Superconducting Super Collider Laboratory

High Frequency Breakdown Voltage

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Abstract

This report contains information about the effect of frequency on the breakdown voltage of an air gap at standard pressure and temperature, 76 mm Hg and 0°C, respectively. The frequencies of interest are 47 MHz and 60 MHz. Additionally, the breakdown in vacuum is briefly considered.

The breakdown mechanism is explained on the basis of collision and ionization. The presence of the positive ions produced by ionization enhances the field in the gap, and thus determines the breakdown. When a low-frequency voltage is applied across the gap, the breakdown mechanism is the same as that caused by the DC or static voltage. However, when the frequency exceeds the first critical value $f_c$, the positive ions are trapped in the gap, increasing the field considerably. This makes the breakdown occur earlier; in other words, the breakdown voltage is lowered. As the frequency increases two decades or more, the second critical frequency, $f_{cc}$, is reached. This time the electrons start being trapped in the gap. Those electrons that travel multiple times across the gap before reaching the positive electrode result in an enormous number of electrons and positive ions being present in the gap. The result is a further decrease of the breakdown voltage. However, increasing the frequency does not decrease the breakdown voltage correspondingly. In fact, the associated breakdown field intensity is almost constant (about 29 kV/cm). The reason is that the recombination rate increases and counterbalances the production rate, thus reducing the effect of the positive ions' concentration in the gap.

The theory of collision and ionization does not apply to the breakdown in vacuum. It seems that the breakdown in vacuum is primarily determined by the irregularities on the surfaces of the electrodes. Therefore, the effect of frequency on the breakdown, if any, is of secondary importance.

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1.0 INTRODUCTION

The breakdown voltage between electrodes in air at atmospheric pressure is discussed when the frequency of the applied voltage varies. The frequency range of interest is 47–100 MHz, especially the two frequencies of 47 MHz and 60 MHz. Additionally, the breakdown in vacuum is briefly mentioned. However, before the effect of frequency on the breakdown voltage is examined, it is advisable to have an overview of the static or DC breakdown mechanism.

2.0 THE STATIC OR DC BREAKDOWN MECHANISM

Assume a uniform field $E$ is applied to the electrodes, and somehow there is a free electron in the gap. Under the influence of the field $E$, the electron moves toward the anode, gaining more and more energy on the way. If the field $E$ is strong, the electron has enough energy to ionize an air molecule by collision, thereby freeing another electron. The second electron travels to the anode together with the first one, leaving behind it a positive ion. Then the two electrons collide with other air molecules, yielding even more electrons and positive ions. The process goes on and on. When the electrons reach the anode, they have a long tail of positive ions whose density is highest at the anode (Figure 1).

Figure 1. Distribution of Charge Carriers in an Avalanche and Their Contribution to the Applied Uniform Electric Field. $E_1 > E_2; E_2 < E; E_3 > E$.\(^1\)

Note that an electron is much lighter than a positive ion, so the electron moves faster in the same field. Because most of the electrons produced are swept into the anode, the ionization of electrons alone cannot cause a breakdown.\(^2\)

When the applied field is higher, positive ions are allowed to ionize the air modules by collision—producing more electrons and positive ions—or to knock electrons out of the negative cathode. Those newborn electrons are enormous, and they in turn yield more electrons and ions through collision.

Being slower, the positive ions seem to be stationary while the electrons are crossing the gap. The ions then form a space charge near the anode. The field caused by this accumulation of positive ions increases the field in the gap. Consequently, the gap current grows rapidly and leads to breakdown. "...The criterion of breakdown may depend on the concentration of positive ions, rather than on the total number formed within the gap. As a result, the concentration of electrons in the electron avalanche will play a major part in the initiation of breakdown."\(^3\)
3.0 THE EFFECT OF FREQUENCY ON THE BREAKDOWN VOLTAGE

3.1 The Critical Frequency

When the frequency of the applied voltage varies, we have to consider the distances the positive ions and electrons can travel before the cycle reversal—that is, in a half cycle of the voltage. Before we go further, let's define mobility. The mobility of a positive ion is its velocity in an electric field. Mobility varies directly with the field intensity \( E \), so the mobility constant is defined as the velocity in cm/sec at atmospheric pressure and 0°C per V/cm of electric field. A positive ion has the mobility constant of 1.32, so in the field of 30,000 V/cm it has the velocity of \( 30,000 \times 1.32 \), or 39,600 cm/sec. However, the mobility of an electron does not vary directly with the field, so it does not have the mobility constant. Its mobility is a function of \( E/p \), where \( E \) is the field intensity, and \( p \) is the pressure. According to Compton in Physical Review of 1923, an electron has a velocity of 13,800,000 cm/sec at \( E \), equal to 30,000 V/cm. In the same condition, the electron moves 348 times faster than the positive ion in this example.

