PROBABILISTIC ASSESSMENT OF
DYNAMIC SYSTEM PERFORMANCE

By

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Accurate prediction of dynamic system failure behavior can be important for the reliability and risk analyses of nuclear power plants, as well as for their backfitting to satisfy given constraints on overall system reliability, or optimization of system performance. Global analysis of dynamic systems through investigating the variations in the structure of the attractors of the system and the domains of attraction of these attractors as a function of the system parameters is also important for nuclear technology in order to understand the fault-tolerance as well as the safety margins of the system under consideration and to insure a safe operation of nuclear reactors. Such a global analysis would be particularly relevant to future reactors with inherent or passive safety features that are expected to rely on natural phenomena rather than active components to achieve and maintain safe shutdown. Conventionally, failure and global analysis of dynamic systems necessitate the utilization of different methodologies which have computational limitations on the system size that can be handled. Using a Chapman-Kolmogorov interpretation of system dynamics, a theoretical basis is developed that unifies these methodologies as special cases and which can be used for a

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comprehensive safety and reliability analysis of dynamic systems. It is shown that the theoretical basis leads to generic (i.e. system independent) algorithms which reduce the computational limitations in applications. The practicality of these algorithms in nuclear engineering is demonstrated with example nonlinear systems from reactor dynamics. The results of these applications show that, for dynamic systems, the new algorithms can speed up: a) interval reliability analysis by a factor of 6 and reduce the storage requirements by a factor of 20, b) global analysis by a factor 20 and reduce the storage requirement by a factor of 2.5, and, c) sensitivity analysis by a factor proportional to the number of different probability density functions (pdf) that may need to be considered if a unique pdf cannot be identified to represent the stochasticity in the system parameters. The computational advantages of these algorithms increase their range of applicability to include larger systems.

**PROBABILISTIC ASSESSMENT OF
DYNAMIC SYSTEM PERFORMANCE**

DISSERTATION

**Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University**

by

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*** * * * ***

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A handwritten signature in black ink, appearing to be 'E. J. Miller', is written over a horizontal line.

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To My Parents

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NOMENCLATURE

Abbreviations

ADMIRA	:	Advanced Dynamic Methodology In Risk Analysis
BOA	:	Basin Of Attraction
BWR	:	Boiling Water Reactor
CCMT	:	Cell-to-Cell Mapping Technique
CDF	:	Cumulative Distribution Function
CPU	:	Computational Processor Unit
DETAM	:	Dynamic Event Tree Analytical Methodology
DOA	:	Domain Of Attraction
DPVSM	:	Discrete Process Variable State Markov
DYLAM	:	DYnamic Logical Analytical Methodology
ES	:	Example System
FDP	:	Failure Per Demand
GCM	:	Generalized Cell Mapping
HL	:	High Level
HP	:	High Pressure
HPCS	:	High Pressure Core Spray
L	:	Level
LL	:	Low Level
LOCA	:	Loss Of Coolant Accident

LP	:	Low Pressure
LPCI	:	Low Pressure Core Injection
MDTC	:	Multiple Domicile Transient Cell
MTTF	:	Mean Time To Failure
NRC	:	Nuclear Regulatory Commission
OSU-DM	:	Ohio State University-Dynamic Methodology
P	:	Pressure
PCS	:	Process Control System
PG	:	Persistent Group
PVSS	:	Process Variable State-Space
RCIC	:	Reactor Core Isolation Cooling
RV	:	Reactor Vessel
SBLOCA	:	Small Break Loss Of Coolant Accident
SCM	:	Simple Cell Mapping
SRV	:	Safety Relief Valve
SU	:	Structural Unit
pdf	:	probability density function

Roman Symbols

A	:	absorption probability matrix
B	:	forcing coefficient
I	:	identity matrix
IM	:	image matrix

M	:	transition probability matrix
M'	:	transition from transient to transient cells matrix
Q	:	transition from transient to persistent cells matrix
T	:	transition between persistent cells matrix
F	:	CDF
I	:	total number of subcells in a given cell
Im	:	statistical importance
J	:	total number of cells in the nominal control region
K	:	period number
N	:	total number of component state combinations
P, Pr	:	probability function
PK	:	probability at time t
PL	:	probability at time $t+\tau$
S	:	a set of cells or a set of component state combinations
R	:	nominal control region
$\bar{R}$	:	complement of R in E
V	:	computational cell
c, g, h, q	:	conditional probabilities
d	:	failure-per-demand probability
dv, dv'	:	infinitesimal volume element in E
e	:	step function in E
f	:	time derivative function
i, i', j, j'	:	process variable states
n, n'	:	component state combinations
p	:	probability distribution

r	:	return time
t	:	time variable
v	:	volume of a cell
w	:	pdf
x, x'	:	process variable vectors
$\bar{x}$	:	mean value of x
$\tilde{x}$	:	arrival process variable vector

Greek/Mathematical Symbols

Δt	:	time interval
Γ	:	total number of cells including Top Events
Φ	:	neutron flux
Θ	:	null matrix
α, α'	:	system parameter vectors
δ	:	Kronecker delta function
ε	:	a small number
η	:	system condition vector
λ	:	failure rate
μ	:	repair rate
ρ	:	transition probability
σ^2	:	variance
τ	:	modeling time step or mapping step
θ	:	overall flux reactivity coefficient

$\in$: member of
 $\subset$: subset

Superscripts

∞ : at time infinity

Subscripts

ff : failed-off or failed-closed
 fo : failed-on or failed-open
 i, i' : specified subcell numbers
 j, j' : specified cell numbers
 l : l^{th} component of a vector
 m : a specified system parameter vector or failure mode
 $\max$: maximum value
 γ : a specified Top Event

CHAPTER I

INTRODUCTION

1.1 Overview

Dynamic systems, such as process control systems (PCS), are often used in nuclear power plants to maintain the magnitudes of continuous state process variables (e.g. reactor vessel pressure, temperature and water level) within allowed ranges. In that respect, accurate prediction of dynamic failure behavior can be important for the reliability and risk analyses of nuclear power plants, as well as for their backfitting to satisfy given constraints on overall system reliability, or optimization of system performance. In the failure modeling of dynamic systems, not only the operating and failed component states have to be identified and their effect on the process variables assessed, but also possible feedback effects between the components and the process variables have to be accounted for.

The most widespread modeling approach to the failure modeling of dynamic systems is systematic fault-tree construction for specified undesired events (Top Events) using: a) a static (i.e. time-independent) graphical [1-9] or graphical/tabular [6] representation of system operation, and, b) an arbitrary discretization of the continuous process variable state space (PVSS) through qualitative classification (e.g. small, moderate or large) of changes in the magnitude of the system disturbances [1-4, 7, 8]. The resulting fault-tree can be analyzed by a variety of techniques [8] to quantify the Top Event properties (e.g. cumulative distribution function, probability density function, s-importance of component failure with respect to a

Top Event). Although this approach allows convenience in the reliability modeling and analysis of PCS with a large number of components and process variables, some concerns are:

1. Static representation of system operation can cause difficulties in the correct description of the propagation of system disturbances through the control loops [5, 10-12].
2. Arbitrary discretization of PVSS may lead to the omission of some failure mechanisms in model construction due to inconsistencies in the definition of the allowed range for the controlled variables [13].
3. System model verification in a self-consistent manner is usually impossible and may require using other dynamic methodologies (e.g. Monte Carlo approach based on direct system simulation [14]).

Another concern with the failure modeling or probabilistic assessment of dynamic system performance in general is that nonlinear systems appear in many engineering disciplines, including nuclear power plants, and there has been ample experimental [15-30] and theoretical [17, 19, 21, 22, 28, 31-33] evidence in the recent years that the qualitative and quantitative features of the time evolution of nonlinear dynamic systems can change drastically with a small change in system parameters and/or initial conditions. For example, it has been shown that space-independent Xenon oscillations are linearly unstable at high neutron flux, even with a negative power coefficient of reactivity (θ), if the magnitude of θ is too small [34]. These linearly unstable oscillations can be triggered by a very small change in flux reactivity coefficient θ [35, 36] and typically develop into stable limit cycle when nonlinear neutronics and Xenon burnout are accounted for in the model [34]. In a boiling water reactor (BWR), the change in θ may arise from random fluctuations in recirculation flow rate or malfunction

of some of the recirculation pumps during operation. Subsequently, global analysis of non-linear dynamic system behavior through investigating the variations in the structure of the attractors of the system, the domains of attraction (DOAs) of these attractors as a function of the system parameters become very important for nuclear technology in order to understand the fault-tolerance as well as the safety margins of the system under consideration and to insure a safe operation of nuclear reactors. Such a global analysis would be particularly relevant to future reactors with inherent or passive safety features that are expected to rely on natural phenomena rather than active components to achieve and maintain safe shutdown. An attractor is a compact region in the system state-space that is asymptotically reached in time by the dynamical system [22]. An attractor that is not a finite set of points, a limit cycle, a piecewise smooth surface or bounded by a piecewise smooth closed surface volume is called a strange attractor [22]. Strange attractors are important for systems with chaotic behavior. The DOA of an attractor is a range of values of initial conditions (or a set of points in the system state-space) that eventually lead to the attractor [22].

Global analysis by the direct numerical integration of the system equations can be a very time consuming, if not an infeasible task, particularly if the asymptotic system behavior is chaotic [26, 32, 37]. Also, uncertainties on the system parameters (e.g. due to possible experimental error or random fluctuations) may affect the attractor boundaries, "blurring" the DOAs [29]. While numerous fast techniques have been proposed to determine the stability of the equilibrium states and periodic solutions of nonlinear systems as a function of the system parameters [22, 23, 24, 28, 31, 32, 38, 39], only two methods are found in the literature that are suitable for global analysis in the sense described above when the system has higher than two degrees of freedom. These methods are: a) global perturbation of invariant

manifolds [33], which is mainly analytical and will be referred to as Wiggins method, and, b) the cell-to-cell mapping technique (CCMT) [40, 41], which is a heuristic numerical technique.

Sections 1.2 and 1.3 respectively describe the currently available methodologies for dynamic failure modeling and global analysis of dynamic systems in more detail, including their advantages and disadvantages. Section 1.4 provides motivation for the dissertation research in view of the limitations of these methodologies and lists the objectives of the dissertation research. Finally, Section 1.5 describes the dissertation organization.

1.2 Literature Review on the Failure Modeling of Dynamic Systems

Historically, the first attempt to eliminate Concern 1 in Section 1.1 was made by Kumamoto and Henley [10] who proposed the use of dynamic tabular data to represent system operation in discrete time. The data are based on the predetermined effect of component action on the process variables for all possible changes in the component inputs and component states. The tabular data are then manipulated (in a similar manner to decision tables [42]) to identify the combination of component states that lead to system failure. However, the description of component response to system disturbances still involves an arbitrary discretization of PVSS (i.e. "high", "low", "don't care") with this approach and Concerns 2 and 3 in Section 1.1 are still valid. In a variation of this approach, Kumamoto, Henley and Inoue [5] assume a linear relationship between component input and output variables to remove the arbitrariness in PVSS discretization. Although this assumption has justification for proportional controllers, its validity is not clear for on/off controllers or proportional-integral controllers. Again, the model cannot be verified in a self-consistent manner (Concern 3 in Section 1.1).

Following these attempts, several other methodologies were proposed which explicitly account for the time element in system operation in the failure modeling of dynamic systems. These methodologies are called dynamic methodologies and can be grouped under three categories:

- State transition (Markov) models
- Dynamic event tree approach
- Direct system simulation (analog Monte Carlo)

The following sub-sections present the dynamic methodologies developed to date and discuss their advantages as well as their limitations.

1.2.1 State Transition Models

State transition (or Markov) models represent the time evolution of system dynamics as possible transitions between system states. The system states can be continuous or discrete, however, transition between states are assumed to occur only at the end of a given time interval. Also, the transition probabilities should not depend on the history of system operation (i.e. the system is memoryless). The model yields the probability of finding the system at a given state and at a given time. Markov models are most commonly used in reliability and safety analyses when the system states are discrete and the transitions between the system states are statistically dependent. Two recent examples of such applications are given by Walker [43] and Dhillon [44] where the system states can be described purely in terms of the hardware and operator states.

Using a Chapman-Kolmogorov interpretation of system dynamics, Devooght and Smidts [45] show that Markov modeling techniques can be extended to systems whose state-space

includes both process variables with continuous states and components with discrete states. In this case, the Markov model is a set of first order partial differential equations having time dependence, the dynamical system variables are the independent variables, and the probability of finding the system at a given state and a given time is the dependent variable. The differential equations are coupled through the component failure rates and are solved using Monte Carlo techniques. This approach avoids computer storage limitations that usually plague Markov models but it is not clear how the method can be applied if the component failure data are given as failure-per-demand. Also, since the equations are coupled through the component failure rates, every change in the component failure data (for example, for sensitivity analysis purposes) requires a new solution which can be quite time consuming.

A variation of this technique that uses discrete process variable states (i.e. magnitude intervals) had been independently developed earlier by Aldemir [46] and illustrated on a simple tank system. A more detailed description of this discrete process variable state Markov (DPVSM) methodology will be given in Chapter 2. The main difference between DPVSM and the methodology proposed by Devooght and Smidts is that DPVSM uses a recurrence relation in discrete time to calculate probability of finding system at a given state [46] instead of partial differential equations [45]. The coefficients in the recurrence relation are products of two conditional probabilities: a) probability that the process variables move from state j' to j within $[t, t+\Delta t]$ for given component state at t , and, b) probability that given the component state changes from n' to n within $[t, t+\Delta t]$ for given transition between process variable states. The first type of conditional probabilities are determined from the system equations and constitute the process model. The second type of conditional probabilities are obtained from the given component failure data and control laws. This approach allows decoupling the description of system evolution (scenario generation) and probabilistic

evaluation of scenario contribution to Top Events. Subsequently, once the conditional probability of transitions between process variable states are determined, the same process model can be used with different component failure data to investigate the Top Event sensitivity to component failure data. A derivative of DPVSM has been also developed by Hassan and Aldemir (OSU-DM [47]) for use with simulators which do not allow fault injection at arbitrary time points. The OSU-DM has been applied to determine the reliability of the feed-bleed cooling of a BWR core during 60 minutes following a small-break which incapacitates the reactor core isolation cooling system [47]. In its existing form, OSU-DM is system specific, the controlled variables are limited to 2 and so is the number of components that can be modeled.

Markov models eliminate all Concerns 1, 2 and 3 mentioned in Section 1.1 and present some advantages in handling:

- time-dependent system parameters,
- operator behavior dependent on the current hardware and process variable states or changes in the process variable states, and,
- statistical dependencies arising from history independent interactions of hardware-hardware, hardware-operator, hardware-process variable, control law-process variable changes.

Modeling the statistical dependencies between the control laws and process variable changes may require the discrete process variable representation scheme (i.e. DPVSM or OSU-DM). This representation scheme is also useful for utilization of failure-per-demand data as reported, however, suffers worse than some other approaches regarding limitations on system

size due to computational limitations. The other difficulty with Markov models is that operator and/or hardware behavior dependent on system history cannot be represented.

1.2.2 Dynamic Event Tree Approach

The dynamic event tree approach tracks possible branchings in system evolution at specified time intervals following an initiating event through simulations. The process physics model for the dynamic event tree approach consists of a set of parametric equations which describe the system operation under normal and abnormal conditions. Then, these equations are coupled to a "driver" to simulate system evolution under given initial conditions and as a function of changing components states in time. The simulations stop when a specified number of time intervals or a Top Event is reached. The Top Events (i.e. undesirable events) are defined in terms of magnitude intervals of the process variables. The sum of scenario (event sequence) probabilities leading to a Top Event gives the probability that this Top Event will occur during the specified time interval.

The DYLAM (Dynamic Logical Analytical Methodology) is the first computational tool that has been developed using this approach [48]. Izquierdo et al. [49] show how DYLAM can be coupled to a generic simulation tool for nuclear power plant analysis. Two variations of DYLAM are DETAM [50] and ADMIRA [51]. DETAM allows a more general treatment of stochastic variability in operator behavior and ADMIRA allows on-line modification of plant status. In situations where operator actions and state of knowledge are not considered and on-line modification of plant status is not allowed, then DYLAM and its derivatives are conceptually identical. An application of DETAM to a pressurized water reactor steam generator tube rupture is given in [50].

The techniques using dynamic event trees have almost all the capabilities of the techniques using Markov models. DYLAM and its variations can be viewed as an alternate representation of an extended Markov model. Also, due to the dynamic nature of the system simulation under all possible components states and non-arbitrary definition of Top Events in PVSS, Concerns 1, 2 and 3 in Section 1.1 are again eliminated. One computational advantage of these techniques is that, if the future system behavior does not depend on the history of system operation, they require very little computer storage compared to Markov models. However, such applications require the development of very fast simulators and analyses have to be repeated every time component failure data are changed. Also, for the representation of history dependent operator or component behavior, all generated event sequences need to be saved. In these situations, the dynamic event tree approach also becomes very storage-limited.

1.2.3 Direct System Simulation

In contrast to the dynamic event tree approach, direct system simulation techniques do not track all possible pre-specified scenarios but rather build a pattern of system responses to an initiating event and determine the probability of specified outcomes by repeated sampling from the infinite number of possible branchings. Using this technique on the tank system of [46], Marseguerra [52] has shown that when the reliability characteristics of the plant strongly depend on the process, the results obtained from direct system simulation and the classical event-tree/fault-tree approach are quite different. Another example of direct system simulation is the failure analysis of the decay heat removal system of a liquid metal fast breeder reactor by Nakada and his co-workers [53]. For this system, the failure rates are

time and event sequence dependent. Direct system simulation has been also applied to obtain estimates of the probability of human error [54].

Almost all types of hardware-operator-process variable interactions of interest and time/process variable/event sequence dependent failure rates can be modeled using direct simulation techniques. They have much lower computer storage requirements compared to Markov models if the definition of system states is based on component states and discrete process variable states. Also, they may require less computer time compared to the dynamic event tree approaches with proper variance reduction techniques. However, variance reduction techniques are usually problem specific and complicate the model. Also, Monte Carlo techniques may not sample from rare event sequences and subsequently miss their consequences. Another problem is that the results give the integrated effect of the event sequences simulated so it is usually very difficult to obtain information about the individual event sequences leading to a given Top Event. Finally, the analysis has to be repeated if the failure data are changed so they may not be suitable for sensitivity analyses.

1.3 Methodologies for Global Analysis of Dynamic Systems

As indicated in Section 1.1, an additional concern in the failure modeling or probabilistic assessment of dynamic system performance in general is system non-linearities which may lead to drastic changes in system behavior with a very small change in system parameters or initial conditions. The objectives of global analysis are to determine: a) how the asymptotic system behavior changes with changes in system parameters, and, b) which initial conditions lead to what types of asymptotic system behavior. The following two sub-sections describe the existing most general procedures which can be used for the global analysis of non-linear dynamical systems.

1.3.1 Wiggins Method [33]

Wiggins method uses system equations linearized about the invariant manifold to investigate the stability of the manifold. Within the context of global analysis, an invariant manifold is a region in the state-space of the dynamical system with the property that a trajectory starting from this region remains within the region throughout the course of its evolution. The invariant manifold which consists of fixed points in the state-space can be obtained by solving for the zeroes of the coupled nonlinear relations (equations) that govern the dynamics of the system. When the invariant manifold has a more general structure (e.g. consists of orbits or surfaces in the state-space), then a prior knowledge of state-space topology and/or a detailed knowledge of the orbit structure of the nonlinear ordinary differential equations of the system are required in order to locate the invariant manifold.

The linearization of system equations about invariant manifolds consisting of fixed points is straightforward. Also, when the invariant manifold consists of a periodic orbit or quasi-periodic orbit, the invariant manifold can be represented by a graph of a function and the linearization of the system equations can be obtained through coordinate transformation. On the other hand, if the invariant manifold is more general, its representation by a graph of a function and linearization of system equations may be tedious. Once the linearized system equations are obtained, investigating the stability of the manifold involves the computation of Lyapunov type numbers or exponents through perturbations of the linearized system equations. A stable manifold represents an attractor for the system.

The Wiggins method is very powerful and straightforward if all the invariant manifolds consist of fixed points in the state-space. However, the method requires the differentiability of the governing equations with respect to the system parameters (at least twice and six-fold in some cases) and also some prior knowledge of state-space topology. When the invariant

manifolds do not only consist of fixed points but also contain periodic and chaotic trajectories, global analysis becomes a tedious job if not impossible. For example, Wiggins method can prove that a system has a chaotic attractor by using an analytical technique developed by Melnikov [55] but cannot determine the envelope of the chaotic attractor or its statistical properties which can be important in the reliability and safety assessment of system operation.

1.3.2 CCMT Methodology

The CCMT is a heuristic procedure for the global analysis of non-linear dynamic systems using a Markov chain in discrete time to represent the system dynamics [40]. One of the most attractive features of CCMT is computational speed coupled with relatively wide range of applicability. For example, it has been shown that CCMT can determine the attractors in the 4-dimensional state-space of a coupled two degree-of-freedom van der Pol system 200 times faster than direct numerical integration [56]. It has been also shown that the statistical properties of the strange attractor of a "stretch-contraction-reposition" map [57] can be determined by the CCMT 8 times faster than the direct integration method for the same accuracy [58]. The CCMT is particularly useful if the attractor is a strange (i.e. chaotic) attractor. Recently, it has been shown that nuclear reactors can exhibit such chaotic behavior [56, 57].

In a manner similar to DPVSM (see Section 1.2.1), the CCMT models dynamic system evolution as a mapping or probability of trajectory transitions between specified computational cells (dynamic variable magnitude intervals) that partition the system state-space. This probabilistic aspect of CCMT makes it naturally suited for the global analysis of systems with stochastic parameters [40, 59, 60], provided that the probability density functions (pdfs) for these system parameters are known [40, 58]. Such stochasticity may arise from the

uncertainty on the observed monitor data during plant operation or possible error in the experimental determination of system parameters, or due to component failure.

In spite of the attractive features of CCMT, some practical difficulties associated with its implementation are the following:

- The procedures proposed to determine the mapping rules or the cell-to-cell trajectory transition probabilities are heuristic schemes and it is not always clear what assumptions are implicit in these schemes.
- The identification of a unique joint pdf for the system parameters may be very difficult. In that respect, several analyses may need to be carried out with different pdfs to study the impacts of the choice of the pdf on the system behavior. The CCMT methodology as originally proposed requires the Markov model transition probabilities to be recalculated every time the pdf is changed.
- The computational requirements (i.e. computational speed and storage) can be still prohibitive for practical systems, in general.

1.4 Motivation

In order to insure safe and reliable operation of any system, it is important to identify both:

1. the possible accident scenarios, their consequences and their likelihood and the importance of their contribution to risk, and,
2. the envelope of system parameters and initial conditions that lead to acceptable system performance or those that lead to unacceptable system performance.

Conventionally, the objective of failure analysis is to obtain only the type of information in Item 1 above. The need for the type of information in Item 2 arises from uncertainties on the parameters and/or initial conditions, including possible error on the experimentally determined quantities used in the design model and other stochastic parameter changes during operation (e.g. noise), as well as uncertainty on the monitored data. Obtaining this type of information is usually regarded as a design problem (i.e. fault-tolerance or robustness of design), but it is not clear that either type of information can be obtained without the other. In the case of BWR response to temperature perturbations, for example, Ref.[59] shows that, under certain set of initial conditions, there exists a critical value for the clad-to-coolant heat transfer coefficient beyond which the system response is chaotic. In that respect, a small change in the heat transfer coefficient due to stochastic changes in the local coolant properties can have just as undesirable consequences as the consequences of a large change due to recirculation pump trip (such as in the Lasalle incident), depending on how close the system is to these critical conditions at the time of the initiating event. System closeness to the critical conditions, in turn, depends on the magnitude of the uncertainties on other system parameters and initial conditions. The need for the type of information in Item 2 would again be particularly relevant for reactors with passive or inherent safety features.

The dynamic methodologies described in Section 1.2 (i.e. state transition models, dynamic event tree approach and direct system simulation) eliminate the concerns associated with the applicability of the conventional event-tree/fault-tree approach for the failure analysis of dynamic systems in view of their capability of: a) explicitly accounting for the time element in system evolution, b) allowing quantitative definition of Top Events in terms of process variable magnitudes instead of qualitative definitions (such as "high", "low"), and, c) self-consistent verification (e.g. by using smaller process variable magnitude intervals with

DPVSM). On the other hand, only the integrated effect of the event sequences are obtained using direct system simulation and the integrated effects reveal no information on the contribution of different scenarios to the undesirable consequences (Top Events) for risk management or design improvement. Furthermore, rare event sequences may not be sampled and subsequently their consequences may be ignored in direct simulation. The continuous state transition model developed by Devooght and Smidt [45] is effective in determining mean system failure times and failure modes, but it is again difficult to determine what scenarios or event sequences contribute to the failure mode using this approach and the approach cannot be used with failure-per-demand data. For history independent failure data, the dynamic event tree approach [60] and the DPVSM [46] have similar capabilities and yield the information in Item 1 above. Among the two methodologies described in Section 1.3, only CCMT provides all the information indicated in Item 2 and is algorithmically similar to DPVSM approach.

In view of the above arguments and the limitations and difficulties associated with DPVSM (Section 1.2.1) and CCMT (Section 1.3.2) the objectives of this study are the following:

1. Develop a theoretical basis that unifies the DPVSM, OSU-DM and CCMT as special cases and which can be used for a comprehensive safety and reliability analysis of dynamic systems.
2. Develop generic (i.e. system independent) algorithms which also reduce the computational limitations in applications.
3. Demonstrate the practicality of these algorithms in nuclear engineering with example nonlinear systems from reactor dynamics.

1.5 Dissertation Organization

The dissertation is organized in the following manner: Chapter II provides an overview of the DPVSM and OSU-DM and outlines the difficulties associated with their implementation. An overview of CCMT and a discussion of the sources of ambiguities associated with the implementation procedure of CCMT to systems with stochastic parameters are also provided in Chapter II. The theoretical basis towards Objective 1 of this study is developed in Section 3.1. Sections 3.2 and 3.3 present the algorithms that arise from this theoretical basis and reduce the computational limitations in applications. The generalization of OSU-DM and procedures of implementing the new algorithms are presented in Chapter IV. Chapter V validates and benchmarks the new algorithms and demonstrates the flexibility and ease of their implementation. Chapter V also discusses some computational aspects of the algorithms and provides comparisons to the originally proposed algorithms of the CCMT and OSU-DM. Chapters VI, VII and VIII give example applications. The conclusions of the study and recommendations for future work are given in Chapter IX.

CHAPTER II

AN OVERVIEW OF DPVSM, OSU-DM AND CCMT

This chapter provides the background for the unified approach developed in Chapter III. The overview of DPVSM and OSU-DM are given in Sections 2.1 and 2.3, respectively. Sections 2.2 and 2.4 outline the difficulties associated with their implementation. Section 2.5 provides an overview of CCMT. Sources of ambiguities and practical difficulties associated with CCMT are discussed in Section 2.6. Finally, Section 2.7 describes the differences and similarities between DPVSM, OSU-DM and CCMT.

2.1 Overview of DPVSM [46]

Failure model construction with DPVSM first requires choosing a time interval Δt and partitioning the PVSS into computational cells (or magnitude intervals) V_j ($j=1, \dots, J + \Gamma$), with the cell indices $j=1, \dots, J$ representing the allowed ranges for the process variables and the indices $j=J+1, \dots, J+\Gamma$ corresponding to Top Events. Under normal system operation, the process variables are within the predefined allowed ranges or in the nominal control region. Upon failure of one or more of the components, the process variables may be disturbed and moved out of the nominal control region, or kept in the nominal control region by the action of the remaining operational components according to system control laws. The failure of one or more components does not necessarily mean system failure. System fails when one of the process variables moves out of the nominal control region.

The choice of the cells V_j are based on the following criteria:

1. V_j must be disjoint.
2. Values of the process variables defining the Top Events and the system setpoints must fall on the boundaries of V_j and not within V_j .

Then the following assumptions are made:

1. The components have discrete states and do not change states during $[t, t+\Delta t)$, but possibly at $t+\Delta t$.
2. The probability $p_{n,j}(t)$ that the process variables are in the computational cell V_j at time t with the system components having the state combination n is uniformly distributed over V_j .
3. No two process variable trajectories arrive at the same point in the PVSS at the same time and move in different directions for the same component state combination.
4. The transition probabilities between component state combinations do not depend on the history of system operation.
5. The cells corresponding to Top Events are absorbing cells, i.e. once a system trajectory (or evolution) reaches such a cell, the system remains in that cell forever.

Assumptions 1 and 2 are approximations and necessitated by the discrete nature of the methodology. Assumption 3 implies that once the controlled variables and the states of the components at time t are specified, then the controlled variables are uniquely determined at time $t+\Delta t$. Assumption 4 excludes the possibility that failure-per-demand for a given component may change with the number of previous demands on the component, however component aging can be modeled if it is a function of time only. Assumption 5 excludes the

possibility of the system recovering from a Top Event, thus the cells that represent Top Events corresponding to absorbing states for the system dynamics.

In order to describe failure model construction with DPVSM, some definitions are going to be presented first as follows:

$$p(n, x, t) \equiv \Pr\{n(t)=n, x(t)=x\} \quad (1)$$

$$q(n, x | n', x', \Delta t) \equiv \Pr\{n(t+\Delta t)=n, x(t+\Delta t)=x | n(t)=n', x(t)=x'\} \quad (2)$$

$$g(x | n', x', \Delta t) \equiv \Pr\{x(t+\Delta t)=x | n(t)=n', x(t)=x'\} \quad (3)$$

$$h(n | n', x' \rightarrow x, \Delta t) \equiv \Pr\{n(t+\Delta t)=n | n(t)=n', x(t)=x' \rightarrow x(t+\Delta t)=x\} \quad (4)$$

In Eqs.(1)-(4) $n(t)$ indicates the component state combination at time t ($n=1, \dots, N$) and $x(t)$ is the process variable vector at time t . From the above definitions of $g(x | n', x', \Delta t)$ (Eq.(3)) and $h(n | n', x' \rightarrow x, \Delta t)$ (Eq.(4)) and the concept of conditional probability, it follows that:

$$q(n, x | n', x', \Delta t) = g(x | n', x', \Delta t) h(n | n', x' \rightarrow x, \Delta t). \quad (5)$$

From the definitions of $p(n, x, t)$ (Eq.(1)) and $q(n, x | n', x', \Delta t)$ (Eq.(2)) and assuming that component state is discrete, Ref.[61] shows that the probabilities $p(n, x, t)$ satisfy a recurrence relation given by:

$$\begin{aligned} p(n, x, t+\Delta t) &= \sum_{n'=1}^N \left[\int_R dv' + \int_{\bar{R}} dv' \right] g(x | n', x', \Delta t) h(n | n', x' \rightarrow x, \Delta t) p(n', x', t) \\ &= \sum_{n'=1}^N \left[\int_R dv' + \int_{\bar{R}} dv' \right] q(n, x | n', x', \Delta t) p(n', x', t) \end{aligned} \quad (6)$$

where R and $\bar{R}$ are the nominal control region and its complement in the Euclidean space, E , respectively, and dv' represents the infinitesimal volume element in E .

From the partitioning of the PVSS, the integrals in Eq.(6) can be written as

$$\int_R dv = \sum_{j=1}^J \int_{V_j} dv \quad (7)$$

$$\int_{\bar{R}} dv = \sum_{\gamma=J+1}^{J+\Gamma} \int_{V_\gamma} dv. \quad (8)$$

Let

$$p_{n,j}(t) = \int_{V_j} dv p(n, x, t). \quad (9)$$

Then since $p(n, x, t)$ is uniformly distributed within each V_j (by Assumption 2 above),

$$p(n, x, t) = \sum_{j=1}^J \frac{p_{n,j}(t)}{v_j} e_j(x) \quad (10)$$

where $e_j(x)$ are step functions in E defined as

$$e_j(x) = \begin{cases} 1 & \text{if } x \in V_j \\ 0 & \text{otherwise,} \end{cases} \quad (11)$$

the substitution of Eqs.(7), (8) and (9) into Eq.(6) yields

$$\begin{aligned} p_{n,j}(t+\Delta t) = \sum_{n'=1}^N \bigg[& \sum_{j'=1}^J \int_{V_{j'}} dv \int_{V_{j'}} dv' q(n, x | n', x', \Delta t) p(n', x', t) \\ & + \sum_{\gamma=1}^{J+\Gamma} \int_{V_j} dv \int_{V_\gamma} dv' q(n, x | n', x', \Delta t) p(n', x', t) \bigg]. \end{aligned} \quad (12)$$

Note that the first term of right-hand side of Eq.(12) represents the contribution from the cells inside R , and the second term (i.e., second line of Eq.(12)) is the contribution from the absorbing cells in $\bar{R}$. Since once the system is in an absorbing cell in $\bar{R}$ (i.e. $\mathcal{X} \in V_\gamma$) the system remains in that cell forever (by Assumption 5), it follows that

$$\int_{V_j} dv \int_{V_\gamma} dv' q(n, \mathcal{X} | n', \mathcal{X}', \Delta t) = \delta_{j,j} \delta_{n',n} \quad (13)$$

where $\delta_{n',n}$ is the Kronecker delta, defined as

$$\delta_{n',n} = \begin{cases} 1 & \text{if } n=n' \\ 0 & \text{otherwise.} \end{cases} \quad (14)$$

Substituting Eqs.(10) and (13) into Eq.(12), yields

$$\begin{aligned} p_{n,j}(t+\Delta t) = \sum_{n'=1}^N \left[\sum_{j'=1}^J \frac{p_{n',j'}(t)}{v_{j'}} \int_{V_j} dv \int_{V_{j'}} dv' q(n, \mathcal{X} | n', \mathcal{X}', \Delta t) \right. \\ \left. + \sum_{j'=J+1}^{J+\Gamma} p_{n',j'}(t) \delta_{n',n} \delta_{j',j} \right] \end{aligned} \quad (15)$$

The first and second terms of right-hand side of Eq.(15) represent the contributions from cells inside R and from absorbing cells in $\bar{R}$, respectively. Now we proceed to outline how Ref.[46] proposes to calculate the contribution from cells inside R .

Ref.[46] assumes that a description of system dynamics in the form

$$\frac{dx}{dt} = f_{n'}(\mathcal{X}) \quad (16)$$

is available, where $\mathcal{X} = x(t)$, $n' = n(t)$ and $f_{n'}(\mathcal{X})$ is the vector function of time derivatives of

the process variables for a given component state combination n' . Then, for sufficiently small Δt and under the assumption that the components do not change states within $[t, t+\Delta t)$ but possibly at $t+\Delta t$ (i.e. Assumption 1), we can write

$$x(t+\Delta t) \approx x' + f_{n'}(x')\Delta t, \quad (17)$$

which implies that

$$Pr \{x(t+\Delta t) \in V_j | n(t)=n', x(t)=x'\} = e_j(x' + f_{n'}(x')\Delta t). \quad (18)$$

A closer look at the definition of $g(x | n', x', \Delta t)$ (Eq.(3)) indicates that

$$\begin{aligned} \int_{V_j} dv \ g(x | n', x', \Delta t) &= Pr \{x(t+\Delta t) \in V_j | n(t)=n', x(t)=x'\} \\ &= e_j(x' + f_{n'}(x')\Delta t). \end{aligned} \quad (19)$$

Also, since $p(n, x, t)$ is assumed to be uniformly distributed within each V_j (by Assumption 2), $h(n | n', x' \rightarrow x, \Delta t)$ can be written as

$$h(n | n', x' \rightarrow x, \Delta t) = \sum_{j=1}^J \sum_{j'=1}^J h(n | n', j' \rightarrow j, \Delta t) e_j(x) e_{j'}(x'). \quad (20)$$

where $h(n | n', x' \rightarrow x, \Delta t)$ represents the transition probability between component states for given transition between process variable states as defined by Eq.(4). The substitution of Eq.(20) into Eq.(5) and integration over V_j and also over $V_{j'}$ yields

$$\int_{V_j} dv \int_{V_{j'}} dv' \ q(n, x | n', x', \Delta t) = h(n | n', j' \rightarrow j, \Delta t) \int_{V_{j'}} dv' \int_{V_j} dv \ g(x | n', x', \Delta t) \quad (21)$$

for any $j=1, \dots, J$ and a substitution of Eqs. (18) and (19) into Eq.(21) leads to

$$\int_{V_j} dv \int_{V_{j'}} dv' q(n, x | n', x', \Delta t) = h(n | n', j' \rightarrow j, \Delta t) \int_{V_{j'}} \frac{dv'}{v_{j'}} e_j(x' + f_{n'}(x') \Delta t). \quad (22)$$

Eqs.(22) and (15) show that $p_{n,j}(t)$ can be recursively determined as follows:

$$\begin{aligned} p_{n,j}(t+\Delta t) = \sum_{n'=1}^N \left[\sum_{j'=1}^J p_{n',j'}(t) h(n | n', j' \rightarrow j, \Delta t) \int_{V_{j'}} \frac{dv'}{v_{j'}} e_j(x' + f_{n'}(x') \Delta t) \right. \\ \left. + \sum_{j'=J+1}^{J+\Gamma} p_{n',j'}(t) \delta_{n',n} \delta_{j',j} \right]. \end{aligned} \quad (23)$$

Eq.(23) can be expressed in a more compact form as

$$p_{n,j}(t+\Delta t) = \sum_{n'=1}^N \sum_{j'=1}^{J+\Gamma} g(j | j', n') h(n | n', j' \rightarrow j, \Delta t) p_{n',j'}(t) \quad (24)$$

where $g(j | j', n')$ represents process variable transition probability from cell $V_{j'}$ to V_j , given that system component state combination at t is n' and is formally defined as

$$g(j | j', n') = \begin{cases} \int_{V_{j'}} \frac{dv'}{v_{j'}} e_j(x' + f_{n'}(x') \Delta t) & \text{if } V_{j'} \subset R \\ \delta_{j',j} & \text{otherwise.} \end{cases} \quad (25)$$

Since

$$\sum_{n=1}^N \sum_{j=1}^{J+\Gamma} g(j | j', n') h(n | n', j' \rightarrow j, \Delta t) = 1 \quad (26)$$

and

$$\sum_{n=1}^N \sum_{j=1}^{J+\Gamma} p_{n,j}(t) = 1, \quad (27)$$

Eqs.(24) and (25) define a Markov chain. If component failures and repairs are statistically independent, $h(n|n', j' \rightarrow j, \Delta t)$ is the product of the transition probabilities between each component states, $c_k(n_k|n'_k, j' \rightarrow j, \Delta t)$, i.e.

$$h(n|n', j' \rightarrow j, \Delta t) = \prod_k c_k(n_k|n'_k, j' \rightarrow j, \Delta t) \quad (28)$$

where n'_k and n_k indicate the state of component k at times t and $t+\Delta t$, respectively. The logic in determining $c_k(n_k|n'_k, j' \rightarrow j, \Delta t)$ for a component k with a given failure rate λ_k and a given repair rate μ_k from the control laws is illustrated in Refs.[46, 47]. Section 6.3 illustrates the determination of $h(n|n', j' \rightarrow j, \Delta t)$ when the failure data are given as both failure rates and probability of failure per demand.

Once $p_{n,j}(t)$ are obtained, then the CDF (cumulative distribution function) and pdf (probability distribution function) can be respectively calculated from:

$$F_\gamma(t) = \sum_{j \in S_\gamma} \sum_n p_{n,j}(t) \quad (29)$$

$$w_{n,\gamma}(t) = \frac{1}{\Delta t} [p_{n,\gamma}(t+\Delta t) - p_{n,\gamma}(t)] \quad (30)$$

where S_γ represents the set of cells in PVSS that correspond to Top Event γ . Similarly, the statistical importance [7] of component failure in mode m to Top Event γ is obtained from:

$$(Im)_{m,\gamma} = \sum_{n \in S_m} w_{n,\gamma} / \sum_n w_{n,\gamma} \quad (31)$$

where S_m represents the set of component state combinations that contains a component failure in mode m .

