplified by the DDH model. Values for the ρ - and ω -meson coupling constants, h_{ρ}^{pp} and h_{ω}^{pp} , are determined from comparison with the measured asymmetries at 13.6 MeV, 45 MeV, and 221 MeV.

PACS. 21.30.-x Nuclear forces – 24.80.+y Nuclear tests of fundamental interactions and symmetries – 25.40.Cm Elastic proton scattering

1 Introduction

The parity violating longitudinal asymmetry in proton-proton pp elastic scattering is calculated in the lab-energy range 0-350 MeV. Contemporary, realistic strong interaction potentials such as the Argonne V_{18} [1], the Bonn 2000 [2], and the Nijmegen I [3] models are combined with a weak-interaction potential comprised of ρ - and ω -meson exchanges as exemplified by the well known DDH model [4]. The full scattering problem in the presence of the strong parity-conserving, the Coulomb, and the weak parity-non-conserving forces is solved.

The meson-exchange terms in the weak-interaction are assumed to have the same cut-off as in the Bonn 2000 strong interaction. Values for the ρ - and ω -meson coupling constants, h_{ρ}^{pp} and h_{ω}^{pp} , are determined from comparison with the measured asymmetries at 13.6 MeV [5], 45 MeV [6], and 221 MeV [7].

2 Parity Non-Conserving Potentials

The DDH parity-nonconserving interaction can be defined as

$$V_{NN}^{PV}(ij) = i \frac{g_{\pi} f_{\pi}}{2\sqrt{2}} \frac{1}{m} \tau_i \times \tau_j \cdot \hat{z} (\sigma_i + \sigma_j) \cdot [\mathbf{p}_{ij}, Y_{\pi}(r_{ij})] - g_{\rho} \left(h_{\rho}^0 \tau_i \cdot \tau_j + \frac{1}{2} h_{\rho}^1 (\tau_i + \tau_j) \cdot \hat{z} + \frac{h_{\rho}^2}{(2\sqrt{6})} (3\tau_i \cdot \hat{z} \tau_j \cdot \hat{z} - \tau_i \cdot \tau_j) \right) \times$$

$$\left(\frac{1}{m} (\sigma_i - \sigma_j) \cdot \{\mathbf{p}_{ij}, Y_{\rho}(r_{ij})\} + i \frac{1 + \kappa_{\rho}}{m} (\sigma_i \times \sigma_j) \cdot [\mathbf{p}_{ij}, Y_{\rho}(r_{ij})] \right) - g_{\omega} [h_{\omega}^0 + \frac{1}{2} h_{\omega}^1 (\tau_i + \tau_j) \cdot \hat{z}] \times \left(\frac{1}{m} (\sigma_i - \sigma_j) \cdot \{\mathbf{p}_{ij}, Y_{\omega}(r_{ij})\} + i \frac{1 + \kappa_{\omega}}{m} (\sigma_i \times \sigma_j) \cdot [\mathbf{p}_{ij}, Y_{\omega}(r_{ij})] \right) - \frac{1}{2m} (g_{\omega} h_{\omega}^1 - g_{\rho} h_{\rho}^1) (\tau_i - \tau_j) \cdot \hat{z} \times (\sigma_i + \sigma_j) \cdot \{\mathbf{p}_{ij}, Y_{\rho}(r_{ij})\} - \frac{1}{2m} g_{\rho} h_{\rho}^1 i (\tau_i \times \tau_j) \cdot \hat{z} \times (\sigma_i + \sigma_j) \cdot [\mathbf{p}_{ij}, Y_{\rho}(r_{ij})].$$

DDH emphasizes the major difficulties in attempting to provide reliable estimates for these weak parity-violating couplings:

- the large S-P factorization term due to the dependence upon the absolute size of the current u,d quark masses;
- enhancement factors associated with the re-normalization group treatment of the effective weak Hamiltonian;
- use of a relativistic vs a non-relativistic quark model;
- the size of the sum-rule contribution to pion emission due to SU(3) breaking;
- the size of the vector meson vs pion emission amplitudes due to SU(6) breaking effects;
- etc.

The DDH weak coupling constants quoted in units of the “sum rule” value of 3.8×10^{-8} are given in Table 1.

