

Recent results using all-point quark propagators

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Pseudofermion methods for extracting all-point quark propagators are reviewed, with special emphasis on techniques for reducing or eliminating autocorrelations induced by low eigenmodes of the quark Dirac operator. Recent applications, including high statistics evaluations of hadronic current correlators and the pion form factor, are also described.

1. Introduction

In many cases the extraction of multipoint hadronic correlators of physical interest requires enormous statistics, which, given the difficulty of unquenched simulations in QCD, makes it critical to extract the maximum physical content of each available configuration. Recently the pseudofermion approach [1] to the calculation of all-point quark propagators has been substantially improved [2] by the use of mode-shifting, effectively eliminating autocorrelation problems and allowing the accurate extraction of all-point quark propagators for arbitrary sources and sinks. In this talk, some applications of these methods will be described. We will describe results for (a) fits of current correlators to chiral Lagrangian forms, (b) studies of the pion form-factor on $10^3 \times 20$ unquenched lattices (with 200 MeV pions), and (c) the problem of disconnected parts in isoscalar channels.

2. Methodology

We begin from a bosonic pseudofermion field ϕ_{ma} with action (m a lattice site, a the spin-color index, Q the Wilson or clover operator):

$$\begin{aligned} S(\phi) &= \phi^\dagger Q^\dagger Q \phi \\ &= \phi^\dagger H^2 \phi, \quad H \equiv \gamma_5 Q = H^\dagger \end{aligned}$$

For fixed background gauge field A , simulating the field ϕ produces the correlator (where $\langle\langle$

$O \rangle\rangle$ means the average of O relative to the measure e^{-S}):

$$\begin{aligned} \langle\langle \phi_{ma} \phi_{nb}^* \rangle\rangle_{S(\phi)} &= (H^{-2})_{ma,nb} \\ \langle\langle \phi_{ma} (\phi^\dagger H)_{nb} \rangle\rangle_{S(\phi)} &= (H^{-1})_{ma,nb} \\ &= (Q^{-1} \gamma_5)_{ma,nb} \end{aligned}$$

Unfortunately, low eigenmodes of H frequently result in very long autocorrelations for the low momentum parts of hadronic correlators. These correlations can be removed (or substantially reduced) by shifting the guilty IR modes into the UV: define a shifted hermitian Wilson-Dirac operator, with λ_i , $i = 1, \dots, N$ the N lowest eigenvalues, and \mathbf{v}_i corresponding eigenvectors:

$$\begin{aligned} H_s &\equiv H + \sum_{i=1}^N \delta_i \mathbf{v}_i \mathbf{v}_i^\dagger, \\ \delta_i &\equiv \lambda_i^{(s)} - \lambda_i \end{aligned}$$

Take $\lambda_i^{(s)} = \text{sign}(\lambda_i)$: this evacuates the IR part of the quark operator, greatly reducing low momentum autocorrelations (see Fig. 1). Then, we reconstruct the correct all-point propagator by adding in the contribution from shifted modes:

$$\begin{aligned} H_{ax,by}^{-1} &= \langle\langle \phi_{ax} \phi_{by} \rangle\rangle - \sum_{i=1}^N \Delta_i \mathbf{v}_{i,ax} \bar{\mathbf{v}}_{i,by} \\ \Delta_i &\equiv 1 - \frac{1}{\lambda_i^2} \end{aligned}$$

Accurate mode-shifted all-point propagators can be obtained even for *very* light quarks, when conjugate gradient inversion dies. Moreover, the