2.2 Difficulties Associated with the Implementation of DPVSM

Some difficulties associated with the implementation of the DPVSM approach described in Section 2.1 are the following:

- Since system dynamics are described by a first-order Taylor expansion of system equations (see Eq.(17)), Δt must be very small. Then unless the time interval of interest for the failure analysis is also small, the computational time required for the analysis can be long.
- For a small Δt the system dynamics may remain in the same cell. This condition will offer no valuable information on the system dynamics and should be avoided, by either increasing Δt or decreasing the cell size. Increasing Δt will increase the error in evaluating system dynamics. On the other hand, decreasing the cell size will lead to storage deficiencies.
- For nonlinear systems, a first-order Taylor expansion of system equations may not be appropriate.

2.3 Overview of OSU-DM [47]

The OSU-DM is a derivative algorithm of DPVSM. The main difference between OSU-DM and DPVSM is that DPVSM assumes an explicit knowledge of the system equations under normal and abnormal conditions whereas OSU-DM can be used with an existing simulator for the system. The simulator may use graphical or tabular data to describe the process physics instead of equations and may not even allow to change component states (e.g. for fault-injection) once the simulation starts. The software for the implementation of OSU-DM consists of a driver and the simulator. The driver generates the possible

component failure scenarios during the time interval of interest for failure analysis by considering component failures at every modeling time step τ . Then the simulator is used to obtain the system trajectories in the process variable space for each scenario (or until the process variables fall outside the allowed range). These trajectories are stored as a data base. This approach also eliminates the concerns attributed to the use of Eq.(17) to determine system evolution as discussed in Section 2.2.

Failure model construction with OSU-DM is similar to that with DPVSM except for the computation of the conditional probabilities for transitions between the process variables states, i.e.,

$$g(j|j', n') \equiv \Pr\{x(t+\tau) \in V_j | n(t) = n', x(t) = x' \in V_{j'}\}, \quad (32)$$

where now τ is not necessarily a small time step as far as the system dynamics is concerned (but still small compared to component's mean time to failure). Since the system equations are not necessarily explicitly known and may be implicit in the simulator, the $g(j|j', n')$ are not obtained from Eq.(25), but by simulating the system evolution with initial conditions that $n(t) = n'$, and $x(t) = x' \in V_{j'}$, or by integrating Eq.(16) over τ , i.e.

$$x(t+\tau) = \int_t^{t+\tau} f_{n'}(x'(t')) dt'. \quad (33)$$

Then in a similar manner to the derivation of Eq.(25) it can be shown that

$$g(j|j', n') = \begin{cases} \int_{V_{j'}} \frac{dv'}{v_{j'}} e_j \left(\int_t^{t+\tau} f_{n'}(x'(t')) dt' \right) & \text{if } V_j \subset R \\ \delta_{j'j} & \text{otherwise.} \end{cases} \quad (34)$$

Under Assumption 3 in Section 2.1, the cells V_j can be chosen small enough that they contain at most one trajectory point for each component state combination. Then, Eq.(34) reduces to

$$g(j|j', n') = \begin{cases} 1 & \text{if } x(t+\tau) \in V_j \text{ for } x(t) \in V_{j'} \text{ and } n(t) = n' \\ 0 & \text{otherwise,} \end{cases} \quad (35)$$

where $x(t+\tau)$ is found from Eq.(33) or from the simulator.

Choosing V_j too small may lead to computational limitations, mainly due to increased storage. On the other hand, if V_j are large there may be several trajectory segments departing from a $V_{j'}$ at time t with the same component state n' and arriving in IC number of V_j at $t+\tau$, depending on the location of the controlled variables within $V_{j'}$ at t . In this situation OSU-DM assigns equal weight to each transition. A cell with multiple departure trajectories is assumed to be partitioned in equal-volume sub-cells in such a way that only one trajectory departs from each subcell and hence Eq.(34) becomes

$$g(j|j', n') = \begin{cases} \frac{1}{IC} & \text{if } x(t+\tau) \in V_j \text{ for } x(t) \in V_{j'} \text{ and } n(t) = n' \\ 0 & \text{otherwise.} \end{cases} \quad (36)$$

Then, $p_{n,j}(t+\tau)$ and other probability quantities can be determined from Eq.(24) and Eqs.(29)-(31), respectively. In the computation of $p_{n,j}(t)$ from Eq.(24), the following conditions must be satisfied at every time step to bound roundoff error propagation:

$$\sum_n \sum_j p_{n,j}(t)=1, \text{ and,} \quad (37)$$

$$\sum_{n'} \sum_{j'} g(j|j',n')h(n|n',j' \rightarrow j)=1. \quad (38)$$

2.4 Practical Difficulties Associated with OSU-DM

The OSU-DM eliminates concerns associated with the use of first-order Taylor expansion (Eq.(17)) of the system equations to describe the process physics. However, some practical difficulties associated with the implementation of the algorithm of Ref.[47] are the following:

- It is not always clear what assumptions are implicit in determining $g(j|j',n')$ by Eq.(36). For example, if we denote by $I_{j'}(n')$ the number of system trajectories originating from cell $V_{j'}$ at t with component state n' , and by $N_{jj'}(n')$ the number of system trajectories arriving in cell V_j at $t+\tau$ and which have originated from $V_{j'}$ at t with component state n' , it is not clear how Eq.(34) could be used to determine $g(j|j',n')$ for the case when $N_{jj'}(n')$ is larger than 1 and $I_{j'}$ is larger than IC .
- In its existing form the algorithm is system specific. The controlled variables are limited to two and so is the number of components that can be modeled. System specific component data and control inputs (such as setpoints) and component specific state transition probability calculations are hardwired to the algorithm.
- Computational requirements (i.e., time and memory) may become important or even prohibitive for large systems with more than two controlled variables of interest or more than two components. In that respect, adaptation of the algorithm to super-computing and vectorization is essential.

- Partitioning of PVSS is manual which can be time consuming.

We now proceed to overview CCMT and discuss the sources of ambiguities associated with its implementation.

2.5 Overview of CCMT [40, 41]

Like DPVSM, CCMT also assumes that a description of system dynamics in the form of a set of first-order differential equations is available, i.e.

$$\frac{dx}{dt} = f(x, \alpha) \quad (39)$$

where x is a the continuous state process variable (or dynamical variable) vector as before and α is the system parameter vector (implicitly assumed to be continuous). We will further assume that the system is autonomous, i.e. the vector function $f(x, \alpha)$ does not explicitly depend on t . This assumption does not lead to loss of generality, since a non-autonomous system can be converted to an autonomous system by letting t be an additional dynamic variable.

In order to illustrate the philosophy of CCMT, first let $\alpha = \alpha_m$ be a fixed vector. Then for every trajectory departure point $x(t) = x'$ in the state-space, the arrival point at time $t + \tau$ can be uniquely determined from

$$\bar{x}(x', \alpha_m) = \int_t^{t+\tau} f(x(t'), \alpha_m) dt' \quad (40)$$

by the numerical integration of Eq.(39). Now partition the state-space region of interest, R , into cells V_j , ($j=1, \dots, J$) each of which is a specified set of magnitude intervals for the dynamical variables, as with DPVSM and OSU-DM. In the original formulation of CCMT,

the region outside R is represented by a single sink cell V_{J+1} . The CCMT regards system evolution in time as a mapping between these cells. The mapping rules can be also interpreted as the conditional probabilities $g_{j,j'}(\tau)$, i.e. the probability of system moving to V_j during the time interval $[t, t+\tau]$, given that the departure point is within $V_{j'}$ at time t . The time period τ is called the mapping step. The CCMT calculates $g_{j,j'}(\tau)$ using either simple cell mapping (SCM) or generalized cell mapping (GCM). For SCM, a single departure point is chosen in each cell $V_{j'}$ (usually the center) and the arrival point is determined from Eq.(40) to find the image cell. Since there is one departure point from $V_{j'}$ and the arrival point is uniquely determined from Eq.(40) once the departure point is specified, $g_{j,j'}(\tau) = 1$ if the arrival point is within V_j . Otherwise, $g_{j,j'}(\tau) = 0$. In SCM terminology, a cell is said to be a periodic cell of period K if it maps onto itself in K mapping steps (i.e. in a time period $K\tau$). This definition implies that the sink cell is also a periodic cell with $K=1$. If a cell is not periodic, then it is a transient cell. The GCM allows multiple departure points from each cell. Subsequently a cell can have several image cells with GCM. In this case, two recipes proposed in Ref.[40] to find $g_{j,j'}(\tau)$ are:

1. Find the portion of the volume of $V_{j'}$ that maps to V_j during $[t, t+\tau]$ using point-to-point mapping (i.e. from Eq.(40)) and define $g_{j,j'}(\tau)$ as the ratio of this volume to the total volume of $V_{j'}$.
2. Further partition each cell $V_{j'}$ into $I_{j'}$ equal-volume subcells and choose one departure point in each subcell ("interior and boundary sampling"). If $x'_{i,j'}$ ($i=1, \dots, I_{j'}$) is a point in the i^{th} subcell of cell $V_{j'}$, then

$$g_{j,j'}(\tau) = \frac{N_{j,j'}(\alpha)}{I_{j'}} \quad (41)$$

where $N_{j,j'}(\alpha)$ is the number of system trajectories arriving in cell V_j which have originated from cell $V_{j'}$ for a given α and is again determined by integrating Eq.(39) over $[t, t+\tau]$ with $x(t) = x'_{1,j'}, \dots, x'_{I,j'}$.

In GCM terminology, a cell V_i is said to communicate with cell V_j if the system starting from $V_{j'}$ eventually visits cell V_j and also if the system starting from V_j eventually visits cell $V_{j'}$. A cell V_j is called a persistent cell if all the system trajectories starting from V_j eventually return to V_j . Otherwise, V_j is a transient cell. The set of persistent cells that communicate with each other form a persistent group (PG). Each PG corresponds to an asymptotically stable solution of the system (stationary or periodic) or to a strange attractor. The set of cells that lead to one specific PG are called single domicile cells. The single domicile cells leading to a PG constitute the basin of attraction (BOA) for that PG. The BOA is the GCM terminology for DOA. The DOA and BOA will be used interchangeably in the rest of this thesis. Cells that lead to more than one PG are called multiple domicile cells.

A SCM and a GCM are called compatible if, for every cell, the single image of the SCM is a member of the set of multiple images of GCM. For both SCM and GCM, $g_{j,J+1}(\tau) = 1$ if $j=J+1$ and 0 otherwise, since V_{J+1} is a sink cell by definition. Sections 2.5.1 and 2.5.2 show how the $g_{j,j'}$ are used to determine the PGs, BOAs and their statistical properties.

2.5.1 Determination of the Limiting Probability Distribution of PGs

Once the transition probabilities $g_{j,j'}(\tau)$ are found, the probability, $P_j(t+\tau)$, that the dynamic variables are within cell V_j ($j=1, \dots, J+1$) at time $t+\tau$ is obtained from the relation

$$P_j(t+\tau) = \sum_{j'=1}^{J+1} g_{j,j'}(\tau) P_{j'}(t). \quad (42)$$

Since the system has to be at some point in the state-space at any time and the V_j partition the state-space, we must have

$$\begin{aligned} \sum_{j=1}^{J+1} P_j(t) &= 1 \quad \text{for all } t, \\ \sum_{j=1}^{J+1} g_{jj'} &= 1 \quad \text{for all } j'=1, \dots, J+1 \end{aligned} \quad (43)$$

and thus Eqs.(41) and (42) define a Markov chain in discrete time. Then

$$P_j^\infty \equiv \lim_{k \rightarrow \infty} P_j(k\tau) \quad (44)$$

yields the limiting probability distribution of PGs [40]. For fixed initial conditions and system parameters, τ/P_j^∞ can be interpreted as the mean duration between trajectory visits to cell V_j [62]. The P_j^∞ are also useful in determining the statistical properties of chaotic attractors. For example, the asymptotic mean value, $\bar{x}$, of the location of the system in the state-space can be found from

$$\bar{x} = \sum_{j=1}^J x_j P_j^\infty \quad (45)$$

where x_j is a point within cell V_j . Similarly, the variance of the dynamic variable x_l from its mean value $\bar{x}_l$ can be found from

$$\sigma_l^2 = \sum_{j=1}^J (x_{lj} - \bar{x}_l)^2 P_j^\infty. \quad (46)$$

where x_{lj} and $\bar{x}_l$ are the l^{th} components of the vectors x_j and $\bar{x}$, respectively. When the system parameters are stochastic variables, then Eq.(41) is modified as

$$g_{jj'}(\tau) = \sum_{m=1}^M \frac{N_{jj'}(\alpha_m)}{I_{j'}} \tilde{P}(\alpha_m) \quad (47)$$

where α_m ($m=1,\dots,M$) are a set of selected discrete values of the parameter vector and $\tilde{P}(\alpha_m)$ are the associated probabilities determined from the given joint pdf for the system parameters [40, 63, 64]. Then the analysis is carried out as described above for fixed α through Eqs.(42), (43) and (44). A second procedure proposed by [65] for the determination of $g_{jj'}(\tau)$ is to approximate $P(x, t+\tau|x', t)$, the transition probability density from x' to x in the state space within $[t, t+\tau]$, by a some assumed distribution and determine the $g_{jj'}$ from

$$g_{jj'}(\tau) = \int_{V_j} dx P(x, t+\tau|x', t) \quad (48)$$

where $x_{j'}$ denotes the center of cell $V_{j'}$ (see Eq.(5) of [65]). Compared to Recipe 2, this approach has been found to lead to substantial decrease in the computation time for the determination of $g_{jj'}(\tau)$ for systems under external and parametric Gaussian white noise excitations.

2.5.2 Determination of PGs, BOAs and Their Absorption Probabilities

In order to determine the PGs, the CCMT first selects a SCM compatible with the given GCM for the system. Such an SCM can be selected by picking one among the multiple image cells of a cell V_j and regarding this image as the single image cell of V_j . Then the mapping evolution of each cell in the selected SCM is followed to determine if the cell is periodic or transient. After having determined the periodic cells for the selected SCM, the search for PGs starts by choosing a cell that is periodic under the selected SCM and following its mapping evolution to generate a cell sequence while considering all the possible multiple images of the cell for the given GCM. If the cell sequence leads to a cell that is known to be a transient cell or a persistent cell of a previously discovered PG, then all the cells in

the sequence are tagged as transient. If the sequence leads to a new persistent cell, the sequence is continued to locate all the cells the new persistent cell leads to. These cells are tagged as the members of the new PG. All the other cells in the sequence are tagged as transient. The tagging rules are based on some properties of finite Markov chains given in [40].

In order to locate the domiciles of the transient cells (i.e. the PGs that the transient cells lead to) a pre-image array is constructed for the cells in PGs. A pre-image cell of a cell V_j is a cell which has V_j as one of its images. Since the domicile of a transient cell is also the domicile of all its pre-image cells, the pre-image array is used to march backwards in time to locate the cells that have their domiciles as the PG under consideration. The transient cells can be single domicile cells (i.e. leading to only one PG) or multiple domicile cells (i.e. leading to more than one PG). As indicated before, single domicile cells constitute BOAs. Multiple domicile cells delineate the boundaries of the BOAs.

Once all the PG cells are determined and the BOAs consisting of single domicile cells are identified, the absorption probabilities of multiple domicile transient cells (MDTCs) in a given PG are calculated. For this purpose, the cell space has to be rearranged in such a way that persistent cells are listed first, followed by the transient cells. A subsequent simultaneous rearrangement of rows and columns of the transition probability matrix M , whose elements consist of the $g_{jj'}$ obtained from GCM, leads to the normal form of M given by

$$M = \begin{bmatrix} M' & T \\ \Theta & Q \end{bmatrix} \quad (49)$$

where Θ is a null matrix and the submatrices M' , T and Q describe, respectively, the transitions: a) between persistent cells, b) from transient to persistent cells, and, c) from transient to transient cells. Then it can be shown that [66] the absorption probability of a multiple

domicile transient cell V_j in a persistent cell V_i is the element (i,j) of the matrix $A = T(I - Q)^{-1}$ where I is the identity matrix.

2.6 Practical Difficulties Associated with CCMT

Some practical difficulties associated with the CCMT as described above are the following:

- Due to the heuristic nature of the GCM algorithm, it is not always clear what assumptions are implicit in determining the cell-to-cell transition probabilities $g_{j,j'}(\tau)$. For example, it is not clear what would be the consequences of using an unequal number of subcells to partition the V_j with Recipe 2 in Section 2.5. Similarly, in the implementation of CCMT to systems with stochastic parameters, both the justification and validation of the procedures used to determine $g_{j,j'}(\tau)$ have been application specific (such as when the parameters are uniformly distributed within specified ranges [63] or when parametric excitations are in the form of Gaussian white noise [65]) and it is again not clear how the $g_{j,j'}(\tau)$ will be determined from a given pdf for system parameters in general.
- It may be difficult to identify a unique pdf to represent the uncertainty on the system parameters and several analyses may need to be carried out with different pdfs to study the impacts of the choice of the pdf on the system behavior. Equation (47) implies that the transition probabilities need to be recalculated every time the pdf is changed.
- Determining the inverse of $(I - Q)$ can be very time consuming and usually requires large memory [67].

- Constructing pre-image arrays for the PGs in the determination of BOAs (Section 2.5.2) can again lead to large memory requirements.

We now proceed to compare DPVSM, OSU-DM and CCMT and outline similarities and differences between the two methodologies.

2.7 Comparison of DPVSM, OSU-DM and CCMT

As indicated in Section 2.3, OSU-DM is a derivative of DPVSM. However, two differences are the following:

1. The DPVSM assumes that the system equations are available and calculates the transition probabilities $g(j|j',n')$ between process variable states from Eq.(25) using a first-order Taylor expansion of $x(t+\Delta t)$ around t , which is only valid if the arrival time in cell j is close to the departure time from j' (or for small Δt). The OSU-DM can determine $g(j|j',n')$ from Eqs.(33) and (35) or (33) and (36) using an existing simulator without the explicit knowledge of system equations making assumptions on arrival times.
2. The DPVSM can find the attractors of the system, however OSU-DM does not have this capability.

While the overall philosophies of OSU-DM and CCMT are also similar, the main differences are the following:

1. The CCMT assumes statistical independence between the system parameter (or component) state transitions and process variable state transitions and subsequently cannot model components which may change states during $[t, t+\tau]$ and may fail upon demand, depending on the process variable transitions during $[t, t+\tau]$. The OSU-DM accounts

for such dependence through $h(n | n', x' \rightarrow x, \Delta t)$.

2. The cell-to-cell transition probabilities are calculated differently. However, for a single component state, OSU-DM reduces to the SCM technique of CCMT.
3. The CCMT regards all the region outside R as a single sink cell. The OSU-DM uses several sink cells to distinguish between the different failure modes of the system (i.e. Top Events).
4. The CCMT can determine both the attractors of the system and the BOAs of the attractors. The OSU-DM is not designed for such a purpose.

Now we proceed in Chapter III to show how all the three methodologies DPVSM, OSU-DM and CCMT can be derived from a Chapman-Kolmogorov interpretation of system evolution. Chapter III also shows how this interpretation leads to new and more efficient algorithms for a safety and reliability oriented global analysis of dynamic systems.

CHAPTER III

THEORETICAL BASIS FOR A COMPREHENSIVE APPROACH TO THE RELIABILITY AND SAFETY ANALYSIS OF DYNAMIC SYSTEMS

This chapter presents a theoretical basis that unifies DPVSM, OSU-DM and CCMT towards a comprehensive approach to the reliability and safety analysis of dynamic systems. Section 3.1 presents a Chapman-Kolmogorov interpretation of system evolution. Sections 3.2 and 3.3 show how DPVSM, OSU-DM and CCMT can be all derived from this interpretation. Sections 3.4 and 3.5 show that this interpretation leads to new algorithms that reduce the computational time in determining limiting probability distribution of the attractors, as well as in identifying the BOAs and quantifying their absorption probabilities in PGs.

3.1 A Chapman-Kolmogorov Interpretation of System Evolution

Consider the system

$$\frac{dx}{dt} = f(x, \eta) \quad (50)$$

where η is the system condition vector which can be a set of component states and/or a set of system parameters and x is a vector whose elements are the process state variables as before. For a given system condition vector at time t ($\eta(t) = \eta'$), a given set of process variables $x(t) = x'$ and under the assumption that the components do not change states within $[t, t+\tau)$ but possibly at $t+\tau$, the system state at $t+\tau$ can be uniquely determined from

$$x(t+\tau) = \int_t^{t+\tau} f(x(t'), \eta') dt'. \quad (51)$$

Equation (51) shows that the probability, $P(x, \eta, t+\tau) d\eta dx$, that the system is within a small volume element $d\eta dx$ around a specified point $\{ \eta, x \}$ in the product space $x \times \eta$ at time $t+\tau$ can be uniquely determined from $\eta(t) = \eta'$, and $x(t) = x'$, which implies that the system under consideration has the Markov Property in $x \times \eta$. The Markov Property can be expressed through the Chapman-Kolmogorov equation [68] as

$$P(x, \eta, w | x'', \eta'', u) = \int d\eta' \int dx' P(x, \eta, w | x', \eta', v) \\ \times P(x', \eta', v | x'', \eta'', u), \quad 0 \leq u \leq v \leq w \quad (52)$$

where u, v, w denote time and $P(A|B)$ denotes the conditional probability that event set A occurs given that event set B occurs (i.e. transition probability from event set B to event set A). If for discrete system states, the integration over η in Eq.(52) is replaced by summation.

Now replace w and v in Eq.(52) with $t+\tau$ and t , respectively, to obtain:

$$P(x, \eta, t+\tau | x'', \eta'', u) = \int d\eta' \int dx' P(x, \eta, t+\tau | x', \eta', t) \\ \times P(x', \eta', t | x'', \eta'', u), \quad 0 \leq u \leq t \leq t+\tau \quad (53)$$

Integrating Eq.(53) over all possible $0 \leq u \leq t$, x'' and η'' we get

$$P(x, \eta, t+\tau) = \int d\eta' \int dx' P(x, \eta, t+\tau | x', \eta', t) P(x', \eta', t), \quad (54)$$

which also shows that

$$\begin{aligned} \int d\eta \int dx P(x, \eta, t) &= 1 \\ \int d\eta \int dx P(x, \eta, t+\tau | x', \eta', t) &= 1. \end{aligned} \quad (55)$$

From the concept of conditional probability

$$P(x, \eta, t+\tau | x', \eta', t) = P(x, t+\tau | x', \eta', t) P(\eta, t+\tau | x(t)=x', \eta(t)=\eta', x(t+\tau)=x). \quad (56)$$

Also, once $x(t)$ and $\eta(t)$ are specified, the location of the dynamic variables in the state-space at $t+\tau$ is uniquely determined from Eq.(51) and subsequently

$$P(x, t+\tau | x', \eta', t) = \delta(x - \bar{x}(x', \eta')) \quad (57)$$

where $\delta(\cdot)$ denotes the Dirac delta function and

$$\bar{x}(x', \eta') = \int_t^{t+\tau} f(x(t'), \eta') dt' \quad (58)$$

is obtained from the numerical integration of Eq.(50) with $x(t)=x'$ and fixed $\eta(t)=\eta'$ as in Eq.(51). The substitution of Eqs.(56) and (57) into Eq.(54) yields

$$\begin{aligned} P(x, \eta, t+\tau) &= \int d\eta' \int dx' \delta(x - \bar{x}(x', \eta')) P(\eta, t+\tau | x(t)=x', \eta(t)=\eta', x(t+\tau)=x) \\ &\quad \times P(x', \eta', t). \end{aligned} \quad (59)$$

Sections 3.2 ad 3.3 show how Eq.(51) leads to DPVSM, OSU-DM and CCMT.

3.2 Theoretical Basis for DPVSM and OSU-DM

In order to obtain DPVSM and OSU-DM from Eq.(59), we assume that the elements of the system condition vector are discrete variables, i.e.

$$\eta(t) = n(t). \quad (60)$$

Then Eq.(59) becomes

$$P(x, n, t + \tau) = \sum_{n'=1}^N \int dx' \delta(x - \bar{x}(x', n')) P(n, t + \tau | x(t) = x', n(t) = n', x(t + \tau) = x) \times P(x', n', t). \quad (61)$$

where,

$$\bar{x}(x', n') = \int_t^{t+\tau} f(x(t'), n') dt'. \quad (62)$$

Equation (4) shows that $P(n, t + \tau | x(t) = x', n(t) = n', x(t + \tau) = x)$ in Eq.(61) and $h(n | n', x' \rightarrow x, \Delta t)$ are the same if $\tau = \Delta t$. Substituting Eq.(4) into Eq.(61) and letting n' and n denote the component state combinations at t and $t + \Delta t$, respectively, yields

$$P(x, n, t + \tau) = \sum_{n'=1}^N \int dx' \delta(x - \bar{x}(x', n')) h(n | n', x' \rightarrow x, \tau) P(x', n', t). \quad (63)$$

Integrating Eq.(63) over V_j and approximating $P(x, n, t)$ and $h(n | n', x' \rightarrow x, \tau)$ using Eqs.(10) and (20), respectively, yields Eq.(24) but with $g(j | j', n')$ as given by Eq.(34) (or more precisely with the argument of the function $e_j(x)$ in Eq.(34) replaced by $\bar{x}(x', n')$ as defined by Eq.(62)) instead of Eq.(25) and Δt in $h(n | n', j' \rightarrow j, \Delta t)$ replaced by τ . This result shows that DPVSM is a derivative of the Chapman-Kolmogorov interpretation based on a first-order Taylor expansion of the system equations (i.e. Eqs.(16) or (39)).

Since the OSU-DM uses Eq.(33) or the system simulator to determine the system evolution in time, Eq.(33) is the same as Eq.(62) and the reasoning process used for DPVSM above shows that Eqs.(24) and (34) also applies to OSU-DM. However, two special cases of

Eq.(34) need to be considered to resolve the first difficulty indicated in Section 2.4 regarding the implementation of OSU-DM. If for a given component state vector there exist only one trajectory leaving a cell, then Eq.(34) leads to Eq.(35). In case of multiple trajectory departure points from a cell $V_{j'}$ which arrive in IC distinct cells V_j , Eq.(34) assigns equal probability to each transition and Eq.(34) reduces to Eq.(36). This procedure is similar to "sampling" approach of Ref.[40]. However, the situation where there are two or more of the multiple trajectories leaving a given cell $V_{j'}$ at given t and for a given n' and arriving in the same cell V_j is not accounted for by either Eq.(35) or (36). In this situation, it is not clear how $g(j|j',n')$ can be determined from Eq.(34). For this special case, we propose to calculate $g(j|j',n')$ in a manner equivalent to CCMT calculation (i.e. Eq.(41)) from

$$g(j|j',n') = \frac{N_{j,j'}(n')}{I_{j'}} \quad (64)$$

where $N_{j,j'}(n')$ is the number of system trajectories arriving in cell V_j which have originated from cell $V_{j'}$ for a given n' and $I_{j'}$ is the total number of trajectories leaving cell $V_{j'}$. As it will be shown in the next section, Eq.(64) is a Monte Carlo scheme to approximate Eq.(34). Calculating $g(j|j',n')$ using Eq.(64) eliminates the first difficulty with OSU-DM presented in Section 2.4. The second and third difficulties associated with the generalization of OSU-DM and computational requirements will be addressed in Chapter IV.

3.3 Theoretical Basis for CCMT

In order to derive the CCMT algorithm from Eq.(59), we replace η with α to obtain

$$P(x, \alpha, t+\tau) = \int d\alpha' \int dx' \delta(x - \bar{x}(x', \alpha')) P(\alpha, t+\tau | x(t)=x', \alpha(t)=\alpha', x(t+\tau)=x) \\ \times P(x', \alpha', t) \quad (65)$$

where now

$$\bar{x}(x', \alpha') = \int_t^{t+\tau} f(x(t'), \alpha') dt'. \quad (66)$$

Assume that the changes in the dynamic system variables and system parameters are statistically independent and the pdf for the system parameters, $P(\alpha)$, is given. The assumption of statistical independence includes, for example, random fluctuations of the system parameters around some mean value but excludes the situations where the magnitude of the fluctuations may be influenced by the changes in the dynamical variables. Then

$$P(\alpha, t+\tau | x(t)=x', \alpha(t)=\alpha', x(t+\tau)=x) = P(\alpha(t+\tau)) = P(\alpha) \quad (67)$$

and

$$P(x, \alpha, t) = P(x, t) P(\alpha). \quad (68)$$

The substitution of Eqs.(67) through (68) into Eq.(65) and integration over all possible α yields

$$P(x, t+\tau) = \int d\alpha' \int dx' \delta(x - \bar{x}(x', \alpha')) P(\alpha') P(x', t). \quad (69)$$

In order to find the cell-to-cell transition probabilities, partition the state-space region of interest for the analysis (i.e. R) into computational cells V_j , ($j=1, \dots, J$) and let V_{J+1} be the sink cell as before. Then approximate $P(x, t)$ as

$$P(x, t) = \sum_{j=1}^J \frac{P_j(t) e_j(x)}{v_j}, \quad e_j(x) = \begin{cases} 1 & \text{if } x \in V_j \\ 0 & \text{otherwise} \end{cases}, \quad P_j(t) \equiv \int_{V_j} dx P(x, t) \quad (70)$$

where v_j is the volume of the cell V_j . Since $P_j(t)$ ($j=1, \dots, J$) is the probability that $x(t)$ is

within V_j by definition, Eq.(70) assumes that the location of the dynamical variables within a cell V_j is uniformly distributed over that cell at all times. Substitution of Eq.(70) into Eq.(69) and integration over V_j yields

$$P_j(t+\tau) = \sum_{j'=1}^{J+1} g_{jj'}(\tau) P_{j'}(t) \quad (71)$$

where

$$g_{jj'}(\tau) = \begin{cases} \frac{1}{v_{j'}} \int d\alpha' \int_{V_{j'}} dx' e_j(\tilde{x}(x', \alpha')) P(\alpha') & \text{if } V_{j'} \in R \\ 1 & \text{if } j'=j=J+1 \\ 0 & \text{if } j'=J+1 \text{ and } j \neq J+1 \end{cases} \quad (72)$$

and the function $e_j(x)$ is as defined in Eq.(70).

A comparison of Eqs.(41), (42), (47) and (70) through (72) leads to the following observations:

1. An implicit assumption in CCMT in the determination of cell-to-cell transition probabilities from Eq.(41) or Eq.(47) is that the probability of finding the dynamic variables around a given point in a given cell is uniformly distributed over that cell. For fixed $\alpha=\alpha'$, the $g_{jj'}(\tau)$ improve with decreasing size of $V_{j'}$ and increasing number of subcells per $V_{j'}$.
2. Recipe 1 of CCMT (Section 2.5) approximates Eq.(72) for fixed $\alpha=\alpha'$ by finding the region in $V_{j'}$ for which $e_j(\tilde{x}(x', \alpha'))=1$
3. Since both $1/I_{j'}$ in Eq.(41) and $dx'/v_{j'}$ in Eq.(72) can be interpreted as the probability distributions of the dynamical variables within $V_{j'}$ under the assumption of uniform distribution, the interior and boundary sampling process of Eq.(41) or the Recipe 2 in

Section 2.5 can be regarded as a Monte Carlo scheme to approximate the integral over $V_{j'}$ in Eq.(72).

4. If Eq.(47) is regarded as the extension of the Monte Carlo approach of Recipe 2 to the system parameter space, the simplest interpretation of $\tilde{P}(\alpha_m)$ ($m=1,\dots,M$) in Eq.(47) is that it is $1/M$, with α_m selected by sampling over $P(\alpha)$ with no further assumptions. Other possible ways of selecting α_m for the Monte Carlo evaluation of the integral in Eq.(72) over the parameter space, corresponding interpretations of $\tilde{P}(\alpha_m)$ and a discussion of the associated error magnitudes can be found in [69]. If α_m are chosen independent of $P(\alpha)$ as [40], [63], and [64] seem to imply, then the integral over the parameter space in Eq.(72) is being evaluated by quadratures rather than Monte Carlo techniques. For this integral, integration by quadratures carries the assumption that each α_m is uniformly distributed within some region of the system parameter space and $\tilde{P}(\alpha_m)$ need to be interpreted as the integral of $P(\alpha)$ over these regions (see Section 3.2).

Observation 1 is intuitively obvious and is demonstrated in [40] by numerical experiments. From Observations 2 and 3, the SCM is seen to be a special case of Recipe 2 as expected, i.e. using a single point to represent the integrand in Eq.(72) for the Monte Carlo evaluation of the integral over $V_{j'}$. Observations 2 and 3 also imply that if $V_{j'}$ or its subcells have different sizes, Recipe 1 is more appropriate than Recipe 2 for the determination of $g_{jj'}(\tau)$. It is interesting to note that, if the integration over $V_{j'}$ in Eq.(72) is performed using quadratures instead of the Monte Carlo scheme of Recipe 2, the numerator in Eq.(41) is replaced by the sum of the volumes of $V_{j'}$ subcells whose image cell is V_j , the denominator by $v_{j'}$ (i.e. the total volume of $V_{j'}$) and Eq.(41) reduces to Recipe 1. Observation 4 suggests several possibilities for selecting α_m and quantifying $\tilde{P}(\alpha_m)$ in Eq.(47).

3.4 Determining the Limiting Probability Distribution of PGs

As indicated earlier, Eq.(47) requires the $g_{j,j'}(\tau)$ to be recomputed every time $P(\alpha)$ is changed (and hence the trajectory arrival points from Eq.(40)). On the other hand, it will be later shown in Chapter V that the determination of the trajectory arrival points can account for 96% of the computational time spent in determining the PGs and their probability distributions and 41% of the total time for the global analysis. Using the theoretical basis provided by Section 3.3, this recomputation process can be avoided with the following approach: In an analogous manner to Eq.(70), partition the system parameter space into M cells $\tilde{V}_m$ ($m=1,\dots,M$) and approximate $P(\alpha)$ by a set of piecewise constant functions as

$$P(\alpha) = \sum_{m=1}^M \frac{\tilde{P}_m e_m(\alpha)}{\tilde{v}_m}, \quad e_m(\alpha) = \begin{cases} 1 & \text{if } \alpha \in \tilde{V}_m \\ 0 & \text{otherwise} \end{cases}, \quad \tilde{P}_m \equiv \int_{\tilde{V}_m} d\alpha P(\alpha) \quad (73)$$

where $\tilde{v}_m$ is the volume of the cell $\tilde{V}_m$. Equations (71), (72) and (73) show that

$$P_j(t+\tau) = \sum_{j'=1}^{J+1} \sum_{m=1}^M \tilde{g}_{j,j',m}(\tau) \tilde{P}_m P_{j'}(t) \quad (74)$$

where now the cell-to-cell transition probabilities are defined separately for each fixed $\alpha_m \in \tilde{V}_m$ as

$$\tilde{g}_{j,j',m}(\tau) = \begin{cases} \frac{1}{v_{j'}} \int_{v_{j'}} dx' e_j(\tilde{x}(x', \alpha_m)) & \text{if } V_{j'} \in R \\ 1 & \text{if } j'=j=J+1 \\ 0 & \text{if } j'=J+1 \text{ and } j \neq J+1 \end{cases} \quad (75)$$

Since α_m are fixed and hence independent of the choice of $P(\alpha)$, Eq.(74) can be alternatively expressed as

$$P_{j,m}(t+\tau) = \sum_{j'=1}^{J+1} \tilde{g}_{j,j',m}(\tau) P_{j',m}(t) \quad (76)$$

$$P_j(t) = \sum_{m=1}^M \tilde{P}_m P_{j,m}(t). \quad (77)$$

Then the transition probabilities $\tilde{g}_{j,j',m}(\tau)$ need to be calculated only once to find $P_{j,m}(k\tau)$ ($k=0,1,2,\dots$) iteratively from Eq.(76) and these $P_{j,m}(k\tau)$ can be used with $\tilde{P}_m$ corresponding to different $P(\alpha)$ to find $P_j(k\tau)$ from Eq.(77). As before, P_j^∞ (see Eq.(44)) yields the probability distribution of PGs and the results can be improved by the increasing number of $\tilde{V}_m$ used to partition the system parameter space. While the approaches of [40, 63, 64] (i.e. Recipe 2 in Section 2.5) also use a selected set of α_m to represent the parameter space and the approach of [65] also leads to computational time savings in the determination of $g_{j,j'}(\tau)$, the procedure described by Eqs.(73)-(77) is substantially different from these approaches for the following reasons:

- In the previous implementations of CCMT to systems with stochastic parameters, the procedures used have been somewhat application specific (i.e. Gaussian white noise as indicated in Section 2.5.1). Equations (73)-(77) can be used with any given $P(\alpha)$.
- The Markov chain defined by Eq.(76) yields the conditional probability that $\mathbf{x}(t) \in V_j$ given $\alpha = \alpha_m$ (i.e. $P_{j,m}(t)$). These conditional probabilities are independent of the pdf describing the stochasticity in system parameters and can be used with $\tilde{P}_m$ corresponding to different $P(\alpha)$ as indicated above. The Markov chain defined by Eq.(42) yields the probability that $\mathbf{x}(t) \in V_j$ (i.e. $P_j(t)$) directly and subsequently $P_j(t)$ are $P(\alpha)$ dependent as can be also seen from Eq.(77). Equations (42), (47), (76) and (77) show that Recipe 2 in Section 2.5 is equivalent to computing the cell-to-cell transition probabilities from

$$g_{j,j'}(\tau) = \sum_{m=1}^M \tilde{g}_{j,j',m}(\tau) \bar{P}_m$$

for each $P(\alpha)$ separately. The quantities $\tilde{g}_{j,j',m}$ in Eq.(75) correspond to the ratio $N_{j,j'}(\alpha_m)/I_{j'}$ in Eq.(47) and subsequently Recipe 2 recalculates the $\tilde{g}_{j,j',m}(\tau)$ every time $P(\alpha)$ is changed while they need to be only calculated once with Eqs.(73)-(77).

- Section 2.5.1 and Eq.(69) show that the exact expression for $P(x, t+\tau|x', t)$ used by Ref.[65] is

$$P(x, t+\tau|x', t) = \int d\alpha' \delta(x - \tilde{x}(x', \alpha')) P(\alpha').$$

In the approximation of $P(x, t+\tau|x', t)$ to determine $g_{j,j'}(\tau)$, Ref.[65] assumes that $P(x, t+\tau|x', t)$ is some specified distribution, with the mean and variance as obtained from the system equations given that the system departs from some fixed $x' = x_0$ at t (see Eq.(5) and Example 1 of Ref.[65]). However, Eqs. (75)-(77) above only approximate $P(\alpha')$ (by Eq.(73)) and account for the exact variation of $P(x, t+\tau|x', t)$ with the departure point x' through the function $e_j(\tilde{x}(x', \alpha_m))$.