Table 1. Original DDH weak coupling constants

Coupling	DDH Range	DDH "best" value
f_π	[0, 30]	12
h_ρ^0	[30,-81]	-30
h_ρ^1	[-1, 0]	-0.5
h_ρ^2	[-20,-29]	-25
h_ω^0	[15,-27]	-5
h_ω^1	[-2, -5]	-3

The DDH approach has been generally successful in describing a wide variety of data involving the weak interaction effects in nucleon and nuclear systems. However, the determination of weak meson-exchange couplings from measurements in nuclei [8] is difficult because the observables depend on the nuclear wave functions.

The most fundamental appearance of the weak interaction in the purely hadronic sector is parity violation in the two nucleon system, where the strong interaction is well represented. For pp scattering, the parity non-conserving weak interaction becomes

$$v^{PV} = -\frac{g_\rho h_\rho^{pp} m_\rho}{4\pi m} \left\{ (\sigma_1 - \sigma_2) \cdot \{\mathbf{p}, Y(m_\rho r)\} \right. \\ \left. + i(1 + \kappa_\rho)(\sigma_1 \times \sigma_2) \cdot [\mathbf{p}, Y(m_\rho r)] \right\} \\ - \frac{g_\omega h_\omega^{pp} m_\omega}{4\pi m} \left\{ (\sigma_1 - \sigma_2) \cdot \{\mathbf{p}, Y(m_\omega r)\} \right. \\ \left. + i(1 + \kappa_\omega)(\sigma_1 \times \sigma_2) \cdot [\mathbf{p}, Y(m_\omega r)] \right\},$$

where $\kappa_\omega \approx 0$ in the Bonn model is used to define the strong interaction parameters in the analysis. The coupling constants h_ρ^{pp} and h_ω^{pp} result from evaluating the isospin operators for the pp system in the first DDH parity non-conserving interaction above.

Since the pp longitudinal asymmetry depends only on h_ρ^{pp} and h_ω^{pp} , the energy dependence of this process alone can determine the two parameters. However, the energy dependence of the partial waves [9] enhances the sensitivity of this determination. Below 50 MeV the 1S_0 - 3P_0 partial wave dominates and the two spin terms add coherently, such that the asymmetry is approximately proportional to

$$h_\rho^{pp} g_\rho (2 + \kappa_\rho) + 2h_\omega^{pp} g_\omega.$$

At energies close to 225 MeV the 3P_2 - 1D_2 partial wave dominates (because the 1S_0 - 3P_0 partial wave is near zero) and the two spin terms subtract, such that the asymmetry is approximately proportional to

$$h_\rho^{pp} g_\rho \kappa_\rho.$$

Consequently, the data near 225 MeV are more sensitive to h_ρ^{pp} alone, and the data at 13.6 MeV and 45 MeV can determine the linear combination of h_ρ^{pp} and h_ω^{pp} .

3 Results

The energy dependence of the pp asymmetry can help dial in the relative contribution of the ρ and ω , and thus provide enough constraints to determine the h_ρ^{pp} and h_ω^{pp} weak

coupling constants. With the the recent publication [7] of the Triumf results at 221 MeV, there now exist enough data from 13.5 MeV to 221 MeV with the accuracy to make this determination.

A comparison of our theoretical calculations [10] with the available total asymmetry (A) data is shown in Figure 1. The calculated nuclear asymmetries are obtained

Fig. 1. Asymmetry in pp scattering as a function of energy

by retaining in the partial-wave expansion all channels with J up to $J_{\max} = 8$. The curves labeled AV18, BONN, and NIJ-I all use the DDH potential with the coupling constants h_ρ^{pp} and h_ω^{pp} determined by a rough fit to data (the AV18 is used in the fitting procedure). There is very little sensitivity to the input strong-interaction potential, as seen by comparing the top three curves labeled AV18, BONN, and NIJ-I. The results are generally similar to those of earlier calculations [11,12].

Also presented in Figure 1 are AV18 (DDH) and Reid-SC (DDH), where the label (DDH) corresponds to using the "best" estimates for the h_ρ^{pp} and h_ω^{pp} coupling constants [4]. Notice that these two curves differ significantly from those using the DDH-adj parameters obtained by fitting to the pp asymmetry data.

A summary of parameter values used for the ρ and ω components of the strong- and weak-interactions in this study [10] along with the weak coupling constants of the original DDH model [4] is provided in Table 2.