As the frequency increases, a point is reached at which the ions cannot reach the cathode before this electrode reverses its polarity. The ions remain in the gap, and because the avalanche continues, they form a space charge with growing density. The space charge then distorts the field in the gap. According to the above breakdown criterion suggested by Pim, it is the space charge that lowers the breakdown voltage in comparison with the DC breakdown value. The frequency at which the breakdown voltage begins to decrease below the static value is called the first "critical frequency," \( f_c \).

If the frequency is raised even more, another phenomenon occurs. At this time, the electrons themselves cannot have enough time to travel to the positive electrode in the half cycle. Assume one electron ionizes once for every thousand collisions with air molecules. Because the mean free path of an electron (the distance the electron travels before colliding with a molecule) at atmospheric pressure is about 0.00005 cm, the electron will ionize 20 times per centimeter. The number of new electrons produced by that single electron is given by \( e^{20d} \), where \( d \) is the gap width in centimeters. Because that electron cannot reach the positive electrode during the previous half cycle, it has to travel the distance of \( d \) cm once more to reach the new positive electrode. It will produce a total of \( e^{20d} \) new electrons and the same number of positive ions. The density of the already existing positive ions space charge will then be highly increased and will in effect distort the field in the gap considerably. Therefore, the breakdown voltage is expected to be lowered even more. The corresponding frequency is called the second "critical frequency," \( f_{cc} \). With a further increase in frequency, however, the electrons become trapped in the gap, as do the positive ions, and the electrons cancel out the space charge of positive ions. In other words, the recombination rate goes up and finally counterbalances the production rate. This reduces the field intensity in the gap. Consequently, the breakdown voltage begins to rise toward the static value.

3.2 The Critical Gap Width

3.2.1 Previous Investigations

On considering the effect of the frequency on the breakdown voltages, researchers are also interested in the "critical gap" in which, at a given frequency, the high-frequency breakdown voltage begins to decrease below the static value as the gap width is increased. Experiments have shown the existence of this critical gap.

Misere and Luft, working on 10-cm-sphere electrodes at frequencies around 450 KHz, came up with the curves shown in Figure 2. For comparison, the breakdown values at 50 Hz are included on the graph. The critical gap is approximately 1.5 cm. Note that gaps smaller than the critical gap have breakdown values almost identical to DC values.

Curves obtained by Bright showed the similar proofs of critical gaps when he studied short gaps in nitrogen (See Figure 3.)
Figure 2. Gap Width versus Breakdown Voltage Obtained by Misere and Luft. (10-cm-Kugeln Symmetrische Anordnung: 10-cm-sphere gap symmetrically arranged.)

Figure 3. Breakdown Voltage Curves in Nitrogen as a Function of Gap Length between Spheres of 2.0-cm diameter.
Recalling the explanation for the critical frequency in Section 3.1, we can reason in the same way for the effect on the breakdown values as gap width increases. At a given frequency, a point is reached where the gap is so long that positive ions cannot travel to the positive electrode during the half cycle of the voltage. They become trapped in this critical gap and form the space charge, the field of which increases the field in the gap; then the breakdown voltage begins to decrease from the static values. This critical gap width, \( g_c \), is associated with the critical frequency, \( f_c \). This gap is clearly shown in the experimental curves issued by Reukema.² (Figure 4).

Reukema has worked on gaps up to 2.5 cm (1 in.) in the range of 20–425 KHz. It is observed that there is not much change in the breakdown voltage versus frequency until it approaches 20 KHz. Then there is a gradual decrease in the breakdown values in the range of 20–60 KHz. However, there is little change as the frequency rises to 425 KHz, which is the highest test frequency. In fact, on the graph we find a single curve for the whole bunch of frequencies in the range of 60.6–425.530 KHz. In other words, the breakdown voltage increases with constant gradient as the gap width increases. (In this case the voltage gradient approximates 17 kV/cm.)