In principle, all the non-zero P_j^∞ obtained from Eqs.(76) and (77) correspond to persistent cells. On the other hand, some transient cells may be inadvertently included in the predicted PG structure, since only a finite number of iterations (i.e. k) can be used to approximate P_j^∞ from Eqs.(76) and (77). For example, if the convergence criterion (or the stopping rule) is

$$\max_j \frac{|P_{j,m}(k\tau) - P_{j,m}((k-1)\tau)|}{P_{j,m}(k\tau)} \leq \varepsilon_1, \quad (78)$$

where ε_1 is a specified small number, a transient cell V_j may satisfy the criterion if it takes a

long time (i.e. many τ s) for a trajectory starting from this cell to be absorbed in a persistent group, depending on how small ϵ_1 is. This problem is not relevant to the implementation of the CCMT as described in Section 2.5.2, since the persistent and transient cells are determined by starting from a cell that is periodic under a selected SCM compatible with the given GCM and following its mapping evolution while considering all the possible multiple images of the cell for the given GCM. However, the problem can be eliminated by comparing the approximate mean trajectory return times for the cells (i.e. $r_j = \tau/P_{j,m}(k\tau)$) to the maximum return time. Since a trajectory starting from a subcell of cell V_j (which has a total of I_j subcells) can at most visit all the other subcells in R before returning to the same subcell, this maximum return time is

$$r_{\max} = \tau \sum_{j=1}^J I_j. \quad (79)$$

Subsequently, if $r_j > r_{\max}$ (or $P_{j,m}(k) < 1/\sum_{j=1}^J I_j$), then V_j is a transient cell. A few transient cells may still remain within the predicted PGs due to numerical roundoff even after the application of this criterion, but such cells can be eliminated during the determination of BOAs and their absorption probabilities, as described in the next section.

3.5 Determination of PGs, BOAs and Absorption Probabilities

For the determination of the PGs, the new algorithm also follows the mapping evolution of the cells (Section 2.5.2). The main difference is that the new algorithm does not need to use SCM to determine the starting set of cells towards the identification of the PGs as CCMT described in Section 2.5.2 does, since the persistent cells are identified earlier as described in Section 3.4.

The approach used in the determination of BOAs and their absorption probabilities, however, is again fundamentally different from the CCMT approach described in Section 2.5.2 and does not require either the creation of pre-image arrays or the inversion of the matrix $(I - Q)$ (see Section 2.5.2). For this purpose, the new algorithm uses the k -step transition probabilities, $\tilde{p}_{j,i,m}(k+1)$, from V_i to V_j for fixed α_m . A recurrence relationship for these probabilities (k -step transition probabilities) can be obtained by letting $w=u+(k+1)\tau$, $v=u+\tau$ and $\eta = \alpha$ in Eq. (52) to get

$$P(x, \alpha, u+(k+1)\tau | x'', \alpha'', u) = \int d\alpha' \int dx' P(x, \alpha, u+(k+1)\tau | x', \alpha', u+\tau) \\ \times P(x', \alpha', u+\tau | x'', \alpha'', u). \quad (80)$$

Then for a fixed value of $\alpha'' = \alpha = \alpha_m$, Eq.(80) becomes

$$P(x, \alpha_m, u+(k+1)\tau | x'', \alpha_m, u) = \int d\alpha' \int dx' P(x, \alpha_m, u+(k+1)\tau | x', \alpha', u+\tau) \\ \times \delta(\alpha' - \alpha_m) P(x', u+\tau | x'', \alpha_m, u), \quad (81)$$

which yields

$$P_m(x, u+(k+1)\tau | x'', u) = \int dx' P_m(x, u+(k+1)\tau | x', u+\tau) P_m(x', u+\tau | x'', u), \quad (82)$$

where, for notational convenience, the subscript m indicates that all the quantities are being evaluated at $\alpha = \alpha_m$. In an analogous manner to Eq.(57), we have

$$P_m(x', u+\tau | x'', u) = \delta(x' - \tilde{x}(x'', \alpha_m)) \quad (83)$$

where

$$\tilde{x}(x'', \alpha_m) = \int_u^{u+\tau} f(x(t), \alpha_m) dt. \quad (84)$$

Substituting Eq.(83) into Eq.(82) we get

$$P_m(x, u+(k+1)\tau | x'', u) = \int dx' \delta(x' - \bar{x}(x'', \alpha_m)) P_m(x, u+(k+1)\tau | x', u+\tau). \quad (85)$$

Now, we partition the state-space region of interest for the analysis (i.e. R) into computational cells V_j and approximate $P_m(x, u+(k+1)\tau | x', u+\tau)$ in analogous manner to Eq.(70), i.e.

$$P_m(x, u+(k+1)\tau | x'', u) = \sum_{j=1}^J \sum_{j''=1}^J \frac{\tilde{p}_{j,j'',m}(k+1) e_j(x) e_{j''}(x'')}{v_j v_{j''}}, \quad (86)$$

where

$$\tilde{p}_{j,j'',m}(k+1) \equiv \int_{V_j} dx \int_{V_{j''}} dx'' P_m(x, u+(k+1)\tau | x'', u) \quad (87)$$

and

$$P_m(x, u+(k+1)\tau | x', u) = \sum_{j=1}^J \sum_{j'=1}^J \frac{\tilde{p}_{j,j',m}(k+1) e_j(x) e_{j'}(x')}{v_j v_{j'}}, \quad (88)$$

where

$$\tilde{p}_{j,j',m}(k+1) \equiv \int_{V_j} dx \int_{V_{j'}} dx' P_m(x, u+(k+1)\tau | x', u). \quad (89)$$

Substitution of Eqs.(86) and (88) into Eq.(85), integration over V_j and $V_{j''}$ and replacing j'' with i yields

$$\tilde{p}_{j,i,m}(k+1) = \sum_{j'=1}^{J+1} \tilde{g}_{j',i,m}(\tau) \tilde{p}_{j,j',m}(k) \quad (90)$$

where $\tilde{g}_{j',i,m}(\tau)$ is as given by Eq. (75). Then if γ indicates a PG, Eq.(90) shows that

$$\rho_{\gamma,i,m}(k+1) \equiv \sum_{j \in \gamma} \tilde{p}_{j,i,m}(k+1) = \sum_{j'=1}^{J+1} \tilde{g}_{j',i,m}(\tau) \rho_{\gamma,j',m}(k), \quad (91)$$

and subsequently, for fixed α_m , the absorption probability of trajectories starting from cell V_i in PG γ is

$$\rho_{\gamma,i,m}^{\infty} = \lim_{k \rightarrow \infty} \rho_{\gamma,i,m}(k). \quad (92)$$

To determine $\rho_{\gamma,i,m}^{\infty}$, the iterations with Eq.(91) are carried out until

$$\frac{|\rho_{\gamma,i,m}(k+1) - \rho_{\gamma,i,m}(k)|}{\rho_{\gamma,i,m}(k+1)} < \varepsilon_2 \quad (93)$$

for every i , where ε_2 is some specified small number, and then $\rho_{\gamma,i,m}^{\infty}$ is set to be equal to $\rho_{\gamma,i,m}(k+1)$. If $\rho_{\gamma,i,m}^{\infty}$ is non-zero for a cell V_i , then PG γ is a domicile of V_i . For single domicile transient cells, $\rho_{\gamma,i,m}^{\infty}$ is either 1 or 0. If $\rho_{\gamma,i,m}^{\infty} = 1$ (or more precisely $\rho_{\gamma,i,m}^{\infty} \geq 1 - \varepsilon_3$, where ε_3 is again a small specified number to account for numerical round-off), the cell V_i is a member of the BOA for PG γ . If V_i is a multiple domicile transient cell, then $\rho_{\gamma,i,m}^{\infty}$ is a number between 0 and 1 (or more precisely $\varepsilon_3 < \rho_{\gamma,i,m}^{\infty} < 1 - \varepsilon_3$) and indicates the absorption probability of trajectories starting from cell V_i in the PG γ . If there is uncertainty on α represented by a given $P(\alpha)$, then the absorption probability of V_i in PG γ is

$$\rho_{\gamma,i}^{\infty} = \sum_{m=1}^M \rho_{\gamma,i,m}^{\infty} \tilde{P}_m \quad (94)$$

where $\tilde{P}_m$ is as defined in Eq.(73).

The computational advantages of this approach will be illustrated in Chapter V.

CHAPTER IV

IMPROVEMENT OF IMPLEMENTATION PROCEDURES

Chapter III provided a theoretical basis that unifies DPVSM, OSU-DM and CCMT as a single methodology that can be used for a comprehensive safety and reliability analysis of dynamic process systems. As indicated in Section 1.4, such a comprehensive analysis requires identifying both

1. the possible accident scenarios, their consequences and their likelihood and the importance of their contribution to risk, and,
2. the envelope of system parameters and initial conditions that lead to acceptable system performance.

Chapter III also facilitates the identification process by providing fast computational procedures to investigate how the structure of the attractors (or the PGs) and BOAs and the limiting probability distribution of the PGs are affected by the uncertainty on system parameters.

The focus of this chapter is machine level implementation of the computational procedures that arise from the theoretical basis provided in Chapter III. Section 4.1 describes the generalization of the OSU-DM software to include systems other than the one studied in [47] and mechanization of the input preparation process. Section 4.2 describes how the new and the improved algorithms for determining PGs, BOAs and the limiting probability distribution of the PGs are implemented to obtain vectorizable and very fast procedures requiring much less storage. Section 4.3 shows how these procedures can be also used to determine

the set of initial conditions that lead to safe operation. The implementation procedures will be illustrated in Chapter V where the validation of the procedures are also discussed.

4.1 Generalization and Improvement of Existing OSU-DM Software

As indicated in Section 2.4, a practical difficulty with the implementation of OSU-DM is that the existing software is system specific. Also, partitioning of PVSS is manual and computational requirements may become prohibitive for systems with more than two controlled variables of interest or more than two components.

The OSU-DM has been developed for applications in situations where the system equations (i.e. Eqs.(16) or (50)) have been incorporated into a simulator and their analytical form is not readily available. We will denote by Type I simulator a simulator whose input structure allows fault injection at arbitrary time points. Otherwise, the simulator will be denoted as Type II simulator. This section presents the generalization and improvement of OSU-DM software for applications with Type II simulators. For Type I simulators, the implementation scheme has a lot of parallel with the improvement and the generalization of the implementation procedures for determining PGs, BOAs and the limiting probability distribution of the PGs and will be presented in Section 4.2.

The generalization and improvement of existing OSU-DM software for Type II simulators involved the following tasks:

1. Development of a mechanized and generalized algorithm for partitioning the PVSS into computational cells, and calculating the cell-cell-transition probabilities $g(j|j',n')$ (ALG-G).

2. Development of a subroutine for calculating the transition probabilities $h(n|n', j' \rightarrow j, \tau)$ (ALG-H).
3. Improvement and generalization of the algorithm for calculating $p_{n,j}(t)$ (ALG-P).

For each problem, the user has to provide an input file that contains the following:

1. The dimension of PVSS.
2. Minimum and maximum values of each variable that define the region of interest.
3. Number of magnitude intervals to be considered for each PVSS axis.
4. System setpoints for each controlled variable.

The ALG-G uses Input 1 and Input 2 to fix the region of interest, R . Then R is partitioned into computational cells using Inputs 3 and 4 and Criteria 1 and 2 presented in Section 2.1 for the partitioning of PVSS. A number is assigned to each cell inside R from 1 to J . Each region in the state-space corresponding to a Top Event is automatically represented by one large cell and a number from $J+1$ to $J+I$, sequentially. Then ALG-G searches through the data base consisting of time/component state/process variable magnitude records obtained from the simulator for the event scenarios generated by the OSU-DM driver and converts these records to time/component state/cell number records. The algorithm proceeds in the same manner as described in Ref.[47] for calculating $g(j|j', n')$'s, except that Eqs.(35) and (36) have been substituted by Eq.(64) which is a Monte Carlo scheme to approximate Eq.(34) (See Ref.[47] for more details about the procedure of implementing the $g(j|j', n')$ calculations). The ALG-G searches the time/component state/cell number records and determines the total number of departure points, $I_{j'}(n')$, leaving cell $V_{j'}$ and the number of these trajectory segments, $N_{j,j'}(n')$, that arrive in V_j for a given component state combination n' . Then

the cell-to-cell transition probabilities are calculated by dividing $N_{jj'}(n')$ by $I_{j'}(n')$ as suggested by Eq.(64) and stored on disk. A listing of ALG-G is provided in Appendix A.

The original OSU-DM algorithm for the determination of $p_{n,j}(t)$ reads all the $g(j|j',n')$ from the disk and stores them in active memory [47]. In the application of OSU-DM to determine the reliability of the feed-bleed of a BWR following LOCA [47], the $g(j|j',n')$ arrays account for more than 95% of the storage used by the algorithm. It then follows that by eliminating reading the whole data base into active memory, storage limitations can be reduced tremendously. The new algorithm ALG-P only reads the relevant $g(j|j',n')$ at each time step. This improvement in storage management leads to modeling larger number of components and controlled variables. Similarly, instead of calculating $h(n|n',j' \rightarrow j, \tau)$ and $p_{n,j}(t)$ in the same monolithic structure as the original OSU-DM algorithm does, ALG-H obtains the component state transitions as a function of time and trajectory transitions from the time/component state/cell number records and determines the $h(n|n',j' \rightarrow j, \tau)$ from the system specific information supplied by the user in the form of a subroutine. Since failure and repair data and possible failure modes are different for different systems, the user must provide this subroutine for each specific application, following the procedure and guidelines described in Ref.[47]. Then the $p_{n,j}(t)$ calculations are performed according to the recurrence relation given by Eq.(24).

The results of Tasks 1-3 have led to an improved and generalized OSU-DM. However, the user is still required to provide a driver for event sequence (scenario) generation for the simulator. The driver for Type II simulator cannot be generalized since each simulator has its own input/output format and interaction mode for the input of event sequences.

4.2 Implementation of the Improved Algorithms for Determining PGs, Their Limiting Probability Distributions and BOAs

The implementation of the improved algorithms for determining PGs, their limiting probability distributions and BOAs towards the identification of the envelope of system parameters and initial conditions that lead to acceptable system performance required the development of three procedures:

1. A procedure for discretizing the state-space into computational cells V_j and generating the system trajectory segments in the PVSS that describe how the system moves between V_j (ALG-D).
2. A fast (vectorizable) and storage efficient procedure to calculate limiting probability distributions for PGs (ALG-L).
3. A procedure for determining the PGs, BOAs and their absorption probabilities in PGs (ALG-B).

4.2.1 The Procedure ALG-D

The input data required for ALG-D are the following:

1. The dimension of PVSS.
2. Minimum and maximum values of each variable that define the region of interest.
3. Number of magnitude intervals to be considered in each PVSS axis.
4. System setpoints for each controlled variable.
5. Number of subintervals for each interval specified by Input 3.

6. Simulation time interval (τ).
7. Parameter values (α_m) and/or component state combinations (n) to be considered.

The discretization of PVSS with ALG-D is performed in a similar manner to the discretization process with ALG-G (Section 4.1), using Inputs 1 through 4. Then ALG-D proceeds as follows:

1. Partition each V_j obtained from the CVSS discretization process into subcells using Input 5.
2. Fix α_m and/or n using Input 7.
3. Assume that the process variables at $t=k\tau$ are at the center of each subcell (departure points) and find the location of the process variables at time $t=(k+1)\tau$ (i.e. arrival points) from Eq.(33) and/or (40) as appropriate.
4. Translate the departure and arrival points to corresponding cell numbers j .
5. Store the results in a data base, with each record consisting of the departure cell number followed by the arrival cell number for each initial condition (i.e. center of a subcell) and α_m and/or n .

This structure of the data base leads to tremendous storage savings, as will be explained in Sections 4.2.2 and 4.2.3. A listing of ALG-D, accompanied by step-by-step comments, is provided in Appendix A.

4.2.2 The Procedure ALG-L

The procedure ALG-L calculates the limiting probability distributions for PGs and needs the following inputs:

1. Total number of cells in the PVSS.
2. Number of subcells considered in each cell.
3. Convergence criteria ϵ_1 in Eq.(78).

Inputs 1 and 2 are obtained from the ALG-D inputs. Input 3 is an additional input that needs to be specified by the user. ALG-L reads the data base generated by ALG-D and arranges the entries in a $IM(J \times JS)$ matrix where each entry $IM(j', js)$ corresponds to the image (i.e. arrival cell) of cell $V_{j'}$ (departure cell) starting from subcell js . Then, denoting $p_{j,m}(t+\tau)$ by $PL(j)$ and $p_{j,m}(t)$ by $PK(j)$ in Eq.(76) for notational convenience and substituting $\tilde{g}_{j,j',m}(\tau)$ from Eq.(47) into Eq.(76) we obtain

$$PL(j) = \sum_{j'=1}^{J+1} \frac{N_{jj'}}{I_{j'}} PK(j') = \frac{1}{I_j} \sum_{j'=1}^{J+1} N_{jj'} PK(j'), \quad (95)$$

where $N_{jj'}$ denotes the number of trajectories that arrive at cell V_j given that they have originated from cell $V_{j'}$ and can be calculated from

$$N_{jj'} = \sum_{js=1}^{JS} \delta_{j,IM(j',js)}. \quad (96)$$

Substituting Eq.(96) into Eq.(95), we get

$$PL(j) = \frac{1}{I_j} \sum_{j'=1}^{J+1} \sum_{js=1}^{JS} \delta_{j,IM(j',js)} PK(j'). \quad (97)$$

Using Eq.(97) to calculate $PL(j)$ (i.e. $p_{j,m}(t+\tau)$ for fixed m) is very attractive since the $\tilde{g}_{j,j',m}(\tau)$ need not to be calculated or stored explicitly. Storing the $\tilde{g}_{j,j',m}(\tau)$ requires a $J \times J \times J$ matrix compared to only $J \times JS$ matrix for the storage of the images. Now we proceed to show how this new method can also be vectorized for supercomputing.

Figure 1 shows a flowchart of the probability calculations. The algorithm proceeds as follows:

1. Assign equal probabilities to all the cells in R at $t=0$, i.e.

$$PK(j) = \frac{1}{J} \quad (98)$$

2. Set the probabilities of all the cells to zero ($PL(j) = 0$)
3. Set $js = 1$
4. Set $j' = 1$
5. Set $PL(IM(j', js)) = PL(IM(j', js)) + PK(j')$
6. Increment j' by 1 until $j' = J + 1$
7. Increment js by 1 until $js = JS$
8. Set $PK(j) = PL(j) / JS$
9. Repeat steps 2-8 until the convergence criterion (i.e. Eq.(78)) is met.

Steps 3 through 7 are two independent DO loops to evaluate the double summation in Eq.(97). Step 8 is the division of the result by I_j which is equal to JS . Steps 1, 2 and 8 are vectorized over the index j . The computer program listing of ALG-L is provided in Appendix A. Note that although the above algorithm assumes fixed system parameters (α_m) and component states (n), the same algorithm can be also used with a slight modification in situations where there are stochastic transitions between α_m and/or n . In these situations, the $h(n|n', j' \rightarrow j, \tau)$ that indicate the probability of transitions should be calculated in a separate subroutine and multiplied by $PK(j')$ to determine $PL(j)$.

Fig.1 Flowchart for Calculating the PG Limiting Probability Distribution.

4.2.3 The Procedure ALG-B

As indicated in Section 2.5.2, the CCMT first selects a SCM compatible with the given GCM for the system to determine the PGs. Then, the the mapping evolution of each cell in the selected SCM is followed to determine if the cell is periodic or transient. After having determined the periodic cells for the selected SCM, the search for PGs starts by choosing a cell that is periodic under the selected SCM and following its mapping evolution to generate a cell sequence while considering all the possible multiple images of the cell for the given GCM. The selection of a compatible SCM and determining whether a cell is periodic or transient may be time consuming. Instead, ALG-B makes use of the limiting PG probability distributions found by ALG-L to distinguish between periodic and transient cells as described in Section 3.4. Then, each periodic cell is put to the persistence test based on some properties of finite Markov chains (as described in Section 2.5.2 and Ref.[40]) to determine the PG's. Since the persistence test has not been altered, the reader is referred to the original implementation of CCMT (Ref.[40]) where a detailed description of the persistence test is given.

In order to determine the BOAs and their absorption probabilities in PGs, ALG-B requires Inputs 1 and 2 of ALG-L (Section 4.2.2), the convergence criterion ϵ_2 of Eq.(93) and a small number ϵ_3 to account for roundoff error (Section 3.5) as input. Like ALG-L, ALG-B also reads the data base generated by ALG-D and arranges the entries in a matrix $IM(J \times JS)$. Then, denoting $\rho_{y,i,m}(k+1)$ by $PL(i)$ and $\rho_{y,j',m}(k)$ by $PK(j')$ in Eq.(91) again for notational convenience and substituting $\tilde{g}_{j,j',m}(\tau)$ from Eq.(47) into Eq.(91) we obtain

$$PL(i) = \sum_{j'=1}^{J+1} \frac{N_{j',i}}{I_i} PK(j') = \frac{1}{I_i} \sum_{j'=1}^{J+1} N_{j',i} PK(j') \quad (99)$$

where $N_{j,i}$ denotes the number of trajectories that arrive at cell V_j given that they have originated from cell V_i . Replacing j by j' and j' by i in Eq.(96) and substituting the result into Eq.(99) we get

$$PL(i) = \frac{1}{I_i} \sum_{j'=1}^{J+1} \sum_{js=1}^{JS} \delta(j' - IM(i, js)) PK(j') \quad (100)$$

which simplifies to

$$PL(i) = \frac{1}{I_i} \sum_{js=1}^{JS} PK(IM(i, js)). \quad (101)$$

Again, calculating $\rho_{\gamma,i,m}(k)$ using Eq.(101) leads to a vectorizable algorithm over the index i , and less storage as compared to calculating $\bar{g}_{j,j',m}(\tau)$ explicitly. Then ALG-B carries out the calculation of $\rho_{\gamma,i,m}(k)$ in the same way as described in Section 4.2.2 for ALG-L, except for the initialization Steps 1 and 2. Instead, ALG-B uses the following initializations

1. Set

$$PK(j) = \begin{cases} 1 & \text{if } V_j \in \gamma \\ 0 & \text{otherwise} \end{cases}$$

2. Set

$$PL(j) = \begin{cases} 1 & \text{if } V_j \in \gamma \\ 0 & \text{otherwise} \end{cases}$$

where γ indicates a cell belonging to a PG. Then ALG-B follows Steps 3 through 9 in Section 4.2.2 until the convergence criterion (i.e. Eq.(93)) is met. When this criterion is

met, the cells are classified according to the following criteria:

1. If $PL(j) = 1$ (or $PL(j) \geq 1 - \epsilon_3$), then cell V_j is BOA of PG γ .
2. If $0 < PL(j) < 1$ (or $\epsilon_3 < PL(j) < 1 - \epsilon_3$), then cell V_j is multiple domicile cell.
3. If $PL(j) = 0$ (or $PL(j) \leq \epsilon_3$), then cell V_j is not transient to PG γ .

4.3 Determining the Set of Initial Conditions That Lead to Acceptable Operation

For safety reasons or the establishment of the level of robustness of the system, determining the set of initial conditions under which the magnitudes of the process variables always stay within specified intervals may be more important than determining the PGs and BOA absorption probabilities the PGs. In this case, the periodic cells need not to be checked for persistence and the PGs need not to be identified. Also, only the absorption probabilities into the sink cell (which would represent the unacceptable operation range) need to be calculated in order to classify cells. Then if $PL(j)$ is the converged $\rho_{\gamma j, m}(k+1)$ as before, with γ indicating the sink cell, we have the following classification rules:

1. If $PL(j) = 0$ (or $PL(j) \leq \epsilon_3$), then system operation starting from cell V_j is acceptable (since V_j is not transient to the sink cell).
2. If $PL(j) = 1$ (or $PL(j) \geq 1 - \epsilon_3$), then system operation starting from cell V_j is unacceptable (since V_j is part of the BOA for the sink cell).
3. If $0 < PL(j) < 1$ (or $\epsilon_3 < PL(j) < 1 - \epsilon_3$), then system operation starting from cell V_j leads to unacceptable operation with probability $PL(j)$ and to acceptable operation with probability $1 - PL(j)$.

For this special application, the algorithm remains the same, except that the classification of periodic cells into PGs and the calculations of the absorption probabilities into the PG's are by-passed using GOTO statements in the computer program.

CHAPTER V

VALIDATION AND BENCHMARK

This chapter validates and benchmarks the procedures developed in Chapter IV. Section 5.1 describes Example System-1 (ES-1) used for illustrating the implementation of the new algorithm for OSU-DM (Section 5.2). The results and the validation of the algorithm are also given in Section 5.2. Section 5.3 presents Example System-2 (ES-2) which is used to benchmark the new procedures for determining the PGs, their limiting probability distributions and the BOAs (Section 5.4). Section 5.5 summarizes the findings of this chapter.

5.1 Example System-1

The Example System-1 (ES-1) under consideration is part of the emergency core cooling system of BWR/6 and has been also used for the work reported in Ref.[47]. Figure 2 shows the ES-1 layout, including some other components used for energy removal from the core under normal and emergency conditions. The ES-1 consists of: a) the high pressure core spray (HPCS) system which includes the HPCS pump, a jockey pump, a spray sparger, an injection valve (F004 in Fig.2) and the associated system piping, valves, controls and instrumentation, b) one safety relief valve (SRV1), and, c) the condensate storage tank and a motor operated valve (F001 in Fig.2). Following a small-break loss-of-coolant accident (SBLOCA) which incapacitates the reactor core isolation cooling (RCIC) system, the reactor vessel (RV) water level (L) decreases, but the RV pressure (P) increases, since the enthalpy

Fig.2 ES-1 Layout.

production in the RV coolant inventory due to core decay heat generation usually exceeds the enthalpy loss through the break. Under normal operation, SRV1 opens when P exceeds 1103 psi and both P and L start decreasing due to steam flow through SRV1. SRV1 closes when $P < 1103$ psi. When $L \leq -36$ in (measured with respect to some reference point in the RV), the HPCS pump turns on and the valves F001 and F004 open to inject makeup water from the condensate storage tank into the RV. If the condensate storage tank level is below a preset limit, F015 opens for the HPCS pump to obtain water from the suppression pool. The HPCS pump stays on and the valve F001 stays open after the first demand, but F004 closes when $L > +55$ in. The valve F012 provides by-pass into the suppression pool when F004 is closed. The function of the jockey pump is to keep the HPCS line pressurized. The HPCS system is not operable when $P < 300$ psi. SRV1 and F004 continue their cyclic operation until the low pressure core injection (LPCI) system needs to be initiated (i.e. when $P < 300$ psi and $L < -148$ in).

5.2 Benchmark and Validation of the Improved OSU-DM

Figure 3 shows the simulation of normal ES-1 operation for the break size (0.04 ft^2) considered by Ref.[47]. The simulation was carried out using a code which has been benchmarked against an actual BWR/6 simulator for a number of accident conditions, including SBLOCAs [70]. Figure 3 indicates that ES-1 can maintain the RV pressure and water level within $300 \leq P \leq 1110$ psi and $-148 \leq L \leq 60$ in for at least 60 minutes following 0.04 ft^2 break in the RCIC piping under normal operation. The Top Events associated with ES-1 malfunction are the same as the ones considered by Ref.[47] which are: a) $P > 1110$ psi (high pressure), b) $P < 300$ psi (low pressure), c) $L > +60$ in (high level), and, d) $L < -148$ in (low level). The Top Events do not have direct safety implications. For example, if SRV1 fails to open at $P = 1103$

Fig.3 Response of the ES-1 to 0.04 ft² Size SBLOCA Under Normal Operation.

psi, SRV2-SRV10 and SRV11-SRV19 in Fig.2 are set to open at $P=1113$ psi and $P=1123$ psi, respectively. Similarly, if $L < -148$ in due to HPCS system failure while $P > 300$ psi, the SRVs can be opened by operator action to reduce P to enable the LPCI operation. However, the Top Events reflect the demand for SRV2-SRV19 and the LPCI following the SBLOCA and subsequently these Top Event frequencies are needed to quantify the probability of RV overpressure and core uncover following the SBLOCA in the probabilistic risk assessment of the overall plant.

Although the ES-1 contains several valves, motors and pumps with associated instrumentation and piping, only their combined effect on the makeup water delivery and steam discharge capability is relevant for the prediction of pressure and water level evolution in the RV. Subsequently, Fig.2 implies that the ES-1 can be regarded to consist of two structural unit (SUs) or macrocomponents, with the valve F001 and the whole HPCS constituting SU1 and SU2 being SRV1 (see Fig.4). Each SU was assumed to have 3 possible states, i.e. good (i.e. operating normally), failed-on (for SU1) or failed-open (for SU2) and failed-off (for SU1) or failed-closed (for SU2). Table 1 shows the possible SU state combinations and Table 2 gives the SU failure rates data as reported in Ref.[47].

For the determination of $h(n|n',j' \rightarrow j, \tau)$ and $g(j|j',n',\tau)$, the P - L space was partitioned into 140 cells (i.e. V_j) as shown in Fig.5 to represent the allowed P and L ranges and the Top Events. A τ value of one minute was used for the determination of $h(n|n',j' \rightarrow j, \tau)$ and $g(j|j',n',\tau)$ as in Ref.[47]. The $\bar{x}(x',n')$ in Eq.(62) were generated by the same simulator used to obtain Fig.3. SBLOCA occurs at $t=0$, with the RV pressure and water level at SBLOCA occurrence being $P=1039.4$ psi and $L=+36$ in.

Figure 6 shows the variation of Top Event CDFs for the SBLOCA break size of 0.04 ft^2 . These results are the same as obtained by Ref.[47] and thus the generalized algorithm is

Fig.4 ES-1 Modularization.

Table 1 Possible SU State Combinations

SU States		SU State Combination Number (n)
SU1	SU2	
g	g	1
g	ff	2
g	fo	3
ff	g	4
ff	ff	5
ff	fo	6
fo	g	7
fo	ff	8
fo	fo	9

g:good fo: failed-on ff: failed-off

Table 2 SU Failure Rates as Reported in Ref.[47]

Macrocomponent	Failure Mode	Failure Rate (/hour)
SU1	Fails Off	$3.0\text{E-}3 (\lambda_{1ff})$
SU1	Fails On	$1.0\text{E-}5 (\lambda_{1fo})$
SU2	Fails Closed	$3.0\text{E-}4 (\lambda_{2ff})$
SU2	Fails Open	$1.0\text{E-}5 (\lambda_{2fo})$

Fig.5 P-L Space Partitioning for the Determination of Transition Probability Between Component States and Computational Cells V_j .

Fig.6 Top Events CDFs for ES-1 (Break Size = 0.04 ft²).

validated and benchmarked. The regions of constant CDF or the plateaus in Fig.6 originate from the time delay between SU failure and Top Event occurrence as determined by the PCS dynamics (see Fig.3). For example, Fig.6 shows that it takes at least 16 and 18 minutes for low pressure or low level conditions to occur, respectively, following the SBLOCA and SU failures. Ref.[47] has concluded that there may be competition between Top Events not only due to the dependence of Top Event occurrence on the order of component failure but also on the exact time of component failure and/or the exact magnitudes of the process variables at the time of component failure. The competition between Top Events will be addressed in more detail in Chapter VI.

The results in Fig. 6 were obtained using the State of Ohio CRAY-YMP, except for the simulations to generate the $\bar{x}(x',n')$ for the determination of $g(j|j',n',\tau)$ (see Eqs.(33) and (34) or (62) and (64)). These simulations were performed on VAX-8550 due to some internal functions in the simulator which could not be found in the CRAY library. The data base obtained from the simulations was transferred to CRAY-YMP environment to determine the Top Event CDFs. Table 3 compares the storage requirements and CPU time for both the OSU-DM algorithm as proposed in [47] and the improved algorithm described in Section 4.1. Table 3 shows that the new algorithm leads to higher computational efficiency, since the CPU time savings are by a factor of 6 and the storage savings exceed a factor of 20. Both the storage and CPU requirements have a direct impact on the number of components and controlled variables that can be modeled and the number of cells and the time intervals (or number of time steps) that can be considered in the failure analysis.

The automated discretization scheme has not been used in the above implementation in order to avoid altering the space discretization considered by Ref.[47]. The automated discretization scheme leads to more storage savings, if implemented, since each Top Event

Table 3 Comparison of Computation Requirements on a CRAY Y-MP For ES-1

Quantities Determined	OSU-DM as Originally Proposed	Improved Algorithm
$g(j', n')$ from Eq.(64)	50 s	20 s
Top Events CDFs at 60 minutes	540 s	55 s
Total CPU Time	590 s	85 s
Total Storage	130,000 entries	6,600 entries

can be represented by a single large cell. The testing of the discretization scheme on the state-space of ES-1 yielded Fig.7. Comparison of Figs.5 and 7 shows that the number of Top Events cells can be reduced from 52 to only 4 and that the total number of cells that represent the whole state-space can be reduced from 140 to 86. In terms of storage requirements, this translates to an additional factor of 2 savings.

The above results show that the new procedures developed in Section 4.1 for the implementation of OSU-DM lead to higher computational efficiency than the original OSU-DM algorithm. The automated algorithms also offer the user a lot of ease in implementing the methodology to other systems (or simulators), as it will be discussed in Chapter 6.

5.3 Example System-2

The Example System-2 (ES-2) chosen to illustrate the increase in the computational efficiency in determining the PGs, their limiting probability distributions and the BOAs due to the improved computational procedures presented in Sections 4.2 and 4.3 is the second order forced Duffing oscillator under harmonic forcing used in [40] and [71], i.e.

$$\frac{d^2x}{dt^2} + \alpha_1 \frac{dx}{dt} + \alpha_2 x + x^3 = B \cos(t) \quad (102)$$

or

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= B \cos(t) - (\alpha_1 x_2 + \alpha_2 x_1 + x_1^3) \end{aligned} \quad (103)$$

where x_1 and x_2 are the normalized displacement and velocity, respectively (i.e. dynamic variables), B is the normalized forcing amplitude, and α_1 and α_2 are the system parameters.

Fig.7 Discretization of CVSS Using the Mechanized and Generalized Algorithm.

Reference [40] shows that for $\alpha_1=0.25$, $\alpha_2=0.02$ and $B=8.5$, the PGs of the system of Eq.(103) consist of: a) three points corresponding to a third order subharmonic, and, b) a strange attractor.

For the purposes of illustration, α_1 and B in Eq.(103) were fixed as $\alpha_1=0.25$, $B=8.5$ and the following two cases were considered:

Case 1: α_2 is uniformly distributed over $0.005 \leq \alpha_2 \leq 0.035$, i.e.

$$P(\alpha_2) = \begin{cases} \frac{1}{0.035-0.005} & \text{if } 0.005 \leq \alpha_2 \leq 0.035 \\ 0 & \text{otherwise} \end{cases} \quad (104)$$

Case 2: α_2 is normally distributed over $0.005 \leq \alpha_2 \leq 0.035$, with mean value $\alpha_2=0.02$ and 90% confidence, which corresponds to a standard deviation of 0.00912 around this mean value, i.e.

$$P(\alpha_2) = \frac{e^{-\frac{(\alpha_2 - 0.02)^2}{2(0.00912)^2}}}{0.00912 (2\pi)^{1/2}} \quad (105)$$

The region of interest in the state-space for the analysis (i.e. R) was chosen to be $\{1.5 \leq x_1 \leq 4.2, -3 \leq x_2 \leq 6\}$ as in [40] and $150 \times 150 = 22,500$ uniform sized cells (i.e. V_j) were used to partition R (also as in [40]). For both Cases 1 and 2 above, the uncertainty range for α_2 was partitioned using 3 cells $\tilde{V}_m$ (see Section 3.4). Table 4 shows the definition of $\tilde{V}_m$, including other data (i.e. $\alpha_{2,m}$, $\tilde{v}_m$ and $\tilde{P}_m$) needed for Eqs.(73), (75) and (77). Since by definition,

$$\tilde{P}_m = \int_{\tilde{V}_m} d\alpha_2 P(\alpha_2), \quad (106)$$

Table 4 The Definition of $\tilde{V}_m$ and the Corresponding Data For Eqs.(73), (75) and (77)

Cell or Data Type	m=1	m=2	m=3
$\tilde{V}_m$	$0.005 \leq \alpha_2 \leq 0.015$	$0.015 \leq \alpha_2 \leq 0.025$	$0.025 \leq \alpha_2 \leq 0.035$
$\alpha_{2,m}$	0.01	0.02	0.03
$\tilde{v}_m$	$0.015-0.005=0.01$	$0.025-0.015=0.01$	$0.035-0.025=0.01$
$\tilde{P}_m$ from Eq.(104) and (106)	1/3	1/3	1/3
$\tilde{P}_m$ from Eq.(105) and (106)	0.269	0.462	0.269

the $\bar{P}_m$ in Table 4 were found from Eqs.(104) and (105) by integrating $P(\alpha_2)$ over the respective α_2 ranges given in Table 4. The transition probabilities $\tilde{g}_{j,j',m}(\tau)$ in Eq.(75) were determined by:

- using $\tau=2\pi$ to obtain a cell-to-cell mapping that is stationary in time (since $B \cos(t) = B \cos(t+\tau)$),
- using a fourth order Runge-Kutta scheme for the integration of Eq.(103) over $[t, t+\tau]$ with internal time step size of $2\pi/60$ to find the $\bar{x}(x', \alpha_{2,m})$ defined in Eq.(66) with $\alpha_{2,m}$ as given in Table 4, and,
- dividing each cell V_j ($j=1, \dots, J=22,500$) into 9 uniform subcells and choosing the departure point $x'=[x_1 \ x_2]^T$ to be the mid-point of each subcell to approximate the integral in Eq.(75).

5.4 Benchmark and Validation of the Improved CCMT

Figures 8-10 show the probability distribution of the PGs of the example system for $\alpha_2=0.01, 0.02, 0.03$, respectively, when $\alpha_1=0.25, B=8.5$ in Eq.(102). Fig.8-10 data were obtained from Eq.(74) recursively, using the convergence criterion $\epsilon_1=0.001$ in Eq.(78) for all $j=1, \dots, J+1=22,501$ (in about 120-150 iterations). Figs.8-10 show that while there is little change in the overall shape of the PGs with changing α_2 (compare regions marked 3 in Figs.8-10 and also regions marked 1 and 4), the limiting probability distribution of the PGs can change substantially. For example, comparison of the regions marked 2 in Figs.8 and 9 show that there is two orders of magnitude of change in P_j^∞ of some cells in this region. The change increases to three orders of magnitude with increasing α_2 (compare regions marked 2 in Figs.8 and 10).

Fig.8 Probability Distribution P_j^∞ of the Persistent Groups for ES-2 ($\alpha_2=0.01$).

- $1.0 \times 10^{-2} \leq P_i^\infty \leq 1.0 \times 10^{-1}$
- × $1.0 \times 10^{-3} \leq P_i^\infty \leq 1.0 \times 10^{-2}$
- + $1.0 \times 10^{-4} \leq P_i^\infty \leq 1.0 \times 10^{-3}$
- $1.0 \times 10^{-5} \leq P_i^\infty \leq 1.0 \times 10^{-4}$
- $1.0 \times 10^{-6} \leq P_i^\infty \leq 1.0 \times 10^{-5}$

Fig.9 Probability Distribution P_j^∞ of the Persistent Groups for ES-2 ($\alpha_2=0.02$).

- $1.0 \times 10^{-2} \leq P_i^\infty \leq 1.0 \times 10^{-1}$
- × $1.0 \times 10^{-3} \leq P_i^\infty \leq 1.0 \times 10^{-2}$
- + $1.0 \times 10^{-4} \leq P_i^\infty \leq 1.0 \times 10^{-3}$
- $1.0 \times 10^{-5} \leq P_i^\infty \leq 1.0 \times 10^{-4}$
- $1.0 \times 10^{-6} \leq P_i^\infty \leq 1.0 \times 10^{-5}$

Fig.10 Probability Distribution P_j^∞ of the Persistent Groups for ES-2 ($\alpha_2=0.03$).

