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Abstract

Petrophysical heterogeneity in the South Wasson Clear Fork (SWCF) reservoir and other shallow-water platform carbonates in the Permian Basin and elsewhere is composed of a large-scale stratigraphically controlled component and a small-scale poorly correlated component. The large-scale variability exists as a flow-unit scale petrophysical layering that is laterally persistent at interwell scales and produces highly stratified reservoir behavior. Capturing the rate-enhancing effect of the small-scale variability requires carefully controlled averaging procedures at four levels of scaleup. Porosity can be easily scaled using arithmatic averaging procedures. Permeability, however, requires carefully controlled power-averaging procedures. Effective permeability is increased at every scaleup level.

Introduction

The South Wasson Clear Fork (SWCF) model construction process involves several explicit or implied levels of scaleup. Discussion of the modeling steps can be clarified by establishing a terminology for the scaleup levels:

*Level 0* will refer to petrophysical heterogeneity at scales smaller than the volume of investigation of the well-log porosity measurements. Measurements on core plugs are included in level 0.
**Level 1** will refer to heterogeneity scaled up to the porosity-log investigation volume. Scaleup to level 1 is implied by the use of well-log data as the basis of statistical analysis and modeling. Because the porosity logs are presumed to be accurately calibrated arithmetic averages of the level 0 heterogeneity, this scaleup level for porosity modeling is automatically provided by the log data itself. However, careful consideration of the effects of level 0 heterogeneity in level 1 effective permeabilities is required because effective permeabilities are not arithmetic averages.

**Level 2** will refer to well-log data scaled up to flow-unit thicknesses, but with the lateral scale unchanged. For porosity modeling at SWCF this scaleup level was obtained by a simple arithmetic average of the well-log porosity data within each flow unit at each well. For permeability modeling a nonarithmetic average was required.

**Level 3** will refer to level 2 data scaled up laterally to the grid blocks of the reservoir flow model. In the SWCF model each flow unit was represented by one layer of grid cells, so scaleup from level 2 to level 3 did not involve any vertical scale change. As with the previous scaleup levels, porosity scaleup was accomplished with a simple arithmetic average, but permeability scaleup required a carefully controlled nonarithmetic averaging procedure.

The scaleup level numbers will be used as subscripts in various mathematical symbols to simplify notation. Thus, core-plug permeability data will be denoted by $k_0$, individual well-log porosity measurements by $\phi_1$, average flow-unit porosities at each well by $\phi_2$, grid-block effective permeabilities by $k_3$, and so on.

**Porosity Scaleup and Grid Construction**

Construction of a porosity grid can be easily accomplished by (1) conditional stochastic simulation of the Gaussian transformed residuals using the semivariogram model described above, followed by (2) reverse application of the Gaussian transform to obtain a conditional simulation of $r_\phi$, which is in turn followed by (3) replacement of the trends to obtain a conditional simulation of $\phi_2$ using
\[ \phi_{2g} = r_{\phi 2g} s_{\phi 2g} + \phi_{2g} \]  \hspace{1cm} (1)

However, this simulation of \( \phi_2 \) should not be used directly in flow modeling for two reasons. The first reason is that what is required for flow modeling is porosity at scaleup level 3, the grid cells, not scaleup level 2, the vertical flow-unit averages of well-log scale data. Although \( \phi_2 \) can be easily simulated on the grid centers, and each grid block may be filled with the corresponding constant value for display purposes, the values do not actually represent grid-block averages. Each grid-block porosity obtained in this manner represents a single sample of a level 2 porosity average from the center of the block. Level 2 porosities are averages over much smaller volumes than the required level 3 grid-block averages; therefore, the variance of \( \phi_2 \) is too large.

The second problem with using a simulation of \( \phi_2 \) directly in SWCF flow modeling is that it contains a component of variability generated from the second structure in the semivariogram model. This component of \( \phi_2 \) is not an artifact of the stochastic simulation. It is an actual spatial property of the well-log data set, but it represents random errors in well-log calibration that are not thought to be present in the reservoir itself.

Scaleup from level 2 to level 3 and removal of the second semivariogram structure can be accomplished in a single step by performing the stochastic simulation on a refined grid followed by averaging within each grid block. The level 2 porosity data already represents a vertical flow-unit average, so refinement is only required in the two lateral directions. This refinement was accomplished with a 10 by 10 subdivision of each grid block. The averaging was performed more conveniently using \( r_{\phi 2} \) rather than \( \phi_2 \) because the geostatistical software can compute the block averages of the stochastically simulated variable automatically producing a conditional block stochastic simulation of \( r_{\phi 3} \). The conditional simulation of \( \phi_3 \) was obtained from

\[ \phi_{3g} = r_{\phi 3g} s_{\phi 3g} + \phi_{3g} \]  \hspace{1cm} (2)

using \( s_{\phi 2} \) and \( \phi_2 \) as approximations for the trends \( s_{\phi 3} \) and \( \phi_3 \).
Models constructed with more traditional geostistical approaches do not resolve the important flow-unit-scale petrophysical layering described by the geological/petrophysical models. Quantification and modeling of the petrophysical layering requires a high-resolution sequence-stratigraphic framework and careful modeling of lateral trends at the flow-unit scale.