Table 2. Parameter values used for coupling of the ρ - and ω -meson to the nucleon in the strong- and weak-interaction models

	$g_\alpha^2/4\pi$	κ_α	$10^7 h_\alpha^{pp}$ (DDH-adj)	$10^7 h_\alpha^{pp}$ (DDH-orig)	A_α (GeV/c)
ρ	0.84	6.1	-22.3	-15.5	1.31
ω	20.	0.	+5.17	-3.04	1.50

In the fit to data h_{ρ}^{pp} is essentially determined by the 221 MeV measurement, and to reproduce the data at 13.6 MeV and 45 MeV h_{ω}^{pp} must be opposite in sign to h_{ρ}^{pp} . Nonetheless, the values extracted for h_{ρ}^{pp} and h_{ω}^{pp} are still compatible with the “reasonable” ranges quoted by DDH.

The DDH-adjusted values for coupling constants h_{ρ}^{pp} and h_{ω}^{pp} are somewhat insensitive to the ingredients of our calculational procedure as illustrated in Figure 1. Although the angular distribution of the longitudinal asymmetry does depend sensitively on the Coulomb interaction at small angles, the total asymmetry (integrated over angles) does not. In any case, our calculation [10] includes the effects of the Coulomb potential. The sensitivity to the cut-off in the weak-interaction potential [10] is more significant and could affect the determination of these coupling constants. The use of a cut-off in the weak-interaction corresponding to the one used in the strong interaction was introduced earlier [12].

Curves of constant total χ^2 obtained by analyzing the experimental pp data with the AV18 model, and ρ - and ω -meson strong-interaction couplings in the DDH potential from the Bonn-2000 model are plotted in Figure 2. The curves indicate surfaces of total $\chi^2 = 1, 2, 3, 4$, and 5 for various values of h_{ρ}^{pp} and h_{ω}^{pp} . The low-energy data put a tight constraint on linear combinations of h_{ρ}^{pp} and h_{ω}^{pp} . The Triumf data point at 221 MeV then fixes h_{ρ}^{pp} .

Fig. 2. Curves of constant total χ^2 obtained by analyzing the experimental pp data with the AV18 model, and ρ - and ω -meson strong-interaction couplings in the DDH potential from

4 Conclusions

The DDH-adjusted values for coupling constants h_{ρ}^{pp} and h_{ω}^{pp} reported here [10] are fairly reliable and somewhat insensitive to the ingredients of our calculation. For a more precise determination of these weak-coupling constants, the use of the cut-off prescription for the short-range weak interaction should be examined further. The use of a cut-off is appropriate for the strong force, but it is not so well motivated for the short-range weak interaction. A measurement of the pp longitudinal asymmetry at an energy between 50 and 221 MeV would also be useful.

Future few-nucleon weak-interaction experiments involving both neutrons and protons can help tie down the weak coupling constants, including that for the weak π -meson exchange, which introduces the parity violating isospin-changing triplet-triplet transition. The empirical determination of the meson weak-coupling parameters from the few-nucleon sector, where strong-interaction effects are reasonably under control, can lead to a better understanding of weak-interaction effects in nuclei, where strong-interaction effects are problematic.

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The most fundamental appearance of the weak interaction in the purely hadronic sector is parity violation in the two nucleon system, where the strong interaction is well represented. For pp scattering, the parity non-conserving weak interaction becomes

$$\begin{aligned}
 v^{PV} = & -\frac{g_{\rho}}{4\pi} \frac{h_{\rho}^{pp}}{m_{\rho}} \left\{ (\sigma_1 - \sigma_2) \cdot [\mathbf{p}, Y(m_{\rho}r)] \right. \\
 & \left. + i(1 + \kappa_{\rho})(\sigma_1 \times \sigma_2) \cdot [\mathbf{p}, Y(m_{\rho}r)] \right\} \\
 & -\frac{g_{\omega}}{4\pi} \frac{h_{\omega}^{pp}}{m_{\omega}} \left\{ (\sigma_1 - \sigma_2) \cdot [\mathbf{p}, Y(m_{\omega}r)] \right. \\
 & \left. + i(1 + \kappa_{\omega})(\sigma_1 \times \sigma_2) \cdot [\mathbf{p}, Y(m_{\omega}r)] \right\},
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