As the gap width increases, the electrons in turn are caught in the gap. We expect further decrease in the breakdown value at the second critical gap, \( g_{ce} \), which is associated with the critical frequency, \( f_{ce} \). Indeed, in the above experiment Reukema² also predicted that there would be further decrease in breakdown values at frequency equal to 6 MHz for a 1-cm gap, which accounts for \( f_{ce} \) and \( g_{ce} \).

Pim³ has worked on parallel plane gaps up to 1 mm with a field of frequencies between 100 MHz and 300 MHz (Figure 5). Breakdown voltages versus gap width at various frequencies at atmospheric pressure are shown. Breakdown voltage follows the normal rising curve, which agrees quantitatively with the above values obtained by Reukema (10–15% lower than the low-frequency values), until the critical gap is achieved. Then the breakdown voltage drops suddenly, passes a minimum, and finally rises with a constant gradient.
Figure 5. Variation of Ultra-High-Frequency Breakdown Voltage with Gap Width at Atmospheric Pressure.

The corresponding curves of breakdown field intensity versus the gap width are shown in Figure 6. Gaps smaller than the critical gap have a breakdown field 10–15% lower than the 50 Hz values. At the critical gap, the field decreases almost double, finally arriving at a constant value of 29 kV/cm as the gap increases at all frequencies.³

Figure 6. The Electrical Breakdown Strength of Air at Ultra-High Frequencies.³
The above theory is further confirmed by the experiment of Rohde and Wedemeyer. They worked on 1-cm-sphere gap spacing 2 mm apart. The breakdown voltage is made relative to its value at 50 Hz for comparison (Figure 7).

![Figure 7. Ratio of High-Frequency Breakdown Voltage to Static Breakdown Voltage as a Function of Frequency for an Air Gap.](image)

Two observations are worth noting:

1. Around 20–40 KHz the breakdown potential starts reducing. This must be associated with the critical frequency, $f_c$.

2. When the frequency increases, as expected, the breakdown potential reduces further, passes a minimum at around 1 MHz, and rises toward the static value. Note that the breakdown voltage is finally becoming greater than the static value.

3.2.2 Extrapolation

Knowledge of the critical gap permits the calculation of the critical gaps in the frequency range of interest (47–100 MHz) from Pim’s curves (Figure 5).

Analyzing the curve, we find that the critical gap, at a given frequency, must lie on the curve KA. In other words, the gap’s coordinates will satisfy the equation of the curve KA. We are going to approximate a straight line for the curve KA using “the least square curve fitting” method based on empirical data from Pim’s curve. We choose eight points on the curve KA: A, B, C, D, E, F, G, and K, corresponding to breakdown points at 100 MHz, 125 MHz, 150 MHz, 175 MHz, 200 MHz, 225 MHz, 250 MHz, and 300 MHz, respectively (Table 1).
TABLE 1. GAP WIDTHS AND BREAKDOWN VOLTAGES FOR EIGHT POINTS ON CURVE KA.

<table>
<thead>
<tr>
<th>POINT</th>
<th>GAP WIDTH (cm)</th>
<th>BREAKDOWN VOLTAGE (rms V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.040</td>
<td>1427</td>
</tr>
<tr>
<td>B</td>
<td>0.031</td>
<td>1200</td>
</tr>
<tr>
<td>C</td>
<td>0.240</td>
<td>1013</td>
</tr>
<tr>
<td>D</td>
<td>0.020</td>
<td>907</td>
</tr>
<tr>
<td>E</td>
<td>0.017</td>
<td>833</td>
</tr>
<tr>
<td>F</td>
<td>0.015</td>
<td>773</td>
</tr>
<tr>
<td>G</td>
<td>0.014</td>
<td>733</td>
</tr>
<tr>
<td>K</td>
<td>0.010</td>
<td>640</td>
</tr>
</tbody>
</table>

A program in Fortran gives the equation of the straight line that most fits the curve KA:

\[
\text{Breakdown Voltage (in rms V)} = 26423.46 \text{ gap}_c \text{ (in cm)} + 375.948 .
\] (1)