**Permeability Scaleup and Grid Construction**

Permeability modeling and scaleup are complicated because effective permeabilities cannot be accurately represented by arithmetic averages. However, considerable simplification of permeability modeling in carbonates is obtained from the observation that effective permeabilities within flow units are generally well approximated by a one-third power average,

\[ k_{\text{eff}} = \left( \frac{\sum k_i^{1/3}}{n} \right)^3, \quad (3) \]

where \( k_{\text{eff}} \) is the effective permeability for a volume of rock, \( k_i \) is the permeability of a small-scale sample within the volume, and \( n \) is the number of such samples. The one-third power average for effective permeability is a theoretical result that applies in three dimensions when the small-scale permeabilities are log-normally distributed, are isotropic, and have isotropic spatial correlations whose correlation ranges are small compared with the size of the averaging volume (Hristopulos and Christakos, 1999). These conditions are well approximated by the small-scale permeabilities within most carbonate flow units where the spatial correlations are weak, most of the variance is concentrated at small scales, and the correlation ranges are only moderately anisotropic (Jennings, 2000; Jennings and others, 2000). In addition, most core-scale permeability data from carbonate outcrops and subsurface reservoirs are approximately log-normally distributed with only moderate directional permeability anisotropy. The validity of a one-third power average for approximating effective permeabilities within a carbonate flow unit has been experimentally verified (Noetinger and Jacquin, 1991).

The use of power averaging greatly simplifies permeability grid construction and scaleup because \( k_{\text{eff}} \) as expressed in equation 3 may be viewed as the cube of an arithmetic average of permeabilities raised to an exponent of 1/3. Thus, a grid of
effective permeabilities at scaleup level 3 may be constructed from well-log-based permeability estimates at level 1 by (1) raising the level 1 data to an exponent of 1/3, (2) applying the same modeling procedure as was used for porosity, and (3) raising the resulting grid values to an exponent of 3.

For this approach to work correctly the well-log-based permeability estimates must themselves represent effective permeabilities at scaleup level 1. However, the porosity-permeability transform used in this study was developed with a regression on the logarithms of core-plug measurements at scaleup level 0 that predicts the geometric average of permeability for a given porosity. This geometric average can be corrected to provide an approximate one-third power average of plug permeabilities within the well-log investigation volume with the following formula (Aitchison and Brown, 1969):

$$k_{\text{eff}} = k_g \exp\left(\frac{\sigma^2}{6}\right), \quad (4)$$

where $k_g$ is a geometric average obtained by applying the porosity-permeability transform to a well-log porosity measurement, and $\sigma^2$ is the variance of the natural logarithms of the plug-scale permeabilities within the well-log investigation volume. The formula is approximate in that the plug-scale permeabilities are assumed to be log-normally distributed within any well-log measurement volume. The second term on the right-hand side of this formula can be regarded as a simple multiplicative correction to the porosity-permeability transform. For South Wasson Clear Fork (SWCF) modeling $\sigma^2$ was estimated as the fraction of the variance of the logarithms of the core-plug permeabilities from well 7531 that was not explained by a regression with the corresponding well-log porosities. Application of this variance in equation 4 produces a multiplicative correction, $k_{\text{eff}} = 2.22 \, k_g$, that was assumed to be constant throughout the reservoir (fig. 1).

These resulting well-log effective permeability estimates were used for modeling and scaleup using the procedure described above. The trends and residuals of $k^{1/3}$ were modeled with the same procedures applied to porosity. The $k^{1/3}$ residuals after Gaussian transformation had very nearly the same semivariogram as porosity. Cross plots of the simulated permeability and porosity for each scaleup level indicate a systematic effective permeability increase with each scaleup step (fig. 1). The largest permeability increase was produced at the smallest scale, scaling up from core-plug permeabilities to well-log effective permeabilities.
Equation 4 is not only useful in approximate scaleup calculations, it also summarizes nicely the dependence of flow rate on small-scale permeability heterogeneity and helps in understanding the observations from figure 1. In three dimensions the effective permeability is dependent not only on the average of the permeability logarithms, but also on the variance of the permeability logarithms. Effective permeability increases systematically with increasing variance for a given geometric average, and we should expect effective permeabilities to increase with scale as additional variance is incorporated. Furthermore, in carbonates we should expect the largest increases to occur at the smallest scales because that is where most of the spatial variance is concentrated. Therefore, the plug-to-well-log scaleup step is likely to be the most important, but neglecting any component of permeability variance in scaleup calculations will produce a scaleup estimate that is too small, leading to a corresponding flow-rate underestimate in reservoir performance predictions.

References
Nomenclature

Variables

\( k \) = permeability
\( k_{\text{eff}} \) = effective permeability
\( k_g \) = geometric average permeability
\( n \) = number of samples
\( r \) = residual
\( s \) = moving standard deviation
\( \phi \) = porosity
\( \bar{\phi} \) = moving average of porosity
\( \sigma^2 \) = variance

Subscripts

0, 1, 2, 3 = scaleup levels
\( g \) = grid point index
\( \phi \) = porosity
Figure 1. Core-plug porosity and permeability data from SWCF well 7531 (points) and power-law porosity-permeability correlations at scaleup levels 0, 1, 2, and 3.