Final Report for: Fundamental study of long-short interfacial wave interactions with application to flow regime development

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The long waves that cause slugs almost always form more slowly than short waves and linear stability always predicts the growth rate for long waves is much less than short waves. However, at many conditions above neutral stability, long waves dominate the wave field. The question then is why? The work described in the 1995 proposal was intended to answer this question. The major goals of the previous work were to understand how long (compared to the channel depth) waves form in stratified systems (and are not usually predicted correctly by linear theories) in consideration with the short waves that were usually present in these systems that were correctly predicted by linear theories. The major motivation was the formation of slugs and roll waves in pipeline flows.

As part of this work, 3 different studies were undertaken.

1. Linear interaction for unsteady flows.

Data shown in figure 1 illustrate the phenomena mentioned above about how long waves dominate the wave field even though short waves have a higher growth rate. While many nonlinear theories have been proposed, it was first deemed necessary to examine if there was a linear response in the system that could cause the long wave growth. The hypothesis was that once short waves grow to a finite amplitude, the gas flow over them causes pressure fluctuations that could affect the linear growth rate of long waves. The physical situation would be a traveling oscillatory pressure field added to the steady base state flow. Because the gas flows are difficult to keep precisely constant when disturbances are present, these wave induced pressure variations of specific wavelength could lead to some degree of pressure fluctuations within the entire.

\[1\] Of course short waves also affect the average pressure drop and thus the depths of the liquid and gas. This alteration of effective base state for the growth of long waves is noted. However, this change of base state does not affect the qualitative shape of the linear growth curve.
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gas flow. Pressure fluctuations are often present to some degree in our experiments so we chose to study the problem of linear stability of gas-liquid laminar flow with an imposed pressure fluctuation as the first model of this phenomena. Further, if the blower, pump or compressor has some sort of flow oscillation, this is the type of disturbance that would be present. A paper has been written and submitted to Physics of Fluids on this study. (Effect of mode-linking on interfacial stability in stratified flow, http://www.nd.edu/~mjm/modelink.pdf) The primary conclusion is that there can be a strong stabilization or destabilization of the interfacial waves caused by interaction of different modes (e.g., the upstream interfacial mode if it is excited or gas and liquid internal modes) if the pressure oscillation frequency is the appropriate natural frequency defined as the difference between the interfacial wave speed and the speed of the second most unstable mode divided by the wavelength of the interfacial mode. Figure 2 shows a set of results for gas-liquid flow where the magnitude of the oscillation is 10% of the mean. What happens physically is that for the correct oscillation frequency, $\Omega_{nat}$, two different modes which usually travel at different speeds (or even directions) travel at the same speed for a significant portion of a wave period. This allows them to interact. The degree of interaction between the downstream interfacial waves can be significant interaction with the upstream mode and figure 2 shows that short waves can be significantly dampened. A slight mismatch at 0.9 $\Omega_{nat}$ or 1.1 $\Omega_{nat}$ reduces the effect to some extent. The largest effect predicted by the theory is that if a shear mode in the gas
phase is unstable and the oscillation is chosen correctly, there could be a very large enhancement of wave growth. Of course in this case the gas shear mode is not likely to be present because the gas phase should be fully turbulent, with an entire spectrum of modes present. To return to the specific issue with oscillations in the flow, while the effect is potentially significant for short waves, it does not explain any effect on long waves.

Figure 2. Plot of linear growth rate showing the effect of oscillation on the linear growth of the interfacial mode and the gas shear mode.

2. Wave evolution in oil-water channel flows

The gas-liquid system is the most important industrial class of fluid-fluid flows but has the disadvantage of the gas phase usually being turbulent. This makes exact comparisons between theory and experiment impossible. Consequently, we have done experiments using water and a light hydrocarbon oil. This enables a wide range of phenomena similar to gas-liquid flows without the complication of turbulence.