The critical gap at a given frequency also matches the following equation given by Pim:

\[
g_{ce} = \frac{2kE}{7wp} \left[ 6 - \sqrt{15 - \frac{21pE^2}{4k^2}} \right], \tag{2}
\]

where

\[k = \text{proportional factor between velocity and } (E/p)\]
\[v = k \cdot (E/p)\]
\[k = 3.9 \times 10^5 \text{ (cm/sec) (mm Hg) (V/cm)}\]
\[E = \text{peak electric field (V/cm)}\]
\[w = 2\pi f\]
\[p = \text{gas pressure (mm of mercury) (p = 760 mm Hg at atmospheric pressure)}\]
\[B = \text{loss coefficient representing electron loss due to recombination, spread of electron}\]
\[B = 0.0116 \text{ in DC condition}^3 \]
\[B = 0.0133 \text{ at atmospheric pressure and}\]
\[E = 34 \text{ kV/cm},^3 B \text{ is proportional to gas pressure. Assume } B = 0.0133 \text{ in our calculation.}\]
\[A = \text{empirical data} = 1.58 \times 10^{-5} \text{ (mm Hg)/(V/cm)^2.}\]

Of course, we have:

\[
\text{Breakdown Voltage} = \frac{E_{gce}}{\sqrt{2}}, \tag{3}
\]

because the breakdown voltage value is an rms value, and \(E\) is the peak value.
A solution to \( g_{ce} \) is obtained by solving the three simultaneous equations (1), (2), and (3). (See Table 2.) This is done in a program in Fortran using iteration methods as follows:

1. Guess a value for \( E \) in V/cm.
2. Use this \( E \) value to calculate \( g_{ce} \) in Eq. (2).
3. Use this value of \( g_{ce} \) to calculate the breakdown voltage in Eq. (1).
4. Use this value of breakdown voltage to calculate \( E \) in Eq. (3).
5. Compare the value of \( E \) in Step 4 and the guessed value in Step 1. If the difference is greater than 0.01, let \( E \) equal \( E_1 \), and repeat Steps 2-5.

### Table 2. Critical Gap at Given Frequencies in the Range of 47–100 MHz.

<table>
<thead>
<tr>
<th>FREQUENCY (MHz)</th>
<th>CRITICAL GAP (cm)</th>
<th>BREAKDOWN VOLTAGE (rms V)</th>
<th>BREAKDOWN FIELD INTENSITY (peak kV/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>0.064</td>
<td>2066</td>
<td>45.67</td>
</tr>
<tr>
<td>50</td>
<td>0.060</td>
<td>1972</td>
<td>46.16</td>
</tr>
<tr>
<td>60</td>
<td>0.051</td>
<td>1728</td>
<td>47.74</td>
</tr>
<tr>
<td>70</td>
<td>0.045</td>
<td>1554</td>
<td>49.28</td>
</tr>
<tr>
<td>75</td>
<td>0.042</td>
<td>1485</td>
<td>50.03</td>
</tr>
<tr>
<td>80</td>
<td>0.040</td>
<td>1424</td>
<td>50.77</td>
</tr>
<tr>
<td>90</td>
<td>0.036</td>
<td>1322</td>
<td>52.20</td>
</tr>
<tr>
<td>100</td>
<td>0.033</td>
<td>1240</td>
<td>53.60</td>
</tr>
</tbody>
</table>

Finally, we use the same method, "the least square curve fitting," to:

1. Establish the approximate line for the 50-Hz-breakdown voltage versus gap width in frequencies 47–100 MHz.
2. Establish the approximate line for the "limit" line \( K'A' \) so that we can predict the gap range, at a given frequency, in which the breakdown voltage is reduced substantially.

All the parts of curves obtained by extrapolation are included on the original curves from Pim for comparison (Figures 8–10).
Figure 8. Variation of Ultra-High-Frequency Breakdown Voltage with Gap Width at Atmospheric Pressure (with Extrapolated Values).

Figure 9. Electrical Breakdown Strength of Air at Ultra-High Frequencies (with Extrapolated Values).
4.0 BREAKDOWN IN VACUUM

Breakdown in vacuum cannot be explained by the above theory, which states that breakdown is caused by the ionization electrons, positive ions, and molecules through collisions; that is, unless there are gas molecules, no breakdown can occur. When the pressure and gas density in a system are so low that the electron mean free path is much larger than the spacing of conductors, electron multiplication by impact ionization of the gas molecules cannot take place. Meek also agrees on this point.