*Talk presented by J. Yoo

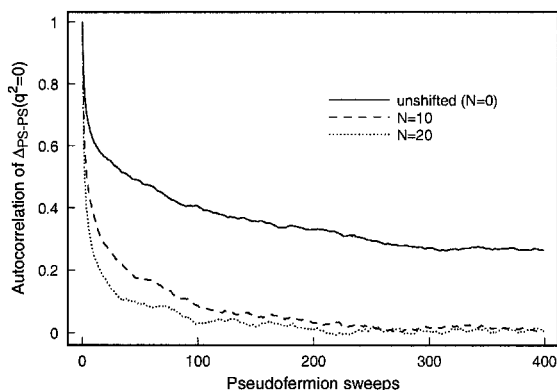


Figure 1. Reduction of autocorrelation from mode-shifting

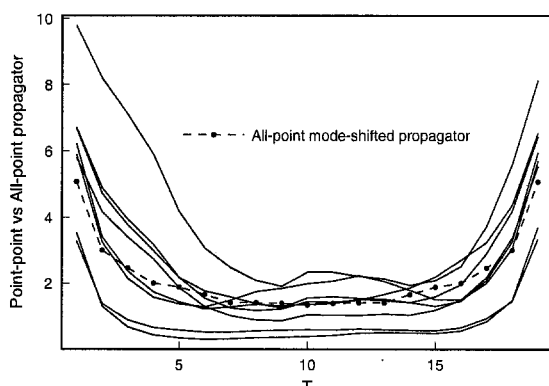


Figure 2. Comparison of single vs all-point propagators

all-point propagator represents a great increase in statistics over conventional one-point propagators: e.g. on a $10^3 \times 20$ lattice, the all-point is the average of 20000 (admittedly highly correlated) conventional propagators (see Fig. 2).

3. Applications

3.1. Fits of current correlators

The large statistics provided by all-point propagators allows a high precision calculation of hadronic current correlators, which can then be fit to the predictions of chiral perturbation theory, ultimately allowing us to extract parameters of the chiral Lagrangian directly from these

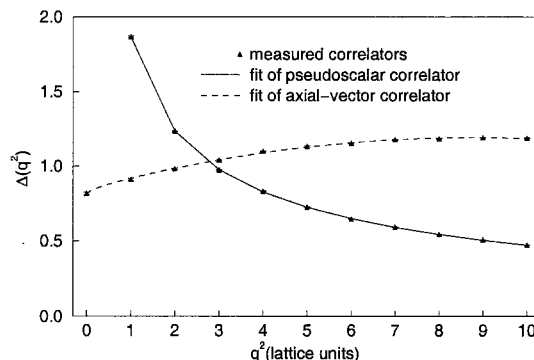


Figure 3. Fits of pseudoscalar and axial-vector correlators from all-point measurements

correlators. Fits of this sort from pseudoscalar-pseudoscalar and axial vector-axial vector correlators are shown in Fig.3 (using 800 unquenched configurations generated on large coarse lattices with the truncated determinant algorithm (TDA) [3,5]). The values for M_π and F_π extracted from these fits are consistent with those obtained by conventional fits using smeared-local correlators, but we also obtain an accurate reading of the size of higher order terms in the chiral expansion.

From chiral perturbation theory, we have:

$$\begin{aligned} \Delta_{psps}(q^2) &= \frac{A_1}{q^2 + A_2} + A_3 + A_4 q^2 + A_5 (q^2)^2 \\ \Delta_{axax}(q^2) &= \frac{q^2 F_\pi^2}{q^2 + M_\pi^2} + A_1 + A_2 q^2 + A_3 (q^2)^2 \end{aligned}$$

The fits to these formulas are shown in Fig.3: for example, for the axial vector case we find that the one-loop chiral contribution A_2 is extracted with a statistical accuracy of $< 1\%$:

$$\begin{aligned} F_\pi &= 0.187 \pm 0.011, A_1 = 0.8148 \pm 0.0038 \\ A_2 &= 0.0777 \pm 0.0003, A_3 = -0.00442 \pm 0.00002 \end{aligned}$$

3.2. Three-point Functions: the Pion Form Factor

To extract the pion formfactor, we need the following 3-point function:

$$\begin{aligned} J_{t_0 t_1 t_2}(q^2) &= \sum_{\vec{w} \vec{x} \vec{y} \vec{z}} e^{i\vec{q} \cdot (\vec{x} - \vec{y})} f^{\text{sm}}(\vec{z}) f^{\text{sm}}(\vec{w}) \\ \langle \bar{\psi}(\vec{z} + \vec{x}, t_2) \gamma_5 \psi(\vec{x}, t_2) j_0^{\text{em}}(\vec{y}, t_1) \bar{\psi}(\vec{w}, t_0) \gamma_5 \psi(0, t_0) \rangle \end{aligned}$$