Figures 11 and 12 correspond to Case 1 (i.e. Eq.(104)) and Case 2 (i.e. Eq.(105)), respectively. These figures show that there is virtually no change in the PG structure with changing pdf for α_2 (i.e. $P(\alpha_2)$) and a small change in the P_j^∞ for some j (compare regions marked 2 in Figs.11 and 12 and also regions marked 5). The maximum difference between the P_j^∞ in Figs.11 and 12 is 32%. However, a comparison of Figs.8-12 indicates that there is a "fattening" of PGs with uncertainty on system parameters (e.g. compare regions marked 3 in Figs.8 and 11), as expected from previous applications of CCMT to systems with stochastic parameters [63]. The data for Figs.11 and 12 were obtained from Figs.8-10 data and $\tilde{P}_m$ data in Table 4, using Eq.(77). Table 5 gives some statistical properties of the asymptotic system behavior for Cases 1 and 2 which may be also used to assess the impact of the choice of the pdf for α_2 on the structure of PGs. Table 5 data were obtained by using the Fig.11 and Fig.12 data in conjunction with Eqs.(45) and (46).

Figures 13, 14 and 15 show the PGs for $\alpha_2=0.01$, 0.02 and 0.03, respectively, which are obtained by the procedure described in Sections 2.5.2 and 4.2 (i.e. persistence test). The $\alpha_2=0.02$ value is the one considered in Ref.[40]. Comparison of Figs.13-15 to Figs.8-10 illustrates the discussion in Section 3.4 as to how transient cells may appear as part of the PGs depending on the convergence criterion (i.e. Eq.(78)). Figures 16 through 19 show the BOAs, absorption probabilities of the multiple domicile cells into the sink cell, the third subharmonic group and the strange (chaotic) attractor, respectively, for $\alpha_2 = 0.02$. The absorption probabilities and the BOAs are determined using the convergence criteria given in Section 3.4 with $\epsilon_2=0.0001$ and $\epsilon_3=0.01$. The PGs shown in Fig.14, BOA shown in Fig.16 and the absorption probabilities for the multiple domicile cells shown in Figs.17 through 19 are identical to those given in Ref.[40]. This agreement is important for the benchmarking of

- $1.0 \times 10^{-2} \leq P_i^\infty \leq 1.0 \times 10^{-1}$
- × $1.0 \times 10^{-3} \leq P_i^\infty \leq 1.0 \times 10^{-2}$
- + $1.0 \times 10^{-4} \leq P_i^\infty \leq 1.0 \times 10^{-3}$
- $1.0 \times 10^{-5} \leq P_i^\infty \leq 1.0 \times 10^{-4}$
- $1.0 \times 10^{-6} \leq P_i^\infty \leq 1.0 \times 10^{-5}$

Fig.11 Probability Distribution P_j^∞ of the Persistent Groups for ES-2 (α Uniformly Distributed, Case 1).

Fig.12 Probability Distribution P_i^∞ of the Persistent Groups for ES-2 (α Normally Distributed, Case 2).

Table 5 Some Statistical Properties of the ES-2 Asymptotic Behavior
For $\alpha_1=0.25$, $B=8.5$ and $0.005 \leq \alpha_2 \leq 0.035$

Statistical Property ^a	$P(\alpha_2)$ From Eq.(104)	$P(\alpha_2)$ From Eq.(105)
$\bar{x}_1$	2.44600	2.44564
$\bar{x}_2$	0.47363	0.47140
σ_1^2	0.32756	0.32745
σ_2^2	0.06347	0.06312

^a See Eqs.(45) and (46) for the definition of σ_l^2 , $\bar{x}_l$ ($l=1,2$)

- Third order subharmonic attractor (group 2)
- + Strange attractor (group 3)

Fig.13 Persistent Groups for ES-2 ($\alpha_2=0.01$).

- Third order subharmonic attractor (group 2)
- + Strange attractor (group 3)

Fig.14 Persistent Groups for ES-2 ($\alpha_2=0.02$).

- Third order subharmonic attractor (group 2)
- + Strange attractor (group 3)

Fig.15 Persistent Groups for ES-2 ($\alpha_2=0.03$).

- o BOAs for the sink cell
- BOAs for the third order subharmonic group
- + BOAs for the strange attractor
- "blank" Multiple domicile transient cells

Fig.16 Basins of Attraction for ES-2 ($\alpha_2=0.02$).

- $0.75 \leq \rho_{1,i}^{\infty} \leq 0.99$
- × $0.50 \leq \rho_{1,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{1,i}^{\infty} \leq 0.50$
- $0.01 \leq \rho_{1,i}^{\infty} \leq 0.25$

Fig.17 Absorption Probabilities $\rho_{1,j}^{\infty}$ of Multiple Domicile Cells into the Sink Cell (Group 1) for $\alpha_2=0.02$.

$$\bullet \quad 0.75 \leq \rho_{2,i}^{\infty} \leq 0.99$$

$$\times \quad 0.50 \leq \rho_{2,i}^{\infty} \leq 0.75$$

$$+ \quad 0.25 \leq \rho_{2,i}^{\infty} \leq 0.50$$

$$\circ \quad 0.01 \leq \rho_{2,i}^{\infty} \leq 0.25$$

Fig.18 Absorption Probabilities $\rho_{2,j}^{\infty}$ of Multiple Domicile Cells into the Third Order Subharmonic Attractor (Group 2) for $\alpha_2=0.02$.

- $0.75 \leq \rho_{3,i}^{\infty} \leq 0.99$
- × $0.50 \leq \rho_{3,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{3,i}^{\infty} \leq 0.50$
- $0.01 \leq \rho_{3,i}^{\infty} \leq 0.25$

Fig.19 Absorption Probabilities $\rho_{3,j}^{\infty}$ of Multiple Domicile Cells into the Strange Attractor (Group 3) for $\alpha_2=0.02$.

the improved algorithm. The absorption probabilities of the multiple domicile cells for $\alpha_2=0.01$ and 0.03 can be found in Appendix B.

Figures 20 and 21 show the BOAs for $\alpha_2=0.01$ and 0.03 , respectively. Comparison of Figs.16 and 20 illustrates change in the structure of the BOAs with changing α_2 (i.e. from $\alpha_2=0.02$ to $\alpha_2=0.01$). Some multiple domicile cells between the BOAs for the third order subharmonic group and the strange attractor become part of the BOA for the third order subharmonic group (compare the regions marked 1 in Figs.16 and 20), the multiple domicile cell region marked 2 in Fig.20 moves into the BOA for the strange attractor as can be seen in Fig.16 and some multiple domicile cells near the BOA for the sink cell (region marked 3 in Fig.16) become part of the BOA for the third order subharmonic group (see Fig.20). Comparison of Figs.18 and 22 show how the BOA boundaries may get "blurred" when there is uncertainty on the system parameters. The blank cells in the region marked 1 in Fig.18 are part of the BOA for the third order subharmonic attractor when $\alpha_2=0.02$ (see Figs.16), but when α_2 is uniformly distributed over $0.005 \leq \alpha_2 \leq 0.035$ (i.e. Case 1), their probability of being absorbed in this attractor decreases and the probability of being absorbed in the strange attractor increases as shown in Fig.22. A similar trend is seen in the cells of the region marked 2 in Figs.18 and 22.

The probability distribution of the cells being absorbed in the third order subharmonic group for α_2 being normally distributed over $0.005 \leq \alpha_2 \leq 0.035$ (i.e. Case 2) is shown in Fig.23. A comparison of the regions marked 1 and 2 in Figs.22 (Case 1) and 23 (Case 2) indicates that there is a difference in the limiting PG probability distributions. However, this difference does not exceed 17%. On the other hand, the absorption probabilities into the third order subharmonic attractor in Fig.23 are relatively much closer to the absorption probabilities shown in Fig.18 for $\alpha_2=0.02$ than the ones obtained in Fig.22. This observation can

- BOAs for the sink cell
- BOAs for the third order subharmonic group
- + BOAs for the strange attractor
- "blank" Multiple domicile transient cells

Fig.20 Basins of Attraction for ES-2 ($\alpha_2=0.01$).

- BOAs for the sink cell
- BOAs for the third order subharmonic group
- + BOAs for the strange attractor
- "blank" Multiple domicile transient cells

Fig.21 Basins of Attraction for ES-2 ($\alpha_2=0.03$).

Fig.22 Absorption Probabilities $p_{2,j}^{\infty}$ into Third Order Subharmonic Attractor (Group 2) for α_2 Uniformly Distributed (Case 1).

Fig.23 Absorption Probabilities $\rho_{2,j}^{\infty}$ into Third Order Subharmonic Attractor (Group 2) for α_2 Normally Distributed (Case 2).

be explained by Table 4, where $\bar{P}_m$ for $\alpha_2=0.02$ is 0.462 as compared to 0.269 for $\alpha_2=0.01$ and 0.03 in Fig. 23, and 0.333 for each value of α_2 in Fig.22. The absorption probabilities into other attractors are included in Appendix B.

The computation times for the various steps in the implementation of CCMT as originally proposed (Method 1) and the improved algorithm (Method 2) are shown in Table 6. The total CPU times used by Methods 1 and 2 for a single α_2 value is 540 s and 245 s on the State of Ohio Cray Y-MP, respectively. The total CPU time to generate the data for both Figs.11 and 12 using Method 2 was about 12 minutes, with almost all of this time used for the generation of Fig.8-10 data. On the other hand, if Method 1 were used to generate the data for Figs.11 and 12, the data generation time for *each* Fig.11 and 12 would have been 12 minutes (i.e. $3 \times (220+10) = 690$ seconds) and the total data generation time would have doubled. Furthermore, this approach would not have yielded the information in Figs.8-10 which is important to understand the sensitivity of the structure of PGs to changes in system parameters. Table 6 also shows that the new algorithm used for the determination of the PGs, BOAs and their absorption probabilities (Section 4.2) reduces the CPU time by a factor of about 20. The estimated reduction in storage requirements due to this new algorithm is by a factor of 2.5 not counting the storage requirement of the original CCMT to invert the matrix $(I-Q)$.

5.5 Summary

The algorithms developed in Chapter IV have been validated and benchmarked. The two example systems chosen for the validation and benchmark (i.e. ES-1 and ES-2) show that the new algorithms lead to higher computational efficiency than the originally proposed OSU-DM and CCMT algorithms. In the failure analysis of ES-1, the CPU savings are by a

Table 6 Comparison of Computation Times on a CRAY Y-MP for a Single α_2 For ES-2

Quantities Determined	CCMT as Originally Proposed	Improved Algorithm
$g_{ij}(\tau)$ from Eq.(41)	220 s	220 s
Probability Distribution of PGs from $g_{ij'm}(\tau)$	10 s	10 s
PGs, BOAs and Cell Absorption Probabilities into PGs	310 s	15s

factor of 6 and the storage savings exceed a factor of 20. In the global analysis of ES-2, the CPU time is reduced by a factor of 20 and the storage requirements by a factor of 2.5. The combination of CPU time and storage savings have direct impact on the number of components and controlled variables that can be modeled and the number of cells and the time steps that can be considered in the failure analysis and/or the global analysis of PCS, as it will be demonstrated in the next chapters (i.e. Chapters VI, VII and VIII).

CHAPTER VI
EXAMPLE APPLICATION I:
TOP EVENT SENSITIVITY TO UNCERTAINTIES IN SYSTEM DYNAMICS

An attractive feature of dynamic methodologies arising from their capability to explicitly account for the time-element in system evolution is that they can quantify Top Event sensitivity to uncertainties in system dynamics. As indicated in Section 5.2, it has been shown that in process systems with multiple Top Events, there may be competition between Top Events not only due to the dependence of Top Event occurrence on the order of component failure but also on the exact time of component failure and/or the exact magnitudes of the process variables at the time of component failure [47]. Reference [47] shows that such a competition can lead to large differences in the interval reliability of process control systems as predicted by dynamic methodologies and the fault-tree approach. Similarly, Ref.[72] shows that, although the probability of auxiliary feedwater system failure in pressurized water reactors during 30 minutes following a break in the feedwater line can change by one order of magnitude depending on whether operator intervention is accounted for or not, this change cannot be predicted by the fault-tree approach. Ref.[74] shows that even for a very simple tank level control system with a single proportional controller which controls the inflow into the tank, the steady-state probabilities for tank overflow and dryout can be significantly different depending on whether the tank outflow is forced or gravity driven.

A further complication in quantifying Top Event sensitivity to uncertainties in system dynamics is that, for intermittently operated components, the demand frequency for the component may be a stochastic variable not only due to the uncertainty associated with the process physics but also due to the interaction between the process variable evolution and the behavior of the other system components. A common approximation made in these situations with the event-tree/fault-tree approach is to calculate the demand frequency for the component under the assumption that all system components operate normally and then define a failure rate for the component from its given failure-per-demand (FPD) data [75]. In view of the possible dependence of the Top Event occurrence on the exact time of component failure and/or the exact magnitudes of the process variables at the time of component failure as indicated above, it may be necessary to verify that this approximation is acceptable.

While the above arguments give qualitative justification as to why Top Events under consideration in risk and reliability studies may be sensitive to uncertainty in system dynamics and why static failure modeling schemes such as the fault-tree/event-tree approach may not have the capability to quantify the impact of this uncertainty on the statistical properties of the Top Events, very little effort has been spent to investigate the possible extent of the sensitivity and hence the need for dynamic methodologies for its quantification. Using ES-1 (see Section 5.1), this chapter investigates the impact of the uncertainty in BWR/6 dynamics following an SBLOCA on the probabilistic consequences of the SBLOCA, due to SBLOCA classification by the U.S. Nuclear Regulatory Commission (NRC). The U.S. NRC classifies SBLOCAs as [73] Class I if the break in the system piping is between 0.0001 to 0.005 ft², as Class II if the break is between 0.005 to 0.1 ft² and Class III if the break is between 0.1 to 0.2 ft². Since the break size affects the rate of pressure and water level evolution in the reactor vessel, it may subsequently affect demand frequencies for components/subsystems which

are activated to implement mitigating measures following the SBLOCA (e.g. pressure relief, makeup water injection). A change in the rate of pressure and water level evolution may also change the competition between the Top Events associated with the malfunction of the system components/subsystems. Three sets of analyses were carried out with the break sizes given in Table 7 to determine the Top Event CDFs:

1. Dynamic approach and the component failure data as reported in the literature.
2. Dynamic approach and converting the FPD data to failure rates as suggested in Ref.[75]
3. Fault-tree approach and converting the FPD data to failure rates as suggested in Ref.[75].

Section 6.1 describes the ES-1 behavior under normal operation as a function of break size, the Top Events under consideration and the computational approach. Section 6.2 shows how the failure data used in the analyses were obtained. Section 6.3 illustrates the determination of $h(n|n',j' \rightarrow j, \Delta t)$ from these data. Fault-trees for the Top Events of the system under consideration are presented in Section 6.4. Section 6.5 presents the dynamic methodology results and compares them to the results obtained by the fault-tree approach. Finally, Section 6.6 summarizes the results of this chapter.

6.1 System Description

The example system under consideration is the modularized ES-1 shown in Fig.4. Figure 24 shows simulations of the normal ES-1 operation for various break sizes covering Class I-Class III SBLOCAs (see Table 7 for case definitions), using the ES-1 simulator REMO-B [70]. Figure 24 indicates that the ES-1 can maintain the RV pressure and water level within $300 \leq P \leq 1112.5$ psi and $-148 \leq L \leq 80$ in for at least 60 minutes following $0.0004 - 0.1$ ft²

Table 7 SBLOCA Sizes Under Consideration for ES-1

Case	Class	Break Size (ft ²)	Fraction of Pipe Flow Area
D1	I	0.0004	1.00E-4
D2	I	0.0010	2.50E-4
D3	I	0.0020	5.00E-4
D4	I	0.0025	6.25E-4
D5	I	0.0040	1.00E-3
D6	II	0.0050	1.25E-3
D7	II	0.0070	1.75E-3
D8	II	0.0100	2.50E-3
D9	II	0.0200	5.00E-3
D10	II	0.0250	6.25E-3
D11	II	0.0400	1.00E-2
D12	II	0.0600	1.50E-2
D13	II	0.0800	2.00E-2
D14	III	0.1000	2.50E-2

Fig.24 Response of the ES-1 to Various Size SBLOCAs Under Normal Operation.

breaks in the RCIC piping under normal operation. The large pressure drops in Fig.24(b) are due to the HPCS turning on when $L < -36$ in. In view of Fig.24 data, the Top Events associated with ES-1 malfunction are now: a) $P > 1112.5$ psi (high pressure), b) $P < 300$ psi (low pressure), c) $L > +80$ in (high level), and, d) $L < -148$ in (low level).

Since REMO-B does not allow fault injection at arbitrary times interactively, OSU-DM was chosen for the dynamic failure modeling of ES-1. The investigations consisted of observing the variation of ES-1 Top Event CDFs in time during 60 minutes following the SBLOCA with the break sizes shown in Table 7 and the impacts of neglecting the stochasticity in component demand rates (i.e. converting the FPD data to failure rates as suggested in Ref.[75]) on the Top Event CDFs. For the determination of $h(n|n',j' \rightarrow j, \tau)$ and $g(j|j',n',\tau)$, the P - L space was partitioned into 140 cells (i.e. V_j) as shown in Fig.25 to represent the allowed P and L ranges and the Top Events in view of the new Top Event definitions (compare Fig.25 to Fig.5 which reflects the Top Event definitions of Ref.[47]). Several τ values in the 0.6 to 1 minute range were used for the determination of $h(n|n',j' \rightarrow j, \tau)$ and $g(j|j',n',\tau)$ for each break size to assure that the predicted CDFs do not substantially change with reducing τ (i.e. more than 25%-30%) and hence to validate the results.

6.2 Failure Data Preparation

Fig.4 shows that the modularized ES-1 consists of two SUs. SU2 consists of a single component (i.e. SRV1 in Fig.2), but SU1 consists of several active components in the HPCS suction and discharge paths. Failure data for all the ES-1 components shown in Fig.2, except for SRV1 failing-open, are reported as FPD in the literature [76] (see Table 8). Since SU1 consists of several components, failure data for SU1 were obtained by fault-tree analysis with

Fig.25 State-Space Partitioning for the Three Classes of Break Sizes.

Table 8 ES-2 Component Failure Data as Reported in Ref.[76]

Component	Failure Mode	Failure Rate/Probability
Motor Operated Valve	Fails to Open or Close	1.0E-4/demand
Check Valve	Fails Closed	1.0E-4/demand
Safety Relief Valve	Fails Closed	1.0E-5/demand
	Fails Open	1.0E-5/hour
Motor Driven Pump	Fails to Start	1.0E-3/demand
Relay	Fails to Energize	1.0E-4/demand

"SU1 fails-off" and "SU1 fails-on" as Top Events as shown in Fig.26. Only the components in the suction path of HPCS (i.e. the valves F001, F004, F004, F005, F024, F015, F016 and the HPCS pump) are considered in the fault-trees in Fig.26. The jokey pump and the associated components in the charging system are not included in Fig.26 because the SUs are assumed to be available at the time of LOCA initiation, and consequently, the charging system has no contribution to SU1 failure probability in the analyses performed. The failure data for the SU1 and SU2 as obtained from Fig.26 and Table 8 are shown in Table 9.

For the first set of analyses, the SU failure data were used in their given mixed format as shown in Table 9. For the second and third sets of analyses, the FPD data in Table 9 were converted to failure rates by: a) determining the average demand frequency on the SUs during 60 minutes following the SBLOCA from system simulations with both SUs operating normally for all the break sizes given in Table 7, and, b) multiplying the results with the relevant FPD probabilities. Table 10 gives the SU failure data after this conversion as a function of the break sizes under consideration (see Table 7) and indicates that the largest impact of the break size under normal operation is on the SU2 demand frequency, as can be also inferred from Fig.24. The failure rate for SU2 to fail-open ($\lambda_{fo,2}$) does not change in Table 10 with break size, since this value is taken from Table 8 and no conversion is necessary.

6.3 The Determination of Component State Transition Probabilities

Table 11 shows how the $h(n|n',j' \rightarrow j, \tau)$ were obtained for these analyses using a few sample $\{j',j\}$ and $\{n',n\}$ pairs. To illustrate the determination of $h(n|n',j' \rightarrow j, \tau)$ with Table 9 data, consider the situation that $n'=1$ and $j'=58-67$, i.e. both SUs are good at time t (see Table 1), RV water level is above the setpoint for SU1 to turn-on and RV pressure is

* due to Relay malfunction

Fig.26 Structural Unit 1 Fault-Trees.

Table 9 Mixed Format SU Failure Data

Macrocomponent	Failure Mode	Failure Rate/Probability
SU1	Fails Off	$2.4\text{E-}3/\text{demand } (d_{1ff})$
SU1	Fails On	$1.1\text{E-}3/\text{demand } (d_{1fo})$
SU2	Fails Closed	$1.0\text{E-}5/\text{demand } (d_{2ff})$
SU2	Fails Open	$1.0\text{E-}5/\text{hour } (\lambda_{2fo})$

Table 10 SU Failure Data As Failure Rates (sec^{-1})

Case	SU1		SU2	
	failed-off ($\lambda_{ff,1}$)	failed-on ($\lambda_{fo,1}$)	failed-closed ($\lambda_{ff,2}$)	failed-open ($\lambda_{fo,2}$)
D1	1.33E-06	6.11E-07	3.31E-07	2.78E-09
D2	1.33E-06	6.11E-07	3.22E-07	2.78E-09
D3	1.33E-06	6.11E-07	3.19E-07	2.78E-09
D4	1.33E-06	6.11E-07	3.19E-07	2.78E-09
D5	1.33E-06	6.11E-07	3.11E-07	2.78E-09
D6	1.33E-06	6.11E-07	3.06E-07	2.78E-09
D7	1.33E-06	6.11E-07	3.03E-07	2.78E-09
D8	1.33E-06	6.11E-07	2.94E-07	2.78E-09
D9	1.33E-06	6.11E-07	2.36E-07	2.78E-09
D10	1.33E-06	6.11E-07	2.22E-07	2.78E-09
D11	1.33E-06	6.11E-07	1.56E-07	2.78E-09
D12	2.00E-06	9.17E-07	7.22E-08	2.78E-09
D13	2.00E-06	9.17E-07	6.11E-08	2.78E-09
D14	2.00E-06	9.17E-07	5.28E-08	2.78E-09

Table 11 Some Sample SU State Combination Transition Probabilities, $h(n \ln', j' \rightarrow j, \tau)$

$n^{(a)}$	$n^{(a)}$	$j^{(b)}$	$h(n \ln', j' \rightarrow j, \tau)$	
			With Table 4 Data	With Table 5 Data
1	1	58-67	44-53	$(1-d_{ff,1})(1-\lambda_{fo,2\tau})$ $(1-\lambda_{ff,1\tau}-\lambda_{fo,1\tau})(1-\lambda_{fo,2\tau}-\lambda_{ff,2\tau})$
1	2	58-67	44-53	$(1-\lambda_{ff,1\tau}-\lambda_{fo,1\tau})$
1	1	58-67	54	$(1-d_{ff,1})(1-d_{ff,2}-\lambda_{fo,2\tau})$ $(1-\lambda_{ff,1\tau}-\lambda_{fo,1\tau})(1-\lambda_{fo,2\tau}-\lambda_{ff,2\tau})$
1	2	58-67	54	$\lambda_{fo,2\tau}$
2	1	All	All	0
9	9	All	All	0
All	All	85	85	1
All	All	1-84 86-140	85	1

^(a) See Table 7 for definitions^(b) See Fig.25 for definitions

below the setpoint for SU2 to open (see Fig.25). Then SU1 is good-off and SU2 is good-closed from the description of ES operation in Section 5.1. If P and L move to cells $j=44-53$ at time $t+\tau$ (see Fig.25), SU1 will be demanded to turn-on as L falls below the level setpoint and may fail-off with probability $d_{ff,1}$. Note that the probability that SU1 will stay good in this transition is $1-d_{ff,1}$ and not $1-d_{fo,1}-d_{ff,1}$. The demand probability $d_{fo,1}$ is only relevant if SU1 is demanded to turn-off when on but SU1 is good-off initially. Similarly, since there is no demand on SU2 to open in the transition from $j'=58-67$ to $j=44-53$ (Fig.25 shows that P is still below the pressure setpoint for cells $j=44-53$), $d_{ff,2}$ is not applicable for this transition and SU2 can only fail-open during the transition with probability $\lambda_{fo,2}\tau$. Subsequently, Table 11 shows that $h(1|1,j'\rightarrow j,\tau)=(1-d_{ff,1})(1-\lambda_{fo,2}\tau)$ and $h(1|2,j'\rightarrow j,\tau)=0$ for $j'=58-67$, $j=44-53$. If, however, $j=54$, Fig.25 implies that SU2 will be also demanded to open. In this case, SU2 can either fail-close or fail-open with probabilities $d_{ff,2}$ and $\lambda_{fo,2}\tau$, respectively, and $h(1|1,j'\rightarrow j,\tau)=(1-d_{ff,1})(1-d_{ff,2}-\lambda_{fo,2}\tau)$ for $j'=58-67$, $j=54$. In the determination of $h(n|n',j'\rightarrow j,\tau)$ with Table 10 data, SU failure in all modes needs to be considered at all $t=k\tau$, since the $\{j',j\}$ dependency of the FPD data is lost in the conversion process. That is why $h(1|1,j'\rightarrow j,\tau)$ accounts for SUs both failing on or off in the transitions $j'=58-67$ to $j=44-43$ and $j'=58-67$ to $j=54$ with Table 10 data and neither $h(1|1,j'\rightarrow j,\tau)$ nor $h(1|2,j'\rightarrow j,\tau)$ change with changing j . Rows 5 and 6 in Table 11 imply that the SUs are not repaired upon failure and rows 7 and 8 indicate that cell 85 corresponds to a Top Event (i.e. low pressure from Fig.25).

6.4 Fault-Tree Approach [47]

The fault-trees for the ES-1 Top Events, taken from Ref.[47], are shown in Fig.27. These fault-trees are the results of a diagraph analysis which considers the possible SUs failure modes in the modularized ES-1 configuration (see Table 1 and Fig.4) and is based on ES-1 operation under normal and abnormal conditions (Section 5.1), supplemented by REMO-B simulations.

Table 12 shows how the Top Events CDFs based on the fault-tree approach. The Top Events Cdfs are obtained from Fig.27 and assuming that both SUs are initially "good"(i.e. $t=0$ when the LOCA occurs). Table 12 indicates that the SUs failure data should be in failure rates thus only Table 10 data were used for this analysis.

6.5 Results and Analysis

Figs.28-30 and Figs.31-33 show the variation of Top Event CDFs for the SBLOCA break sizes given in Table 7 as predicted from Table 9 and Table 10 data, respectively. For all Cases D1-D14, SBLOCA occurs at $t=0$, with the RV pressure and water level at SBLOCA occurrence being $P=1039.4$ psi and $L=+36$ in. The regions of constant CDF or the plateaus in Figs.28-33 originate from the time delay between SU failure and Top Event occurrence, as well as the dependence of the CDFs in Figs.28-30 on the SU demand frequencies, as illustrated by the following examples:

Example 1: From the description of ES operation given in Section 5.1, it can be readily seen that high pressure Top Event can only occur if SU2 fails-closed. However, high pressure will not occur at the time of SU2 failure but when $P > 1112.5$ psi from the definition of the high pressure Top Event and the CDF will be constant during this time period. The long plateaus in Figs.28(a)-30(a) and the plateaus in Figs.31(a)-33(a) indicate how long it would take

Fig.27 The Fault-Trees for ES-1 Top Events.

Table 12 Top Event CDFs Based on a Fault-Tree Approach [2]

Top Event	CDF ^a
High Pressure	$\lambda_{2,2}y(\lambda_{2t})/\lambda_2$
Low Pressure	$\lambda_{1,3}y(\lambda_{1t})/\lambda_1 + \lambda_{2,3}y(\lambda_{2t})/\lambda_2 - \lambda_{1,3}\lambda_{2,3}y(\lambda_{1t})y(\lambda_{2t})/\lambda_1\lambda_2$
High Level	$\lambda_{1,3}y(\lambda_{1t})/\lambda_1 + \lambda_{1,2}\lambda_{2,2}y(\lambda_{1t})y(\lambda_{2t})/\lambda_1\lambda_2$
Low Level	$\lambda_{1,2}y(\lambda_{1t})/\lambda_1 + \lambda_{2,3}y(\lambda_{2t})/\lambda_2 - \lambda_{2,3}\lambda_{1,2}y(\lambda_{1t})y(\lambda_{2t})/\lambda_1\lambda_2$

$$^a y(t) = 1 - e^{-t}; \lambda_m = \lambda_{m,2} + \lambda_{m,3} (m=1,2)$$

(a)

(b)

(c)

(d)

Fig.28 Variation of the Top Event CDFs for Class I Type SBLOCAs As Obtained From Table 9 Data.

(a)

(b)

(c)

(d)

Fig.29 Variation of the Top Event CDFs for Class II Type SBLOCAs As Obtained From Table 9 Data.

(a)

(b)

(c)

(d)

Fig.30 Variation of the Top Event CDFs for Class II and Class III Type SBLOCAs As Obtained From Table 9 Data.

(a)

(b)

(c)

(d)

Fig.31 Variation of the Top Event CDFs for Class I Type SBLOCAs As Obtained From Table 10 Data.

(a)

(b)

(c)

(d)

Fig.32 Variation of the Top Event CDFs for Class II Type SBLOCAs As Obtained From Table 10 Data.

Fig.33 Variation of the Top Event CDFs for Class II and Class III Type SBLOCAs As Obtained From Table 10 Data.

for the pressure to exceed 1112.5 psi if water level falls below $L=-36$ in and SU1 turns-on at the time of SU2 failure. Note that this time period is about 15-20 minutes for Case D1, increases with increasing SBLOCA size and becomes infinite for $t>10$ minutes in Class III SBLOCAs (i.e. Case D14), in consistence with observations from Fig.24(b). Also, SU2 failing-closed is not relevant with Table 9 data unless there is a demand on SU2 to open (i.e. if $P>1103$ psi) and the Cdf will be again constant between demands. Comparison of Figs.28(a)-30(a) to Figs.31(a)-33(a) indicates that the short plateaus in Figs.28(a)-30(a) reflect the dependence of high pressure Top Event CDFs on the SU2 demand frequency when mixed format data of Table 9 are used. The plateaus in Figs.28(a)-30(a) merge into longer plateaus with increasing SBLOCA size, since SU2 demand frequency decreases with increasing SBLOCA size.

Example 2: Simulations of ES operation show that the high level Top Event can occur either by SU1 failing-on or due to level swell originating from excessive boiling if SU2 fails-closed when SU1 is off. The initial linear rise in Fig.28(c) implies that the high level occurrence during the first few minutes of a Class I SBLOCA initiation is caused by rapid level swell following SU2 failing-closed when P exceeds the pressure setpoint (i.e. 1103 psi), since: a) SU1 cannot fail-on unless it is first on and demanded to turn-off at $L=+55$ in when Table 9 are used, and, b) RV water level needs to reach $L=-36$ in for SU1 to turn-on. After SU1 turns-on, high level cannot occur until SU1 turns-off at $L=+55$ in and SU2 fails-closed (Fig.24(b) shows that pressure decreases when SU1 is on) or SU1 fails-on upon demand at $L=55$ in, which accounts for the first plateau in Fig.28(c). The second plateau in Fig.28(c) CDFs and the linear increases following the plateaus can be explained in a similar manner. Note that the plateau durations in Fig.28(c) are similar to the duration of the long plateaus in Fig.28(a), which is not surprising in view of the coupling between pressure and level changes

that can be seen from Fig.24. In Fig.31(c), we observe that the plateaus of Fig.28(c) disappear when Table 10 data are used, since when FPD data are converted to failure rates, SU1 failing-open needs to be considered at all $t=k\tau$ and not only after it turns-on at $L=-36$ in.

Tables 13 and 14 give the Top Event CDFs at $t=60$ minutes from Figs.28-33 to facilitate comparisons between the cases considered in this study and Fig.34 displays Table 13 and 14 data graphically. The fluctuations in Fig.34 data points with changing break size partly arise from the differences in the τ values for the determination of $g(j|j',n',\tau)$ for different cases (see Section 6.1) and partly from the differences in the final plateau termination points in the CDFs of Figs.28-33. For example, Fig.28(d) shows that if Fig.34 curves were plotted using $t=58$ minute results instead of $t=60$ minute results, then the predicted difference between Cases D1 and D5 would increase from a factor of 2 to a factor of 2.5. In that respect, these fluctuations do not necessarily have a mechanistic relevance. That is why Fig.34 also includes the curves obtained from a non-linear regression analysis of Table 13 and 14 data to show the general trends.

The general trends of Table 14 data in Fig.34 (i.e. using failure probabilities per demand converted to failure rates as given in Table 10) show that the ES Top Event occurrence probabilities during one hour following the SBLOCA can be significantly different depending on whether the SBLOCA is Class I, Class II or Class III. A new observation from Table 14 data in Fig.34 is the existence of a threshold for the SBLOCA break size beyond which some Top Event probabilities change drastically. Table 7 and Fig.34(b) indicates that the low pressure occurrence probability increases by more than one order of magnitude when the break size exceeds 1.5% of pipe flow area in large Class II SBLOCA. A similar threshold and magnitude of change are predicted for the high pressure occurrence probability from both Table 13 and Table 14 data (see Fig.34(a)). The direction of variation in

Table 13 Top Event Occurrence Probabilities for the ES-1 During 60 Minutes
Following the SBLOCA As Obtained From Table 9 Data

Case	Top Event Occurrence Probability			
	HL	LL	HP	LP
D1	2.601E-02	2.397E-03	1.291E-04	3.303E-06
D2	2.494E-02	2.397E-03	1.289E-04	3.002E-06
D3	3.343E-02	3.203E-03	1.383E-04	3.784E-06
D4	3.347E-02	2.394E-03	1.284E-04	3.933E-06
D5	4.511E-02	1.643E-03	2.359E-04	4.440E-06
D6	3.347E-02	2.377E-03	1.182E-04	4.182E-06
D7	4.616E-02	1.469E-03	2.174E-04	4.793E-06
D8	3.456E-02	2.040E-03	1.092E-04	4.997E-06
D9	2.280E-02	2.396E-03	8.915E-05	5.641E-06
D10	5.477E-03	4.779E-03	9.970E-05	5.787E-06
D11	5.477E-03	4.780E-03	3.993E-05	6.948E-06
D12	2.196E-03	2.398E-03	2.005E-05	8.217E-06
D13	3.291E-03	2.398E-03	1.004E-05	5.556E-04
D14	1.100E-03	4.789E-03	2.004E-05	8.517E-06

HL : High Level LL : Low Level HP : High Pressure LP : Low Pressure

Table 14 Top Event Occurrence Probabilities for the ES-1 During 60 Minutes
Following the SBLOCA As Obtained From Table 10 Data

Case	Top Event Occurrence Probability			
	HL	LL	HP	LP
D1	2.552E-03	1.359E-03	1.176E-03	4.066E-06
D2	2.443E-03	2.114E-03	8.520E-04	3.918E-06
D3	2.162E-03	2.083E-03	1.130E-03	4.810E-06
D4	2.451E-03	2.210E-03	1.130E-03	4.972E-06
D5	2.147E-03	2.778E-03	1.095E-03	5.822E-06
D6	2.133E-03	2.699E-03	1.084E-03	5.038E-06
D7	2.125E-03	2.570E-03	1.066E-03	5.698E-06
D8	2.104E-03	2.831E-03	1.042E-03	5.839E-06
D9	2.075E-03	3.035E-03	8.361E-04	6.434E-06
D10	2.046E-03	2.947E-03	7.838E-04	6.587E-06
D11	2.010E-03	2.868E-03	5.489E-04	7.395E-06
D12	2.155E-03	2.906E-03	5.294E-04	8.226E-06
D13	3.197E-03	4.878E-03	2.403E-05	4.445E-04
D14	2.499E-03	5.486E-03	1.799E-05	1.713E-03

HL : High Level LL : Low Level HP : High Pressure LP : Low Pressure

Fig.34 Variation of the Top Event CDFs at 60 Minutes After SBLOCA With Changing Break Size.

the Top Event occurrence probabilities with changing break size is consistent with physical expectations. For example, since the rate of enthalpy addition to the RV coolant inventory decreases with increasing break size so does the demand rate for SU2 to open to relieve pressure. Subsequently, probability of SU2 failing-closed during the time interval of interest and hence the probability of high pressure occurrence decreases with increasing break size (Fig.34(a)). On the other hand, low level occurrence probability increases with increasing break size (Fig.34(d)), since the demand rate for SU1 to turn-on increases with increasing break size. The small change in the low level occurrence probability indicates that this Top Event is not significantly affected by SU2 behavior (i.e. steam discharge rate from the RV) and the impact of the break size on the SU1 cycling time is small. Fig.34(c) results obtained from Table 13 data predict a large variation in high level occurrence probability with changing break size which implies that this Top Event is mainly due to level swell and not due to SU1 failing-on. Level swell can occur following SU2 failing-closed when P exceeds the pressure setpoint as indicated in *Example 2* above.

As far as the impact of FPD data conversion to failure rates on the results is concerned (i.e. Table 13 vs. Table 14 results in Fig.34), its magnitude also seems to depend on the sensitivity of demand rates for the components whose failures are main contributors to the Top Event to system dynamics. First, note that Table 13 and 14 data yield almost identical results for low pressure occurrence in Fig.34(b). The explanation is that since the change in low pressure occurrence is large with changing break size, the main contributor to this Top Event is SU2 failing-open (and not SU1 failing-on which also sharply reduces RV pressure as shown in Fig.24(b)) and Table 9 data for SU2 failing-open is already in the form of failure rate. Similarly, the difference between Table 13 and Table 14 results in Fig.34(d) is small because the impact of the break size on the SU1 cycling time (and hence the demand rate for

SU1 to turn-on) is small as indicated above. On the other hand, the difference between Fig.34(a) results obtained from Table 13 and Table 14 data are large, because high pressure can only occur if SU2 fails-closed and the demand rate for SU2 is sensitive to break size as can be seen from the large variation in the results obtained from Table 13 data in Fig.34(a) with changing break size. A similar explanation holds for Fig.34(c) since high level occurs mainly due to level swell following SU2 failing-closed when P exceeds the pressure setpoint. Fig.34(c) also implies that SU2 demand rate to open is indeed substantially affected by the SU1 behavior and this demand rate is higher under failed SU1 operation for Class I and most of Class II SBLOCAs. The lower high pressure probability predicted by Table 13 data in Fig.34(a) compared to Table 14 data reflects the competition between high pressure and high level Top Events and implies that, when SU2 fails-off, high level is often reached before high pressure due to very rapid level swell.

Table 15 shows the Top Event occurrence probabilities within 60 minutes following the LOCA as obtained from Table 12 (i.e. fault-tree approach) with Table 10 data. Table 15 shows that the fault-tree results compare relatively well with Table 14 results which use Table 10 data for: a) the high level Top Event of Cases C1-C11, and, b) the high pressure Top Event in most of the cases (highest discrepancy is one order of magnitude for case D14). Otherwise, there are large differences between the two sets of results. The worst discrepancies are for Cases C1-C12 in low pressure occurrence probability (i.e. by a factor of 300 to 550).