In reality, breakdown is observed at all times in vacuum if the field intensity is very high. Sparks appear in good vacuum (10^{-6}-10^{-7} mm Hg) when the field intensity is 100-1000 kV/cm.

In the absence of gas ionization, breakdown can occur from the electrodes' effects. Milikan, Eyring, and Mackeowen studied these sparks [in vacuum 10^{-6}-10^{-7} mm Hg] and came to the conclusion that they resulted from intense localized electron emission at the surfaces of electrodes due to the 'pulling of electrons out of metal.' In fact, there is a protrusion or an irregularity on the surface of the electrode where the field intensity is much higher than that at any other point. At this point, the electron can get out of the surface forming the current; the local temperature rises, so the gas forms a film at the electrode, and the metal point vaporizes. Then there are enough gas and metal vapors to be ionized, forming the path for the breakdown. After the breakdown, the irregularity is destroyed. Another irregularity with the higher breakdown potential will be the next target. Consequently, electrodes that have been flashed in vacuum many times will have higher breakdown potential. In other words, the smoother the surfaces of the electrodes, the higher the breakdown potential.

At first, it was believed that high field intensity reduced the potential barriers of metal atoms so that the electrons were liberated. If this were true, the effect would have to be temperature-dependent. But experiments conducted above 1500°C showed that the results were independent of temperature. Later a theory was developed in which all electrons, including the degenerate ones, can leak out under the influence of the external high field. These electrons consist of not only the ones in the outermost band, but the degenerate ones in the inner bands as well. Therefore, it was concluded that the results are not temperature-dependent.
5.0 CONCLUSION

In summary, the critical gap may be determined at any desired frequency. The two frequencies of interest are 47 MHz and 60 MHz, at which the estimated critical gaps are 0.64 mm and 0.512 mm, respectively, and the estimated breakdown potentials are 2066 rms V and 1728 rms V, respectively. It is suggested that at gaps smaller than the critical gap, the value of breakdown stress should be 10–15% lower than the static value. For gaps much longer than the critical gap, the value of the breakdown stress should be 29 kV/cm.³

Since the breakdown in vacuum originates from irregularities on the surfaces, these irregularities are dominant factors. Moreover, the vacuum breakdown cannot be explained by the theory of ionization due to collisions. The effect of frequency on the breakdown in this case, if any, is of secondary importance.
REFERENCES


APPENDIX

C THIS PROGRAM DETERMINES THE EQUATION OF A STRAIGHT LINE
C REPRESENTING A NUMBER OF EMPIRICAL DATA.

C VARIABLES USED.
   INTEGER N, I, INDEX
   REAL SUMX, SUMY, SUMXY, SLOPE, B, RESSUM,
      X(20), Y(20), ESTY(20), RES(20), RESMAX

C
C
C NUMBER OF DATA USED
READ(1,*) N
C PRINT THE HEADING AND THE COLUMNS OF THE OUTPUT
WRITE(5,100) 'THE LEAST SQUARES LINE FITTING'
100 FORMAT('-','20X','A26')
WRITE(5,200) 'NUMBER OF EXPERIMENTAL POINTS: ', N
200 FORMAT('0','20X','A33','I2')
WRITE(5,300) 'EXPERIMENTAL', 'EXPERIMENTAL',
              'ESTIMATED Y', 'RESIDUAL'
300 FORMAT('0',A12.13X,A12,13X,A9,15X,A9)
WRITE(5,400) 'X-VALUE ', 'Y-VALUE ', 'Y-VALUE '
400 FORMAT(1X,'A7',18X,'A7',18X,'A7')