Figure 3 shows a wave regime map, for the oil water system. The thick lines denote the different regions based on visual observations. The dashed line is long wave linear stability theory. The thin solid line, which is very close to the stable-2D boundary, gives the onset of short wavelength waves. The other branch of the short-wave line, which cuts across the map almost horizontally, gives the conditions where all wavenumbers down to 0 wavenumber are unstable. It is seen that linear stability theory does an excellent job of predicting the onset of 2-D waves using the "short-wave" theory.
There is also relatively good agreement between the long wave theory and the observed onset of "slow waves" (longer waves that appear to be centered within the oil phase). These waves do disturb the interface and show up in the low frequency range of wave spectra. It is this transition to long waves, when short waves are already unstable, that is the focus of this part of the study. Based on the gas-liquid results shown in figure 1, it is desired to examine this transition in detail and try to determine the mechanism for the formation of the longer waves and why they dominate the wave field.

Figure 4 shows measurements of the interfacial wave spectrum at conditions where long waves are stable and only a range of short waves are unstable – and observed. Note that the experimental wave spectrum is peaked at 2 Hz, essentially the same as the linear stability prediction and there is no energy at low frequencies. There are two wave modes that are predicted by linear theory to be important in this system. The solid line linear theory prediction represents an internal mode within the oil phase. For these conditions it is close to being unstable, but it is not unstable.

Figure 5 shows how the increase of the water flow rate causes the wave spectrum to change character completely so that long waves are dominating. Note that this occurs even though the long waves have only very low growth rates. This dramatic shift of the
dominant waves from short waves to longer waves is observed even though the change in the linear theory predictions is very minor. The only change is that long waves are now unstable. Because the data appeared to be at odds with the linear theory, and to

![Figure 4](image-url)

**Figure 4.** Growth rate of two modes and measured spectrum at Re₀=3, Reₘ = 700

![Figure 5](image-url)

**Figure 5.** Growth rate of two modes and measured spectrum at Re₀=3, Reₘ = 1200

investigate the nature of possible nonlinear interactions that could lead to such a large increase in the energy at small frequencies, we have done numerical integration of the complete system of Navier-Stokes equations and boundary conditions for this system. To allow for a solution within available computer time, we have used a Chebyshev–Spectral expansion of the equations and boundary conditions and used the linear eigenfunctions to represent the nonlinear terms. The entire system included up to cubic order. A total of 40 wave modes were used. This formulation is equivalent to the one used by Sangalli et al. (1997) except that a center manifold projection was not done in the
present case. The system of 40 waves with all possible cubic and quadratic interactions was integrated in space until a steady state was obtained – unless one was not found. For figure 6, a steady wave spectra is obtained for $Re_w = 650$ (i.e., close to figure 4) and 850, and the dominant wave corresponds well to the peak in the linear growth. However, for $Re_w = 1200$ (close to figure 5), increasing amounts of energy are accumulating in the low frequency region. Both linear and nonlinear reasons for this major qualitative difference in behavior exist. First, the long wave region has become linearly unstable at $Re_w = 750$. Second there is an increase in the strength of cubic interactions between low frequency modes and modes near the fastest growing peak. Thus, both linear and nonlinear effects work together to cause this dramatic change in the wave behavior.

![Figure 6](image6.png)

![Figure 7](image7.png)

Figure 6. Numerical simulation of interfacial wave spectrum for conditions of figures 4 and 5.

Figure 7. Numerical simulation for conditions of figure 1 (as close as we could do using laminar flow for the simulations)

We have also examined gas-liquid systems such as figure 1. As we mention above, comparison with gas-liquid flows is less exact because of the turbulent gas flow although the interaction coefficients arise because of liquid effects. Figure 7 shows the results from our simulations of gas-liquid channel flow as the liquid Reynolds number is increased. It is seen that an increase in $Re_w$, causes a similar large change in the low frequency wave energy. The mechanism for this system is again the linear instability of long waves and substantial increase in quadratic nonlinear interactions between all of the wave modes and the mean flow (which represents a nonlinear correction to the base flow state).
Another important result from the weakly-nonlinear simulations is an explanation of the lack of visible waves for cases where sufficiently long waves are unstable at any rotation rate for our matched-density, two liquid rotating Couette experiment (Gallagher et al., 1996; http://www.nd.edu/~mjm/vpr_talk.pdf). Gallagher et al. (1996) shows large regions of parameter space where no waves appear even though they are predicted by linear theory. Figure 8 shows snap shots of the spectral evolution at long times where the energy cascade has been set up. Different modes oscillate but there is no preferred wavenumber. The simulations are best viewed as the movie, http://www.nd.edu/~mjm/specsim.mov.