The discrepancies between the dynamic methodology and fault-tree results in Table 15 can be somewhat reduced by using the initiating-event approach [7] in the quantification of the fault-trees. For example, the fault-trees in Fig.27 show that the low pressure Top Event can be caused by either of the two basic events "SU1 fails-on" and "SU2 fails-open", since

Table 15 Top Event Occurrence Probabilities for the ES-1 During 60 Minutes
Following the SBLOCA As Obtained From Table 10 Data and Table 12

Case	Top Event Occurrence Probability			
	HL	LL	HP	LP
D1	2.198E-03	4.781E-03	1.191E-03	2.202E-03
D2	2.197E-03	4.781E-03	1.159E-03	2.202E-03
D3	2.197E-03	4.781E-03	1.148E-03	2.202E-03
D4	2.197E-03	4.781E-03	1.148E-03	2.202E-03
D5	2.197E-03	4.781E-03	1.119E-03	2.202E-03
D6	2.197E-03	4.781E-03	1.101E-03	2.202E-03
D7	2.197E-03	4.781E-03	1.090E-03	2.202E-03
D8	2.197E-03	4.781E-03	1.058E-03	2.202E-03
D9	2.196E-03	4.781E-03	8.492E-04	2.202E-03
D10	2.196E-03	4.781E-03	7.989E-04	2.202E-03
D11	2.195E-03	4.781E-03	5.614E-04	2.202E-03
D12	3.286E-03	7.172E-03	2.599E-04	3.294E-03
D13	3.286E-03	7.172E-03	2.199E-04	3.294E-03
D14	3.285E-03	7.172E-03	1.901E-04	3.294E-03

HL : High Level LL : Low Level HP : High Pressure LP : Low Pressure

either basic event can lead to the reduction of P below 300 psi. On the other hand, simulation results on the P and L evolution under different SU failure scenarios indicate that if SU1 fails-on first, then: a) P needs to be close to 300 psi for PCS to fail by low pressure and PCS fails by high level otherwise (i.e. high level occurs before low pressure), b) Top Event occurrence is also sensitive to the value of L at the time of SU1 failure. If SU2 fails open first, the PCS fails by low pressure almost irrespective of P and L values at the time of SU2 failure, since the pressure reduction due to steam flow out of the RV is much faster than level rise due to SU1 failing on. Consequently, an enabling event needs to be defined in terms of P and L for the PCS to fail by low pressure with SU1 failing on as the initiating event. In principle, the definition of such an enabling event requires simulations like those used for the construction of the Markov failure model and a different enabling event may need to be defined for each type of LOCA. However, if the probability of enabling event occurrence is assumed to be zero for C1-C12 types of LOCAs as a first approximation (comparison of Figs.24(a) and 24(b) shows that P is close to 600 psi when SU1 needs to turn off), then SU2 failing-open becomes the only contributor to PCS failure by low pressure for C1-C12 type LOCAs and the Top Event occurrence probability within 60 minutes following the LOCA reduces to $\approx 10^{-7}$. The new result is in much better agreement with the dynamic methodology predictions given in Table 15, but there is still a factor of 4-7 difference.

6.6 Summary

The results of this investigation indicate that:

- predicted demand rates for nuclear plant systems with safety functions can substantially change with changing system dynamics due the uncertainties on the system parameters, and,

- such effects cannot be accounted for by failure modeling techniques which use static and qualitative plant models.

The results of the study also show that: a) demand rates for nuclear plant systems with safety functions can be largely overestimated if component FPD data are converted to failure rates (see Figs.34(a) and 34(c)), which is the standard procedure when the event-tree/fault-tree approach is used for time-dependent reliability analyses, and, b) the data conversion process does not necessarily lead to conservative results (see Fig.34(c)). An additional new result of the investigation is that dynamic methodologies may be useful in accident classification by identifying the initiating event threshold values which lead to large and sudden changes in the demand rates for systems with safety functions. As indicated in Section 6.5, Fig.34(b) shows that the probability of demand for LPCI actuation (since the HPCS system is not operable below 300 psi) to prevent core uncover during 1 hour following a BWR/6 SBLOCA which incapacitates the RCIC system can change by more than one order of magnitude depending on whether the SBLOCA size is smaller or larger than 1.5% of the pipe flow area. Fig.34(a) gives a similar result regarding the probability of demand for SRV2-SRV19 actuation to prevent RV overpressure. In these respects, this investigation provides new reasons for the need for dynamic methodologies in risk and reliability studies.

From the results of the investigation it is observed that the impact of changes in the process physics or conversion of FPD data to failure rates on the predicted Top Event statistical properties depends on how sensitive the component demand rates are to system dynamics, as well as on the competition between Top Events. These observations imply that dynamic methodologies may be particularly useful in the risk and reliability analysis of systems which have intermittently operated components with statistically dependent demand rates or of systems with phased missions and multiple failure modes. The implementation of the dynamic

methodologies for large systems is often computationally infeasible. However, as this study shows, all the system components need not be explicitly represented for dynamic methodologies and fault-trees can be used to obtain data for an equivalent reduced system. Conversely, dynamic methodologies can be used to provide data for the improved risk and reliability analysis of the overall plant by the event-tree/fault-tree approach, such as the probability of demand for LPCI following the SBLOCA as obtained in this investigation.

The results of this investigation also show that even though conventional fault-tree methodology may not be suitable for quantifying the effects of uncertainties in the process dynamics on the interval reliability of the system, fault-tree predictions can be improved by considering the effects of the differences in component failure sequencing on Top Event occurrence probabilities. However, this approach may need simulation of the PCS behavior under different component failure scenarios and may still lead to large over- or under-estimation of the Top Event occurrence probabilities. In these respects, dynamic methodologies [46-48, 72] which use simulation based process dynamics models are expected to be more effective in the interval reliability analysis of dynamic systems.

CHAPTER VII
EXAMPLE APPLICATION II:
GLOBAL ANALYSIS OF REACTOR DYNAMICS - PART I

It is well known that the asymptotic solutions of the non-linear differential equations encountered in reactor dynamics can change from stable to periodic [36] or from periodic to chaotic [77] with a very small change in system parameters and/or initial conditions. In that respect, determination of the variation of PGs and BOAs as a function of the system parameters (i.e. global analysis) is important for the safety and reliability of reactor operation. The PGs indicate whether the reactor behavior remains within the imposed constraints or not and the BOAs show which initial conditions lead to these PGs.

Two applications are considered in this study to illustrate global analysis of reactor dynamics using the improved CCMT methodology developed in Chapter III. The first application (Part I) presented in this chapter is the analysis of Xenon evolution in a thermal reactor as a function of the overall flux coefficient. The second application (Part II) presented in Chapter VIII is the analysis of BWR dynamics as a function of the heat transfer coefficient from fuel to the coolant.

Section 7.1 describes the Xenon problem and presents the cases to be considered in this investigation. Section 7.2 gives the results for a coarse state-space discretization (i.e. $20 \times 20 \times 20$ cells). A refined discretization of the state-space (i.e. $40 \times 40 \times 40$ cells) is given in Section 7.3. Section 7.4 presents the results obtained by further refining the discretization of

the state-space (i.e. $80 \times 80 \times 80$ cells). Section 7.5 shows the computational advantages of the improved CCMT over the originally proposed CCMT and direct integration. Finally, Section 7.6 summarizes the results of this investigation.

7.1 System Description

The example Xenon problem considered in this chapter is taken from Ref.[78] and will be referred to as Example System 3 (ES-3). The system equations are:

$$\begin{cases} \Lambda \frac{d\Phi}{dt} = \left[\rho_o - \frac{\sigma_x}{c \Sigma_f} X - \theta \Phi \right] \Phi \\ \frac{dX}{dt} = y_x \Sigma_f \Phi - \lambda_x X + \lambda_i I - \sigma_x X \Phi \\ \frac{dI}{dt} = y_i \Sigma_f \Phi - \lambda_i I \end{cases} \quad (107)$$

where Φ is flux, X and I are the ^{135}Xe and ^{135}I concentrations per fuel atom, c converts local xenon absorption to reactivity loss, ρ_o is the reactivity at zero flux and zero xenon poisoning and θ is the overall flux coefficient of reactivity. Other symbols have their conventional definitions (see Table 16). In order to make the problem more general (i.e. reactor independent), the Iodine and Xenon concentrations can be normalized with respect to the reactor macroscopic cross-section Σ_f , to obtain:

$$\begin{cases} \Lambda \frac{d\Phi}{dt} = \left[\rho_o - \frac{\sigma_x}{c} \bar{X} - \theta \Phi \right] \Phi \\ \frac{d\bar{X}}{dt} = y_x \Phi - \lambda_x \bar{X} + \lambda_i \bar{I} - \sigma_x \bar{X} \Phi \\ \frac{d\bar{I}}{dt} = y_i \Phi - \lambda_i \bar{I} \end{cases} \quad (108)$$

Table 16 Parameters' Definitions and Corresponding Values for ES-3

Parameter	Definition	Value
γ_i	Iodine yield	6.2E-2
γ_x	Xenon yield	2.0E-3
λ_i	Iodine decay constant	2.87E-5 /s
λ_x	Xenon decay constant	2.09E-5 /s
σ_x	Xenon microscopic cross section	2.8E6 b
c	Xenon-to-reactivity coefficient	1.49
Λ	Neutron generation mean time	0.083 s
ρ_0	Reactivity at zero flux and zero Xenon concentration	5.0E-4

where now $\bar{X}$ and $\bar{I}$ are the ^{135}Xe and ^{135}I concentrations per fuel atom divided by the macroscopic fission cross-section.

The first attempt to study the effects of Xenon poisoning on reactor dynamics using Eqs.(108) is described in Refs.[35,78]. References [35,78] showed the existence of stable limit cycles and found fixed-point invariant manifolds in the parameter plane $\theta \times \rho_0$, where ρ_0 is the reactivity at zero flux and zero Xenon concentration. From their results, they have concluded that space-independent Xenon oscillations can exist at high as well as low level fluxes depending on θ and ρ_0 . The results also indicated that these oscillations may lead to instabilities, since the study showed the existence of regions in the $\theta \times \rho_0$ plane, where fixed points are stable for small perturbations but unstable for large perturbations.

A later complementary study performed by Ref.[36] using Hopf bifurcation analysis [79] (similar in nature to Wiggins method [33]) has determined the stability boundary in $\theta \times \rho_0$ plane as shown in Fig.35. Both the stability boundary and the regions around the stability boundary consist of only fixed points. On the right-hand side of the stability boundary oscillations are stable for small perturbations (stable region) and on the left-hand side of the stability boundary oscillations are unstable for large perturbations (unstable region).

For the illustration, all the system parameters in Eq.(108) were fixed at the values shown in Table 16, except for θ , which is the parameter of interest for global analysis. The θ range under consideration is $1.0 \times 10^{-16} \leq \theta \leq 7.0 \times 10^{-16} \text{ s cm}^2$. The θ range as chosen includes points from both from the stable and unstable regions in the $\theta \times \rho_0$ plane for the value of ρ_0 given in Table 16 (i.e. $\rho_0 = 5.0 \times 10^{-4}$, or $\log_{10}(\rho_0) = -3.30$). The θ range is divided into three equal size regions as shown in Table 17. Only uniform distribution of θ over its range is considered in this investigation, i.e.

Fig.35 Stability Boundary in θ - ρ_0 Space (From Ref.[79]).

Table 17 The Definition of $\tilde{V}_m$ and the Corresponding Data For Eqs. (73), (75) and (77)

Cell or Data Type	m=1	m=2	m=3
$\tilde{V}_m$	$1.0\text{E-}16 \leq \theta \leq 3.0\text{E-}16$	$3.0\text{E-}16 \leq \theta \leq 5.0\text{E-}16$	$5.0\text{E-}16 \leq \theta \leq 7.0\text{E-}16$
θ_m	$2.0\text{E-}16$	$4.0\text{E-}16$	$6.0\text{E-}16$
$\tilde{v}_m$	$2.0\text{E-}16$	$2.0\text{E-}16$	$2.0\text{E-}16$
$\tilde{P}_m$ from Eq.(104) and (109)	$1/3$	$1/3$	$1/3$

$$P(\theta) = \begin{cases} \frac{1}{(7.0 - 1) \times 10^{-16}} & \text{if } 1.0 \times 10^{-16} \leq \theta \leq 7.0 \times 10^{-16} \\ 0 & \text{otherwise} \end{cases} \quad (109)$$

The region of interest (i.e. R) was defined as:

- $10^8 \leq \Phi \leq 10^{13} \text{ n/cm}^2\text{s}$,
- $3.0 \times 10^{10} \leq \bar{X} \leq 3.0 \times 10^{16} \text{ atoms/cm}$, and
- $2.0 \times 10^{10} \leq \bar{I} \leq 2.0 \times 10^{16} \text{ atoms/cm}$.

The flux range of interest covers the operating range of most thermal reactor from startup to full power. The normalized Iodine and Xenon lower ranges correspond to 1/5 of the equilibrium values of $\bar{I}$ and $\bar{X}$ at the startup flux, respectively. The upper ranges of the normalized Iodine and Xenon are twice the equilibrium values of $\bar{I}$ and $\bar{X}$ at the full power flux, respectively. Three cases are considered for partitioning R :

Case 1: $20 \times 20 \times 20$ equal size cells in the logarithmic axes Φ , $\bar{I}$, and $\bar{X}$

Case 2: $40 \times 40 \times 40$ equal size cells in the logarithmic axes Φ , $\bar{I}$, and $\bar{X}$

Case 3: $80 \times 80 \times 80$ equal size cells in the logarithmic axes Φ , $\bar{I}$, and $\bar{X}$.

For each case, each cell was divided into 8 equal-volume subcells to approximate the integral in Eq.(75). The $x(x', \alpha)$ term in the integral (where $\alpha = \theta$ for the system under consideration) was evaluated by a fourth order Runge-Kutta integration of Eq.(108) with $\tau = 15$ minutes, and the internal integration time step is 5 seconds. The calculations for all the three cases were performed on the Ohio Supercomputing Center CRAY-YMP.

7.2 Case 1 Results

Figures 36-38 show the probability distribution of the cells (i.e. $P_j(\infty)$) in R for Case 1 (i.e. $20 \times 20 \times 20 = 8,000$ cells), integrated over $\bar{I}$, for $\theta = 2.0, 4.0$ and 6.0×10^{-16} , respectively. Figures 39-41 show the state-space projection of the PGs on the $\bar{X}$ - Φ plane for $\theta = 2.0, 4.0$ and 6.0×10^{-16} , respectively. Comparisons of Figures 37 and 38 to Figures 40 and 41 indicate that all the cells shown in Figs.37 and 38 are persistent cells. On the other hand, a comparison of Figures 36 and 39 shows that some of the cells in Fig.36 are extra transient cells inadvertently included in the predicted PG structure, since only a finite number of iterations can be used to approximate P_j^∞ from Eqs.(76) and (77), as indicated in Section 3.4. All the PGs in Figs.39-41 are located in the vicinity of system equilibrium conditions. Figures 39-41 indicate that the region in the state-space that contains the PGs becomes larger with increasing value of θ and that the number PGs vary with the value of θ . For example, in Fig.39 (i.e. $\theta = 2.0 \times 10^{-16}$) the system has only one PG (i.e. cyclic group 3 with a period of $T = 2\tau$) other than the sink cell. However, Fig.41 (i.e. $\theta = 6.0 \times 10^{-16}$) shows that the system has 3 different PGs in addition to the sink cell. Other figures showing the limiting probability distribution of the cells in R integrated over $\bar{X}$ or Φ , and the corresponding PGs projections are given in Appendix C.

Figure 42-44 show cuts of the BOAs around the equilibrium $\bar{I}$ level (i.e. $1.0 \times 10^{14} \leq \bar{I} \leq 2.0 \times 10^{14}$ atoms/cm) for $\theta = 2.0, 4.0$ and 6.0×10^{-16} , respectively. Only Fig.44 indicates the presence of multiple domicile cells. Figures 42-44 show that for $\log(\bar{X}) > 33.5$ (or $\bar{X} > 3.5 \times 10^{14}$ atoms/cm) and $1.0 \times 10^{14} \leq \bar{I} \leq 2.0 \times 10^{14}$ atoms/cm, ES-3 is attracted to the sink cell, irrespective of the initial neutron flux level, and for $\bar{X} < 3.0 \times 10^{14}$ atoms/cm and $1.0 \times 10^{14} \leq \bar{I} \leq 2.0 \times 10^{14}$ atoms/cm, the system approaches asymptotically to one of the PGs within R . Thus, $\bar{X} = 3.5 \times 10^{14}$ atoms/cm corresponds to a stability boundary for

Fig.36 Probability Distribution P_j^∞ , Integrated Over $\bar{I}$, of the Persistent Groups for ES-3 ($\theta = 2.0\text{E-}16$, Case 1).

Fig.37 Probability Distribution P_j^∞ , Integrated Over $\bar{I}$, of the Persistent Groups for ES-3 ($\theta = 4.0\text{E-}16$, Case 1).

$$\textcircled{\text{||}} 1.0 \times 10^{-1} \leq P_j^\infty \leq 1.0$$

$$\times 1.0 \times 10^{-2} \leq P_j^\infty \leq 1.0 \times 10^{-1}$$

$$+ 1.0 \times 10^{-3} \leq P_j^\infty \leq 1.0 \times 10^{-2}$$

$$\circ 1.0 \times 10^{-4} \leq P_j^\infty \leq 1.0 \times 10^{-3}$$

Fig.38 Probability Distribution P_j^∞ , Integrated Over $\bar{I}$, of the Persistent Groups for ES-3 ($\theta = 6.0\text{E-}16$, Case 1).

- Stationary Cells, Group 2 ($T=\tau$)
- + Cyclic Group 3 ($T=2\tau$)
- × Cyclic Group 4 ($T=3\tau$)

Fig.39 State-Space Projection of the PGs on the $\bar{X}$ - Φ Plane for ES-3
($\theta = 2.0E-16$, Case 1).

○ Stationary Cells, Group 2 ($T=\tau$)

+ Cyclic Group 3 ($T=2\tau$)

.x Cyclic Group 4 ($T=3\tau$)

Fig.40 State-Space Projection of the PGs on the $\bar{X}-\bar{\Phi}$ Plane for ES-3
($\theta = 4.0\text{E-}16$, Case 1).

○ Stationary Cells, Group 2 ($T=\tau$)

+ Cyclic Group 3 ($T=2\tau$)

× Cyclic Group 4 ($T=3\tau$)

Fig.41 State-Space Projection of the PGs on the $\bar{X}-\Phi$ Plane for ES-3
($\theta = 6.0\text{E-}16$, Case 1).

- ⊙ BOAs for the sink cell
- BOAs for the stationary cells, group 2 ($T=\tau$)
- + BOAs for the cyclic group 3 ($T=2\tau$)
- × BOAs for the cyclic group 4 ($T=3\tau$)

"blank" Multiple domicile transient cells

Fig.42 Cut of the BOAs at the Equilibrium $\bar{I}$ -Level for ES-3 ($\theta = 2.0\text{E-}16$, Case 1).

Fig.43 Cut of the BOAs at the Equilibrium $\bar{T}$ -Level for ES-3 ($\theta = 4.0\text{E-}16$, Case 1).

- BOAs for the sink cell
- BOAs for the stationary cells, group 2 ($T=\tau$)
- +
- BOAs for the cyclic group 3 ($T=2\tau$)
- ×
- BOAs for the cyclic group 4 ($T=3\tau$)

"blank" Multiple domicile transient cells

Fig.44 Cut of the BOAs at the Equilibrium $\bar{I}$ -Level for ES-3 ($\theta = 6.0E-16$, Case 1).

$1.0 \times 10^{14} \leq \bar{I} \leq 2.0 \times 10^{14}$ atoms/cm. Since all the PGs within R are in the vicinity of the system equilibrium conditions, Figs. 42-44 show that an initial value of $\bar{X} < 2.5 \times 10^{14}$ atoms/cm and $1.0 \times 10^{14} \leq \bar{I} \leq 2.0 \times 10^{14}$ atoms/cm leads to the equilibrium conditions asymptotically, irrespective of the initial flux level. Other cuts of BOAs for Case 1 at different planes of the state-space are provided in Appendix C.

7.3 Case 2 Results

Case 2 corresponds to the first refinement of the state-space discretization, with every subcell of Case 1 now being a cell for Case 2. Thus, the total number of cells in R are $40 \times 40 \times 40 = 64,000$ for Case 2 and each cell is again divided in 8 equal-volume subcells to approximate the integral given by Eq.(75). Figures 45-47 show the probability distribution of the cells in R for Case 2, integrated over $\bar{I}$, for $\theta = 2.0, 4.0$ and 6.0×10^{-16} , respectively, and Figs. 48-50 show the projection of the PGs on the $\bar{X}-\Phi$ plane again for $\theta = 2.0, 4.0$ and 6.0×10^{-16} , respectively. A comparison of Figs.45-47 and 48-50 shows that the cells that have non-zero probabilities in Figs. 45-47 are exactly the PGs obtained using the persistence test for this case and that no transient cells are included in the predicted PG structure as was in Case 1.

Comparing Figs.36-38 to Figs. 45-47, it is observed that while the location of the PGs in the state-space do not change from Case 1 to Case 2, the PGs become thinner in Case 2. These observations indicate that the PGs predicted both in Cases 1 and 2 are indeed covers for the attractors and the covers become more refined with increasing number of cells, as expected. For example, if only one of the 8 subcells of a persistent cell in Case 1 has non-zero limiting probability, then only that subcell would appear as part of the PG in Case 2 whereas the whole cell would be included in the PG for Case 1. The changes in the limiting

- $1.0 \times 10^{-1} \leq P_f^\infty \leq 1.0$
- × $1.0 \times 10^{-2} \leq P_f^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P_f^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P_f^\infty \leq 1.0 \times 10^{-3}$
- $1.0 \times 10^{-5} \leq P_f^\infty \leq 1.0 \times 10^{-4}$

Fig.45 Probability Distribution P_f^∞ , Integrated Over $\bar{I}$, of the Persistent Groups for ES-3 ($\theta = 2.0\text{E-}16$, Case 2).

- $1.0 \times 10^{-1} \leq P_j^\infty \leq 1.0$
- × $1.0 \times 10^{-2} \leq P_j^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P_j^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P_j^\infty \leq 1.0 \times 10^{-3}$
- $1.0 \times 10^{-5} \leq P_j^\infty \leq 1.0 \times 10^{-4}$

Fig.46 Probability Distribution P_j^∞ , Integrated Over $\bar{I}$, of the Persistent Groups for ES-3 ($\theta = 4.0\text{E-}16$, Case 2).

- $1.0 \times 10^{-1} \leq P_j^\infty \leq 1.0$
- × $1.0 \times 10^{-2} \leq P_j^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P_j^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P_j^\infty \leq 1.0 \times 10^{-3}$
- $1.0 \times 10^{-5} \leq P_j^\infty \leq 1.0 \times 10^{-4}$

Fig.47 Probability Distribution P_j^∞ , Integrated Over $\bar{I}$, of the Persistent Groups for ES-3 ($\theta = 6.0\text{E-}16$, Case 2).

Fig.48 State-Space Projection of the PGs on the $\bar{X}-\Phi$ Plane for ES-3
($\theta = 2.0\text{E-}16$, Case 2).

○ Stationary Cells, Group 2 ($T=\tau$)

+ Cyclic Group 3 ($T=2\tau$)

× Cyclic Group 4 ($T=3\tau$)

□ Cyclic Group 5 ($T=4\tau$)

Fig.49 State-Space Projection of the PGs on the $\bar{X}$ - Φ Plane for ES-3 ($\theta = 4.0E-16$, Case 2).

Fig.50 State-Space Projection of the PGs on the $\bar{X}$ - Φ Plane for ES-3
($\theta = 6.0\text{E-}16$, Case 2).

PG probability distributions between Cases 1 and 2 also reflect this refinement. Again, as an example, Fig.46 shows that two of the subcells of the center cell in Fig.37 have non-zero limiting probabilities, one in the range $1.0 \times 10^{-3} \leq P^\infty \leq 1.0 \times 10^{-2}$ and one in the range $1.0 \times 10^{-2} \leq P^\infty \leq 1.0 \times 10^{-1}$. In comparing Figs.39-41 to 48-50, it is also observed that:

1. the cyclic groups with period $T=2\tau$ in Figs.39 and 40 are divided into 2 distinct cyclic groups of periods 3τ and 4τ as shown in Figs.48 and 49,
2. all the cyclic groups in Fig.41 have different periods in Fig.50, and,
3. one of the stationary cells in Fig.41 has disappeared in Fig.50.

Observations 1 and 2 demonstrate how the results obtained from CCMT can be sensitive to the cell size. Observation 3 shows that if the cell size is large, the system may not leave a cell within the modeling time-step τ and the results may become erroneous. The above discussion and observations also indicate how the results obtained from CCMT can be verified/improved.

Cuts of the BOAs for Case 2 around the equilibrium $\bar{I}$ level ($1.18 \times 10^{14} \leq \bar{I} \leq 1.66 \times 10^{14}$ atoms/cm in this case due to the refinement of the partitioning scheme) are shown in Figs.51-53 for $\theta = 2.0, 4.0$ and 6.0×10^{-16} , respectively. Figs.51-53 again show that for $\log(\bar{X}) > 33.5$ or $\bar{X} > 3.5 \times 10^{14}$ atoms/cm is a stability boundary when $1.18 \times 10^{14} \leq \bar{I} \leq 1.66 \times 10^{14}$ atoms/cm. Figures 51-53 also show that for $\log(\bar{X}) < 33.5$ the number of multiple domicile cells decreases with decreasing θ and hence the region of the state-space that leads to ES-3 staying within the imposed constraints increases with increasing θ . This observation is in agreement with the results of Ref.[36], since Fig.35 shows that at $\rho_0 = 5.0 \times 10^{-4}$, the critical value of θ is about 3.0×10^{-16} , above which the reactor operation is stable and below which the reactor operation is unstable.

- BOAs for the sink cell
- BOAs for the stationary cells, group 2 ($T=\tau$)
- + BOAs for the cyclic group 3 ($T=2\tau$)
- × BOAs for the cyclic group 4 ($T=3\tau$)

"blank" Multiple domicile transient cells

Fig.51 Cut of the BOAs at the Equilibrium $\bar{T}$ -Level for ES-3 ($\theta = 2.0\text{E-}16$, Case 2).

- BOAs for the sink cell
- BOAs for the stationary cells, group 2 ($T=\tau$)
- + BOAs for the cyclic group 3 ($T=2\tau$)
- × BOAs for the cyclic group 4 ($T=3\tau$)

"blank" Multiple domicile transient cells

Fig.52 Cut of the BOAs at the Equilibrium $\bar{I}$ -Level for ES-3 ($\theta = 4.0E-16$, Case 2).

Fig.53 Cut of the BOAs at the Equilibrium $\bar{I}$ -Level for ES-3 ($\theta = 6.0\text{E-}16$, Case 2).

Additional figures showing the limiting probability distribution of the cells in R integrated over $\bar{X}$ or Φ and the corresponding PGs projections, as well as other cuts of BOAs at different planes of the state-space for Case 2 are provided in Appendix C. We now proceed to discuss the results of Case 3.

7.4 Case 3 Results

Case 3 corresponds to the second refinement of the state-space discretization. Every subcell in Case 2 now becomes a cell for Case 3 and the total number of cells in R increases to $80 \times 80 \times 80 = 512,000$. The probability distribution of the cells in R , integrated over $\bar{I}$ are shown in Figs.54-56 and the projections of the PGs on the $\bar{X}-\Phi$ plane are shown in Figs.57-59 for $\theta = 2.0, 4.0$ and 6.0×10^{-16} , respectively. A comparison of Figs.54-56 and 57-59 shows that the cells located at $\log(\bar{X})=33.3$ and $\log(\Phi)<24.5$ in Figs.54-56 are transient cells appearing as persistent cells. Otherwise, the PGs maintain their shape and location compared to Cases 1 and 2, except for the regions marked 1 and 2 in Figs.55 and 56 which will be addressed later in this section. Some other differences between Case 1 and 2 results are the following:

1. The cyclic group 4 with period $T=3\tau$ in Fig.48 has disappeared in Fig.57.
2. Some of the cyclic groups in Figs.49 and 50 have different periods in Figs.58 and 59.
3. The stationary cells in Figs.49 and 50 have disappeared in Figs.58 and 59.

The above observations are similar to the observations made in Section 7.3. From these observations, it follows that the cell size for Case 3 may again be large and a second refinement is needed for the verification/improvement of the results.

- $1.0 \times 10^{-1} \leq P_j^\infty \leq 1.0$
- × $1.0 \times 10^{-2} \leq P_j^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P_j^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P_j^\infty \leq 1.0 \times 10^{-3}$
- △ $1.0 \times 10^{-5} \leq P_j^\infty \leq 1.0 \times 10^{-4}$
- $1.0 \times 10^{-6} \leq P_j^\infty \leq 1.0 \times 10^{-5}$

Fig.54 Probability Distribution P_j^∞ , Integrated Over $\bar{I}$, of the Persistent Groups for ES-3 ($\theta = 2.0\text{E-}16$, Case 3).

- $1.0 \times 10^{-1} \leq P_j^\infty \leq 1.0$
- × $1.0 \times 10^{-2} \leq P_j^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P_j^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P_j^\infty \leq 1.0 \times 10^{-3}$
- △ $1.0 \times 10^{-5} \leq P_j^\infty \leq 1.0 \times 10^{-4}$
- $1.0 \times 10^{-6} \leq P_j^\infty \leq 1.0 \times 10^{-5}$

Fig.55 Probability Distribution P_j^∞ , Integrated Over $\bar{I}$, of the Persistent Groups for ES-3 ($\theta = 4.0\text{E-}16$, Case 3).

- Stationary Cells, Group 2 ($T=\tau$)
- + Cyclic Group 3 ($T=2\tau$)
- × Cyclic Group 4 ($T=3\tau$)
- Cyclic Group 5 ($T=4\tau$)

Fig.57 State-Space Projection of the PGs on the $\bar{X}$ - Φ Plane for ES-3
($\theta = 2.0E-16$, Case 3).

- Stationary Cells, Group 2 ($T=\tau$)
- + Cyclic Group 3 ($T=2\tau$)
- × Cyclic Group 4 ($T=3\tau$)
- Cyclic Group 5 ($T=4\tau$)

Fig.58 State-Space Projection of the PGs on the $\bar{X}$ - Φ Plane for ES-3
($\theta = 4.0\text{E-}16$, Case 3).

- Stationary Cells, Group 2 ($T=\tau$)
- + Cyclic Group 3 ($T=2\tau$)
- × Cyclic Group 4 ($T=3\tau$)
- Cyclic Group 5 ($T=4\tau$)

Fig.59 State-Space Projection of the PGs on the $\bar{X}-\Phi$ Plane for ES-3
($\theta = 6.0E-16$, Case 3).

Regions 1 and 2 in Figs.58 and 59 consist of a newly discovered cyclic group with period $T=2\tau$ (Fig.58) and a new stationary cell (Fig.59). These regions have the following characteristics:

- A. The limiting probability is about 2 orders of magnitude lower than the probability of the other PGs (see Fig.55 and 56).
- B. The location of regions 1 and 2 is at the edge of the region of interest R ($\log(\bar{X})$ is at the maximum allowable level).

Observation A indicates that both regions 1 and 2 are rarely visited by the other cells in the state-space. Both observations suggest that the persistence of cells comprising regions 1 and 2 should be verified. Integration of Eqs.(108) starting from the subcells in regions 1 and 2 showed that all the subcells in regions 1 and 2 eventually lead to the sink cell and confirmed that regions 1 and 2 are not part of the PGs. The emergence of regions 1 and 2 was found to be caused by the sensitivity of system dynamics to the modeling time-step τ and cell size in regions 1 and 2. For region 1, most of the trajectories did not leave the departure cell within τ and few other trajectories mapped to the neighboring cell after one τ . For region 2 none of the trajectories left the cell during one τ . These results again show that a more refined partitioning of the state-space may be needed for better results.

Cuts of the Case 3 BOAs around the equilibrium $\bar{I}$ level ($1.28 \times 10^{14} \leq \bar{I} \leq 1.53 \times 10^{14}$ atoms/cm) for $\theta = 2.0, 4.0$ and 6.0×10^{-16} are shown in Figs.60-62, respectively. Figs.60-62 show that:

1. for $\log(\bar{X}) > 33.5$, the system dynamics leads to the sink cell regardless of the flux level as in Cases 1 and 2,

Fig.60 Cut of the BOAs at the Equilibrium $\bar{T}$ -Level for ES-3 ($\theta = 2.0\text{E-}16$, Case 3).

- BOAs for the sink cell
- △ BOAs for the cyclic group 3 ($\bar{\tau}=2\tau$)
- × BOAs for the cyclic group 5 ($\bar{\tau}=4\tau$)
- "blank" Multiple domicile transient cells

Fig.61 Cut of the BOAs at the Equilibrium $\bar{\tau}$ -Level for ES-3 ($\theta = 4.0E-16$, Case 3).

- BOAs for the sink cell
 - BOAs for the stationary cells, group 2 ($T=\tau$)
 - + BOAs for the cyclic group 4 ($T=3\tau$)
 - × BOAs for the cyclic group 5 ($T=4\tau$)
- "blank" Multiple domicile transient cells

Fig.62 Cut of the BOAs at the Equilibrium $\bar{T}$ -Level for ES-3 ($\theta = 6.0E-16$, Case 3).

2. there is no region of stability for $\theta=2.0\times 10^{-16}$, but only a narrow region of multiple domicile cells (see Fig.60),
3. the region of stability increases with increasing θ (compare Figs.61 and 62), and
4. the BOA to the sink cell shrinks with increasing θ (compare Figures 60 and 61).

The above observations are still consistent with the findings of Ref.[36]. A comparison of Figs.60-62 to Figs.51-53 shows that the results in Figs.60-62 are consistent with the results of Figs.51-53, except that the region of stability (i.e. BOAs to the PGs) becomes much thinner in Case 3 and the state-space is mainly occupied by the multiple domicile cells. The thinning in the stability region can be explained by the refinement of the partitioning scheme as discussed in Section 7.3.

Since most of the state-space is occupied by the multiple domicile cells in Figs.60-62, we need to determine the absorption probabilities of cells partitioning the state-space into the various PGs to assess the likelihood of a given initial condition to lead to stable or unstable behavior (i.e. ES-3 remaining within R or moving outside R). Figures 63-65 show the absorption probabilities into the sink cell around the equilibrium Iodine level (i.e. $1.28\times 10^{14}\leq I\leq 1.53\times 10^{14}$ atoms/cm) for $\theta=2.0, 4.0$ and 6.0×10^{-16} , respectively. Figures 63 and 64 indicate that most of the multiple domicile cells obtained in Figs.60 and 61 have absorption probability into the sink cell higher than 0.5 and more than 50% of these cells have absorption probability in the range 0.75-0.99. However, for $\theta=6.0\times 10^{-16}$, Fig.65 shows that less than 3% of the multiple domicile cells have absorption probabilities into the sink cell higher than 0.5. These results are consistent with the earlier findings that the stability region grows with increasing value of θ . The difference in absorption probabilities into the sink cell as shown in Figs.63-65 causes "blurring" when θ is a stochastic parameter. Figure 66 shows the cell absorption probabilities into the sink cell around the equilibrium Iodine level using

- $0.75 \leq \rho_{1,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{1,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{1,i}^{\infty} \leq 0.50$
- × $0.01 \leq \rho_{1,i}^{\infty} \leq 0.25$

Fig.63 Absorption Probabilities $\rho_{1,i}^{\infty}$ into the Sink Cell at the Equilibrium $\bar{I}$ -Level Plane for ES-3 ($\theta = 2.0E-16$, Case 3).

- $0.75 \leq \rho_{1,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{1,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{1,i}^{\infty} \leq 0.50$
- × $0.01 \leq \rho_{1,i}^{\infty} \leq 0.25$

Fig.64 Absorption Probabilities $\rho_{1,j}^{\infty}$ into the Sink Cell at the Equilibrium $\bar{T}$ -Level Plane for ES-3 ($\theta = 4.0\text{E-}16$, Case 3).

- $0.75 \leq \rho_{1,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{1,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{1,i}^{\infty} \leq 0.50$
- × $0.01 \leq \rho_{1,i}^{\infty} \leq 0.25$

Fig.65 Absorption Probabilities $\rho_{1,j}^{\infty}$ into the Sink Cell at the Equilibrium $\bar{I}$ -Level Plane for ES-3 ($\theta = 6.0\text{E-}16$, Case 3).

Fig.66 Absorption Probabilities $\rho_{1,j}^{\infty}$ into the Sink Cell at the Equilibrium $\bar{I}$ -Level Plane for ES-3 (θ Uniformly Distributed, Case 3).

Eq.(94), Eq.(109) and Table 17 data, with θ uniformly distributed over the range $1.0 \times 10^{-16} \leq \theta \leq 7.0 \times 10^{-16}$. The following observations are made from Fig.66:

- A. All the cells with $\log(\bar{X}) > 33.5$ have absorption probability of 1.0.
- B. All the cells with $\log(\bar{X}) < 31.75$ have absorption probabilities higher than 0.5.

Observation A means that all cells with $\log(\bar{X}) > 33.5$ are single domicile transient cells to the sink cell regardless of the value of θ . Observation B shows the "blurring" in the BOAs. Then, only a narrow region in the $\bar{X}-\Phi$ plane exists where the system dynamics has a probability higher than 0.5 to lead to stable behavior. The "blank" strip in Fig.66 corresponds to cells with a probability of 1.0 to steer the system to an attractor within R .

7.5 Computational Aspects

The computational requirements based on one value of θ in Case 3 are presented in Table 18 for comparison purposes, which shows that the algorithm for determining PGs, BOAs and absorption probability requires a storage capacity of 10.8 MW (i.e. 10.8 megawords or 10.8×10^6 entries). In that respect, although the results of the previous section indicates that a further refinement of the state-space discretization is needed for a reliable global analysis of ES-3, such a refinement was not possible since the users of the State of Ohio CRAY-YMP are limited to 16.0MW memory. In order to overcome this obstacle, the global analysis of ES-3 could be shifted towards investigating the system dynamics in a sub-space or a smaller region of the state-space. The total number of cells could be maintained at 512,000, but the cell size would become much smaller than in Case 3. Then, the user would be required to only change the minimum and the maximum values of the controlled variables in the input file.

Table 18 Computational Requirements for Method 2 Based on one Value of θ (Case 3)

Quantities Determined	CRAY-YMP CPU Time	Storage Requirement
Images	9,293 s	6.2E06 entries
Limiting Probabilities	205 s	8.3E06 entries
PGs, BOAs and Absorption Probabilities	217 s	10.8E06 entries

If this study used CCMT as originally proposed (Method 1), the storage requirement would have been a factor of 3 higher than that required by the new approach presented in Chapter 3 (Method 2) for ES-3 and it would not have been possible to perform the Case 3 studies. Table 18 also shows that the total CRAY-YMP CPU time used by Method 2 for one value of θ in Case 3 is $9,293+205+217=9,715$ s (≈ 2.7 h), where 95% of this time is used to determine the image cells. If the analysis presented in this chapter were performed by direct integration, $8 \times 80 \times 80 \times 80 = 4,096,000$ integrations (also based on one value of θ in Case 3) of the system equations (i.e. Eq.(108)) over a period of 40 to 300τ would be required to cover all the points in the discretized state-space considered by this study. The CRAY-YMP CPU time would be $40 \times 9,293 = 375,720$ s ≈ 104 h to $300 \times 9,293 = 2,787,900$ s ≈ 775 h (as compared to only 2.7 h for Method 2), even if no CPU time were spent in establishing that asymptotic behavior reached.

7.6 Summary

This chapter illustrates the implementation of the improved CCMT methodology presented in Chapter III on a three degree-of-freedom system from reactor dynamics (i.e. ES-3) and the computational advantage of this methodology over both the original version of CCMT and global analysis by direct integration. This implementation shows that the improved methodology can be a very efficient tool for the study of reactor dynamics, especially under uncertainty on the system parameters and allows a lot of flexibility to the user. Most of the observations made in Sections 7.2-7.4 regarding the BOAs of ES-3 are based on one plane in the state-space around the equilibrium Iodine level. Cuts of BOAs at other planes provided in Appendix C show the generality of these observations. Other ES-3 specific results of this chapter are:

- there exist regions in state-space which lead to stable and unstable ES-3 behavior,
- for Iodine levels around the equilibrium level, the region of stability is narrow and becomes larger with increasing value of θ (consistent with Ref.[36] findings), and,
- the results depend on the cell size and show improvement with decreasing cell size.

Now, we proceed to Part II of global analysis of reactor dynamics (Chapter VIII).