C
C READ THE EMPIRICAL DATA INTO ARRAYS X(I), Y(I)
READ(1,*) (X(I), Y(I), I =1,N)
C FIND THE EQUATION OF THE STRAIGHT LINE Y = A * X + B
C A IS THE SLOPE; B IS THE Y-INTERCEPT
C SUMX = X1 + X2 + ....+ XN
C SUMY = Y1 + Y2 + ....+ YN
C SUMXY = X1Y1 + X2Y2 + ....+ XNYN
C SUMSQX = X1XX1 + X2X2 + ....+ XNXN
C SLOPE A = ( SUMXXSUMY - FLOAT(N)~SUMXY )/(SUMXXSUMY - FLOAT(N)XXSUMSQX )
C Y-INTERCEPT B = ( SUMY - SLOPE3CSUMX )/ FLOAT(N)
C COMPUTE THE RESIDUAL OF EACH POINT, WHICH IS THE
C DIFFERENCE BETWEEN THE ACTUAL AND ESTIMATED VALUES OF Y
RESSUM = 0.000
DO 500 I =1,N
   SUMX = SUMX + X(I)
   SUMY = SUMY + Y(I)
   SUMXY = SUMXY + X(I)*Y(I)
   SUMSQX = SUMSQX + X(I)XX2
500 CONTINUE
SLOPE = ( SUMXXSUMY - FLOAT(N)XXSUMXY )/
        ( SUMXXSUMY - FLOAT(N)XXSUMSQX )
B = ( SUMY - SLOPEXXSUMX )/ FLOAT(N)
C COMPUTE THE RESIDUAL OF EACH POINT, WHICH IS THE
C DIFFERENCE BETWEEN THE ACTUAL AND ESTIMATED VALUES OF Y
RESSUM = 0.000
DO 600 I =1,N
   ESTY(I) = SLOPEXX(I) + B
   RES(I) = Y(I) - ESTY(I)
   RESSUM = RESSUM + RES(I)XX2
600 CONTINUE
C FIND THE LEAST ACCURATE POINTS WHOSE RESIDUALS ARE MAXIMUM.
C INDEX INDICATES THE SUBSCRIPTS OF THESE POINTS IN RES(I)
C RESMAX IS THE CORRESPONDING VALUE OF RES(INDEX)
RESMAX = ABS(RES(I))
INDEX = 1
DO 700 I = 1,N
    IF ( ABS(RES(I)) .GT. RESMAX ) THEN
        RESMAX = ABS(RES(I))
        INDEX = I
    ENDIF
700   CONTINUE
C PRINT THE RESULTS.
   DO 800 I = 1,N
      WRITE(5,900) X(I), Y(I), ESTY(I), RES(I)
800 CONTINUE
   WRITE(5,1000) 'THE LINEAR EQUATION IS: '
1000 FORMAT('0',20X,A23)
   WRITE(5,2000) 'Y = ', SLOPE, 'X + ', B
2000 FORMAT(1X,20X,A5,F12.3,A5,F8.3)
   WRITE(5,3000) 'THE LEAST ACCURATE POINT IS: '
3000 FORMAT('0',20X,A29)
   WRITE(5,4000) 'X = ', X(INDEX), ' Y = ', Y(INDEX)
4000 FORMAT(20X,A5,F8.3,A10,F8.3)
   WRITE(5,5000) 'THE RESIDUAL SUM IS: ', RESSUM
5000 FORMAT('0',20X,A22,F8.3)
STOP
END
THE LEAST SQUARES LINE FIT

NUMBER OF EXPERIMENTAL POINTS: 8

<table>
<thead>
<tr>
<th>EXPERIMENTAL VALUE</th>
<th>EXPERIMENTAL Y-VALUE</th>
<th>ESTIMATED Y-VALUE</th>
<th>RESIDUAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0400</td>
<td>1427.000</td>
<td>1432.887</td>
<td>-5.887</td>
</tr>
<tr>
<td>.0310</td>
<td>1200.000</td>
<td>1195.076</td>
<td>4.924</td>
</tr>
<tr>
<td>.0240</td>
<td>1013.000</td>
<td>1010.112</td>
<td>2.888</td>
</tr>
<tr>
<td>.0200</td>
<td>907.000</td>
<td>904.418</td>
<td>2.582</td>
</tr>
<tr>
<td>.0170</td>
<td>833.000</td>
<td>825.147</td>
<td>7.853</td>
</tr>
<tr>
<td>.0150</td>
<td>773.000</td>
<td>772.300</td>
<td>.700</td>
</tr>
<tr>
<td>.0140</td>
<td>733.000</td>
<td>745.877</td>
<td>-12.877</td>
</tr>
<tr>
<td>.0100</td>
<td>640.000</td>
<td>640.183</td>
<td>-.183</td>
</tr>
</tbody>
</table>

THE LINEAR EQUATION IS

\[ Y = 26423.460 \times X + 375.948 \]

THE LEAST ACCURATE POINT IS:

\( X = .014 \quad Y = 733.000 \)

THE RESIDUAL SUM IS: 301.918

THIS IS THE DATA TABLE TO APPROXIMATE THE EQUATION OF THE STRAIGHT LINE REPRESENTING THE BREAKDOWN vs GAPS.