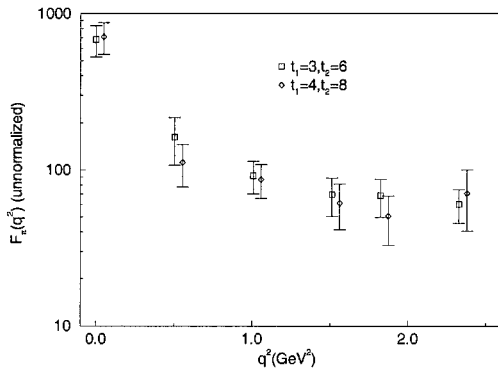


Figure 4. Connected and Disconnected contributions to Pion Form Factor

For large time ($t_0 = 0 \ll t_1 \ll t_2$) (using optimized smearing functions f^{sm}):

$$J_{t_1 t_2}(\vec{q}) \rightarrow (M_\pi + E(q)) e^{-M_\pi t_1 - E(q) t_2} F_\pi(q^2)$$

Of course, one needs this convergence before the signal disappears! This requires a large ensemble. The results for $F_\pi(q^2)$ using times $(t_1, t_2) = (3, 6)$ and $(4, 8)$ are shown in Fig. 4, using mode-shifted all-point propagators for 65 configurations of unquenched $10^3 \times 20$ lattices at $M_\pi \simeq 200$ MeV generated by the TDA method. We expect that a useful result for the pion form factor will emerge from an ensemble of a few hundred configurations, which are presently being generated.

3.3. Disconnected parts and the η'

Calculations of the η' mass from isoscalar correlators suffer from the need to extract disconnected (annihilation) graphs which require all-point propagators. Some examples of connected and disconnected contributions to the isoscalar correlator computed with mode-shifted all-point propagators on 5 configurations of unquenched 6^4 lattices are shown in Fig. 5. The statistical errors of the connected and disconnected contributions are *comparable* using all-point propagators, but the fluctuations are large from one gauge configuration to the next, requiring a large ensemble before an accurate η' mass can be obtained from the subtracted amplitude. However, the difficulties of disconnected configurations can be avoided entirely by extracting the η' mass from scalar isovector correlators which are dominated at large time

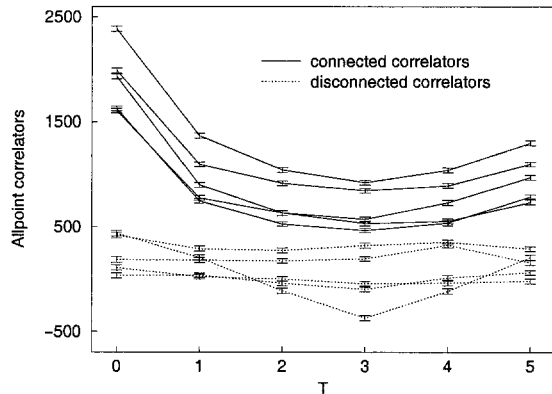


Figure 5. Connected/Disconnected allpoint correlators

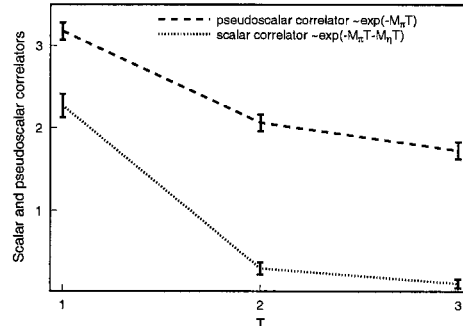


Figure 6. Scalar and Pseudoscalar Correlators for Unquenched 6^4 Lattices

by a $\eta' - \pi$ two-body s-wave state. The isovector scalar and pseudoscalar correlators extracted from 70 fully unquenched 6^4 lattices ($a=0.36$ F) with two degenerate light sea-quarks ($M_\pi \simeq 330$ MeV) are shown in Fig. 6: the corresponding η' mass is $\simeq 735$ MeV.

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