CHAPTER VIII
EXAMPLE APPLICATION II:
GLOBAL ANALYSIS OF REACTOR DYNAMICS-PART II

In this chapter, the global analysis of nonlinear BWR reactor dynamics, governed by temperature and void feedback, is addressed. References [80, 81] show that BWRs are susceptible to reactivity instabilities when operated at high-power and low-flow conditions; conditions which do not occur at normal BWR operation but may occur for some accident scenarios. Also, Refs.[77] and [78] show that BWR dynamics at high-power and low flow-conditions can undergo limit cycle oscillations which are very sensitive to both initial conditions and system parameters.

Section 8.1 presents the five degree-of-freedom model that represents the BWR dynamics. Section 8.2 discusses input preparation for global analysis of reactor dynamics for the model under consideration. The results of the study are discussed in Section 8.3. Section 8.4 presents some computational aspects of the methodology described in Chapter 3 as implemented on this BWR model and Section 8.5 summarizes the results of this investigation.

8.1 Representation of BWR Dynamics

Reference [82] has shown that BWR dynamics can be represented by a five degree-of-freedom system or a set of five first order differential equations in time which will be referred to as Example System 4 (ES-4). The ES-4 couples the neutronic and thermal-hydraulic representation of BWR dynamics through reactivity feedbacks and has been

validated against large detailed BWR codes and experiments. Reference [82] provides a physical explanation of the model and contains quantitative as well as qualitative analyses of BWR dynamics.

The neutronics of ES-4 are governed by

$$\frac{dn(t)}{dt} = \frac{(\rho(t)-\beta)}{\Lambda} n(t) + \lambda c(t) + \frac{\rho(t)}{\Lambda}, \quad (110)$$

and,

$$\frac{dc(t)}{dt} = \frac{\beta}{\Lambda} n(t) - \lambda c(t), \quad (111)$$

where $n(t)=(N(t)-N_0)/N_0$ denotes the fractional change in the neutron population with $N(t)$ and N_0 corresponding to the neutron population at time t and initial neutron population (at steady-state), respectively, $c(t)=(C(t)-C_0)/C_0$ is the fractional change in the delayed neutron precursors with $C(t)$ and C_0 corresponding to the delayed neutron precursor concentration at time t and the initial precursor concentration (at steady state), respectively, $\rho(t)$ is the feedback reactivity and the rest of the parameters are as defined in Table 19. The thermal-hydraulics of ES-4 are governed by

$$\frac{dT(t)}{dt} = Q [n(t)+\Delta(t)] - a_3 T(t), \quad (112)$$

and,

$$\frac{d^2\gamma(t)}{dt^2} + a_2 \frac{d\gamma(t)}{dt} + a_1 \gamma(t) = \alpha K_0 T(t), \quad (113)$$

where $T(t)$ represents the change in fuel temperature, $\gamma(t)$ corresponds to the relative change in coolant density, $\Delta(t)=-\delta N(t)/N_0$ represents the relative excess in neutron population with

Table 19 Parameters' Definitions and Corresponding Values for ES-4

Parameter	Definition	Value
Q	heat generation parameter	25.044 K/s
a_1	model parameter 1	6.8166 /s ²
a_2	model parameter 2	2.2494 /s
a_3	model parameter 3	0.2325 /s
k_0	nominal fuel-to-coolant/moderator heat transfer coef.	-0.01318 /K
Γ_1	coolant-moderator density reactivity coef.	0.15
Γ_2	Doppler coefficient of reactivity	-2.61E-05 /K
Λ	neutron generation time	4.0E-05 s
β	delayed precursor fraction	0.0056
λ	decay constant	0.08 /s

$\delta N(t)$ corresponding to the increase in neutron population, α is an inverse measure of flow conditions and the other parameters are as defined in Table 19. The k_0 is a negative quantity (see Table 19) and subsequently increasing α leads to decreased heat transfer. The neutronic and the thermal-hydraulic systems are coupled through the reactivity feedback

$$\rho(t) = \Gamma_1 \gamma(t) + \Gamma_2 T(t) \quad (114)$$

where Γ_1 is the coolant/moderator density reactivity coefficient and Γ_2 is the Doppler coefficient of reactivity. The ES-4 model (i.e. Eqs.(110)-(114)) does not account for external controls (e.g. reactivity insertion through control rods).

Equations (110)-(114) constitute a lumped-parameter representation of BWR dynamics with the dependent variables denoting spatially-averaged quantities [77]. Initially (i.e. $t \leq 0$), the excess in neutron population is zero (i.e. $\Delta(t)=0$ for $t \leq 0$) and the reactor is assumed to be critical, i.e.

$$\left\{ \begin{array}{l} n(t) = 0 \\ c(t) = 0 \\ T(t) = 0 \\ \gamma(t) = 0 \\ \frac{d\gamma(t)}{dt} = 0 \\ \rho(t) = 0. \end{array} \right. \quad (115)$$

For $t > 0$ the relative neutron population (i.e. $n(t)$) is increased by the amount $\Delta(t)$ and the system becomes dynamical. When the relative excess neutron population is a constant value (i.e. $\Delta(t)=\Delta$ for $t > 0$), ES-4 has two equilibria given by

$$\left\{ \begin{array}{l} n = -1 \\ c = -\beta/\lambda\Lambda \\ T = (\Delta-1)Q/a_3 \\ \gamma = \frac{\alpha K_0(\Delta-1)Q}{a_1 a_3} \\ \frac{d\gamma}{dt} = 0 \\ \rho = \left[\frac{\Gamma_1 \alpha K_0}{a_1} + \Gamma_2 \right] \frac{(\Delta-1)Q}{a_3} \end{array} \right. \quad (116)$$

and,

$$\left\{ \begin{array}{l} n = -\Delta \\ c = \frac{-\beta\Delta}{\lambda\Lambda} \\ T = 0 \\ \gamma = 0 \\ \frac{d\gamma}{dt} = 0 \\ \rho = 0 . \end{array} \right. \quad (117)$$

The equilibrium condition given by Eq.(116) corresponds to zero neutron population (and hence a shutdown without external means), while the equilibrium condition given by Eq.(117) describes steady-state reactor operation.

Reference [77] indicates that the equilibrium condition given by Eq.(116) is unstable. For the reactor to remain stable in this condition, not only the neutron population should be zero but also the reactivity should be non-positive. However, the sign of the expression in Eq.(116) for ρ depends on the signs and magnitudes of the system parameters. For the data given in Table 19 (for Vermont Yankee), Eq.(116) yields

$$\begin{cases} \rho < 0 & \text{when } \Delta > 1 \\ \rho = 0 & \text{when } \Delta = 1 \\ \rho > 0 & \text{when } \Delta < 1. \end{cases} \quad (118)$$

Case 1 ($\Delta > 1$ or $\delta N < -N_0$) has no physical meaning since the neutron population cannot be physically decreased by more than N_0 . Case 2 ($\Delta = 1$ or $\delta N = -N_0$) corresponds to imposed shutdown, i.e. the neutron population is decreased to zero level by external means which maintain the reactor at that level all the times. With no external controls, Case 3 ($\Delta < 1$ or $\delta N > -N_0$) indicates that the reactor is unstable at this equilibrium point, since any small disturbances about the equilibrium will result in increase in neutron population and hence divergence from the equilibrium condition.

Reference [77] also shows that ES-4 behavior changes from stable to periodic to aperiodic (chaotic) behavior about the equilibrium condition given by Eq.(117) with increasing α . Since α is an inverse measure of flow conditions, it follows that for low-flow conditions the reactor becomes unstable. The low-flow conditions may occur as a result of failure or malfunction of the recirculation pumps (as it did in La Salle incident). From an operational point of view, it is important to know which initial and flow conditions and location in the state-space will lead to stable operation of the reactor without exceeding the constraints imposed by the regulators. Although unstable, the equilibrium condition given by Eq.(116) (i.e. shutdown condition) does not pertain to practical operation of a power reactor. When a reactor is shutdown, measures are taken for the reactor to stay shutdown (e.g. external insertion of negative reactivity).

8.2 Problem Statement and Input Preparation

Reference [77] has shown that for $\Delta=-0.1$ the amplitude of $n(t)$ oscillations can range from -1 (i.e. zero neutron population) to 15 (i.e. $16 N_0$) and the excess fuel temperature can range from -20°C to 60°C . These amplitudes exceed by far the constraints imposed by regulators. For this study, we are going to restrict the allowable operational range (i.e. R) to

$$\left\{ \begin{array}{l} -0.07 \leq n(t) \leq 0.3 \\ 155.0 \leq c(t) \leq 200.0 \\ -3.0 \leq T(t) \leq 5.0 \\ -0.012 \leq \gamma(t) \leq 0.014 \\ -0.006 \leq \frac{d\gamma}{dt} \leq 0.007 \end{array} \right. \quad (119)$$

The condition $n(t) \leq 0.3$ corresponds to a maximum allowable overpower of 30% (i.e. $N(t) = 1.3 N_0$).

For this study, Δ was chosen to be $\Delta=-0.1$, with the other system parameters as given in Table 19. The α range of interest is $1.5 \leq \alpha \leq 2.5$. The α is assumed to be uniformly distributed over this range, i.e

$$P(\alpha) = \left\{ \begin{array}{ll} \frac{1}{2.5-1.5} & \text{if } 1.5 \leq \alpha \leq 2.5 \\ 0 & \text{otherwise} \end{array} \right. \quad (120)$$

and the α range of interest is represented by three regions as shown in Table 20. The state-space of interest R is as given by Eq.(119) and is divided into $12 \times 12 \times 12 \times 12 \times 12 = 248,832$ cells. Each cell is further divided into $2 \times 2 \times 2 \times 2 \times 2 = 32$ subcells in order to approximate the

Table 20 The Definition of $\tilde{V}_m$ and the Corresponding Data For Eqs. (73), (75) and (120)

Cell or Data Type	m=1	m=2	m=3
$\tilde{V}_m$	$1.50 \leq \alpha \leq 1.80$	$1.80 \leq \alpha \leq 2.20$	$2.20 \leq \alpha \leq 2.50$
$\alpha p a_m$	1.65	2.00	2.35
$\tilde{v}_m$	0.30	0.40	0.30
$\tilde{P}_m$ from Eq.(104) and (109)	3/10	2/5	3/10

integrals in Eqs.(110)-(114) and Eq.(75). The internal integration time step (δt) is 0.005 s and the simulation time (τ) is 0.1 s.

8.3 Results and Analysis

The procedure for analysis is again as described in Section 4.2. Figures 67, 68 and 69 show the limiting probability distributions integrated over c , γ and $\frac{d\gamma}{dt}$ for $\alpha = 1.65, 2.00$ and 2.35, respectively. Also, Figures 70, 71 and 72 show the projections of the PGs over the $n-T$ plane for $\alpha = 1.65, 2.00$ and 2.35, respectively. The $n-T$ plane was chosen to illustrate the results of this study because only neutron population (i.e. power level) and temperature are directly measured and monitored in the reactor control room.

Figures 70, 71 and 72 show that there is only one PG for each value of α . The PG is periodic and its period increases with α , but its location in the $n-T$ plane does not change with α . Figures 70, 71 and 72 show that the PG period is 11τ , 12τ and 13τ , respectively. Also, the location of the PG is in the vicinity of the equilibrium condition $n=-\Delta=0.1$, $T=0.0$ for all the three values of α . A comparison of Figs.67, 68 and 69 to Figs.70, 71 and 72 indicates that all the persistent cells have limiting probability higher than 10^{-3} . The cells that have limiting probability lower than 10^{-3} in Figs.67, 68 and 69 turn out to be extra transient cells. It is also observed that the limiting probability distribution for the persistent cells (only) changes with the value of α . For example, the limiting probability for the cell located at $n=0.1$ and $T=0.0$ is $10^{-2} \leq P^\infty \leq 10^{-1}$ both in Fig.67 and 68, but the limiting probability for this cell becomes $10^{-3} \leq P^\infty \leq 10^{-2}$ when α increases to 2.35 (Fig.69). A comparison of the limiting probabilities in Figs.67, 68 and 69 indicates a maximum difference of 25%. Figure 73 shows the limiting probability distribution for the persistent cells when α is uniformly

Fig.67 Probability Distribution P_j^∞ , Integrated Over c , γ and $d\gamma/dt$, of the Persistent Groups for ES-4 ($\alpha = 1.65$).

$$\odot \quad 1.0 \times 10^{-2} \leq P^{\infty} \leq 1.0 \times 10^{-1}$$

$$\circ \quad 1.0 \times 10^{-3} \leq P^{\infty} \leq 1.0 \times 10^{-2}$$

$$\triangle \quad 1.0 \times 10^{-4} \leq P^{\infty} \leq 1.0 \times 10^{-3}$$

$$\times \quad 1.0 \times 10^{-5} \leq P^{\infty} \leq 1.0 \times 10^{-4}$$

$$+ \quad 1.0 \times 10^{-6} \leq P^{\infty} \leq 1.0 \times 10^{-5}$$

Fig.68 Probability Distribution P_j^{∞} , Integrated Over c , γ and $d\gamma/dt$, of the Persistent Groups for ES-4 ($\alpha = 2.00$).

$$\odot \quad 1.0 \times 10^{-2} \leq P^{\infty} \leq 1.0 \times 10^{-1}$$

$$\bigcirc \quad 1.0 \times 10^{-3} \leq P^{\infty} \leq 1.0 \times 10^{-2}$$

$$\triangle \quad 1.0 \times 10^{-4} \leq P^{\infty} \leq 1.0 \times 10^{-3}$$

$$\times \quad 1.0 \times 10^{-5} \leq P^{\infty} \leq 1.0 \times 10^{-4}$$

$$+ \quad 1.0 \times 10^{-6} \leq P^{\infty} \leq 1.0 \times 10^{-5}$$

Fig.69 Probability Distribution P_j^{∞} , Integrated Over c , γ and $d\gamma/dt$, of the Persistent Groups for ES-4 ($\alpha = 2.35$).

Fig.70 State-Space Projection of the PGs on the n - T Plane for ES-4 ($\alpha = 1.65$).

Fig.71 State-Space Projection of the PGs on the n - T Plane for ES-4 ($\alpha = 2.00$).

Fig.72 State-Space Projection of the PGs on the n - T Plane for ES-4 ($\alpha = 2.35$).

- $1.0 \times 10^{-2} \leq P^\infty \leq 1.0 \times 10^{-1}$
- $1.0 \times 10^{-3} \leq P^\infty \leq 1.0 \times 10^{-2}$
- △ $1.0 \times 10^{-4} \leq P^\infty \leq 1.0 \times 10^{-3}$
- × $1.0 \times 10^{-5} \leq P^\infty \leq 1.0 \times 10^{-4}$
- ⊕ $1.0 \times 10^{-6} \leq P^\infty \leq 1.0 \times 10^{-5}$

Fig.73 State-Space Projection of the PGs on the n - T Plane for ES-4 (α Uniformly Distributed).

distributed, using Eqs.(120) and (94) and Table 20 data. Although Fig.73 appears to be the same as Figs.67 and 68 when considering the cells with limiting probabilities larger than 10^{-3} , there is about 30% difference in actual magnitude of the limiting probabilities. The difference in magnitude of the limiting probabilities between Figs.73 and 69 increases to about 60%.

The BOA cuts shown in Figs.74, 75 and 76 correspond to the BOAs for $\alpha=1.65$, 2.00 and 2.35, respectively, at $c=175.0$, $\gamma=0.0$ and $\frac{d\gamma}{dt}=0.0$ (i.e. equilibrium values of c , γ and $\frac{d\gamma}{dt}$). A comparison of Figs.74, 75 and 76 shows that the BOA for the cyclic PG (which is represented by the cells located at $T=0.0$ in Figs.70, 71 and 72) is the same for all the three values of α under consideration. This means that as long as $-0.5^{\circ}\text{C} \leq T \leq 0.5^{\circ}\text{C}$ (taking in consideration the cell size in the temperature axis), and γ , $\frac{d\gamma}{dt}$ and c are in the vicinity of their equilibrium values, the neutron oscillations are bounded within the allowed operational range R defined by Eq.(119). Figures 74, 75 and 76 show that the region containing the BOA for the sink cell (or the unallowed range of operation) becomes larger with increasing value of α . All the cells located at $T \leq -1.0^{\circ}\text{C}$ and $T \geq 2.0^{\circ}\text{C}$ are BOAs for the sink cell independently of the neutron population level. This observation implies that BWR behavior may be very sensitive to fuel excess temperature for the α values considered in this study.

The absorption probabilities of the MDTCs (i.e. multiple domicile transient cells) into the sink cell in the $n-T$ plane for $c=175.0$, $\gamma=0.0$ and $\frac{d\gamma}{dt}=0.0$ are shown in Figs.77, 78 and 79 for $\alpha=1.65$, 2.00 and 2.35, respectively. Figures 77, 78 and 79 show that all the MDTCs for a given α value have a high absorption probability into the sink cell ($0.75 \leq \rho_{1j}^{\infty} \leq 0.99$). The difference between the absorption probabilities in Figs.77, 78 and 79 is less than 10%.

- BOAs for the sink cell
- × BOAs for the cyclic group ($T=11\tau$)

"blank" Multiple domicile transient cells

Fig.74 Cut of the BOAs in the n - T Plane, at the Equilibrium c , γ and $d\gamma/dt$, for ES-4 ($\alpha = 1.65$).

- BOAs for the sink cell
- × BOAs for the cyclic group ($T=12\tau$)
- "blank" Multiple domicile transient cells

Fig.75 Cut of the BOAs in the n - T Plane, at the Equilibrium c , γ and $d\gamma/dt$, for ES-4 ($\alpha = 2.00$).

- BOAs for the sink cell
- × BOAs for the cyclic group ($T=13\tau$)
- "blank" Multiple domicile transient cells

Fig.76 Cut of the BOAs in the n - T Plane, at the Equilibrium c , γ and $d\gamma/dt$, for ES-4 ($\alpha = 2.35$).

$$\odot \quad 0.75 \leq \rho_{1,i}^{\infty} \leq 0.99$$

$$\circ \quad 0.50 \leq \rho_{1,i}^{\infty} \leq 0.75$$

$$\times \quad 0.25 \leq \rho_{1,i}^{\infty} \leq 0.50$$

$$+ \quad 0.01 \leq \rho_{1,i}^{\infty} \leq 0.25$$

Fig.77 Absorption Probabilities $\rho_{1,j}^{\infty}$ into the Sink Cell in the n - T Plane, at the Equilibrium c , γ and $d\gamma/dt$, for ES-4 ($\alpha = 1.65$).

- $0.75 \leq \rho_{1,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{1,i}^{\infty} \leq 0.75$
- × $0.25 \leq \rho_{1,i}^{\infty} \leq 0.50$
- + $0.01 \leq \rho_{1,i}^{\infty} \leq 0.25$

Fig.78 Absorption Probabilities $\rho_{1,j}^{\infty}$ into the Sink Cell in the n - T Plane, at the Equilibrium c , γ and $d\gamma/dt$, for ES-4 ($\alpha = 2.00$).

$$\bullet \quad 0.75 \leq \rho_{1,i}^{\infty} \leq 0.99$$

$$\circ \quad 0.50 \leq \rho_{1,i}^{\infty} \leq 0.75$$

$$\times \quad 0.25 \leq \rho_{1,i}^{\infty} \leq 0.50$$

$$+ \quad 0.01 \leq \rho_{1,i}^{\infty} \leq 0.25$$

Fig.79 Absorption Probabilities $\rho_{1,j}^{\infty}$ into the Sink Cell in the n - T Plane, at the Equilibrium c , γ and $d\gamma/dt$, for ES-4 ($\alpha = 2.35$).

Figure 80 shows the absorption probabilities into the sink cell when α is uniformly distributed over the range $1.50 \leq \alpha \leq 2.50$. Figure 80 indicates that all the cells with $T \geq 1.7^\circ\text{C}$ and $T \leq -1.7^\circ\text{C}$ are BOAs to the sink cell due to their high absorption probabilities. The region occupied by the cells with $0.5 \leq T \leq 1.7^\circ\text{C}$ and $-1.7 \leq T \leq -0.5^\circ\text{C}$ consists of MDTCs with high absorption probability into the sink cell ($0.75 \leq \rho_{1,j}^\infty \leq 0.99$). Since only one PG is identified (i.e. the cyclic group in Figs. 70-72), the absorption probability of the MDTCs into this PG would be $1 - \rho_{1,j}^\infty$ and the narrow region in Fig.80 with $-0.5 \leq T \leq 0.5^\circ\text{C}$ corresponds to the BOA to this PG. The difference between the absorption probabilities of Fig.80 and Figs.77-79 is less than 15%. Comparing Figs.76 and 80, it is seen that some cells of the BOA to the sink cell in Fig.76 have absorption probabilities of less than unity into the sink cell in Fig.80 (i.e. probabilities in the range $0.75 \leq \rho_{1,j}^\infty \leq 0.99$). This observation again demonstrates the blurring of BOAs with uncertainty on the system parameters. The BOA to the cyclic PG shown in Figs.70, 71 and 72 is not affected in this case with uncertainty on the system parameters.

8.4 Computational Aspects

The computational requirements based on one value of α are shown in Table 21. Table 21 shows that the algorithm for determining PGs, BOAs and absorption probabilities requires a storage capacity of 13.7 MWords. As mentioned in Section 7.5, the users of the State of Ohio CRAY-YMP are limited to 16.0 MWords. Thus, refinement of the results obtained in Section 8.3 by, for example, using 4 subcells instead of 2 subcells for the determination of cell-to-cell transition probabilities was not possible since such a refinement requires at least $4^5/2^5 \times 13.7 \text{ MWords} = 438.4 \text{ MWords}$ of storage. However, it should be mentioned that it would not be possible to obtain the results reported in Section 8.3 with the CCMT algorithm

- ⊗ $0.99 \leq \rho_{1,i}^{\infty} \leq 1.00$
- $0.75 \leq \rho_{1,i}^{\infty} \leq 0.99$
- △ $0.50 \leq \rho_{1,i}^{\infty} \leq 0.75$
- × $0.25 \leq \rho_{1,i}^{\infty} \leq 0.50$
- ⊕ $0.01 \leq \rho_{1,i}^{\infty} \leq 0.25$

Fig.80 Absorption Probabilities $\rho_{1,j}^{\infty}$ into the Sink Cell in the $n-T$ Plane, at the Equilibrium c , γ and $d\gamma/dt$, for ES-4 (α Uniformly Distributed).

Table 21 Computational Requirements for ES-4 Based on one Value of α

Quantities Determined	CRAY-YMP CPU Time	Storage Requirement
Images	1,975 s	8.9E06 entries
Limiting Probabilities	382 s	10.8E06 entries
PGs, BOAs and Absorption Probabilities	217 s	13.7E06 entries

as originally proposed, since the algorithm would require about 47 MWords ($\approx 3.5 \times 13.7$ MWords) of random access memory storage. Also, a search of the same dynamic variable and parameter range of interest (i.e. as given by Eq.(119) and Table 20) using the same discretization scheme (i.e. 2×12 initial values per dynamic variable and 3 α values) would require $3 \times 2^5 \times 12^5 = 23,887,872$ integrations of the ES-4 system equations (i.e. Eqs.(110)-(114)) over a period of 60 to 400 τ (i.e. approximate time for the system to reach asymptotic behavior depending on the initial conditions and α value). Since the integration of these equations over 1 τ takes about 2.5×10^{-4} s of CRAY YMP CPU time, the total CPU time needed would be between $2.5 \times 10^{-4} \times 60 \times 23,887,872 = 355,500$ s ≈ 100 h to $2.5 \times 10^{-4} \times 400 \times 23,887,872 = 2,366,667$ s ≈ 660 h, as compared to only $1975 + 382 + 217 = 2574$ s ≈ 0.72 h (see Table 21).

8.5 Summary

This chapter demonstrates how the improved algorithms presented in Chapter 4 allow global analyses that would have been computationally infeasible using either direct integration or the original CCMT algorithm. The system specific results for ES-4 show that BWR stability may be very sensitive to fuel excess temperature under some low-flow conditions (i.e. for high α values). For small change in fuel temperature ($-0.5^\circ\text{C} \leq T \leq 0.5^\circ\text{C}$), the reactor remains in stable behavior. A possible physical explanation for this behavior is that while the Doppler coefficient of reactivity for the system is negative (i.e. Γ_1 in Table 19), the coolant/moderator density coefficient is positive (i.e. Γ_2 in Table 19). When the change in fuel temperature exceeds 0.5°C (or the heat transferred to the coolant increases), the positive feedback from the coolant/moderator exceeds the negative Doppler feedback and the overall reactivity feedback becomes positive, leading to unstable reactor dynamics which may

necessitate shutdown by external means. However, due to the relatively coarse state-space partitioning scheme used in this study, these results need to be verified against results obtained using a finer partitioning scheme. Also the flow conditions (i.e. α values) considered in this study will not occur under normal operating conditions, but may occur for some accident scenarios.

CHAPTER IX

CONCLUSION AND RECOMMENDATIONS FOR FUTURE WORK

9.1 Conclusion

This dissertation achieved all its objectives stated in Section 1.4. The theoretical basis developed in Chapter III is directed to Objective 1 of the dissertation. Using a Chapman-Kolmogorov interpretation of system dynamics, this theoretical basis:

- unifies the DPVSM, OSU-DM and CCMT as special cases, and,
- can not only account for the impact of the complex hardware/process variable/human interaction encountered in PCS on the system reliability and safety but also for the effects of the uncertainties on the predicted probabilistic system performance.

The latter feature of the theoretical basis is particularly relevant to future reactors with passive or inherent safety features which may rely on natural phenomena rather than active components to achieve safe shutdown. The impact of process uncertainties on the reliability and safety of these reactors will be much more significant than it is for the current systems.

The theoretical basis presented in Chapter III lead to the development of new, generic (i.e. system independent) algorithms described in Chapter IV which reduce the computational limitations in applications towards Objective 2 of this dissertation. The new algorithms also allow:

1. automatic discretization of the state-space region of interest,
2. utilization of a data base to describe the PCS physics and automatic organization of the data base,
3. utilization of the same data base with varying SU failure data or probability distribution of the stochastic parameters for sensitivity analysis to assess the impact of uncertainties in the failure data or the probability distribution of the stochastic parameter,
5. utilization of failure data as reported (i.e. failure per demand and/or failure rate) without conversion failure rates as required in obtaining time dependent Top Event statistics with conventional techniques, and,
6. self-consistent model improvement by the systematical refinement of the state-space discretization.

The new algorithms were validated and benchmarked in Chapters V and VI using ES-1 and ES-2. In consistence with the previously reported work in the literature, the results of Chapters V and VI indicated that in PCS with multiple Top Events:

- Top Event occurrence may not only depend on the order of component failure but also on the exact time of component failure and/or the exact magnitudes of the process variables at the time of component failure,
- competition between Top Events can lead to large differences in the interval reliability of process control systems as predicted by the dynamic methodology and the fault-tree approach,
- fault-tree predictions can be improved by considering the effects of the differences in component failure sequencing on Top Event occurrence probabilities, however, this approach may need simulation of the PCS behavior under different component failure

scenarios and may still lead to large over- and under-estimation of the Top Event occurrence probabilities.

The results of Chapter VI also showed that:

- demand rates for nuclear plant systems with safety functions can be largely overestimated if component FPD data are converted to failure rates, which is the standard procedure when the event-tree/fault-tree approach is used for time-dependent reliability analyses,
- the data conversion process does not necessarily lead to conservative results, and,
- dynamic methodologies may be useful in accident classification by identifying the initiating event threshold values which lead to large and sudden changes in the demand rates for systems with safety functions.

In these respects, Chapter VI provided new reasons for the need for dynamic methodologies in risk and reliability studies. The practical utility of the new algorithms in nuclear engineering were demonstrated by using two non-linear systems from reactor dynamics (i.e. Chapters VII and VIII) towards Objective 3. For example, the results of Chapter VIII show that these algorithms yield the finding that even small perturbations in fuel temperature may lead to undesirable BWR behavior under low-flow conditions that do not occur under normal operating conditions, but possibly for some accident scenarios. The algorithms also allow quantifying the likelihood of such behavior if there is uncertainty on the observed flow conditions (e.g. fluctuation in the rate of heat transfer to the coolant in ES-4 through a change in α).

From a computational viewpoint, the illustrations in Chapters V through VIII demonstrate that the new algorithms:

- can speed up PCS interval reliability analysis by factor of 6 and reduce the storage requirements by a factor of 20 compared to the original version of OSU-DM (see Table 3) and hence increase the range of applicability of dynamic methodologies to include larger systems,
- can speed up the identification of initial conditions and/or parameter values that lead to desirable and/or undesirable system behavior (through the BOAs) by a factor of 20 and reduce the storage requirements by a factor of 2.5 compared to CCMT as originally proposed (see Table 6),
- for systems with stochastic parameters, reduce the computational time needed to determine the probability distribution of PGs by a factor proportional to the number of pdfs that need to be considered if a unique pdf cannot be identified to represent the stochasticity in the system parameters, and,
- are much faster than direct integration (a factor of 50-400 reduction in CPU time).

9.2 Recommendations For Future Work

Although the new algorithms are computationally much more advantageous than the originally proposed OSU-DM and CCMT, the new algorithms may still not be computationally feasible if:

1. the system has a large number of ungroupable components and thus modularization is not feasible,
2. validation of the results through refinement of state-space discretization requires a large storage space, and,

3. system behavior during a long time period is of interest and the system model does not allow interactively changing system parameters during arbitrary times.

Conditions 1 and 2 may lead to storage limitations, while Condition 3 may lead to CPU limitations. However, since most of the CPU time is spent in generating the images that yield the cell-to-cell transition probabilities and which are obtained by integrating the system equations over τ for given initial conditions (e.g. see Table 21) and since the integration of the system equations for a given set of initial conditions can be performed independently of the other initial conditions, it is expected that CPU limitations can be greatly reduced using parallel or distributed computing. Distributed computing may also reduce the storage limitations without compromising appreciably from CPU savings.

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APPENDIX A

Listing of Computer Programs

```

C      *      disc.f (ALG-G)
C      *
C      * THE PROGRAM disc.f (ALG-G) IS WRITTEN TO ORGANIZE THE
C      * CONTROLLED VARIABLE SEQUENTIAL TRAJECTORY DATA IN
C      * A TABULAR FORM SUITABLE FOR MARKOV MODEL CONSTRUCTION.
C      *
C      * THE PROGRAM disc.f HAS AN AUTOMATIC DISCRETIZATION
C      * SCHEME BASED ON A RECTANGULAR MESH. THEN, FOR EACH
C      * TRAJECTORY, THE PROGRAM ASSIGNS THE DEPARTURE AND THE
C      * ARRIVAL CELL NUMBERS ACCORDINGLY.
C      *
C      * THE PROGRAM disc.f CHECKS FOR CONSISTENCY IN THE
C      * TRAJECTORY DATA AND HANDLES MULTIPLE TRAJECTORIES IN
C      * A GIVEN TIME STEP FOR THE SAME SU STATE COMBINATIONS.
C      *
C      * TRAJECTORIES ORIGINATING IN ANY OF THE ABSORBING STATES
C      * ARE AUTOMATICALLY REJECTED, AND UNIFORM WEIGHT IS
C      * ASSIGNED TO ALL TRAJECTORIES.
C      *
C      * INPUTS FOR THIS PROGRAM ARE :
C      *
C      *   - nd : NUMBER OF STATE VARIABLES OR NUMBER
C      *         OF DIRECTIONS TO BE CONSIDERED
C      *
C      *   - ncvc : TOTAL NUMBER OF NON-ABSORBING CELLS
C      *             IN THE CVSS; i.e. TOTAL NUMBER OF
C      *             CELLS IN THE REGION OF INTEREST
C      *             EXCLUDING THE TOP EVENTS
C      *
C      *   - ncsc: TOTAL NUMBER OF COMPONENT STATE
C      *             COMBINATIONS
C      *
C      * FOR EACH PROBLEM, THESE INPUTS HAVE TO BE CHANGED
C      * BELOW IN THE parameter DECLARATION STATEMENTS.
C      *
C      * THREE INPUT FILES HAVE TO BE GENERATED TO RUN THIS
C      * PROGRAM :
C      *
C      *   siml.dat : THIS FILE CONTAINS THE CONTROLLED
C      *                 VARIABLE SEQUENTIAL TRAJECTORY DATA
C      *                 AS OBTAINED FROM THE SIMULATIONS
C      *
C      *   dimn.dat : THIS FILE CONTAINS THE DISCRETIZATION
C      *                 INFORMATION FOR EACH DIRECTION OR
C      *                 STATE VARIABLE. THIS FILE HAS TO BE
C      *                 CREATED BY THE USER PRIOR TO RUNNING
C      *                 THIS PROGRAM. THE FILE CONSISTS OF

```

```

C      *      "nd" (see definition of "nd" above)
C      *
C      *      LINES. EACH LINE HAS 4 ENTRIES,
C      *      WHERE FIRST ENTRY IS THE DIRECTION
C      *      SEQUENTIAL NUMBER (id(*)), THE
C      *      SECOND ENTRY SPECIFIES THE NUMBER
C      *      OF CELLS (OR DIVISIONS, ncd(*)) TO BE
C      *      CONSIDERED IN THAT DIRECTION, THE
C      *      THIRD AND THE FOURTH ENTRIES SPECIFY
C      *      THE MINIMUM AND THE MAXIMUM VALUE OF
C      *      THE CONTROLLED VARIABLE IN THAT
C      *      DIRECTION (x(*,1) and x(*,2)),
C      *      RESPECTIVELY. AN EXAMPLE LINE WOULD
C      *      BE AS FOLLOWS :
C      *
C      *      3  12  10.2  30.7
C      *
C      *      WHERE 3 REFERS TO THE THIRD CONTROLLED
C      *      VARIABLE OR THIRD DIRECTION, 12 REFERS
C      *      12 DIVISIONS OR 12 CELLS TO BE
C      *      CONSIDERED IN THE DISCRETIZATION OF
C      *      THE THIRD DIRECTION, 10.2 AND 30.7
C      *      REFER TO THE MINIMUM AND THE MAXIMUM
C      *      VALUES OF THE THIRD CONTROLLED
C      *      VARIABLE IN THE REGION OF INTEREST.
C      *      IF nd IS EQUAL TO 4, THEN THE FILE
C      *      dimn.dat MUST CONTAIN FOUR LINES.
C      *
C      *      sets.dat : THIS FILE CONTAINS THE INFORMATION
C      *      PERTAINING TO THE SYSTEM SETPOINTS
C      *      IN EACH DIRECTION. THIS FILE HAS TO
C      *      BE CREATED BY THE USER TOO. THIS FILE
C      *      CONSISTS OF nd LINES. FOR EACH LINE,
C      *      THE FIRST ENTRY IS THE DIRECTION
C      *      SEQUENTIAL NUMBER (id(*)), THE SECOND
C      *      ENTRY SPECIFIES THE ABSOLUTE NUMBER OF
C      *      SETPOINTS (nstp) IN THAT DIRECTION,
C      *      FOLLOWED BY THE CONTROLLED VARIABLE
C      *      VALUES ASSIGNED TO THESE SETPOINTS
C      *      STARTING FROM THE SMALLEST TO THE
C      *      LARGEST VALUE. SOME EXAMPLE LINES ARE
C      *      PRESENTED BELOW TO GUIDE THE USER IN
C      *      CREATING THIS FILE. HOWEVER, THE USER
C      *      MUST READ CAREFULLY THESE EXAMPLES
C      *      BEFORE TEMPTING TO CREATE THIS FILE.
C      *
C      *      FIRST EXAMPLE LINE :
C      *      2  0
C      *

```



```

C      *      FOR DISCRETIZING THE STATE-SPACE
C      *
C      *      seq2.dat      : THIS FILE ARRANGES THE CELL-TO-CELL
C      *                      TRANSITION PROBABILITIES. THE FIRST
C      *                      ENTRY OF EACH LINE IN THIS FILE IS THE
C      *                      PRINTED LINE NUMBER, FOLLOWED BY THE
C      *                      TIME STEP NUMBER (it(*)), THE DEPARTURE
C      *                      CELL NUMBER (jp), THE ARRIVAL CELL
C      *                      NUMBER (j), AND THE TRANSITION
C      *                      PROBABILITIES FROM CELL jp TO CELL j
C      *                      FOR EACH OF THE COMPONENT STATE
C      *                      COMBINATIONS.
C      *
C      *      cuts.dat      : THIS IS USED AS A POINTER FILE. THIS
C      *                      CONTAINS THE FIRST LINE NUMBER OF EACH
C      *                      MODELING TIME STEP.
C      *
C      *
C      *      THIS PROGRAM IS ORIGINALLY WITTEN BY
C      *      M. HASSAN      2 JANUARY 1990
C      *
C      *      MODIFIED AND GENERALIZED BY
C      *      M. BELHADJ     14 SEPTEMBER 1991
C      *
C

```

```

parameter(nd=2)

parameter(ncsc=9)
parameter(ncvc=140)
dimension G(ncvc,NCSC)
DIMENSION IFLAG(ncvc)
DIMENSION IW(NCSC)
DIMENSION JV(ncvc),NP(ncvc),N(ncvc),JVP(ncvc)
DIMENSION IT(ncvc),JP(ncvc),J(ncvc)

dimension ncd(nd),x(nd,2),stp(ncvc),id(nd)
dimension idp(nd),yp(nd),y(nd)
dimension xsub(ncvc),drang(ncvc)
dimension dstep(ncvc),a(nd,ncvc)

OPEN(UNIT=2,FILE='seq2.dat',STATUS='NEW')
OPEN(UNIT=3,FILE='cuts.dat',STATUS='NEW')

OPEN(UNIT=10,FILE='seq1.dat',STATUS='OLD')

```



```

OPEN(UNIT=21,FILE='dimn.dat',STATUS='OLD')
OPEN(UNIT=22,FILE='sets.dat',STATUS='OLD')
OPEN(UNIT=23,FILE='rslt.dat',STATUS='new')

DO 20 I=1,ND
c
C   readin input data prepared by the user for discretizing
C   the state-space
c
READ(21,*)K,NCD(K),X(K,1),X(K,2)
20  CONTINUE

DO 30 I=1,ND
c
C   reading the system setpoints to be incorporated in
C   the state-space discretization
c
READ(22,*)NN,NSTP,(STP(JJ),JJ=1,NSTP)
IF(NSTP.GE.NCD(NN))GOTO 1200
c
C   discretizing the state-space
c

XSUB(1)=X(NN,1)
DO 40 K=1,NSTP
XSUB(K+1)=STP(K)
40  CONTINUE
XSUB(NSTP+2)=X(NN,2)
STEP0=(X(NN,2)-X(NN,1))/(NCD(NN)-NSTP)
NTOT=0
DO 50 K=1,NSTP+1
DRANG(K)=XSUB(K+1)-XSUB(K)
NP(K)=INT(DRANG(K)/STEP0)
IF(NP(K).EQ.0) NP(K)=1
DSTEP(K)=DRANG(K)/NP(K)
NTOT=NTOT+NP(K)
50  CONTINUE
IF(NTOT.EQ.NCD(NN)) GOTO 1100
1000 DIFF=0.0
DO 60 K=1,NSTP+1
IF(DSTEP(K).GT.DIFF)THEN
DIFF=DSTEP(K)
NDIFF=K
ENDIF
60  CONTINUE
NP(NDIFF)=NP(NDIFF)+1