<table>
<thead>
<tr>
<th>POINT</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>08</td>
<td>.0400</td>
</tr>
<tr>
<td></td>
<td>1427.0</td>
</tr>
<tr>
<td>A</td>
<td>.0310</td>
</tr>
<tr>
<td></td>
<td>1200.0</td>
</tr>
<tr>
<td>B</td>
<td>.0240</td>
</tr>
<tr>
<td></td>
<td>1013.0</td>
</tr>
<tr>
<td>C</td>
<td>.0200</td>
</tr>
<tr>
<td></td>
<td>907.0</td>
</tr>
<tr>
<td>D</td>
<td>.0170</td>
</tr>
<tr>
<td></td>
<td>833.0</td>
</tr>
<tr>
<td>E</td>
<td>.0150</td>
</tr>
<tr>
<td></td>
<td>773.0</td>
</tr>
<tr>
<td>F</td>
<td>.0140</td>
</tr>
<tr>
<td></td>
<td>733.0</td>
</tr>
<tr>
<td>G</td>
<td>.0100</td>
</tr>
<tr>
<td></td>
<td>640.0</td>
</tr>
</tbody>
</table>
THE LEAST SQUARES LINE FIT

NUMBER OF EXPERIMENTAL POINTS: 8

<table>
<thead>
<tr>
<th>EXPERIMENTAL VALUE</th>
<th>EXPERIMENTAL Y-VALUE</th>
<th>ESTIMATED Y-VALUE</th>
<th>RESIDUAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0629</td>
<td>1350.000</td>
<td>1343.735</td>
<td>6.265</td>
</tr>
<tr>
<td>.0550</td>
<td>1173.000</td>
<td>1180.506</td>
<td>-7.506</td>
</tr>
<tr>
<td>.0479</td>
<td>1033.000</td>
<td>1033.806</td>
<td>-.806</td>
</tr>
<tr>
<td>.0450</td>
<td>980.000</td>
<td>973.886</td>
<td>6.114</td>
</tr>
<tr>
<td>.0425</td>
<td>913.000</td>
<td>922.231</td>
<td>-.9231</td>
</tr>
<tr>
<td>.0350</td>
<td>766.000</td>
<td>767.267</td>
<td>-1.267</td>
</tr>
<tr>
<td>.0325</td>
<td>726.000</td>
<td>715.612</td>
<td>10.388</td>
</tr>
<tr>
<td>.0300</td>
<td>660.000</td>
<td>663.957</td>
<td>-3.957</td>
</tr>
</tbody>
</table>

THE LINEAR EQUATION IS

\[ Y = 20661.940 X + 44.099 \]

THE LEAST ACCURATE POINT IS:

\[ X = .032 \quad Y = 726.000 \]

THE RESIDUAL SUM IS: 344.007

THIS IS THE DATA TABLE TO APPROXIMATE THE EQUATION OF THE STRAIGHT LINE REPRESENTING THE "LIMIT 29kV/cm" CURVE

<table>
<thead>
<tr>
<th>POINT</th>
<th>NUMBER OF POINTS USED</th>
</tr>
</thead>
<tbody>
<tr>
<td>A'</td>
<td>.0629</td>
</tr>
<tr>
<td>B'</td>
<td>.0550</td>
</tr>
<tr>
<td>C'</td>
<td>.0479</td>
</tr>
<tr>
<td>D'</td>
<td>.0450</td>
</tr>
<tr>
<td>E'</td>
<td>.0425</td>
</tr>
<tr>
<td>F'</td>
<td>.0350</td>
</tr>
<tr>
<td>G'</td>
<td>.0325</td>
</tr>
<tr>
<td>K'</td>
<td>.0300</td>
</tr>
</tbody>
</table>
THE LEAST SQUARES LINE FIT