A much more complete description of the results of this work is available in the paper Weakly-nonlinear simulation of planar stratified flows (King & McCready, 2000). (http://www.nd.edu/~mjm/nonlin.pdf). One general conclusion of this study, is that both linear and nonlinear processes act together to cause the very significant changes in the wave spectrum for the long wave region. Thus the idea that flow regime transitions can be predicted with only linear theories is probably not correct.
3. Retrograde stability and subcritical bifurcations

Figure 9 shows linear stability predictions for oil-water channel flow at an angle, \(\pi/37\) (downflow) where some interesting behavior should occur. At this angle, for fixed oil flow rate, the growth rate of the waves at 30-40/m first increases with water flowrate, until \(R_w=180\) and then begins to decrease. The decrease in wave growth with increasing flowrate continues until about \(R_w=300\) when a short wave mode begins to appear. This type of retrograde behavior, where an increase in flow rate causes a reduction in waves, is rather rare in parameter space. In this case it is likely that the increase in water flow rate reduces the thickness of the oil phase making wave formation more difficult.

This retrograde behavior is more interesting because it corresponds to a region of subcritical nonlinear behavior. In terms of our previous work (Sangalli, 1997; Sangalli et al. 1995; Sangalli et al., 1997), we reduce the two-layer governing equations and boundary conditions using an eigenfunction expansion and a center manifold reduction. The resulting Stewart – Landau equation for the amplitude of the dominant mode can be written as

\[ \dot{A} = \lambda A + \beta A \ddot{A}. \]
The Landau coefficient, $\beta$, is stabilizing, hence a supercritical bifurcation if it is $<0$ and destabilizing, hence a subcritical bifurcation. The Landau coefficient is negative at $R_w=180$, becomes positive at $R_w=190$ and then switches back to negative at $R_w=260$. Figure 10 shows this behavior. While regions of subcritical behavior are common, the link with retrograde stability behavior has not been observed before.

These results were presented in the IUTAM Symposium on nonlinear waves and have been published in the conference proceedings.

Figure 10. Landau coefficient components for the conditions of figure 9. The subcritical region occurs between a water Reynolds number of $\sim 190$ - 260 and is caused by the overtone mode becoming destabilizing.
Conclusions

The overall conclusions about flow regime development based on the present study are as follows.

1. Oscillations in pressure and flow rate, due to interfacial waves or a malfunctioning pump can cause significant growth rate changes in short waves within in narrow frequency ranges, but probably do not have a large effect on long waves and thus regime transition.

2. Linear and nonlinear processes act together to cause regime transitions. However, it is almost certain that long waves cannot form solely by nonlinear energy transfer from shorter waves. Thus long wave linear stability analysis gives a reasonable bound on regime transitions caused by growth of a long wave because only linearly unstable modes can grow to significant amplitude.

3. The nonlinear coefficient spectrum contains the information necessary to tell if certain wave interactions will occur to a significant extent (e.g., formation of a low frequency mode, or efficient transfer to shortwaves where dissipation dominates.) Thus further studies of this should provide new insight into flow regime transitions.

References


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D. D. Uphold and M. J. McCready, Formation of large disturbances in separated fluid-fluid flows, ASME, Fall meeting, Dallas, TX, November, 1997.

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Donald D. Uphold, "Linear stability of multifluid flows" -- Ph. D. 1997
Kimberly A. Gifford, "Long-short wave interactions in two-liquid channel flow" -- MS - 1998
Xiaohong Wang, "Numerical simulation of stratified flow" -- MS-2000

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