```

```

        DSTEP(NDIFF)=DRANG(NDIFF)/NP(NDIFF)
        NTOT=NTOT+1
        IF(NTOT.LT.NCD(NN)) GOTO 1000
1100    CONTINUE
        KKK=0
        DO 70 K=1,NSTP+1
        DO 80 KK=1,NP(K)
        KKK=KKK+1
        A(NN,KKK)=XSUB(K)+(KK-1)*DSTEP(K)
80     CONTINUE
70     CONTINUE
        A(NN,KKK+1)=X(NN,2)
30     CONTINUE
c
C     writing the results of the discretization
c
        DO 100 I=1,ND
        WRITE(23,*)(A(I,JJ),JJ=1,NCD(I)+1)
100    CONTINUE
        GOTO 1300
c
C     warnings to the user in case the discretization
C     of the state-space is incorrect due to deficiencies
C     or discrepancies in the input files
c
1200    WRITE(23,*)' '
        WRITE(23,*)'**** Warning ****'
        WRITE(23,*)' '
        WRITE(23,*)' Discretization scheme failed.'
        WRITE(23,*)' The number of setpoints NSTP should be'
        WRITE(23,*)' less than the number of cells NCD in'
        WRITE(23,*)' direction ',nn
        GOTO 9000
1300    CONTINUE

        IK=0
        K=0
        INDX=0

2000    CONTINUE
c
C     reading the simulations data
c
        READ(10,*,END=4000)ISTP,(YP(II),II=1,ND),(Y(II),II=1,ND),NNP,NN
c
C     preparing and rearranging the simulation data
c

```

```

NFLAG=0
I=0
DO 220 I=1,ND
IF(YP(I).LT.A(I,1))GOTO 8000
IF(YP(I).GE.A(I,NCD(I)+1))GOTO 8000
JJ=0
DO 230 JJ=1,NCD(I)
IF(YP(I).LT.A(I,JJ+1).AND.YP(I).GE.A(I,JJ))GOTO 2100
230   CONTINUE
2100  IDP(I)=JJ

IF(Y(I).LT.A(I,1))GOTO 2700
IF(Y(I).GE.A(I,NCD(I)+1))GOTO 2600
DO 240 JJ=1,NCD(I)
IF(Y(I).LT.A(I,JJ+1).AND.Y(I).GE.A(I,JJ))GOTO 2500
240   CONTINUE
2500  ID(I)=JJ
GOTO 2800
2600  ID(I)=NCD(I)+2
NFLAG=1
GOTO 2800
2700  ID(I)=NCD(I)+1
NFLAG=1
2800  CONTINUE
220   CONTINUE

c
C      rearranging the simulation data
c

JJP=0
NCELL=0
DO 260 I=ND,2,-1
NCELL=IDP(I)-1
DO 270 II=1,I-1
NCELL=NCELL*NCD(II)
270   CONTINUE
JJP=JJP+NCELL
260   CONTINUE
JJP=JJP+IDP(1)

IF(NFLAG.EQ.1) GOTO 2200

JJ=0
NCELL=0
DO 280 I=ND,2,-1
NCELL=ID(I)-1
DO 290 II=1,I-1
NCELL=NCELL*NCD(II)

```

```
290    CONTINUE
      JJ=JJ+NCELL
280    CONTINUE
      JJ=JJ+ID(1)

      GOTO 3000

2200   CONTINUE
      IF(ISTP.GT.IT(K)) GOTO 2900
      DO 250 I=1,ND
      IF(ID(I).EQ.NCD(I)+1)THEN
        K=K+1
        IT(K)=ISTP
        JP(K)=JJP
        J(K)=NCVC+I*2-1
        NP(K)=NNP
        N(K)=NN
        IFLAG(K)=0
      ENDIF
      IF(ID(I).EQ.NCD(I)+2)THEN
        K=K+1
        IT(K)=ISTP
        JP(K)=JJP
        J(K)=NCVC+I*2
        NP(K)=NNP
        N(K)=NN
        IFLAG(K)=0
      ENDIF
250    CONTINUE
      GOTO 2000

2900   K=K+1
      GOTO 4100

3000   CONTINUE

      K=K+1
      IT(K)=ISTP
      JP(K)=JJP
      J(K)=JJ
      NP(K)=NNP
      N(K)=NN
      IFLAG(K)=0
      IF(K.EQ.1)GOTO 2000
      IF(IT(K).EQ.IT(K-1)) GOTO 2000
      GOTO 4100

4000   INDX=1
```

```

4100  CONTINUE
      LNS=0
      KSTEP=0
      MSTEP=K-1
      IF(INDX.EQ.1)MSTEP=K

4200  KSTEP=KSTEP+1
      IF(KSTEP.GT.MSTEP)GOTO 4400
      IF(IFLAG(KSTEP).GT.0)GOTO 4200
c
C      calculating the cell-to-cell transition probabilities
c
      JR=JV(KSTEP)
      JRP=JVP(KSTEP)

      NRP=NP(KSTEP)
      NR=N(KSTEP)
      LNS=LNS+1
      JP(LNS)=JRP
      J(LNS)=JR
      DO 300 ICC=1,MSTEP
      IF(IFLAG(ICC).GT.0)GOTO 4300
      IF(JR.EQ.JV(ICC).AND.JRP.EQ.JVP(ICC))THEN
      G(LNS,NP(ICC))=1.0
      IFLAG(ICC)=LNS
      ENDIF
4300  CONTINUE
300   CONTINUE
      GOTO 4200
4400  CONTINUE
      IF(LNS.EQ.1) GOTO 4500
      DO 320 I1=1,LNS
      DO 330 J1=1,NCSC
      IW(J1)=1
330   CONTINUE
      IF(I1.EQ.LNS) GOTO 4500
      I3=I1+1
      DO 340 I2=I3,LNS
      IF(JP(I1).EQ.JP(I2))THEN
      DO 350 KK=1,NCSC
      APT1=ABS(G(I1,KK)-1.0)
      APT2=ABS(G(I2,KK)-1.0)
      IF(APT1.LT.1.0E-5.AND.APT2.LT.1.0E-5)IW(KK)=IW(KK)+1
350   CONTINUE
      ENDIF
340   CONTINUE

      DO 360 I2=I3,LNS

```

```

IF(JP(I1).EQ.JP(I2))THEN
  DO 370 KK=1,NCSC
    KFLAG=0
    APT1=ABS(G(I1,KK)-1.0)
    APT2=ABS(G(I2,KK)-1.0)
    IF(APT1.LT.1.0E-5.AND.APT2.LT.1.0E-5)THEN
      G(I2,KK)=1.0/IW(KK)
      KFLAG=1
    ENDIF
    IF(KFLAG.GT.0)G(I1,KK)=1.0/IW(KK)
370    CONTINUE
    ENDIF
360    CONTINUE

320    CONTINUE
4500    CONTINUE
c
C    writing the results of cell-to-cell transition
C    probabilities
c
DO 380 KK=1,LNS
  IK=IK+1
  WRITE(2,111)IK,IT(KK),JP(KK),J(KK),(G(KK,JK),JK=1,NCSC)
380  CONTINUE
c
C    writing to the pointer file
c
write(3,*)lms

IF(NFLAG.EQ.0)THEN
  IT(1)=IT(K)
  JP(1)=JP(K)
  J(1)=J(K)
  NP(1)=NP(K)
  N(1)=N(K)
  K=1
ENDIF

IF(INDX.EQ.0.AND.NFLAG.EQ.0) GOTO 2000

IF(NFLAG.EQ.1)THEN
  K=0
  DO 450 I=1,ND
    IF(ID(I).EQ.NCD(I)+1)THEN
      K=K+1
      IT(K)=ISTP
      JP(K)=JJP
      J(K)=NCVC+I*2-1

```

```
NP(K)=NNP
N(K)=NN
IFLAG(K)=0
ENDIF
IF(ID(I).EQ.NCD(I)+2)THEN
K=K+1
IT(K)=ISTP
JP(K)=JJP
J(K)=NCVC+I*2
NP(K)=NNP
N(K)=NN
IFLAG(K)=0
ENDIF
450  CONTINUE
ENDIF

8000  continue

IF(INDX.EQ.0) GOTO 2000

111  format(1x,i3,2x,i3,2x,i3,2x,i3,9(2x,f4.2))

CLOSE(3)
CLOSE(2)
9000  CONTINUE
STOP
END
```

```

C      *      cdf.f (ALG-P)
C      *
C      * THE PROGRAM cdf.f (ALG-P) IS WRITTEN TO COMPUTE THE SYSTEM
C      * TOP EVENT CDFs. INPUTS FOR THE PROGRAM ARE THE NUMBER
C      * OF RECORDS IN THE DATABASE, THE NUMBER OF TIME STEPS FOR
C      * WHICH CDFs ARE TO BE COMPUTED, TIME INTERVAL, FAILURE
C      * RATES OF THE SUs, NUMBER OF SYSTEM STATES.
C      *
C      * THE USER HAS TO MODIFY THE ENTRIES FOR ALL THE PARAMETER
C      * DECLARATION STATEMENTS ACCORDINGLY. THE OTHER INPUT FILES
C      * SHOULD BE ALREADY OBTAINED FROM ALG-G ALGORITHM. THE USER
C      * MUST PROVIDE THE SUBROUTINE "FAILD" FOR CALCULATING THE
C      * TRANSITION PROBABILITIES BETWEEN COMPONENT STATES.
C      *
C      *
C      *      THIS PROGRAM WAS ORIGINALLY WRITTEN BY
C      *      M. HASSAN  2 JANUARY 1990
C      *
C      *      MODIFIED, GENERALIZED AND VECTORIZED BY
C      *      M. BELHADJ  FEBRUARY 1992
C      *
C
C      SPECIFY NUMBER OF LINES IN THE INPUT DATAFILE
C      PARAMETER(LINES=3237)
C
C      SPECIFY NUMBER OF TIME STEPS IN THE DATABASE
C      PARAMETER(MAXST=60)
C
C      SPECIFY TIME STEP (MIN)
C      DOUBLE PRECISION ERR,DT
C      PARAMETER(TSTEP=1.0,DT=6.0D01)
C
C      SPECIFY TOTAL NUMBER OF COMPONENT STATES
C      PARAMETER(NCSC=9)
C
C      SPECIFY NUMBER OF NON-ABSORBING CELLS IN CVSS
C      PARAMETER(NCVC=140)
C
C      SPECIFY THE NUMBER OF DIMENSIONS TO BE CONSIDERED
C      PARAMETER(ND=2)
C
C      PARAMETER(NDTOP=ND*2)
C      PARAMETER(NTOT=NCVC+NDTOP)

```



```

C
C   SPECIFY THE INITIAL SYSTEM STATE, CELL # AND
C   COMPONENT STATE COMB.
C   PARAMETER(INj=92,INn=1)
C
INTEGER K1(MAXST)
REAL*8 C,P(NTOT,NCSC),PT(NTOT,NCSC),Q(NTOT,NCSC)
REAL*8 TOT,DUM,GJP,GJPJ,F(NDTOP),W(NDTOP,NCSC)
REAL*8 G(LINES,NCSC),SIM(NDTOP,NCSC)
DIMENSION IT(LINES),JP(LINES),J(LINES)

OPEN(UNIT=3,FILE='seq2.dat',STATUS='OLD')
OPEN(UNIT=4,FILE='cuts.dat',STATUS='OLD')
OPEN(UNIT=10,FILE='cdf.dat',STATUS='NEW')
OPEN(UNIT=11,FILE='w.dat',STATUS='NEW')
OPEN(UNIT=12,FILE='sim.dat',STATUS='NEW')

C
C   DEFINE INITIAL SYSTEM STATE PROBABILITIES
C   TIME=0.0
C   DO 997 NN=1,NCSC
C   DO 997 JJ=1,NTOT
C   P(JJ,NN)=0.0
997 CONTINUE

P(INJ,INN)=1.0

MSTEP=0

5555 CONTINUE

MSTEP=MSTEP+1
read(4,*)k1(mstep)

DO 1000 I=1,K1(MSTEP)
READ(3,*)K,IT(I),JP(I),J(I),(G(I,IN),IN=1,NCSC)
1000 CONTINUE

C
C   TRANSITION MATRIX
C
C   DO 4040 NN=1,NCSC
C   DO 4040 JJ=1,NTOT
C
C   INITIALIZE TRANSITION MATRIX ROW VECTOR
C
C   DO 988 NNPP=1,NCSC

```

```

DO 988 JJPP=1,NTOT
  Q(JJPP,NNPP)=0.0
988  CONTINUE

      DO 4000 JJP=1,NTOT
        DO 4100 NNP=1,NCSC
C
C      FIND G(LP->L,KKP) FROM 'PSEQ5.OUT'
C

      GJP=0.0
      GJPJ=0.0

      DO 2025 IQ=1,K1(MSTEP)
        IF(JP(IQ).EQ.JJP.AND.J(IQ).EQ.JJ)THEN
          GJP=G(IQ,NNP)

          ELSE
            GOTO 2025
          ENDIF
          GJPJ=GJP
2025  CONTINUE
6000  CONTINUE

C
C      DETERMINE C

      CALL FAILD(JJP,JJ,NNP,NN,C)
      Q(JJP,NNP)=GJPJ*C
4100  CONTINUE
      GOTO 4000
4400 IF(JJP.EQ.JJ)Q(JJP,NN)=1.0
4000  CONTINUE
C
C      START P(I,L) CALCULATIONS FOR T+DT
C

      DO 6590 NNN=1,NCSC
      DO 6590 JJJ=1,NTOT
        Q(JJJ,NNN)=Q(JJJ,NNN)*P(JJJ,NNN)
6590  CONTINUE

      PT(JJ,NN)=0.0
      DO 6565 NNN=1,NCSC
      DO 6565 JJJ=1,NTOT
        PT(JJ,NN)=PT(JJ,NN)+Q(JJJ,NNN)
6565  CONTINUE

```

```

4040  CONTINUE
C
      TOT=0.0
      DO 6514 NNN=1,NCSC
      DO 6514 JJJ=1,NTOT
      TOT=TOT+PT(JJJ,NNN)
6514  CONTINUE
C
C
      ERR=1.0-TOT
C
C

      CALL RANDOM(XRAN)
      JRAN=INT(XRAN*NTOT)
      IF(JRAN.EQ.0)JRAN=NCVC
      CALL RANDOM(XRAN)
      NRAN=INT(XRAN*NCSC)
      IF(NRAN.EQ.0)NRAN=NCSC

      PT(JRAN,NRAN)=PT(JRAN,NRAN)+ERR
C
C
      DO 6567 ID=1,NDTOP
      F(ID)=0.0
      IJJ=NCVC+ID
      DO 6568 NNN=1,NCSC
C
C  CALCULATE CUMULATIVE FAILURE PROBABILITY F(t)
C
      F(ID)=F(ID)+PT(IJJ,NNN)

6598  continue
6597  continue

      DO 6567 ID=1,NDTOP
      SWW=0.0
      IJJ=NCVC+ID
      DO 6568 NNN=1,NCSC
C
C  CALCULATE PROBABILITY DISTRIBUTION FUNCTION W(t)
C
      W(ID,NNN)=(PT(IJJ,NNN)-P(IJJ,NNN))/TSTEP
      SWW=SWW+W(ID,NNN)
6568  CONTINUE
      DO 6570 NNN=1,NCSC

```

```

C
C   CALCULATE S-IMPORTANCE
C
      IF(SWW.LE.0.0)GOTO 6666
      SIM(ID,NNN)=W(ID,NNN)/SWW
      GOTO 7777
6666   SIM(ID,NNN)=0.0
7777   CONTINUE
6570   CONTINUE
6567   CONTINUE

C
C   MT REPRESENTS ACTUAL TIME FOR WHICH P(I)'S ARE COMPUTED
C
      TIME=MSTEP*TSTEP

C
C   PRINT RESULTS
C
      WRITE(10,*)'TIME = ',time
      WRITE(10,*)'SUM OF PROB. = ',TOT,' ERROR = ',ERR
      WRITE(10,*)(F(ID),ID=1,NTOP)
      WRITE(10,*)' '
      WRITE(11,*)' '
      WRITE(11,*)'TIME = ',TIME
      WRITE(11,*)' '
      WRITE(12,*)' '
      WRITE(12,*)'TIME = ',TIME
      WRITE(12,*)' '
      DO 220 NNN=1,NCSC
      WRITE(11,*)(W(ID,NNN),ID=1,NDTOP)
      WRITE(12,*)(SIM(ID,NNN),ID=1,NDTOP)
220    CONTINUE

      DO 5686 JJJ=1,NCVC
      DO 5690 NNN=1,NCSC
        P(JJJ,NNN)=PT(JJJ,NNN)
5690    CONTINUE
5686    CONTINUE
      IF(MSTEP.LT.MAXST)GOTO 5555
      STOP
      END

```

```
C      *
C      *
C      *   THIS ALGORITHM AUTOMATICALLY DISCRETISES THE STATE-SPACE
C      *   OF INTEREST, SIMULATES THE SYSTEM DYNAMICS AND ARRANGES
C      *   THE DATABASE (OUTPUT FILE) CELL-TO-CELL TRANSITIONS.
C      *
C      *   THE USER HAS TO MODIFY ALL THE ENTRIES OF THE PARAMETER
C      *   DECLARATION STATEMENTS ACCORDING TO THE SPECIFICATIONS
C      *   OF THE PROBLEM OF INTEREST. ALSO, THE USER HAS TO CHANGE
C      *   THE DIFFERENTIAL EQUATIONS, GOVERNING THE DYNAMICS OF
C      *   THE SYSTEM OF INTEREST, IN THE SUBROUTINE "RKMDF" BELOW,
C      *   THE COMMON BLOCK IN THE MAIN PROGRAM AND THE SUBROUTINE
C      *   WHICH DEFINES THE PARAMETERS OF THE PROBLEM, AND ASSIGN
C      *   THE CORRECT VALUES OF THE PARAMETERS.
C      *
C      *   THE USER CAN SIT BACK AND LET THE PROGRAM DO THE WORK
C      *   FOR HIM. THIS COMPUTER PROGRAM IS MOST ADVANTAGEOUS IF
C      *   USED ON CRAY.
C      *
C      *
C      *
C      *
C      *   PROGRAM DEVELOPED BY
C      *   M. BELHADJ JANUARY 1991
C      *
C      *
C      *   HAVE ANY QUESTIONS, CONTACT ME THROUGH DR. TUNC ALDEMIR
```

COMMON/UNIT1/Y,Y0,t,tau,tmax,h,g1,g2,beta,d,g1
& ,delta,q,zk,a1,a2,a3

DIMENSION Y(9),Y0(9)

```
C
C      enter the number of dimensions (ND) for state-space
C      discretization
```

PARAMETER(ND=5)
parameter(nit=20)

C
C enter the number of discretized intervals (N*) in each
C direction

PARAMETER(N1=12,N2=12,N3=12,N4=12,N5=12,NC=N1*N2*N3*N4*N5)

C
C enter the number of subdivisions inside a cell (NS*) in

```

C      each direction
C
PARAMETER(NS1=2,NS2=2,NS3=2,NS4=2,NS5=2)
PARAMETER(NSC=NS1*NS2*NS3*NS4*NS5)

C
C      enter the minimum allowed value (X*MIN) for each
C      discretized direction
C
PARAMETER(X1MIN=-0.07,X2MIN=155.0,X3MIN=-3.0)
PARAMETER(X4MIN=-0.012,X5MIN=-0.006)

C
C      enter the maximum allowed value (X*MAX) for each
C      discretized direction
C
PARAMETER(X1MAX=0.3,X2MAX=200.0,X3MAX=5.0)
PARAMETER(X4MAX=0.014,X5MAX=0.007)

DIMENSION IM(0:NC,NSC)

OPEN(UNIT=12,FILE='img2.dat',STATUS='new')

C
C      calculate the size of the step (H*) of the discrete interval
C      in each direction
C
T=0.0
TAU=0.005
TMAX=nit*TAU
H=TAU

beta=0.0056
d=4.0e-5
gl=0.08
q=25.044
delta=-0.1
a1=6.8166
a2=2.2494
a3=0.2325
zk=-0.01318
gl=0.15
g2=-2.61e-5

ALPHA=2.00

```

```

TLX1=FLOAT(N1)
TLX2=FLOAT(N2)
TLX3=FLOAT(N3)
TLX4=FLOAT(N4)
TLX5=FLOAT(N5)
H1=(X1MAX-X1MIN)/TLX1
H2=(X2MAX-X2MIN)/TLX2
H3=(X3MAX-X3MIN)/TLX3
H4=(X4MAX-X4MIN)/TLX4
H5=(X5MAX-X5MIN)/TLX5
HS1=H1/FLOAT(NS1)
HS2=H2/FLOAT(NS2)
HS3=H3/FLOAT(NS3)
HS4=H4/FLOAT(NS4)
HS5=H5/FLOAT(NS5)

```

```

C
C   discretize fifth direction
C
DO 31, I5=1,N5
XL5=(I5-1)*H5+X5MIN

```

```

C
C   discretize fourth direction
C
DO 32, I4=1,N4
XL4=(I4-1)*H4+X4MIN

```

```

C
C   discretize third direction
C
DO 33, I3=1,N3
XL3=(I3-1)*H3+X3MIN
c   hs3=(exp(xl3+h3)-exp(xl3))/float(ns3)

```

```

C
C   discretize second direction
C
DO 34, I2=1,N2
XL2=(I2-1)*H2+X2MIN
c   hs2=(exp(xl2+h2)-exp(xl2))/float(ns2)

```

```

C
C   discretize first direction

```

```

C
DO 35, I1=1,N1
XL1=(I1-1)*H1+X1MIN
c   hs1=(exp(xl1+h1)-exp(xl1))/float(ns1)

C
C   assign cell numbers (NN) to the discretized state-space
C
NN=(I5-1)*N1*N2*N3*N4+(I4-1)*N1*N2*N3
&   +(I3-1)*N1*N2+(I2-1)*N1+I1

c   write(5,*)nn

C
C   assign sub-cell numbers (NS) to each cell
C
DO 41, K5=1,NS5
X5=XL5+(FLOAT(K5)-0.5)*HS5
nnss5=(K5-1)*NS1*NS2*NS3*NS4
DO 42, K4=1,NS4
nnss4=(K4-1)*NS1*NS2*NS3
X4=XL4+(FLOAT(K4)-0.5)*HS4
DO 43, K3=1,NS3
nnss3=(K3-1)*NS1*NS2
X3=XL3+(FLOAT(K3)-0.5)*HS3
DO 44, K2=1,NS2
nnss2=(K2-1)*NS1
X2=XL2+(FLOAT(K2)-0.5)*HS2
DO 45, K1=1,NS1
NS=nnss5+nnss4+nnss3+nnss2+k1

C
C   set initial conditions
C
c   X1=exp(XL1)+(FLOAT(K1)-0.5)*HS1
c   X2=exp(XL2)+(FLOAT(K2)-0.5)*HS2
c   X3=exp(XL3)+(FLOAT(K3)-0.5)*HS3

X1=XL1+(FLOAT(K1)-0.5)*HS1

Y0(1)=X1
Y0(2)=X2
Y0(3)=X3
Y0(4)=X4
Y0(5)=X5
T=0.0

```



```

C
C      simulate the system under the given initial condition
C      cell JP, and parameter alpha
C
c      WRITE(*,*)nn,X1,X2,X3

      CALL RKMDNF(ND,ALPHA,nit)
c      WRITE(*,*)Y(1),Y(2),Y(3)
c      IF(Y(1).LT.1.0E-15)Y(1)=1.0E-15
c      IF(Y(2).LT.1.0E-15)Y(2)=1.0E-15
c      IF(Y(3).LT.1.0E-15)Y(3)=1.0E-15
c      IF(Y(4).LT.1.0E-15)Y(4)=1.0E-15
c      IF(Y(5).LT.1.0E-15)Y(5)=1.0E-15
C      XC1=LOG(Y(1))
C      XC2=LOG(Y(2))
C      XC3=LOG(Y(3))
      XC1=Y(1)
      XC2=Y(2)
      XC3=Y(3)
      XC4=Y(4)
      XC5=Y(5)

C
C      determine the cell number of the mapping J
C
      NX1=INT((XC1-X1MIN)/H1)+1
      NX2=INT((XC2-X2MIN)/H2)+1
      NX3=INT((XC3-X3MIN)/H3)+1
      NX4=INT((XC4-X4MIN)/H4)+1
      NX5=INT((XC5-X5MIN)/H5)+1
      IM(NN,NS)=(NX5-1)*N1*N2*N3*N4+(NX4-1)*N1*N2*N3
&      +(NX3-1)*N1*N2+(NX2-1)*N1+NX1
      IF((NX1.GT.N1).OR.(NX1.LT.1))IM(NN,NS)=0
      IF((NX2.GT.N2).OR.(NX2.LT.1))IM(NN,NS)=0
      IF((NX3.GT.N3).OR.(NX3.LT.1))IM(NN,NS)=0
      IF((NX4.GT.N4).OR.(NX4.LT.1))IM(NN,NS)=0
      IF((NX5.GT.N5).OR.(NX5.LT.1))IM(NN,NS)=0

49      CONTINUE
45      CONTINUE
44      CONTINUE
43      CONTINUE
42      CONTINUE
41      CONTINUE
35      CONTINUE
34      CONTINUE
33      CONTINUE
32      CONTINUE
31      CONTINUE

```

```

        write(*,*)'end of simulation'

        DO 60, NK=1,NSC
        IM(0,NK)=0
60      CONTINUE

        DO 70 IC=0,NC
        WRITE(12,*)IC,(IM(IC,NK),NK=1,NSC)
70      CONTINUE

        CLOSE(12)

        STOP
        END

        SUBROUTINE RKMDF(ND,ALPH,nit)
        COMMON/UNIT1/Y,Y0,t,tau,tmax,h,g1,g2,beta,d,gl
        &      ,delta,q,zk,a1,a2,a3
        DIMENSION F(9),Y(9),Y0(9)
C      .... INITIALIZE N, T, Y(i), ICOUNT ....
C
C
        DO 30 I=1,ND
        Y(I)=Y0(I)
30      CONTINUE
        ICOUNT=0

C      YOU HAVE TO ENTER THE RIGHT MATHEMATICAL EXPRESSIONS
C      FOR THE FIRST DERIVATIVES AND INPUT VALUES FOR THE
C      CORRESPONDING CONSTANTS

6      CONTINUE

C      .... COMPUTE DERIVATIVE VALUES ....

        K=0
10      CONTINUE

        Y(6)=Y(4)*g1+Y(3)*g2
        F(1)=Y(1)*((Y(6)-beta)/d)+Y(2)*g1+Y(6)/d
        F(2)=Y(1)*beta/d-Y(2)*g1
        F(3)=(Y(1)+delta)*q-Y(3)*a3

```

```

F(4)=Y(5)
F(5)=Y(3)*alph*zk-Y(4)*a1-Y(5)*a2
F(6)=0.0
F(7)=0.0
F(8)=0.0
F(9)=0.0

```

```

C .... CALL ON THE FOURTH ORDER RUNGE-KUTTA FUNCTION ....
  K=K+1
  M=NRUNGE(ND,K,Y,F,T,H)
  IF(K.EQ.4) GOTO 20
  GOTO 10

20  ICOUNT=ICOUNT+1
    IF(icount.lt.nit) GOTO 6
    return
    END

```

```

FUNCTION NRUNGE(ND,K,Y,F,X,H)
  DIMENSION PHI(100),SAVEY(100),Y(ND),F(ND)

```

```

  KK=K
  IF (KK.EQ.1) GOTO 1
  IF (KK.EQ.2) GOTO 2
  IF (KK.EQ.3) GOTO 3
  IF (KK.EQ.4) GOTO 4

```

```

C .... PASS 1 ....

1  DO 11, J=1,ND
    SAVEY(J)=Y(J)
    PHI(J)=F(J)
    Y(J)=SAVEY(J)+0.5*H*F(J)
11  CONTINUE
    X=X+0.5*H
    GOTO 5

C .... PASS 2 ....

2  DO 22, J=1,ND
    PHI(J)=PHI(J)+2.0*F(J)
    Y(J)=SAVEY(J)+0.5*H*F(J)
22  CONTINUE
    GOTO 5

C .... PASS 3 ....

```

```
3    DO 33, J=1,ND
    PHI(J)=PHI(J)+2.0*F(J)
    Y(J)=SAVEY(J)+H*F(J)
33   CONTINUE
    X=X+0.5*H
    GOTO 5

C    .... PASS 4 ....

4    DO 44, J=1,ND
    Y(J)=SAVEY(J)+(PHI(J)+F(J))*H/6.0
44   CONTINUE

5    CONTINUE
    NRUNGE=KK
    RETURN
    END
```

```

C      *           ALG-L
C      *
C      *   THIS ALGORITHM AUTOMATICALLY CALCULATES THE LIMITING
C      *   PROBABILITY DISTRIBUTION OF THE PERSISTENT CELLS OF THE
C      *   SYSTEM OF INTEREST. THIS PROGRAM USES THE OUTPUT FILE
C      *   OF "ALG-D" ALGORITHM.
C      *
C      *   THE USER HAS TO MODIFY ALL THE ENTRIES OF THE PARAMETER
C      *   DECLARATION STATEMENTS ACCORDING TO THE SPECIFICATIONS
C      *   OF THE PROBLEM OF INTEREST. AGAIN, THIS PROGRAM IS MOST
C      *   ADVANTAGEOUS IF USED ON CRAY ENVIRONMENT. THE USER MAY
C      *   NEED TO CHANGE THE NAME OF THE RANDOM NUMBER GENERATOR
C      *   FUNCTION IF THE PROGRAM IS USED ON ANOTHER COMPUTING
C      *   MACHINE.
C      *
C      *   THE OUTPUT FILE GIVES (IN EACH LINE) THE CELL NUMBER
C      *   FOLLOWED BY ITS LIMITING PROBABILITY. THE FIRST LINE
C      *   OF THE OUTPUT FILE GIVES THE NUMBER OF ITERATIONS USED
C      *   TO SATISFY THE CONVERGENCE CRITERIA.
C      *
C      *   THE USER CAN SIT BACK AND LET THE PROGRAM DO THE WORK
C      *   FOR HIM. THIS COMPUTER PROGRAM IS MOST ADVANTAGEOUS IF
C      *   USED ON CRAY DUE TO VECTORIZATION.
C      *
C      *           PROGRAM DEVELOPED BY
C      *           M. BELHADJ  APRIL 1991
C      *
C      *   HAVE ANY QUESTIONS, CONTACT ME THROUGH DR. TUNC ALDEMIR
C      *

```

```

COMMON/UNIT2/KRAN,XRAN

```

```

C
C      enter the number of dimensions (ND) for state-space
C      discretization
C

```

```

PARAMETER(ND=5)

```

```

C
C      enter the number of discretized intervals (N*) in each
C      direction
C

```

```

PARAMETER(N1=12,N2=12,N3=12,n4=12,n5=12,NC=N1*N2*N3*n4*n5)

```

```

C
C      enter the number of subdivisions inside a cell (NS*) in
C      each direction
C
PARAMETER(NS1=2,NS2=2,NS3=2,ns4=2,ns5=2)
PARAMETER(NSC=NS1*NS2*NS3*ns4*ns5)

DIMENSION IM(0:NC,NSC)
DOUBLE PRECISION PK(0:nc),PL(0:nc),SP1(0:NC),SP2(0:NC)
DOUBLE PRECISION DZERO,DONE,DONEF,DMTEN,CMPL
DOUBLE PRECISION SUM,ERR
PARAMETER (DZERO=0.0D0,DONE=1.0D0,EMTEN=1.0E-10,kmin=10)
PARAMETER (ERMAXS=1.0E-2,ERMAX0=1.0E-4,EM20=1.0e-20)
PARAMETER (KMAX=1000)

OPEN(UNIT=11,FILE='doa2.dat',STATUS='new')
OPEN(UNIT=12,FILE='img2.dat',STATUS='old')

DO 70 IC=0,NC
read(12,*)Ik,(IM(Ik,NK),NK=1,NSC)
70  CONTINUE

CLOSE(12)

KRAN=93567
CMPL=DONE/FLOAT(NSC)

C
C      initialize the probabilities vectors
C
KSTEP=0
irun=0
DONEF=DONE/FLOAT(NC)
DO 120, jP=0,NC
PL(jP)=DONEF
SP1(jP)=DZERO
SP2(jP)=DZERO
120  CONTINUE
PL(0)=DZERO

3000  CONTINUE

SUM=DZERO
DO 150 JP=0,NC
SUM=SUM+PL(JP)
150  CONTINUE

```

```

      IF(SUM.NE.DONE)THEN
      CALL RANDOM(xran)
      MRAN=INT(XRAN*NC)
      ERR=DONE-SUM
      PL(MRAN)=PL(MRAN)+ERR
      ENDIF

      DO 290 J=0,NC
      PK(J)=PL(J)
      PL(J)=DZERO
290    CONTINUE

      KSTEP=KSTEP+1

      do 210 kp=1,nsC
      do 220 jp=0,nC
      pl(Im(jp,kp))=pl(Im(jp,kp))+pk(jp)
220    continue
210    continue

      do 400,j=0,nC
      pl(j)=pl(j)*cmlpl
400    continue

      ERR0=0.0

      IF(IRUN.GT.0) GOTO 4000
      IF(KSTEP.LT.KMIN)GOTO 3000

      ERR0=ABS((PL(0)-PK(0))/PL(0))
      IF(ERR0.LT.ERMAX0) THEN
      IRUN=1
      DO 260 J=0,NC
      SP1(J)=PK(J)
      SP2(J)=0.5*(PL(J)+PK(J))
260    CONTINUE
      ENDIF

      GOTO 3000

4000  CONTINUE

      ERRM=0.0
      ERRP=0.0

      IRUN=IRUN+1
      DO 270 J=0,NC

```

```

      SP1(J)=(SP1(J)*(IRUN-1)+PK(J))/IRUN
      SP2(J)=(SP2(J)*IRUN+PL(J))/(IRUN+1)
270    continue
      do 280 j=0,nc
      IF(SP2(J).GT.EMTEN)THEN
      ERRP=ABS((SP2(J)-SP1(J))/SP2(J))
      IF(ERRP.GT.ERRM)THEN
      ERRM=ERRP
      ENDIF
      ENDIF
280    CONTINUE

      IF(ERRM.LT.ERMAXS)GOTO 5000
      IF(kstep.gt.kmax)GOTO 5000

      GOTO 3000

C
C    PRINT RESULTS
C

5000  continue
      WRITE(11,112)KSTEP,irun,errm
      DO 350,J=0,NC
      IF(SP1(J).GT.emten)WRITE(11,111)J,SP1(J)
350    CONTINUE
      close(11)
111    format(i7,3x,e12.5)
112    format(i7,3x,i7,3x,e12.5)

      STOP
      END

      SUBROUTINE RANDOM
      COMMON/UNIT2/KRAN,XRAN
c      XRAN=RAN(kran)
      xran=ranf( )
      RETURN
      END

```



```

C      *      SEQUENTIAL MANNER, STARTING BY
C      *      ASSIGNING GROUP NUMBER 3 TO THE FIRST
C      *      DISCOVERED GROUP OF PERSISTENT CELLS
C      *      THAT FORM TOGETHER A CYCLIC SEQUENCE.
C      *
C      *      cycltr2.dat:  THIS FILE LISTS THE CELLS THAT ARE
C      *      TRANSIENT TO A GIVEN CYCLIC GROUP.
C      *
C      *      multpr2.dat:  THIS FILE LISTS THE MULTIPLE-DOMICILE
C      *      TRANSIENT CELLS TO A GIVEN GROUP,
C      *      FOLLOWED BY EACH MULTIPLE-DOMICILE
C      *      TRANSIENT CELL ABSORPTION PROBABILITY
C      *      INTO THAT GROUP.
C      *
C      *      linecy2.dat:  THIS FILE CONTAINS POINTING
C      *      INFORMATION PERTAINING TO THE OUTPUT
C      *      FILES cycltr2.dat AND multpr2.dat.
C      *      EACH LINE CONTAINS GROUP NUMBER, THE
C      *      POSITION OF THE LAST LINE NUMBER IN
C      *      multpr2.dat CONTAINING THE MULTIPLE
C      *      DOMICILE TRANSIENT CELLS TO THAT GROUP
C      *      AND THE ABSORPTION PROBABILITY,
C      *      FOLLOWED BY THE LAST LINE NUMBER IN
C      *      cycltr2.dat CONTAINING THE TRANSIENT
C      *      CELLS TO THAT CYCLIC GROUP.
C      *
C      *      FOLLOWED BY ITS LIMITING PROBABILITY. THE FIRST LINE
C      *      OF THE OUTPUT FILE GIVES THE NUMBER OF ITERATIONS USED
C      *      TO SATISFY THE CONVERGENCE CRITERIA.
C      *
C      *      THE USER CAN SIT BACK AND LET THE PROGRAM DO THE WORK
C      *      FOR HIM. THIS COMPUTER PROGRAM IS MOST ADVANTAGEOUS IF
C      *      USED ON CRAY DUE TO VECTORIZATION.
C      *
C      *
C      *      PROGRAM DEVELOPED BY
C      *      M. BELHADJ  OCTOBER 1991
C      *
C      *      HAVE ANY QUESTIONS, CONTACT ME THROUGH DR. TUNC ALDEMIR
C      *
C
C      enter the number of dimensions (ND) for state-space
C      discretization
C
C      PARAMETER(ND=5)

```

```

C
C   enter the number of discretized intervals (N*) in each
C   direction
C
PARAMETER(N1=12,N2=12,N3=12,n4=12,n5=12,NC=N1*N2*N3*n4*n5)

C
C   enter the number of subdivisions inside a cell (NS*) in each
C   direction
C
PARAMETER(NS1=2,NS2=2,NS3=2,ns4=2,ns5=2)
PARAMETER(NSC=NS1*NS2*NS3*ns4*ns5)

PARAMETER(pmax=0.99,pmin=0.01)

COMMON/UNIT3/NSINK,P,P2,IM
DIMENSION IM(0:NC,NSC),IMG(NSC),NG(0:NC),NMAX(nc)
INTEGER B(0:NC)
DIMENSION nsink(nc),p(0:nc),p2(0:nc)

C
OPEN(UNIT=10,FILE='multpr2.dat',STATUS='NEW')
OPEN(UNIT=11,FILE='sinktr2.dat',STATUS='NEW')
open(unit=12,file='stblpr2.dat',status='new')
open(unit=13,file='stbltr2.dat',status='new')
open(unit=14,file='cyclpr2.dat',status='new')
open(unit=15,file='cycltr2.dat',status='new')
open(unit=18,file='linecy2.dat',status='new')

      open(unit=21,file='img2.dat',status='old')

C
C   read data file that defines the mappings of each cell
C
      DO 60, ic=0,nc
      read(21,*)ik,(im(ik,nk),nk=1,nsc)
60      CONTINUE

C
C   initialize group # (NG) and absorption probability (P)
C
      lines=0
      DO 70 IC=0,NC
      NG(IC)=0
      P(IC)=0.0
70      CONTINUE

      write(*,*)'program started'

```

```

C
C   assign the sink cell (0) to group # 1
C
C       NG(0)=1

C
C   determine the cells that belong to group #1, i.e. NSINK(1)
C   in this case, only the sink cell (0) belongs to group # 1
C
C       NSINK(1)=0

C
C   determine the total number of cells in group # 1, i.e., NMAX(1)
C   in this case there is only one cell
C
C       NMAX(1)=1

C
C   calculate the absorption probabilities into the sink cell
C   (group # 1)
C
C       nnmax=nmax(1)
C       CALL PROB1(nnmax,nsteps)
C       write(*,*)nsteps
C       write(11,*)'cells that are transient to sink cell'
C       do 20 ic=1,nc

C
C       determine the basins of attraction to the sink cell
C
C       if(p(ic).ge.pmax)then
C           ng(ic)=-2
C           write(11,*)ic
C       endif

C
C       determine multiple domicile transient cells to the sink cell
C
C       if(p(ic).lt.pmax.and.p(ic).ge.pmin)then
C           ng(ic)=-2
C           lines=lines+1
C           write(10,111)ic,p(ic)
C       endif
20   continue
      close(11)

C
C       npg : group #
C
C       npg=1