NUMBER OF EXPERIMENTAL POINTS: 6

<table>
<thead>
<tr>
<th>EXPERIMENTAL -VALUE</th>
<th>EXPERIMENTAL Y-VALUE</th>
<th>ESTIMATED Y-VALUE</th>
<th>RESIDUAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0400</td>
<td>1573.000</td>
<td>1571.476</td>
<td>1.524</td>
</tr>
<tr>
<td>.0375</td>
<td>1493.000</td>
<td>1502.419</td>
<td>-9.419</td>
</tr>
<tr>
<td>.0350</td>
<td>1440.000</td>
<td>1433.362</td>
<td>6.638</td>
</tr>
<tr>
<td>.0325</td>
<td>1367.000</td>
<td>1364.305</td>
<td>2.695</td>
</tr>
<tr>
<td>.0300</td>
<td>1300.000</td>
<td>1295.248</td>
<td>4.752</td>
</tr>
<tr>
<td>.0275</td>
<td>1220.000</td>
<td>1226.191</td>
<td>-6.191</td>
</tr>
</tbody>
</table>

THE LINEAR EQUATION IS
Y = 27622.770 X + 466.565

THE LEAST ACCURATE POINT IS:
X = .038  Y = 1493.000

THE RESIDUAL SUM IS: 203.277

THIS IS THE DATA TABLE TO APPROXIMATE THE EQUATION OF THE STRAIGHT LINE REPRESENTING THE 50Hz- BREAKDOWN CURVE.

6 ; NUMBER OF POINTS USED.
0.04  1573. ; POINT 1
0.0375 1493. ; POINT 2
0.0350  1440. ; POINT 3
0.0325  1367. ; POINT 4
0.03   1300. ; POINT 5
0.0275  1220. ; POINT 6
This program computes the CRITICAL GAP versus FREQUENCY using the iteration method.


C Guess the initial value of breakdown stress E
WRITE(*,*) 'ENTER THE VALUE OF E'
READ (*,*), E

READ (*,*), F

K = 3.93E+5
A = 1.58E-5
B = 1.33E-2
P = 760.

PI = 3.1416

W = 2 * PI * F

100 G = ((2*K*E)/(7*W*P))*(6-SQRT(15-(21*P**2*B)/(A*E**2)))

V = 26423.460 * G + 375.948

El = V * 1.414 /G

C Create the WHILE DO loop to compare values of guessed E and calculated one after an iteration cycle.

IF (ABS(El - E).GT.0.01) THEN
  E = El
  GO TO 100
ENDIF

WRITE (5,*) 'THE APPLIED FREQUENCY IN HZ: ', F
WRITE (5,*) 'THE CRITICAL GAP IN CM: ', G
WRITE (5,*) ' THE BREAKDOWN VOLTAGE IN VOLT RMS: ', V
WRITE (5,*) 'THE PEAK BREAKDOWN STRESS IN VOLT/CM ', E
STOP
END
<table>
<thead>
<tr>
<th>Applied Frequency (Hz)</th>
<th>Critical Gap (cm)</th>
<th>Breakdown Voltage (V RMS)</th>
<th>Peak Breakdown Stress (V/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>470 000 000.000000</td>
<td>6.397 086E-002</td>
<td>2066.280 0000</td>
<td>45672.660 0000</td>
</tr>
<tr>
<td>500 000 000.000000</td>
<td>6.042 679E-002</td>
<td>1972.633 0000</td>
<td>46160.040 0000</td>
</tr>
<tr>
<td>600 000 000.000000</td>
<td>5.118 431E-002</td>
<td>1728.415 0000</td>
<td>47748.580 0000</td>
</tr>
<tr>
<td>700 000 000.000000</td>
<td>4.459 188E-002</td>
<td>1554.220 0000</td>
<td>49284.020 0000</td>
</tr>
<tr>
<td>750 000 000.000000</td>
<td>4.195 645E-002</td>
<td>1484.583 0000</td>
<td>50032.830 0000</td>
</tr>
<tr>
<td>800 000 000.000000</td>
<td>3.965 079E-002</td>
<td>1423.659 0000</td>
<td>50769.590 0000</td>
</tr>
<tr>
<td>900 000 000.000000</td>
<td>3.580 752E-002</td>
<td>1322.107 0000</td>
<td>52208.540 0000</td>
</tr>
<tr>
<td>1.0 000 000E+008</td>
<td>3.273 082E-002</td>
<td>1240.810 0000</td>
<td>53604.060 0000</td>
</tr>
</tbody>
</table>