```

```

write(18,*)npg,lines

npg=2
write(*,*)'determine the stationary cells'

write(12,*)'determine the stationary cells'

c
c   determine the stationary cells
c
DO 130, IC=1,NC
jsc=0
DO 135, JSC=1,NSC
IF(IM(IC,JSC).NE.IC) GOTO 1350
135  CONTINUE

C
C   assign the stationary cells to group # 2
C
NG(IC)=2
NMAX(2)=NMAX(2)+1
NSINK(NMAX(2))=IC
write(12,*)ic
1350  CONTINUE
130  CONTINUE

close(12)

IF(NSINK(1).NE.0)THEN
DO 137 IC=0,NC
P(IC)=0.0
137  CONTINUE

write(13,*)'determine transient cells that lead to stable cells'

C
C   determine absorption probabilities into group # 2
C   (the stationary cells)
c

nnmax=nmax(2)
CALL PROB1(nnmax,nsteps)
write(*,*)nsteps

do 140 ic=1,nc

C

```

```

C      determine the basins of attraction to group # 2
C
      if(p(ic).ge.pmax)then
      if(ng(ic).le.0)then
      ng(ic)=-2
      endif
      write(13,*)ic
      endif

C
C      determine multiple domicile transient cells to group # 2
C
      if(p(ic).lt.pmax.and.p(ic).ge.pmin)then
      ng(ic)=-2
      lines=lines+1
      write(10,111)ic,p(ic)
      endif

140      continue
      write(18,*)npg,lines
      ENDIF
      close(13)

C
C      determine other cyclic persistent groups
c
      DO 150, IC=1,NC

C
C      NG(ic)=0  -> ic is avirgin cell
C      NG(ic)>0  -> ic belongs to a group of persistent cells
C      NG(ic)=-1 -> ic is under processing
C      NG(ic)=-2 -> ic is a transient cell
C      NG(ic)=-3 -> ic is a candidate cell for a cyclic group
C

C
C      initialize a sequence of virgin cells to be processed
C
      IF(NG(IC).EQ.0) THEN
      B(0)=IC
      NG(IC)=-1
      ICC=0
151      continue
      icc=icc+1
      IJ=1

```

```

iass=0
2000  CONTINUE
      B(ICC)=IM(B(ICC-1),IJ)
      NICC=NG(B(ICC))

      IF (NICC.EQ.0) GOTO 3000

      IF (NICC.GT.0.OR.NICC.EQ.-2)THEN
C
C      The sequence under processing has reached a transient cell
C      or an already discovered group -> all the cells in this
C      sequence are transient
C
      DO 156, IR=0,ICC-1
      NG(B(IR))=-2
156  CONTINUE
      ENDIF

      IF (NICC.GT.0.OR.NICC.EQ.-2) GOTO 1500

C
C      if NICC=-1 then the sequence under processing has reached
C      a cell which is a member of this sequence -> there is
C      a possibility that this sequence of cells form a cyclic
C      group. A further test is needed to confirm the cyclic
C      behavior of this sequence of cells
C
      IF(NICC.EQ.-1) IJ=IJ+1
      IF(nicc.eq.-1.AND.nsc.GE.IJ) GOTO 2000
      write(*,*)'testing for a possible persistent group'

C
C      reinitilize the sequence to test for a possible PG
C

      IF(nicc.eq.-1.and.IJ.GT.nsc) THEN
      iass=0
      ick=B(ICC-1)
      NG(ick)=-3
      nsink(1)=ick

c
c      determine the number of distinct images of cell ick
c

      do 110 jsc=1,nsc
      if(im(ick,jsc).eq.nsink(1))goto 1110
      nsink(2)=im(ick,jsc)
      goto 1111

```

```

1110    continue
110     continue
1111    continue
        ikmax=2
        ng(nsink(2))=-3
        do 154, jsc=1,nsc
        j=0
        do 155 j=1,ikmax
        if(im(ick,jsc).eq.nsink(j))goto 1210
155     continue
        ikmax=ikmax+1
        nsink(ikmax)=im(ick,jsc)
        ng(nsink(ikmax))=-3
1210    continue
154     continue

        ik=1
1600    continue
        ik=ik+1

C
C     determine the number of distinct images of each cell
C     in the group
c
        img(1)=im(nsink(ik),1)
        lkkmax=1
        DO 121, JSC=2,NSC
        j=0
        DO 122, J=1,lkkmax
        IF(IM(nsink(Ik),jsc).EQ.IMG(J)) GOTO 1200
122     CONTINUE
        ikkmax=ikkmax+1
        IMG(Ikkmax)=IM(nsink(Ik),jsc)
1200    CONTINUE
121     CONTINUE

C
C     testing the sequence for a possible group
C
        DO 170 IKK=1,IKKMAX
        IASS=0
        IMTEMP=IMg(ikk)
        NGTEMP=NG(IMTEMP)
        IF(NGTEMP.EQ.-3) GOTO 1700
        IF(NGTEMP.EQ.-1) THEN
        IKMAX=IKMAX+1
        Nsink(IKMAX)=IMTEMP

```



```

NG(IMTEMP)=-3
ENDIF
IF(NGTEMP.EQ.-1) GOTO 1700

C
C   IASS is a flag for the test
C   IASS=0  -> a new group is confirmed
C   IASS=-2 -> the sequence has reached a transient or
C           a persistent cell the sequence is transient
C   IASS=-1 -> a new cell is discovered. The sequence has
C           to be augmented by this cell, and the test
C           has to include this new cell in the analysis.
C           The test continues
C
IF(NGTEMP.GT.0.OR.NGTEMP.EQ.-2)IASS=-2
IF(NGTEMP.EQ.0)IASS=-1
GOTO 5000
1700  CONTINUE
170   CONTINUE
      if(ik.lt.ikmax)goto 1600
5000  CONTINUE
ENDIF

IF(IASS.EQ.0) THEN
C
C   IASS=0 -> a new group is discovered. Assign the cells
C           in this sequence to group # npg
C
NPG=npg+1
nmax(npg)=ikmax
  write(14,*)'cells that are cyclic persistent for group ',npg
DO 300, IT=1,IKMAX
  itc=nsink(it)
  NG(itc)=npg
  write(14,*)itc
300  CONTINUE
DO 138 ICm=0,NC
P(ICm)=0.0
138  continue

C
C   Determine the absorption probabilities into the cyclic
C   group # npg
C

nnmax=nmax(npg)

```

```

write(*,*)'nmax = ',nnmax
CALL PROB1(nnmax,nsteps)
write(*,*)nsteps

DO 310, ICm=1,NC
IF(P(ICm).GE.PMAX)THEN
C
C   determine the basins of attraction to the cyclic PG # npg
C
      if(ng(icm).le.0)then
NG(ICm)=-2
WRITE(15,*)ICm
line2=line2+1
endif
ENDIF

      if(p(icm).lt.pmax.and.p(icm).ge.pmin)then
C
C   determine multiple domicile transient cells to group # npg
C
      lines=lines+1
      ng(icm)=-2
      write(10,111)icm,p(icm)
      endif
310  CONTINUE
      write(18,*)npg,lines,line2
      ENDIF

      IF(IASS.EQ.0) GOTO 1500

      IF(IASS.EQ.-1)THEN
B(ICC)=IMTEMP
DO 180, IT=1,IKMAX
if(ng(nsink(it)).eq.-3)NG(nsink(IT))=IASS
180  CONTINUE
      ng(imtemp)=-1
      ng(b(icc-1))=-1
      ENDIF

      IF(IASS.EQ.-1) GOTO 3000

      IF(NGTEMP.GT.0)THEN
DO 190, IT=0,ICC-1
NG(B(IT))=-2
190  CONTINUE
      ENDIF

      IF (NGTEMP.EQ.-2)THEN

```

```

DO 197, IR=0,ICC-1
NG(B(IR))=-2
197  CONTINUE
ENDIF

DO 199, IR=1,IKMAX
IF(NG(Nsink(IR)).EQ.-3)NG(Nsink(IR))=0
199  CONTINUE
GOTO 1500

```

```

3000  CONTINUE
NG(B(ICC))=-1
1510  CONTINUE
goto 151
1500  CONTINUE
ENDIF
150   CONTINUE

CLOSE(14)
CLOSE(15)
CLOSE(17)
close(10)
111   format(i7,3x,e12.5)

STOP
END

```

SUBROUTINE PROB1(nnmax,nsteps)

```

c
C   SUBROUTINE FOR CALCULATING ABSORPTION PROBABILITIES
c
PARAMETER(N1=12,N2=12,N3=12,n4=12,n5=12,NC=N1*N2*N3*n4*n5)
PARAMETER(NS1=2,NS2=2,NS3=2,ns4=2,ns5=2)
parameter(NSC=NS1*NS2*NS3*ns4*ns5)
parameter(dfive=5.0e-4)
COMMON/UNIT3/NSINK,P,P2,IM
dimension nsink(nc),im(0:nc,nsc),p(0:nc),p2(0:nc)
dimension pt(0:nc),ptt(0:nc)

nsteps=0
do 22 IJ=0,nc
pt(ij)=0.0
ptt(ij)=0.0
22  continue
DO 30 IJ=1,nNMAX

```

```

      P(NSINK(IJ))=1.0
30    CONTINUE
      DNSC=1.0/NSC
50    continue
      nflag=0
      nsteps=nsteps+1
      do 29 ic=0,nc
        ptt(ic)=pt(ic)
        p2(ic)=p(ic)
        p(ic)=0.0
29    continue
      do 26, jsc=1,nsc
        do 27 ic=0,nc
          p(ic)=p(ic)+dnsc*p2(im(ic,jsc))
27    continue
26    continue
      DO 40 IJ=1,NnMAX
        P(NSINK(IJ))=1.0
40    CONTINUE
      if(nsteps.lt.101)goto 2700
      do 31 ic=0,nc
        pt(ic)=((ptt(ic)*(nsteps-101))+p(ic))/(nsteps-100)
31    continue
      do 33 ic=0,nc
        diff=abs(pt(ic)-ptt(ic))
        if(diff.gt.dfive)nflag=nflag+1
        if(nflag.gt.0) goto 2800
33    continue
      if(nsteps.gt.100) goto 2800
2700  continue
      do 28 ic=0,nc
        diff=abs(p(ic)-p2(ic))
        if(diff.gt.dfive)nflag=nflag+1
        if(nflag.gt.0) goto 2800
28    continue
2800  continue
      if(nsteps.gt.1000) goto 5000
      if(nflag.gt.0)goto 50
5000  continue
      if(nsteps.gt.100)then
        DO 44 IJ=0,nc
          P(ij)=pt(ij)
44    CONTINUE
      endif
      RETURN
      END

```

APPENDIX B

Absorptions Probabilities Plots for ES-2

- $0.99 \leq \rho_{1,i}^{\infty} \leq 0.75$
- × $0.75 \leq \rho_{1,i}^{\infty} \leq 0.50$
- + $0.50 \leq \rho_{1,i}^{\infty} \leq 0.25$
- $0.25 \leq \rho_{1,i}^{\infty} \leq 0.01$

Fig.81 Absorption Probabilities $\rho_{1,i}^{\infty}$ of Multiple Domicile Cells into the Sink Cell (Group 1) for $\alpha_2=0.01$.

- $0.99 \leq \rho_{1,i}^{\infty} \leq 0.75$
- × $0.75 \leq \rho_{1,i}^{\infty} \leq 0.50$
- + $0.50 \leq \rho_{1,i}^{\infty} \leq 0.25$
- $0.25 \leq \rho_{1,i}^{\infty} \leq 0.01$

Fig.82 Absorption Probabilities $\rho_{1,j}^{\infty}$ of Multiple Domicile Cells into the Sink Cell (Group 1) for $\alpha_2=0.03$.

- $0.99 \leq \rho_{1,i}^{\infty} \leq 1.00$
- $0.75 \leq \rho_{1,i}^{\infty} \leq 0.99$
- × $0.50 \leq \rho_{1,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{1,i}^{\infty} \leq 0.50$
- $0.01 \leq \rho_{1,i}^{\infty} \leq 0.25$

Fig.83 Absorption Probabilities $\rho_{1,j}^{\infty}$ of Multiple Domicile Cells into the Sink Cell (Group 1) for α_2 Uniformly Distributed (Case 1).

- $0.99 \leq \rho_{1,i}^{\infty} \leq 1.00$
- $0.75 \leq \rho_{1,i}^{\infty} \leq 0.99$
- × $0.50 \leq \rho_{1,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{1,i}^{\infty} \leq 0.50$
- $0.01 \leq \rho_{1,i}^{\infty} \leq 0.25$

Fig.84 Absorption Probabilities ρ_{1j}^{∞} of Multiple Domicile Cells into the Sink Cell (Group 1) for α_2 Normally Distributed (Case 2).

- $0.99 \leq \rho_{2,i}^{\infty} \leq 0.75$
- × $0.75 \leq \rho_{2,i}^{\infty} \leq 0.50$
- + $0.50 \leq \rho_{2,i}^{\infty} \leq 0.25$
- $0.25 \leq \rho_{2,i}^{\infty} \leq 0.01$

Fig.85 Absorption Probabilities $\rho_{2,j}^{\infty}$ of Multiple Domicile Cells into the Third Order Subharmonic Attractor (Group 2) for $\alpha_2=0.01$.

Fig.86 Absorption Probabilities $\rho_{2,j}^\infty$ of Multiple Domicile Cells into the Third Order Subharmonic Attractor (Group 2) for $\alpha_2=0.03$.

- $0.99 \geq \rho_{3,i}^{\infty} \geq 0.75$
- × $0.75 \geq \rho_{3,i}^{\infty} \geq 0.50$
- + $0.50 \geq \rho_{3,i}^{\infty} \geq 0.25$
- $0.25 \geq \rho_{3,i}^{\infty} \geq 0.01$

Fig.87 Absorption Probabilities $\rho_{3,j}^{\infty}$ of Multiple Domicile Cells into the Strange Attractor (Group 3) for $\alpha_2=0.01$.

- $0.99 \leq \rho_{3,i}^{\infty} \leq 0.75$
- × $0.75 \leq \rho_{3,i}^{\infty} \leq 0.50$
- + $0.50 \leq \rho_{3,i}^{\infty} \leq 0.25$
- $0.25 \leq \rho_{3,i}^{\infty} \leq 0.01$

Fig.88 Absorption Probabilities $\rho_{3,j}^{\infty}$ of Multiple Domicile Cells into the Strange Attractor (Group 3) for $\alpha_2=0.03$.

- $0.75 \leq \rho_{3,i}^{\infty} \leq 0.99$
- × $0.50 \leq \rho_{3,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{3,i}^{\infty} \leq 0.50$
- $0.01 \leq \rho_{3,i}^{\infty} \leq 0.25$

Fig.89 Absorption Probabilities $\rho_{3,i}^{\infty}$ of Multiple Domicile Cells into the Strange Attractor (Group 3) for α_2 Uniformly Distributed (Case 1).

- $0.75 \leq \rho_{3,i}^{\infty} \leq 0.99$
- × $0.50 \leq \rho_{3,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{3,i}^{\infty} \leq 0.50$
- $0.01 \leq \rho_{3,i}^{\infty} \leq 0.25$

Fig.90 Absorption Probabilities $\rho_{3,j}^{\infty}$ of Multiple Domicile Cells into the Strange Attractor (Group 3) for α_2 Normally Distributed (Case 2).

Fig.91 Cells With Absorption Probabilities ρ_{3j}^∞ into the Strange Attractor, $0.99 \leq \rho_{3j}^\infty \leq 1.00$, for α_2 Uniformly Distributed (Case 1).

Fig.92 Cells With Absorption Probabilities $\rho_{3,j}^\infty$ into the Strange Attractor, $0.99 \leq \rho_{3,j}^\infty \leq 1.00$, for α_2 Normally Distributed (Case 2).

APPENDIX C

PGs, Probability Distributions, BOAs and
Absorptions Probabilities Plots for ES-3

- ⊙ $1.0 \times 10^{-1} \leq P_j^\infty \leq 1.0$
- × $1.0 \times 10^{-2} \leq P_j^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P_j^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P_j^\infty \leq 1.0 \times 10^{-3}$

Fig.93 Probability Distribution P_j^∞ , Integrated Over $\bar{X}$, of the Persistent Groups for ES-3 ($\theta = 2.0\text{E-}16$, Case 1).

- ⊕ $1.0 \times 10^{-1} \leq P_j^\infty \leq 1.0$
- × $1.0 \times 10^{-2} \leq P_j^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P_j^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P_j^\infty \leq 1.0 \times 10^{-3}$

Fig.94 Probability Distribution P_j^∞ , Integrated Over $\bar{X}$, of the Persistent Groups for ES-3 ($\theta = 4.0\text{E-}16$, Case 1).

$$\textcircled{\cdot} \quad 1.0 \times 10^{-1} \leq P_j^{\infty} \leq 1.0$$

$$\times \quad 1.0 \times 10^{-2} \leq P_j^{\infty} \leq 1.0 \times 10^{-1}$$

$$+ \quad 1.0 \times 10^{-3} \leq P_j^{\infty} \leq 1.0 \times 10^{-2}$$

$$\circ \quad 1.0 \times 10^{-4} \leq P_j^{\infty} \leq 1.0 \times 10^{-3}$$

Fig.95 Probability Distribution P_j^{∞} , Integrated Over $\bar{X}$, of the Persistent Groups for ES-3 ($\theta = 6.0\text{E-}16$, Case 1).

Fig.96 Probability Distribution P_j^∞ , Integrated Over Φ , of the Persistent Groups for ES-3 ($\theta = 2.0\text{E-}16$, Case 1).

$$\oplus \quad 1.0 \times 10^{-1} \leq P^\infty \leq 1.0$$

$$\times \quad 1.0 \times 10^{-2} \leq P^\infty \leq 1.0 \times 10^{-1}$$

$$+ \quad 1.0 \times 10^{-3} \leq P^\infty \leq 1.0 \times 10^{-2}$$

$$\circ \quad 1.0 \times 10^{-4} \leq P^\infty \leq 1.0 \times 10^{-3}$$

Fig.97 Probability Distribution P_j^∞ , Integrated Over Φ , of the Persistent Groups for ES-3 ($\theta = 4.0\text{E-}16$, Case 1).

- $1.0 \times 10^{-1} \leq P^\infty \leq 1.0$
- x $1.0 \times 10^{-2} \leq P^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P^\infty \leq 1.0 \times 10^{-3}$

Fig.98 Probability Distribution P_j^∞ , Integrated Over Φ , of the Persistent Groups for ES-3 ($\theta = 6.0\text{E-}16$, Case 1).

○ Stationary Cells, Group 2 ($T=\tau$)

+ Cyclic Group 3 ($T=2\tau$)

× Cyclic Group 4 ($T=3\tau$)

Fig.99 State-Space Projection of the PGs on the $\bar{I}$ - Φ Plane for ES-3
($\theta = 2.0\text{E-}16$, Case 1).

○ Stationary Cells, Group 2 ($T=\tau$)

+ Cyclic Group 3 ($T=2\tau$)

× Cyclic Group 4 ($T=3\tau$)

Fig.100 State-Space Projection of the PGs on the $\bar{I}$ - Φ Plane for ES-3
($\theta = 4.0\text{E-}16$, Case 1).

○ Stationary Cells, Group 2 ($T=\tau$)

+ Cyclic Group 3 ($T=2\tau$)

× Cyclic Group 4 ($T=3\tau$)

Fig.101 State-Space Projection of the PGs on the $\bar{I}$ - Φ Plane for ES-3
($\theta = 6.0\text{E-}16$, Case 1).

○ Stationary Cells, Group 2 ($T=\tau$)

+ Cyclic Group 3 ($T=2\tau$)

× Cyclic Group 4 ($T=3\tau$)

Fig.102 State-Space Projection of the PGs on the $\bar{I}$ - $\bar{X}$ Plane for ES-3
($\theta = 2.0E-16$, Case 1).

- Stationary Cells, Group 2 ($T=\tau$)
- + Cyclic Group 3 ($T=2\tau$)
- × Cyclic Group 4 ($T=3\tau$)

Fig.103 State-Space Projection of the PGs on the $\bar{I}$ - $\bar{X}$ Plane for ES-3
($\theta = 4.0E-16$, Case 1).

Fig.104 State-Space Projection of the PGs on the $\bar{I}$ - $\bar{X}$ Plane for ES-3
($\theta = 6.0E-16$, Case 1).

- $1.0 \times 10^{-1} \leq P_j^\infty \leq 1.0$
- × $1.0 \times 10^{-2} \leq P_j^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P_j^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P_j^\infty \leq 1.0 \times 10^{-3}$
- $1.0 \times 10^{-5} \leq P_j^\infty \leq 1.0 \times 10^{-4}$

Fig.105 Probability Distribution P_j^∞ , Integrated Over $\bar{X}$, of the Persistent Groups for ES-3 ($\theta = 2.0\text{E-}16$, Case 2).

Fig.106 Probability Distribution P_j^∞ , Integrated Over $\bar{X}$, of the Persistent Groups for ES-3 ($\theta = 4.0\text{E-}16$, Case 2).

Fig.107 Probability Distribution P_j^∞ , Integrated Over $\bar{X}$, of the Persistent Groups for ES-3 ($\theta = 6.0\text{E-}16$, Case 2).

- $1.0 \times 10^{-1} \leq P^\infty \leq 1.0$
- × $1.0 \times 10^{-2} \leq P^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P^\infty \leq 1.0 \times 10^{-3}$
- $1.0 \times 10^{-5} \leq P^\infty \leq 1.0 \times 10^{-4}$

Fig.108 Probability Distribution P_j^∞ , Integrated Over Φ , of the Persistent Groups for ES-3 ($\theta = 2.0\text{E-}16$, Case 2).

Fig.109 Probability Distribution P_j^∞ , Integrated Over Φ , of the Persistent Groups for ES-3 ($\theta = 4.0\text{E-}16$, Case 2).

- $1.0 \times 10^{-1} \leq P_j^\infty \leq 1.0$
- × $1.0 \times 10^{-2} \leq P_j^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P_j^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P_j^\infty \leq 1.0 \times 10^{-3}$
- $1.0 \times 10^{-5} \leq P_j^\infty \leq 1.0 \times 10^{-4}$

Fig.110 Probability Distribution P_j^∞ , Integrated Over Φ , of the Persistent Groups for ES-3 ($\theta = 6.0\text{E-}16$, Case 2).

Fig.111 State-Space Projection of the PGs on the $\bar{I}$ - Φ Plane for ES-3
 ($\theta = 2.0\text{E-}16$, Case 2).

- Stationary Cells, Group 2 ($T=\tau$)
- + Cyclic Group 3 ($T=2\tau$)
- × Cyclic Group 4 ($T=3\tau$)
- Cyclic Group 5 ($T=4\tau$)

Fig.113 State-Space Projection of the PGs on the $\bar{I}$ - Φ Plane for ES-3
($\theta = 6.0\text{E-}16$, Case 2).

Fig.114 State-Space Projection of the PGs on the $\bar{I}$ - $\bar{X}$ Plane for ES-3
($\theta = 2.0E-16$, Case 2).

- Stationary Cells, Group 2 ($T=\tau$)
- + Cyclic Group 3 ($T=2\tau$)
- × Cyclic Group 4 ($T=3\tau$)
- Cyclic Group 5 ($T=4\tau$)

Fig.115 State-Space Projection of the PGs on the $\bar{I}$ - $\bar{X}$ Plane for ES-3
($\theta = 4.0E-16$, Case 2).

Fig.116 State-Space Projection of the PGs on the $\bar{I}$ - $\bar{X}$ Plane for ES-3
($\theta = 6.0E-16$, Case 2).

- $1.0 \times 10^{-1} \leq P_j^\infty \leq 1.0$
- × $1.0 \times 10^{-2} \leq P_j^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P_j^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P_j^\infty \leq 1.0 \times 10^{-3}$
- △ $1.0 \times 10^{-5} \leq P_j^\infty \leq 1.0 \times 10^{-4}$
- $1.0 \times 10^{-6} \leq P_j^\infty \leq 1.0 \times 10^{-5}$

Fig.117 Probability Distribution P_j^∞ , Integrated Over $\bar{X}$, of the Persistent Groups for ES-3 ($\theta = 2.0\text{E-}16$, Case 3).

- $1.0 \times 10^{-1} \leq P_j^\infty \leq 1.0$
- × $1.0 \times 10^{-2} \leq P_j^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P_j^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P_j^\infty \leq 1.0 \times 10^{-3}$
- △ $1.0 \times 10^{-5} \leq P_j^\infty \leq 1.0 \times 10^{-4}$
- $1.0 \times 10^{-6} \leq P_j^\infty \leq 1.0 \times 10^{-5}$

Fig.118 Probability Distribution P_j^∞ , Integrated Over $\bar{X}$, of the Persistent Groups for ES-3 ($\theta = 4.0\text{E-}16$, Case 3).

- $1.0 \times 10^{-1} \leq P^\infty \leq 1.0$
- × $1.0 \times 10^{-2} \leq P^\infty \leq 1.0 \times 10^{-1}$
- + $1.0 \times 10^{-3} \leq P^\infty \leq 1.0 \times 10^{-2}$
- $1.0 \times 10^{-4} \leq P^\infty \leq 1.0 \times 10^{-3}$
- △ $1.0 \times 10^{-5} \leq P^\infty \leq 1.0 \times 10^{-4}$
- $1.0 \times 10^{-6} \leq P^\infty \leq 1.0 \times 10^{-5}$

Fig.119 Probability Distribution P_j^∞ , Integrated Over $\bar{X}$, of the Persistent Groups for ES-3 ($\theta = 6.0\text{E-}16$, Case 3).

Fig.120 Probability Distribution P_j^∞ , Integrated Over Φ , of the Persistent Groups for ES-3 ($\theta = 2.0\text{E-}16$, Case 3).

Fig.121 Probability Distribution P_j^∞ , Integrated Over Φ_j , of the Persistent Groups for ES-3 ($\theta = 4.0\text{E-}16$, Case 3).

Fig.122 Probability Distribution P_j^∞ , Integrated Over Φ , of the Persistent Groups for ES-3 ($\theta = 6.0\text{E-}16$, Case 3).

- o Stationary Cells, Group 2 ($T=\tau$)
- + Cyclic Group 3 ($T=2\tau$)
- x Cyclic Group 4 ($T=3\tau$)
- Cyclic Group 5 ($T=4\tau$)

Fig.123 State-Space Projection of the PGs on the $\bar{I}$ - Φ Plane for ES-3
($\theta = 2.0\text{E-}16$, Case 3).

- o Stationary Cells, Group 2 ($T=\tau$)
- + Cyclic Group 3 ($T=2\tau$)
- × Cyclic Group 4 ($T=3\tau$)
- Cyclic Group 5 ($T=4\tau$)

Fig.124 State-Space Projection of the PGs on the $\bar{I}$ - Φ Plane for ES-3
($\theta = 4.0\text{E-}16$, Case 3).

- Stationary Cells, Group 2 ($T=\tau$)
- + Cyclic Group 3 ($T=2\tau$)
- × Cyclic Group 4 ($T=3\tau$)
- Cyclic Group 5 ($T=4\tau$)

Fig.125 State-Space Projection of the PGs on the $\bar{I}$ - Φ Plane for ES-3 ($\theta = 6.0\text{E-}16$, Case 3).

- Stationary Cells, Group 2 ($T=\tau$)
- + Cyclic Group 3 ($T=2\tau$)
- × Cyclic Group 4 ($T=3\tau$)
- Cyclic Group 5 ($T=4\tau$)

Fig.126 State-Space Projection of the PGs on the $\bar{I}$ - $\bar{X}$ Plane for ES-3
($\theta = 2.0E-16$, Case 3).

- Stationary Cells, Group 2 ($T=\tau$)
- + Cyclic Group 3 ($T=2\tau$)
- × Cyclic Group 4 ($T=3\tau$)
- Cyclic Group 5 ($T=4\tau$)

Fig.127 State-Space Projection of the PGs on the $\bar{I}-\bar{X}$ Plane for ES-3
($\theta = 4.0E-16$, Case 3).

- o Stationary Cells, Group 2 ($T=\tau$)
- + Cyclic Group 3 ($T=2\tau$)
- × Cyclic Group 4 ($T=3\tau$)
- Cyclic Group 5 ($T=4\tau$)

Fig.128 State-Space Projection of the PGs on the $\bar{I}$ - $\bar{X}$ Plane for ES-3
($\theta = 6.0\text{E-}16$, Case 3).

- ⊗ BOAs for the sink cell
- o BOAs for the stationary cells, group 2 ($T=\tau$)
- + BOAs for the cyclic group 3 ($T=2\tau$)
- x BOAs for the cyclic group 4 ($T=3\tau$)

"blank" Multiple domicile transient cells

Fig.129 Cut of the BOAs at the Equilibrium $\bar{X}$ -Level for ES-3 ($\theta = 2.0\text{E-}16$, Case 1).

- ⊕ BOAs for the sink cell
- BOAs for the stationary cells, group 2 ($T=\tau$)
- ⊕ BOAs for the cyclic group 3 ($T=2\tau$)
- × BOAs for the cyclic group 4 ($T=3\tau$)

"blank" Multiple domicile transient cells

Fig.130 Cut of the BOAs at the Equilibrium $\bar{X}$ -Level for ES-3 ($\theta = 4.0E-16$, Case 1).

Fig.131 Cut of the BOAs at the Equilibrium $\bar{X}$ -Level for ES-3 ($\theta = 6.0\text{E-}16$, Case 1).

Fig.132 Cut of the BOAs at the Equilibrium Φ -Level for ES-3 ($\theta = 2.0E-16$, Case 1).

Fig.133 Cut of the BOAs at the Equilibrium Φ -Level for ES-3 ($\theta = 4.0E-16$, Case 1).

- ⊙ BOAs for the sink cell
- BOAs for the stationary cells, group 2 ($T=\tau$)
- ⊕ BOAs for the cyclic group 3 ($T=2\tau$)
- ⊗ BOAs for the cyclic group 4 ($T=3\tau$)
- "blank" Multiple domicile transient cells

Fig.134 Cut of the BOAs at the Equilibrium Φ -Level for ES-3 ($\theta = 6.0E-16$, Case 1).

- BOAs for the sink cell
- BOAs for the stationary cells, group 2 ($T=\tau$)
- + BOAs for the cyclic group 3 ($T=2\tau$)
- × BOAs for the cyclic group 4 ($T=3\tau$)
- BOAs for the cyclic group 5 ($T=4\tau$)

"blank" Multiple domicile transient cells

Fig.135 Cut of the BOAs at the Equilibrium $\bar{X}$ -Level for ES-3 ($\theta = 2.0\text{E-}16$, Case 2).

Fig.136 Cut of the BOAs at the Equilibrium $\bar{X}$ -Level for ES-3 ($\theta = 4.0\text{E-}16$, Case 2).

Fig.137 Cut of the BOAs at the Equilibrium $\bar{X}$ -Level for ES-3 ($\theta = 6.0\text{E-}16$, Case 2).

- BOAs for the sink cell
- BOAs for the stationary cells, group 2 ($T=\tau$)
- + BOAs for the cyclic group 3 ($T=2\tau$)
- × BOAs for the cyclic group 4 ($T=3\tau$)
- BOAs for the cyclic group 5 ($T=4\tau$)
- "blank" Multiple domicile transient cells

Fig.138 Cut of the BOAs at the Equilibrium Φ -Level for ES-3 ($\theta = 2.0E-16$, Case 2).

Fig.139 Cut of the BOAs at the Equilibrium Φ -Level for ES-3 ($\theta = 4.0E-16$, Case 2).

- BOAs for the sink cell
- BOAs for the stationary cells, group 2 ($T=\tau$)
- + BOAs for the cyclic group 3 ($T=2\tau$)
- × BOAs for the cyclic group 4 ($T=3\tau$)
- BOAs for the cyclic group 5 ($T=4\tau$)

"blank" Multiple domicile transient cells

Fig.140 Cut of the BOAs at the Equilibrium Φ -Level for ES-3 ($\theta = 6.0E-16$, Case 2).

Fig.141 Cut of the BOAs at the Equilibrium $\bar{X}$ -Level for ES-3 ($\theta = 2.0\text{E-}16$, Case 3).

- BOAs for the sink cell
- △ BOAs for the cyclic group 3 ($T=2\tau$)
- × BOAs for the cyclic group 5 ($T=4\tau$)
- "blank" Multiple domicile transient cells

Fig.142 Cut of the BOAs at the Equilibrium $\bar{X}$ -Level for ES-3 ($\theta = 4.0E-16$, Case 3).

- BOAs for the sink cell
- BOAs for the stationary cells, group 2 ($T=\tau$)
- + BOAs for the cyclic group 4 ($T=3\tau$)
- × BOAs for the cyclic group 5 ($T=4\tau$)
- "blank" Multiple domicile transient cells

Fig.143 Cut of the BOAs at the Equilibrium $\bar{X}$ -Level for ES-3 ($\theta = 6.0E-16$, Case 3).

Fig.144 Cut of the BOAs at the Equilibrium Φ -Level for ES-3 ($\theta = 2.0E-16$, Case 3).

- BOAs for the sink cell
- △ BOAs for the cyclic group 3 ($T=2\tau$)
- × BOAs for the cyclic group 5 ($T=4\tau$)
- "blank" Multiple domicile transient cells

Fig.145 Cut of the BOAs at the Equilibrium Φ -Level for ES-3 ($\theta = 4.0E-16$, Case 3).

- BOAs for the sink cell
 - BOAs for the stationary cells, group 2 ($T=\tau$)
 - + BOAs for the cyclic group 4 ($T=3\tau$)
 - × BOAs for the cyclic group 5 ($T=4\tau$)
- "blank" Multiple domicile transient cells

Fig.146 Cut of the BOAs at the Equilibrium Φ -Level for ES-3 ($\theta = 6.0E-16$, Case 3).

- $0.75 \leq \rho_{1,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{1,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{1,i}^{\infty} \leq 0.50$
- × $0.01 \leq \rho_{1,i}^{\infty} \leq 0.25$

Fig.147 Absorption Probabilities $\rho_{1,j}^{\infty}$ into the Sink Cell at the Equilibrium $\bar{X}$ -Level Plane for ES-3 ($\theta = 2.0\text{E-}16$, Case 3).

Fig.148 Absorption Probabilities $\rho_{1,j}^{\infty}$ into the Sink Cell at the Equilibrium $\bar{X}$ -Level Plane for ES-3 ($\theta = 4.0\text{E-}16$, Case 3).

- $0.75 \leq \rho_{1,i}^\infty \leq 0.99$
- $0.50 \leq \rho_{1,i}^\infty \leq 0.75$
- + $0.25 \leq \rho_{1,i}^\infty \leq 0.50$
- × $0.01 \leq \rho_{1,i}^\infty \leq 0.25$

Fig.149 Absorption Probabilities $\rho_{1,i}^\infty$ into the Sink Cell at the Equilibrium $\bar{X}$ -Level Plane for ES-3 ($\theta = 6.0\text{E-}16$, Case 3).

- $0.75 \leq \rho_{1,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{1,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{1,i}^{\infty} \leq 0.50$
- × $0.01 \leq \rho_{1,i}^{\infty} \leq 0.25$

Fig.150 Absorption Probabilities $\rho_{1,j}^{\infty}$ into the Sink Cell at the Equilibrium Φ -Level Plane for ES-3 ($\theta = 2.0E-16$, Case 3).

Fig.151 Absorption Probabilities $\rho_{1,j}^{\infty}$ into the Sink Cell at the Equilibrium Φ -Level Plane for ES-3 ($\theta = 4.0\text{E-}16$, Case 3).

- $0.75 \leq \rho_{1,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{1,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{1,i}^{\infty} \leq 0.50$
- × $0.01 \leq \rho_{1,i}^{\infty} \leq 0.25$

Fig.152 Absorption Probabilities $\rho_{1,j}^{\infty}$ into the Sink Cell at the Equilibrium Φ -Level Plane for ES-3 ($\theta = 6.0E-16$, Case 3).

Fig.153 Absorption Probabilities $\rho_{5,j}^{\infty}$ into Group 5 at the Equilibrium $\bar{X}$ -Level Plane for ES-3 ($\theta = 2.0\text{E-}16$, Case 3).

- $0.75 \leq \rho_{5,i}^\infty \leq 0.99$
- $0.50 \leq \rho_{5,i}^\infty \leq 0.75$
- + $0.25 \leq \rho_{5,i}^\infty \leq 0.50$
- × $0.01 \leq \rho_{5,i}^\infty \leq 0.25$

Fig.154 Absorption Probabilities $\rho_{5,j}^\infty$ into Group 5 at the Equilibrium $\bar{X}$ -Level Plane for ES-3 ($\theta = 4.0\text{E-}16$, Case 3).

- $0.75 \leq \rho_{5,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{5,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{5,i}^{\infty} \leq 0.50$
- × $0.01 \leq \rho_{5,i}^{\infty} \leq 0.25$

Fig.155 Absorption Probabilities $\rho_{5,j}^{\infty}$ into Group 5 at the Equilibrium $\bar{X}$ -Level Plane for ES-3 ($\theta = 6.0E-16$, Case 3).

- $0.75 \leq \rho_{5,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{5,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{5,i}^{\infty} \leq 0.50$
- × $0.01 \leq \rho_{5,i}^{\infty} \leq 0.25$

Fig.156 Absorption Probabilities $\rho_{5,j}^{\infty}$ into Group 5 at the Equilibrium Φ -Level Plane for ES-3 ($\theta = 2.0E-16$, Case 3).

- $0.75 \leq \rho_{5,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{5,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{5,i}^{\infty} \leq 0.50$
- × $0.01 \leq \rho_{5,i}^{\infty} \leq 0.25$

Fig.157 Absorption Probabilities $\rho_{5,j}^{\infty}$ into Group 5 at the Equilibrium Φ -Level Plane for ES-3 ($\theta = 4.0E-16$, Case 3).

- $0.75 \leq \rho_{5,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{5,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{5,i}^{\infty} \leq 0.50$
- × $0.01 \leq \rho_{5,i}^{\infty} \leq 0.25$

Fig.158 Absorption Probabilities $\rho_{5,i}^{\infty}$ into Group 5 at the Equilibrium Φ -Level Plane for ES-3 ($\theta = 6.0E-16$, Case 3).

Fig.159 Absorption Probabilities $\rho_{5,j}^\infty$ into Group 5 at the Equilibrium $\bar{I}$ -Level Plane for ES-3 ($\theta = 2.0\text{E-}16$, Case 3).

- $0.75 \leq \rho_{5,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{5,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{5,i}^{\infty} \leq 0.50$
- × $0.01 \leq \rho_{5,i}^{\infty} \leq 0.25$

Fig.160 Absorption Probabilities $\rho_{5,j}^{\infty}$ into Group 5 at the Equilibrium $\bar{I}$ -Level Plane for ES-3 ($\theta = 4.0\text{E-}16$, Case 3).

- $0.75 \leq \rho_{5,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{5,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{5,i}^{\infty} \leq 0.50$
- × $0.01 \leq \rho_{5,i}^{\infty} \leq 0.25$

Fig.161 Absorption Probabilities ρ_{5j}^{∞} into Group 5 at the Equilibrium $\bar{I}$ -Level Plane for ES-3 ($\theta = 6.0E-16$, Case 3).

- $0.99 \leq \rho_{5,i}^{\infty} \leq 1.00$
- $0.75 \leq \rho_{5,i}^{\infty} \leq 0.99$
- $0.50 \leq \rho_{5,i}^{\infty} \leq 0.75$
- + $0.25 \leq \rho_{5,i}^{\infty} \leq 0.50$
- × $0.01 \leq \rho_{5,i}^{\infty} \leq 0.25$

Fig.162 Absorption Probabilities $\rho_{5,j}^{\infty}$ into Group 5 at the Equilibrium $\bar{I}$ -Level Plane for ES-3 (θ Uniformly Distributed, Case 3).